

The nominal interest rate associated with an asset (for example, a deposit in an interest-bearing savings account) is the rate at which the dollar value of the asset increases over time. The corresponding real interest rate is the rate at which the purchasing power of the asset's value increases over time. Inflation (a general increase in the prices of goods and services) can erode the purchasing power of the asset's nominal value, leading to a real interest rate that is less than the nominal interest rate.

For example, suppose you deposit \$100 today (in 2014) in a savings account paying a nominal annual interest rate of 5%, *with interest compounded annually (once per year)*. (There aren't any savings accounts paying 5% interest these days, of course. This is just a hypothetical example.) Next year at this time (in 2015), the nominal value of the asset will be  $100(1+0.05) = \$105$ , in 2015 dollars. If, however, there has been inflation over the course of the year, \$105 in 2015 won't buy as many goods as \$105 would have purchased in 2014. Suppose, for example, that the inflation rate for the year is 3%. Then the purchasing power of the account's 2015 value, measured in 2014 dollars, will be only  $\frac{105}{1.03} = \$101.94$ . The real

interest rate is the rate of appreciation in the asset's purchasing power. Measured in 2014 dollars, the asset's purchasing power increased from \$100 to \$101.94, for an increase of 1.94%. In general; with the nominal and real interest rates and the rate of inflation denoted  $n, r$ , and  $i$

respectively; we have  $(1+r) = \frac{(1+n)}{(1+i)}$ , which yields:  $r = \frac{(1+n)}{(1+i)} - 1$ . In the example above,

$r = \frac{(1+0.05)}{(1+0.03)} - 1 = 0.0194$  or 1.94%. Another, much simpler, formula relating  $n, r$ , and  $i$ ;

$r = n - i$ ; would appear to give a pretty close approximation in this case. Using the numbers above:  $r = 0.05 - 0.03 = 0.02$  or 2.0%  $\approx$  1.94%.

Notice that, in the example above, I specified that interest is compounded annually. If the savings account were associated with a nominal annual interest rate of 5% but interest were compounded semi-annually (twice per year), then interest equal to 2.5% of the account's balance would be added to the account halfway through the year; and again at the end of the year. The year-end nominal balance in the account would then be

$100(1+0.025)(1+0.025) = \$105.06$ , 5.06% greater than the initial deposit. Any interest rate associated with a given compounding basis has an equivalent interest rate for any other compounding basis. This example shows that a 5% annual nominal interest rate with semi-annual compounding is equivalent to a 5.06% annual nominal interest rate with annual compounding.

One can imagine interest being compounded even more frequently: monthly, daily, or even *continuously*; as would be the case if the compounding interval became infinitesimally small.

Interest rates can also be expressed in terms of a continuous compounding basis. At this point the mathematics begins to get a little complicated. (A Google search on “compound interest” will point you to some useful resources if you want to get deeper into it.) But it turns out, for example, that a 5% nominal annual interest rate with annual compounding is equivalent to a 4.88% nominal annual interest rate with continuous compounding. Finally we come back to the simple, “approximate” formula relating real and nominal interest rates and the rate of inflation. When interest and inflation rates are expressed in terms of a continuous compounding basis, the simple formula, namely “ $r = n - i$ ,” holds exactly.