Abstract

We introduce the Diamond-Mortensen-Pissarides (DMP) labor search framework into an otherwise standard Eaton-Gersovitz (1981) sovereign debt model. The interactions between external debt dynamics, domestic labor market outcomes, and time-consistent fiscal policies are analyzed. The tension between the debt and labor market is realized through the link of the per-period balanced government budget constraint. In a quantitative exercise of calibrating to Argentina economy, we find that the incorporation of the frictional labor market improves the model’s ability to generate empirically realistic debt level and default frequency. The quantitative impact of high unemployment benefit on deterring vacancy creation not only outweighs its consumption smoothing effect, but also increases vulnerability to a sovereign debt crisis. Our model can serve as a framework for analyzing various stabilization and social-insurance policies in the context of sovereign debt crisis once properly extended to a dynamic labor search environment.
1. Introduction

Explaining both labor market fluctuations and sovereign debt crises in a unified framework turns out to be a new challenge in International Macroeconomics. The textbook quantitative sovereign debt models typically assume an endowment economy and thus ignore the interaction between domestic labor market and external debt dynamics. Recent exceptions include Na et al. (2018) and Bianchi et al. (2017). They introduce the notion of downward nominal wage rigidity to these models and study optimal government policies to stabilize unemployment and debt. Yet, they are both full-employment models and thus a thorough anatomy of labor market dynamics is missing. In the spirit of Diamond-Mortensen-Pissarides (DMP), we introduce a frictional labor market featuring search and matching frictions to an otherwise standard sovereign debt model and address (i) how default risk and fiscal policy affect the labor market outcomes, and (ii) how a tax-financed unemployment benefit scheme affects default decision and sovereign spreads.

Exploring labor market dynamics for the analysis of sovereign default is motivated by the concurrent evidence of high unemployment and sovereign spreads in the recent Eurozone debt crisis. Schmitt-Grohé and Uribe (2016a) report that the average unemployment rate on the periphery of Europe reached above 13 percent in 2010. This same episode is featured with the dramatic widening in spreads of yields on 10 year sovereign bonds of Greece, Ireland, Italy, Portugal, Spain (GIIPS) and that of Germany. What’s more, models with search and matching frictions, such as Diamond (1981), Pissarides (1985) and Mortensen and Pissarides (1994), have become the workhorse model for equilibrium unemployment analysis. These models capture important labor market dynamics over the business cycle and are widely used to understand how fiscal policy affects unemployment and output. It is then of significant importance to examine the link between fiscal policy, unemployment, and sovereign default risk implied by these models.

The main findings in the paper are as follows. First, a frictional labor market, in combination with default penalty, improve the model’s ability to generate empirically realistic level of debt, default frequency, and the observed volatility of country premium under plausible magnitude of productivity shocks. Second, the quantitative impact of high unemployment benefit on deterring vacancy creation not only outweighs its consumption smoothing effect, the continuous provision of which also increases further borrowing costs and the probability of a sovereign debt crisis. Third, the endogenous unemployment cost of default has contributed to government policy making, but only with a minor effect on the decision to default or not, due to the static labor market setup. Finally, the model cannot simultaneously account for both plausible employment level during normal times and sizable spike-up in unemployment upon default. This finding is robust even if one adopts an alternative calibration strategy which yields nearly 100 percent replacement ratio. As in many other DMP models in the business cycle framework, it reflects the difficulty to get employment to fall during downturns driven by productivity shocks.

Our production economy has four main ingredients. First, a representative household consists of both employed and unemployed workers. This type of aggregation eliminates discrepancy in consumption across household members by providing perfect consumption insurance within the family. Also, the number of unemployed workers in the household represents the total unemployment rate in the economy. Second, debt issuance is conducted by the household while the default decision is up to the central government. As illustrated by Kim and Zhang (2012), decentralized borrowing and centralized default might lead to excessive borrowing by the private sector. This is because individual borrowers are price-takers. They take the country premium as given and thus fail to take into account the effect of their borrowing decisions on the interest rate. To this end, a macroprudential style capital control tax is imposed by the government to help the household internalize the pecuniary externality (Bianchi, 2011; Korinek, 2018; Na et al., 2018). Third, we assume government cannot commit to its future policies and characterize the Markov Perfect Equilibria (MPE). The government conduct time-consistent fiscal

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1. It is calculated as the arithmetic mean of Bulgaria, Cyprus, Estonia, Greece, Ireland, Lithuania, Latvia, Portugal, Spain, Slovenia, and Slovakia.
2. See Wright (2014), Figure 1.
3. Pecuniary externality is not a source of inefficiency if it is merely a general equilibrium effect operated through prices. However, they will induce severe welfare losses if the market is incomplete and there is limited enforcement. For more discussions, see Kim and Zhang (2012) and references therein.
4. The notion of Markov Perfect Equilibria can be illustrated in the following example. Consider the infinitely-repeated
policies which can depend only on the current payoff-relevant state variables. That is, it should take into account the effect of today’s policy on that of tomorrow’s, which in turn shapes the incentives and behavior of today’s private sector due to the rational expectation, and thus the feedback from the private sector to today’s policy making. In equilibrium, the policy making should be consistent over time. Fourth, the baseline model is built with a static labor market. The separation rate of each job-worker pair is one after production and the search and matching process re-start in every period with all household members looking for jobs on the market. As a result, the employment stock is no longer a state variable. This setup facilitates computation and convergence by disentangling the debt and labor market in a way that the dynamics of the latter are purely driven by productivity shocks.

The model is calibrated to match relevant moments of quarterly Argentina data for the 1983-2001 period. I do not have the full dataset of Argentina labor market and thus certain comparisons are missing. The matching function curvature, vacancy cost, and worker’s bargaining power are jointly chosen to match the average unemployment rate, hiring cost to wage ratio, and the replacement ratio in the data. Parameters describing shocks, preferences, and the default penalty are calibrated to match the average debt-to-GDP ratio, frequency of default, and output loss. The model also does a good job at replicating a couple of moments that were not targeted in the calibration, such as the positive correlation between the trade balance and the country premium.

In the calibration, the generated cyclical movements of labor market variables seem not to match those observed in the data. The so-called Shimer’s puzzle (Shimer, 2005) still prevails in the baseline model even if one adopts the calibration strategy proposed by Hagedorn and Manovskii (2008). This is mainly due to the slightly different interpretation of unemployment in the context of a static labor search setup. In the model, the unemployed workers are not those who are actively searching for jobs but those who couldn’t get successful matches with firm’s vacancies. The nature of the residually determined unemployment makes it barely move.

With a fixed threshold associated with default penalty, the model can match neither the level nor the volatility of spreads. There are two reasons. First, Na et al. (2018) argues that with risk-neutral foreign lenders, the average country premium should approximately be equal to the average frequency of default adjusted for the probability of being in financial autarky. Next, as pointed out by Chatterjee and Eyigungor (2012), it is important to allow for a quadratic structure of default punishment in generating realistic volatility of sovereign spreads.

The contribution of the paper can be evaluated in three different dimensions. First, it shows that unemployment dynamics can play an important role in shaping the cost of sovereign default. This channel is already highlighted in Balke and Ravn (2016) and Blake (2017) with a slightly different approach and can complement the imported intermediate goods channel explored by Mendoza and Yue (2012). Second, it links the labor market outcomes to debt market dynamics only through the balanced government budget constraint, i.e., the financing need of unemployment benefit is met by imposing the capital control tax and balanced with the residually determined lump-sum transfer. In other words, it abstracts from any intertwined dynamics between the two markets or any spillover effect. This could be a good reference point for evaluating the cost and benefit of maintaining a balanced government budget constraint during the default episode. Third, it studies the effect of high unemployment benefit in a calibrated framework. Our finding echoes that of Krusell et al. (2010) which shows that the depressing impact of higher unemployment benefit on firm entry outweigh its beneficial effect for consumption smoothing in terms of welfare outcomes. Despite several missing targets from calibration, mostly due to the static labor market setup, our model provides a starting point for incorporating the endogenous disaster dynamics recently explored in the baseline DMP framework by Petrosky-Nadeau, Zhang, and Kuehn (2018).

This paper is related to two large strand of literature. The first strand is the sovereign debt framework of prisoner’s dilemma. Two sub-game perfect equilibria are of interest. One is ‘bad’, nasty, and simple in the sense that both players would choose to defect every period. The other one is ‘good’, meaning that if both players are patient enough, by folk theorem, a triggering strategy gives that both would choose to cooperate until at least one of them deviates, then both would choose to defect thereafter, a tit-for-tat spirit. In this example, the first one is a Markov Perfect Equilibrium while the second one is not.

The source of time-inconsistency does not come from the preferences per se, but from the government budget constraint. See, for example, Klein, Krusell, and Ríos-Rull (2008).
work\textsuperscript{8} pioneered by Eaton and Gersovitz (1981) and formalized and developed by Arellano (2008) and Aguiar and Gopinath (2006). An important generalization in these quantitative applications is to introduce ex-post direct exogenous output costs of default so as to support ex-ante empirically plausible level of external debt. Indeed, as pointed out by Bulow and Rogoff (1989), the threat of credit exclusion alone (i.e. the reputational mechanism) cannot sustain sovereign debt, as long as government can still save after default. This is also consistent with the negligible role of credit market exclusion for the quantitative performance of these models (Uribe and Schmitt-Grohë, 2017). Attempts to endogenize default costs include Mendoza and Yue (2012). They argue that when domestic intermediate goods are imperfect substitutes, the decline in the imported inputs due to credit financing exclusion necessarily leads to efficiency loss. All these models only deal with real debt. Phan (2017) recently shows that reputation alone can sustain nominal sovereign debt when markets are incomplete. Mechanisms other than output loss or reputation considerations to sustain debt borrowing are highlighted by Sandleris (2008). He argues that information revelation is crucial to enforce debt repayment as the repayment/default decision is interpreted as a signal used by the government to communicate information to domestic and foreign agents about the fundamentals of the economy.

The second related strand literature is the labor search and matching framework started from Diamond-Mortensen-Pissarides (DMP). Recent work incorporating DMP models into real business cycle analysis includes Merz (1995), Andolfatto (1996) and den Haan et al. (2000). They show that the quantitative performance of the real business cycle model can be improved significantly when DMP model is embedded into it. However, Shimer (2005) finds that the standard DMP model cannot generate observed business-cycle-frequency movements in unemployment and job vacancies in response to productivity shocks of a plausible magnitude when the model is calibrated to U.S. data, a.k.a., the Shimer puzzle\textsuperscript{9}. The reason, as Shimer (2005) argues, is that wage is so flexible that most productivity increase are absorbed by higher wages. As a result, the incentive for vacancy creation is eliminated and little impact is on unemployment, vacancies, and job-finding rate. Solutions such as introducing wage rigidity, comparably high outside options for workers, and procyclical vacancy costs are proposed\textsuperscript{8}. In our model, the wage determination by Nash bargaining protocol is preserved since it is consistent with the wage flexibility suggested from the micro data.

Our paper is related to Na et al. (2018), Bianchi et al. (2017), Balke and Ravn (2016), and Balke (2017) who also explicitly introduce a labor market into the Eaton and Gersovitz (1981) model. In Na et al. (2018), the government finds it optimal to devalue its nominal exchange rate upon default to lower the real value of wages, thereby reducing involuntary unemployment inflicted by the downward nominal wage rigidity. In this way, the Twin Ds (Reinhart, 2002), i.e., the sovereign defaults are accompanied by large devaluations of the nominal exchange rate, is rationalized as an optimal policy outcome. In Bianchi et al. (2017), the option of currency devaluation is not available either because the economy is operating at a fixed exchange rate or equivalently a member of a currency union, and the way to stabilize the labor market and the whole economy is to increase government spending. However, such an expansionary fiscal policy comes at a cost of increasing sovereign spreads and the stabilization effects are shown to be highly non-linear in the severity of the recession. In this sense, their paper joins the recent policy debate of austerity v.s. stimulus for Eurozone crisis management. Other papers along the same line include Arellano and Bai (2014) and Cuadra, Sanchez and Sapriza (2011). All these models assume Walrasian labor markets.

Balke and Ravn (2016) also incorporate the DMP labor search framework into a sovereign debt model to evaluate time-consistent fiscal policy \textsuperscript{.} There are three main differences between theirs and our work. First, they follow the conventional approach of centralized borrowing while we assume that the government retains only the decisions to default and to conduct fiscal policy and let private households borrow \textsuperscript{.} In their setup, borrowing and default decisions are always optimal as long as the government

\textsuperscript{6}For handbook chapters, see Aguiar and Amador (2014) and Aguiar, Chatterjee, Cole, and Stangebye (2016). For an excellent textbook treatment, see Uribe and Schmitt-Grohë (2017). For recent reviews on the politics and economics aspect of sovereign default, see Hatchondo and Martinez (2010) and Hatchondo, Martinez, and Sapriza (2007a), respectively. For empirical evidence, see Tomz and Wright (2013). For discussion on the solution methods in quantitative sovereign debt models, see Hatchondo, Martinez, and Sapriza (2010).

\textsuperscript{7}Most early business cycle models cannot get employment to fall enough in downturns driven by productivity shocks. This same problem in the generic business cycle models is inherited by search models, except it’s even worse.

\textsuperscript{8}See, for example, Shimer (2004), Hagedorn and Manovskii (2008), and Petrosky-Nadeau and Zhang (2017).
is benevolent. This is because it internalizes that the interest rate faced by the country in international financial markets depends on its net external debt position (Uribe and Schmitt-Grohé, 2017). Second, they focus on moral hazard concerns that affect worker’s willingness to search for jobs, as most literature on optimal unemployment insurance (UI) does. On the contrary, Krusell et al. (2010) highlights the trade-off between insurance and job creation in a Bewley type incomplete market model embedded with labor search frictions. Our work complements the latter by using a representative household setup in a sovereign debt model. Third, the unemployment benefit is a policy instrument in their framework while we arbitrarily calibrate it at a fixed level. They show that it is optimal to reduce unemployment benefit during the recession to encourage more active search for jobs so as to support employment. However, this result is neither consistent with the real world policy practice nor the prescriptions offered from the optimal UI literature.

Our work is also related to the emerging literature on optimal unemployment insurance over the business cycle. Recent papers include Nakajima (2012), Landais et al. (2013), Jung and Kuester (2015), Mitman and Rabinovich (2015), and Pei and Xie (2017). Unlike rest of the papers, Nakajima (2012) allows agents to borrow and save. He quantifies the effect of ongoing UI benefit extensions on the unemployment rate using a calibrated structural model that features job search and consumption-saving decisions, skill depreciation, and UI eligibility. Mitman and Rabinovich (2015) characterize optimal cyclical behavior of unemployment insurance and shows that it should rise on impact when the economy is hit by a negative productivity shock but then fall during the recovery. They assume government can commit to future policies while Pei and Xie (2017) relax this assumption and examine the time-consistent policy both qualitatively and quantitatively. Both Landais et al. (2013) and Jung and Kuester (2015) introduce shocks to the worker’s bargaining power that are negatively correlated with productivity. Their results, on the contrary, suggest that the optimal variation in unemployment benefits is quantitatively small and short-lived.

This reminder of the paper is organized as follows. Section 2 describes the model and characterizes optimal government policies. Section 3 contains the calibration strategy and quantitative analysis. Section 4 discusses extension. Section 5 concludes.

2. The Model

The model is a Diamond-Mortensen-Pissarides (henceforth, DMP) model embedded in an otherwise standard quantitative sovereign debt framework à la Eaton and Gersovitz (1981). Time is discrete and extended into an infinite horizon. We consider a small open economy populated by a representative household, a representative firm, and a government. In each period the household can issue a non-state-contingent bond to the risk-neutral lenders in the international financial market.

2.1 The Household

The representative household consists of a continuum of workers of measure unity. Workers can be either employed, in which case they earn market wage $w_t$, or unemployed, and receive unemployment benefit $b$ from the government. In the following static labor market case, we assume $b$ is not a policy instrument but instead a fixed parameter to be calibrated. The household also owns the firm and receives aggregate dividend $D_t$. In addition, it can borrow and lend in the international financial market by issuing a non-state-contingent bond at price $q_t$. Here we assume that the household can commit to repay their debt that are subject to an ad hoc natural debt limit. Household members pool income together and achieve full consumption insurance within the family. Thus each member consumes the same level of consumption, denoted by $C_t$. Let $N_t$ denote the number of employed household members. We assume the household are risk averse and are trying to maximize utility

$$E \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma} - 1}{1 - \sigma},$$

We refer interchangeably to workers or household members.
subject to the budget constraint

\[ C_t + d_t = w_t N_t + b(1 - N_t) + (1 - \tau_t) q_t d_{t+1} - T_t + D_t \]  

(1)

where \( \tau_t \) is a macro-prudential or capital control tax on debt imposed by the government, and \( T_t \) is the lump-sum tax (subsidy if negative). Consumption Euler equation is thus

\[ (1 - \tau_t) q_t C_{t+1}^{-\sigma} = \beta E_t C_t^{-\sigma} \]

The labor market contract is assumed to last for one period only. In other words, at the beginning of each period all household members are unemployed and are searching for jobs in the labor market. Some members find jobs and others do not. At the end of the period, all worker-job pairs separate and the whole search and matching process restart in the next period and so on. The number of jobs is given by

\[ N_t = M(1, v_t) \]  

(2)

where \( v_t \) are the number of vacancies posted by the firm. The matching function is constant returns to scale, strictly increasing and concave, and satisfies the property: \( M(1, v_t) \leq v_t \) due to the uncoordinated nature of the search process. The probability that an unemployed worker finds a job (the job-finding rate) is thus \( f_t^u = m_t \) and the probability that an empty vacancy is filled (the job-filling rate) is \( f_t^v = m_t/v_t \). Labor market tightness is pinned down by \( v_t \) only and we have

\[ f_t^u = M(1, v_t) = v_t f(v_t), \quad f_t^v = \frac{M(1, v_t)}{v_t} = f(v_t) \]

note that \( f'(v_t) < 0 \). That is, a relatively tighter labor market makes it easier for a worker to find a job but harder for a firm to get its vacancy filled. As a result, the number of unemployed workers is given by

\[ u_t = 1 - N_t = 1 - v_t f(v_t) \]

2.2 The Firm

The representative firm posts vacancy \( v_t \) to hire workers and produce output according to a linear form technology. Production is subject to an aggregate productivity shock. The firm’s profit maximization problem is

\[ \max_{v_t} \tilde{z}_t N_t - w_t N_t - v_t k \]

subject to equation (2), where \( \tilde{z}_t \) is the aggregate productivity and \( k \) is the per vacancy cost, respectively. As will be specified later, we adopt the following form of matching function as in den Haan et al. (2000):

\[ M(1, v_t) = \frac{v_t}{(1 + v_t)^{1/\iota}} \]

The choice of the matching technology over the commonly used Cobb-Douglas specification is to guarantee that the job-finding rate and the job-filling rate are always between 0 and 1. Solve the firm’s problem and we get the number of vacancies

\[ v_t = \left[ \left( \frac{k}{\tilde{z}_t - w_t} \right)^{-\iota} - 1 \right]^{1/\iota} \]

2.3 Nash Bargaining Wages

The firm and the worker bargain over the wage rate \( w_t \) according to the Nash bargaining rule. That is, a matched worker and firm choose \( w_t \) to maximize the joint product of their individual payoffs from producing minus their “outside options”, i.e.,

\[ \max_{w_t} (w_t - b)^\eta (z_t - w_t)^{1-\eta} \]
where $\eta \in [0, 1]$ represents the relative bargaining power of the worker. As $\eta$ approaches to zero, for example, the firm gets all the bargaining power. Simple algebra gives

$$w_t = (1 - \eta) b + \eta \tilde{z}_t$$

that is, the equilibrium market wage rate is a weighted average of the marginal product of labor, $\tilde{z}_t$, and the worker’s outside option, $b$. Intuitively, the higher the worker’s outside option, or the greater her bargaining power, the higher the wage rate she can get.

### 2.4 Government

The government sets the level of debt tax and decides whether to default or not. Following Na et al. (2018), we assume that in each period the country can be either in good financial standing or bad financial standing. Let $I_t$ be a relevant binary variable and satisfies

$$I_t = \begin{cases} 1, & \text{good financial standing at } t \\ 0, & \text{bad financial standing at } t \end{cases}$$

Then if $I_t = 1$, the country can choose whether to repay the debt or default. If it defaults in period $t$, it immediately enters into bad financial standing and $I_t = 0$. As standard in the sovereign debt literature, a defaulted country incurs both output loss and reputation cost. Denote $\tilde{z}_t$ the productivity level net of the loss associated with default:

$$\tilde{z}_t = \begin{cases} z_t & \text{if } I_t = 1 \\ z_t - L(z_t) & \text{otherwise.} \end{cases}$$ (3)

In addition, a defaulted country is excluded from any access to the international financial market for a finite period of time (i.e. the reputation cost). In other words, when $I_t = 0$, the country regains good standing in the next period with a constant and exogenous probability of $\lambda$, and maintains its bad financial standing status with probability $1 - \lambda$. Thus, we have

$$1 - I_t \Delta t + 1 = 0$$ (4)

Also when $I_t = 0$, the government simply confiscates any debt payment to the international lenders and rebates the proceeds to the household via either lump-sum transfers or unemployment benefit, or both. Therefore, the government’s sequential budget constraint is given by

$$b (1 - N_t) = \tau_t q_t \Delta t + (1 - I_t) \Delta t + T_t$$ (5)

By a no-arbitrage condition, the equilibrium rate of return of sovereign bonds should be equal to the risk-free world interest rate adjusted for the probability of default, i.e.

$$I_t \left[ q_t - \frac{\mathbb{E}_t I_{t+1}}{1 + r^*} \right] = 0$$

### 2.5 Competitive Equilibrium

Combining (1), (3), (4) and (5) along with the firm’s per period profit of $D_t = \tilde{z}_t N_t - w_t N_t - v_t k_t$ yields the following good-market-clearing condition

$$C_t = I_t [q_t \Delta t - d_t] + [z_t - (1 - I_t) L(z_t)] N_t - v_t k$$ (6)

Assume that $\ln z_t$ obeys the law of motion,

$$\ln z_t = \rho \ln z_{t-1} + \mu_t$$

where $\mu_t$ is an i.i.d. innovation with mean zero and variance $\sigma_\mu^2$, and $| \rho | \in [0, 1)$. We are now ready to define a competitive equilibrium.
Definition 1. Given processes \( \{z_t, T_t, \tau_t, I_t\} \), the initial condition \( d_0 \), and \( b \), a competitive equilibrium consists of stochastic processes \( \{C_t, N_t, d_{t+1}, u_t, v_t, \theta_t, w_t, q_t\} \) satisfying

\[
C_t = I_t [q_t d_{t+1} - d_t] + [z_t - (1 - I_t) L(z_t)] N_t - v_t k, \quad \text{(7)}
\]

\[
(1 - \tau_t) q_t C_t^{-\sigma} = \beta E_t C_{t+1}^{-\sigma}, \quad \text{(8)}
\]

\[
v_t = \left\{ \frac{k}{(1 - \eta) (z_t - b)} \right\}^{-\frac{1}{\sigma}} - 1 \quad \text{and} \quad \text{(9)}
\]

\[
N_t = \frac{v_t}{(1 + v_t)^{1/\sigma}}, \quad \text{(10)}
\]

\[
u_t = 1 - N_t, \quad \text{(11)}
\]

\[
w_t = (1 - \eta) b + \eta z_t, \quad \text{(12)}
\]

\[
(1 - I_t) d_{t+1} = 0, \quad \text{(13)}
\]

and

\[
I_t \left[ q_t - \frac{E_t I_{t+1}}{1 + r^*} \right] = 0, \quad \text{(14)}
\]

2.6 Optimal Government Policy

We focus on Markov perfect equilibria in which a benevolent government chooses the level of tax on debt to maximize households’ welfare subject to implementability conditions. The lump-sum tax is residually determined in a way to balance the per period government budget constraint.

If the country is in good financial standing in period \( t \), \( I_{t-1} = 1 \), the value of continuing to service the external debt, denoted as \( V^c (z_t, d_t) \), i.e., the value of setting \( I_t = 1 \), is given by

\[
V^c (z_t, d_t) = \max_{(C_t, d_{t+1})} \left\{ \frac{C_t^{1-\sigma} - 1}{1 - \sigma} + \beta E_t V^g (z_{t+1}, d_{t+1}) \right\}
\]

subject to the government budget constraint (5), the resource constraint (7), and two implementability constraints (11) and (12), where \( V^g (z_{t+1}, d_{t+1}) \) denotes the value of being in good financial standing. The value of being in bad financial standing in period \( t \), denoted as \( V^d (z_{t+1}) \), is given by

\[
V^d (z_t) = \max_{(N_{t+1})} \left\{ \frac{(z_t N_t - v_t k)^{1-\sigma} - 1}{1 - \sigma} + \beta E_t \left[ \lambda V^g (z_{t+1}, 0) + (1 - \lambda) V^d (z_{t+1}) \right] \right\}
\]

subject to equation (5), (11) and (12).

In any period \( t \) in which the country is in good financial standing, it has the option to either continue to service the debt obligations or to default. It follows that the value of being in good standing in period \( t \) is given by

\[
V^g (z_t, d_t) = \max \left\{ V^c (z_t, d_t), V^d (z_t) \right\}
\]

so the default set is

\[
B (d_t) = \left\{ z_t : V^c (z_t, d_t) < V^d (z_t) \right\}
\]

and the bond price schedule is

\[
q (z_t, d_{t+1}) = \frac{1 - \Pr \{ z_{t+1} \in B (d_{t+1}) \mid z_t \}}{1 + r^*}
\]
3. Quantitative Exercise

I solve the model numerically using discrete state space value function iterations, see Appendix 6.1 for details. The model is calibrated to the Argentine economy in which one period corresponds to a quarter. The intertemporal elasticity of substitution, $\sigma$, is set at 2 which is in line with much of the related literature. The world interest rate, $r^*$, is set to 1 percent per quarter. The probability of reentry, $\lambda$, is calibrated so that the average exclusion period is 7.5 years.

The curvature parameter in the matching function, $\iota$, is set to be 3.46 as in Pei and Xie (2017). I calibrate the fixed per unit vacancy cost, $k$, at 0.04 so that the model can reproduce a realistic quarterly unemployment rate of 7.8 percent, which is not far from the average unemployment rate of 8.3 percent from December 2002 to March 2018. The magnitude of vacancy cost corresponds to 7.5 percent of quarterly wages, which seems empirically plausible in emerging economies like Argentina given relatively high barriers to entry for firms.

The worker’s bargaining power, $\eta$, is set at 0.64, which is also the wage elasticity to labor productivity. Notice that in the current setup, the separation rate of worker-job pair is equal to one. As a result, wage does not depend on labor market tightness. I pick the value of $b = 0.30$ and the resulting replacement ratio is 40.72 percent that is close to Shimer (2005).

I assume that aggregate productivity follows an $AR(1)$ process for the logarithm of $z$ with an auto-correlation of 91.59 percent per quarter and a standard deviation of 2.71 percent. These moments imply that output is of 7.27 percent standard deviation. Following Arellano (2008), I assume that when the country is in bad standing, it loses any productivity above a certain threshold $\bar{z}$, i.e.,

$$z_t - L(z_t) = \begin{cases} z_t & \text{if } z_t < \bar{z} \\ \bar{z} & \text{if } z_t \geq \bar{z}. \end{cases}$$

Then I estimate $\bar{z}$ to be 0.9. Finally, the subjective discount factor, $\beta$, is calibrated at 0.854. Together with the rest of the parameter values, the model produces the following three equilibrium implications: (i) the average debt-to-GDP ratio in periods of good financial standing is about 54.7 percent per quarter; (ii) the frequency of default is 2.7 times per century, and (iii) the average output loss is 11 percent per year conditional on being in financial autarky. Table 1-3 summarize the values of parameters and some other selected empirical and theoretical first and second moments.

From Table 3 we can see that the model only explains half of the observed average country premium in Argentina (3.5 percent versus 7.4 percent per year). As highlighted in Na et al. (2018), international lenders are risk-neutral so that the average country premium should approximately be the same as the average frequency of default. In this sense, there is no way for the model to explain both moments at the same time unless they are equal to each other in the data.

Another couple of unrealistic moments replicated by the model are the volatility and countercyclicality of the country premium. In my model, the default penalty is set similarly as in Arellano (2008) and Balke and Ravn (2016) in which the country loses any productivity above a certain threshold $\bar{z}$. Compared to the quadratic loss function specified by Chatterjee and Eyigungor (2012), it induces lower volatility of the country premium and reduces its correlation with output. Despite the shortfalls, the positive correlation between the trade balance and the country premium does match closely with the data. The intuition, as highlighted by Uribe and Schmitt-Grohé (2017), is because international lenders would demand that the country at risk of default make an effort to improve its financial situation by at least paying part of the interest due.

Table 4 and 5 display the first and second moments of several labor market variables generated from the model. First note that the average job-finding rate is more than twice as large as the job-filling rate. Recall that the total measure of workers looking for jobs at the beginning of each period is one. The lack of accumulation of employment stock thus calls for a constantly tighter labor market to deliver realistic average unemployment level. This leads to an empirically implausible vacancy rate that is larger than one. In Balke and Ravn (2016), they circumvent this issue by introducing individual search cost. Also, my model fails to generate observed cyclical movements in terms of unemployment and vacancy. As Shimer (2005) suggests, this is because wage is too flexible with respect to productivity. Intuitively,}

\footnote{Data source: CEIC, see https://www.ceicdata.com/en/indicator/argentina/unemployment-rate.
TABLE 1: EXTERNALLY CALIBRATED PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r^*$</td>
<td>0.01</td>
<td>risk-free return</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.0333</td>
<td>prob. of reentry</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2</td>
<td>risk aversion</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.9159</td>
<td>autocorrelation</td>
</tr>
<tr>
<td>$\sigma_\rho$</td>
<td>0.0271</td>
<td>standard deviation of $\mu$</td>
</tr>
<tr>
<td>$k$</td>
<td>0.04</td>
<td>vacancy cost</td>
</tr>
</tbody>
</table>

TABLE 2: INTERNALLY CALIBRATED PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.854</td>
<td>discount factor</td>
</tr>
<tr>
<td>$\bar{z}$</td>
<td>0.90</td>
<td>post-default productivity</td>
</tr>
<tr>
<td>$b$</td>
<td>0.30</td>
<td>unemployment benefit</td>
</tr>
<tr>
<td>$\iota$</td>
<td>3.46</td>
<td>matching parameter</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.64</td>
<td>worker’s bargaining power</td>
</tr>
</tbody>
</table>

in good times when productivity is high, the wage level is also high so that firms have no incentive to create more vacancies. To solve the Shimer’s puzzle, one way is to use the calibration strategy proposed by Hagedorn and Manovskii (2008, HM). In particular, they argue that the flow value of unemployment activities should equal that of employment. This makes wage de facto rigid. Firms cannot adjust wage downward and thus reduce vacancy creation during the downturn driven by productivity shocks. However, the experiment with HM’s calibration of the current model still couldn’t generate sufficiently volatile job market flows. In our baseline model, unemployed workers are defined as those who fail to find a job in the labor market, which is largely distinct from the usual definition in the DMP model of people who are actively looking for jobs. The extension to a dynamic labor market is discussed in Section 4.

Figure 1 reports the dynamics of the model economy around a typical default episode under optimal policy. This is obtained by stochastic simulations of the model for one million periods. The first 0.1 million periods are discarded and I compute the paths of the key variables of both debt and labor markets in three year windows around the time of default. Notice that, as predicted by the model, the lump-sum transfer, $T_t$, becomes negative at default. This is because when the government chooses to default, it immediately confiscates any payments of households to international lenders and returns the proceeds to households via income subsidies (Uribe and Schmitt-Grohé, 2017). After default, as long as the country is in bad financial standing, the government has to impose a lump-sum tax so as to meet the need of financing the unemployment benefits.

In regard to the fiscal policy, the macroprudential type capital control tax increases the effective country interest-rate premium and helps private households internalize the pecuniary externality. The net stock of external debt is going down as the country is on the verge of the debt crisis though. Upon default, however, the unemployment increases less than half percent.

TABLE 3: RESULTS AND COMPARISON: DEBT MARKET

<table>
<thead>
<tr>
<th>Source of statistics</th>
<th>Default frequency</th>
<th>Debt-to-Y ratio</th>
<th>Average spread</th>
<th>$SD$ of spread</th>
<th>Corr of spread and Y</th>
<th>Corr of spread and TB-to-Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>2.6</td>
<td>58.0</td>
<td>7.4</td>
<td>5.5</td>
<td>-0.64</td>
<td>0.72</td>
</tr>
<tr>
<td>Model</td>
<td>2.7</td>
<td>54.6</td>
<td>3.6</td>
<td>4.5</td>
<td>-0.47</td>
<td>0.74</td>
</tr>
</tbody>
</table>

Note: All data are from Uribe and Schmitt-Grohé (2017), except the standard deviation of country premium is from Arellano (2008).
Figure 1: A Typical Default Episode Under Optimal Policy
TABLE 4: RESULTS: LABOR MARKET

<table>
<thead>
<tr>
<th>Source of statistics</th>
<th>Average job-finding rate (monthly)</th>
<th>Average job-filling rate (monthly)</th>
<th>Average replacement ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>0.74</td>
<td>0.31</td>
<td>40.71%</td>
</tr>
</tbody>
</table>

TABLE 5: SUMMARY STATISTICS: CYCLICALITY FROM THE CALIBRATED MARKOV ECONOMY

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Productivity</th>
<th>Unemployment</th>
<th>Vacancy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard deviation</td>
<td>0.065</td>
<td>0.007</td>
<td>0.043</td>
</tr>
<tr>
<td>Correlation Matrix</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. Discussion

The baseline model assumes a static labor market which leads to two unsatisfactory outcomes. One is that the unemployment is not persistent. As a result, the model cannot generate plausible cyclicality in terms of labor market flows. It also gives rise to an empirically unrealistic tight labor market. The other drawback is that the labor and debt market are disentangled in the sense that the dynamics of the former is purely driven by productivity shocks. To examine the interactions between the two markets, we need to extend our analysis to a dynamic labor market. As in Petrosky-Nadeau, Zhang, and Kuehn (2018, PZK), the representative firm is now facing a different employment constraint. The firm’s problem becomes

\[
\max_{v_t} \mathbb{E} \sum_{t=0}^{\infty} \frac{\beta^t A_t}{A_0} (\tilde{z}_t N_t - w_t N_t - v_t k)
\]

subject to

\[
N_{t+1} = (1 - s) N_t + M (u_t, v_t)
\]

and

\[
v_t \geq 0
\]

where \( A_t = C_t^{-\sigma} \) and \( \beta A_{t+1} \) denotes the stochastic discount factor of the firm inherited from the representative household. Firm’s intertemporal job creation condition is thus given by

\[
\frac{k}{f(\theta_t)} - \mu_t = \mathbb{E}_t \frac{\beta A_{t+1}}{A_t} \left\{ \tilde{z}_{t+1} - w_{t+1} + (1 - s) \left[ \frac{k}{f(\theta_{t+1})} - \mu_{t+1} \right] \right\}
\]

(15)

where \( \mu_t \) is the Lagrangian multiplier associated with the constraint \( v_t f(\theta_t) \geq 0 \). PZK suggests that because of the occasionally binding vacancy constraint, there exists highly nonlinear dynamics that calls for a globally nonlinear algorithm to solve the model accurately, though they find in simulations it is very rare for the vacancy constraint to bind.

Also note that both sides of equation (15) depend on the tuple \( \{z_t, d_t, N_t\} \), in which employment level \( N_t \) is a state variable. This not only makes it difficult for the numerical algorithm to converge, but could also induce complicated government non-commitment problem if we consider the unemployment benefit, \( b \), as a policy instrument. Due to Nash bargaining, wage depends on the worker’s outside option. The strategic interaction between the government and the private sector in general would lead to multiple equilibria. In Pei and Xie (2017), they specify an exogenous wage setting to facilitate numerical convergence and generate enough unemployment variance. Although in this way, they admit that the macro channel emphasized in Hagedorn, Karahan, Manovskii, and Mitman (2013) is muted since benefit policies does not affect wages.
5. Conclusion

We introduce one additional element into the Eaton and Gersovitz (1981) model: labor market is frictional. This elaboration on the standard quantitative sovereign debt model can equivalently be described as a DMP labor search and matching model with aggregate productivity shocks and with one extra ingredient: sovereign borrowing and default risk. We find that with static labor search and matching setup, the model is feasible to solve despite a number of new elements relative to earlier models with productivity shocks and virtually no propagation.

Our positive findings are that the incorporation of a frictional labor market can help the model generate plausible debt level at good times and overall default frequency that fits the data. It also enables the model to replicate a couple of first and second moments that are not targeted. Specifically, it explains the observed volatility of the country premium and its positive correlation with the trade balance. The simulation of a typical default episode under optimal fiscal policy shows that the imposed debt tax helps private households internalize the increased sensitivity of the sovereign spreads with respect to debt in the run up to default.

Our model can serve as a framework for analyzing various stabilization and social-insurance policies in the context of financial crisis. The fixed level of unemployment benefit during the recession or the recovery of the economy helps the household to smooth consumption, but comes at two different kinds of cost: it deters firm’s vacancy creation by leveraging the worker’s outside option, and at the same time, increases the government’s financing burden as it needs to maintain a balanced budget constraint even during the downturns. It would be interesting to extend our analysis to a dynamic frictional labor market with persistent unemployment and intertemporal job creation decisions by firms. In particular, Petrosky-Nadeau, Zhang, and Kuehn (2018) recently explore the dynamics of endogenous rare disasters generated by the standard search model by solving it accurately with a globally nonlinear algorithm. The presence of an occasionally binding vacancy constraint would help propagate and amplify the effect of productivity shocks and default penalty on the fluctuations of unemployment and vacancies.

One dimension in which the baseline model is not empirically satisfactory is in its inability to simultaneously generate realistic average employment at good times and enough unemployment volatility over the business cycle frequency, even after one adopts the alternative calibration strategy proposed by Hagedorn and Manovskii (2008). Another drawback is the relatively low subjective discount factor. Future versions of the present settings ought to introduce long-term bond and possibly allow the government to optimally choose both the duration and level of unemployment benefit in a time-consistent fashion.

References


6. Appendix

6.1 Algorithm and method

I use discrete state space value function iterations and Brute-force grid search method\textsuperscript{11} in MATLAB to numerically solve for $V^c$, $V^d$ and $V^g$ over $Z \times D$. Here are the algorithm in steps:

1. Use 200 grid points each to cover the values of $z_t$ and $d_t$, respectively, so the state space is $Z \times D$, let $\pi$ denote the transition probability matrix of $z_t$, the values of debt lie between 0 and 0.6 with equally spaced grid points. The transition probability matrix is estimated using the method proposed by Schemitt-Grohé and Uribe (2014). Alternatively, one can follow either Tauchen (1986) or Rouwenhorst (1995) to estimate it.

2. Specify values of parameters, construct two block arrays for $d'$ and $z'$ over $Z$ for each grid point in $D$, respectively (the dimension is $200 \times 200 \times 200$), also construct a matrix for $z$ over $D$ (the dimension is $200 \times 200$);

3. Initialize guesses (zeros) for value functions $V^c$, $V^d$ and $V^g$ on the grid over $Z \times D$, let $i = 0$;

4. Define indicator function $df = 1_{\{vc < vb\}}$, start with guesses of bond price

$$q = \frac{1 - \pi \cdot df}{1 + r^\pi}$$

5. For each $d$ and $z$, compute $V_{i+1}$ by brute-force grid search;

6. Check whether $\varepsilon$ is less than $TOL = 10^{-8}$;

7. Run simulations of the model under optimal policy for 1.1 million quarters and discard the first 0.1 million quarters;

8. Identify default episodes that are of interest.

6.2 Calibration using Hagedorn and Manovskii (2008, HM)

In this section I report the calibration results from the baseline model using the strategy proposed by Hagedorn and Manovskii (2008). For values of parameters, four of them are updated: the subjective discount factor, the worker’s bargaining power, the matching parameter, and the level of unemployment benefit. The standard deviation of output of all times now becomes 1.43 percent, with the quarterly hiring cost to wage ratio of 7.22 percent and average labor market tightness (or vacancy rate) of 0.69. Table 6-9 summarize the rest results.

From Table 6, we can see that the HM calibration strategy gives rise to sizable improvement in terms of the variation of vacancy rate. The intuition, is that a commensurate value of unemployment benefit to wage rate makes workers indifferent between working on the market and staying at home. As a result, small variations in productivity would cause large fluctuations in labor market flows. However, the average unemployment rate now jumps up to an unrealistic level of 91.39%.

\textsuperscript{11}The programming code has largely benefited from the textbook supplemental material from Uribe and Schmitt-Grohé (2017) and Na et al. (2018).
### TABLE 6: UPDATED CALIBRATION VALUES OF PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.863</td>
<td>discount factor</td>
</tr>
<tr>
<td>$b$</td>
<td>0.35</td>
<td>unemployment benefit</td>
</tr>
<tr>
<td>$\iota$</td>
<td>0.307</td>
<td>matching parameter</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.64</td>
<td>worker’s bargaining power</td>
</tr>
</tbody>
</table>

### TABLE 7: RESULTS AND COMPARISON: DEBT MARKET

<table>
<thead>
<tr>
<th>Source of statistics</th>
<th>Default frequency</th>
<th>Debt-to-Y ratio</th>
<th>Average spread</th>
<th>SD of spread</th>
<th>Corr of spread and Y</th>
<th>Corr of spread and TB-to-Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>2.6</td>
<td>58.0</td>
<td>7.4</td>
<td>5.5</td>
<td>-0.64</td>
<td>0.72</td>
</tr>
<tr>
<td>Model</td>
<td>2.6</td>
<td>56.2</td>
<td>3.6</td>
<td>6.3</td>
<td>-0.43</td>
<td>0.68</td>
</tr>
</tbody>
</table>

*Note:* All data are from Uribe and Schmitt-Grohé (2017), except the standard deviation of country premium is from Arellano (2008).

### TABLE 8: RESULTS: LABOR MARKET

<table>
<thead>
<tr>
<th>Source of statistics</th>
<th>Average job-finding rate (monthly)</th>
<th>Average job-filling rate (monthly)</th>
<th>Average replacement ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>9.19%</td>
<td>4.40%</td>
<td>91.36%</td>
</tr>
</tbody>
</table>

### TABLE 9: SUMMARY STATISTICS: CYCLICALITY FROM THE CALIBRATED MARKOV ECONOMY

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Productivity</th>
<th>Unemployment</th>
<th>Vacancy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$z$</td>
<td>$u$</td>
<td>$v$</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.065</td>
<td>0.008</td>
<td>0.125</td>
</tr>
<tr>
<td>Correlation Matrix</td>
<td>$z$</td>
<td>1</td>
<td>-0.812</td>
</tr>
<tr>
<td></td>
<td>$u$</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>$v$</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>