A Two-Country Model of Technology Sharing (Draft Version)

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Abstract

This paper develops a two-country general equilibrium model of technology sharing to study world trade patterns. A technologically advanced country enjoys the benefit of comparative advantage due to her access to superior technology. The static model presented in this paper comes out with structural equations that show if a country with advanced technology shares her exclusive technology know-how with a technologically less-advanced country, up to a certain level, both of them will get higher gains from trade. A novel contribution of this paper is to determine the endogenous technology sharing that provides the highest level of utility to the technology leader.

1 Introduction

Recent macroeconomic literature has concentrated on the advancement of capital technology (i.e., automated technology, robotics, artificial intelligence, machine learning, etc.) in the production process (Zeira (1998), Autor (2015)). Conceptual frameworks are developed to investigate how capital embodied technologies substitute human labor, why this might or might not drive to a lower level of employment, and the contribution of labor share to the national income (Acemoglu and Restrepo (2018a,b)). These are the closed-economy models of labor replacing technology. There are substantial differences in the utilization of these technologies across the countries. Interesting scenarios will appear if we allow these countries to trade with each other. Transferring technology from one country to another plays a vital role in determining the pattern of world trade. Technology sharing is a process by which one country gains access to another's information on technical know-how to utilize it successfully into the production process (Derakhshani (1983),Maskus (2004)).

The paper aims to develop a fully worked-out model of technology sharing in which the usage of cheaper technology determines the pattern of trade. The paper's novelty lies in the determination of sharing cutoff, which ensures the optimality of utility by sharing technology. The formal model postulates a world of two countries: technology leader Home and technology follower Foreign. Countries are identical in terms of factor endowment, while technology possession is different. The adoption of exclusive technologies by Foreign is what gives rise to trade.

The model has several interesting implications. There is no fixed pattern of trade- goods exported by Home eventually becomes export of Foreign instead after sharing technologies. Capital returns will be higher in Home, even if capital productivity is the same in both countries. It happens because Home will share capital technology until she gets a higher return. However, wages across the borders will be the same, while labor demand will be higher in Foreign. Sharing technology should be limited. If Home shares technique more than the limit, it will create a situation for Home, which can be considered as revert to the autarky. It can be a possible explanation of the USA-China trade war: so-called forced technology transfer. This term says if the USA wants to operate in China, the USA has to part their technology know-how. If the USA shares all her exclusive technologies with China, there will be no reason for trade as both countries will be identical in terms of factor and technology endowment. To get the highest gains from trade, the USA has to find out how much of the technologies and which technologies should be shared. The static model presented in this paper answers this question.

The model in this paper has some standard features with Acemoglu and Restrepo (2018b), a very first step in building a conceptual framework, which discusses how automated technologies (i.e., machines, artificial intelligence, etc.) replace human labor and impacts of this consequence on labor and employment. The authors start their comprehensive theory of automation with a static model where the economy has a fixed capital, and technology is exogenous. Their theory implies that automation always reduces employment, labor share in national income and reduces wages under certain conditions; in contrast, the creation of new tasks has an exact impact on these factors. Zeira (1998) builds a model of automation that says highly productive countries only adopt technological innovations while the adoption of technology triggers differences in productivity between countries. However, Aghion et al. (2017) presents a model based on the work of Zeira to investigate the effects of increasing usage of automation on economic growth. Another important insight of this paper is the prediction of singularity due to the non-rivalry of knowledge. Acemoglu and Restrepo (2018a) talk about two countervailing forces of automation: displacement effect- substituting human labor by robots and therefore causing downward pressure on wages and productivity effect- reducing the cost of production through using robots and rising the wages by lowering consumption price. The famous paper of Krusell et al. (2000) shows that the substitutability between capital and unskilled labor is higher than that of between capital and skilled labor, where economic growth led by new and efficient technologies rises wage inequality increasingly. Susskind et al. (2017) develops a model of automated technology that shows the full immiseration of labor due to the emergence of "advanced capital," which can replace labors in performing tasks. Acemoglu and Restrepo (2017) study the impact of using industrial robots on the US labor market empirically.

Samuelson (2004) constructed a two-country and two-good Ricardian trade model where a technological improvement for the production of a commodity, whose comparative advantage belongs to the developing nation and international demand for that commodity is empirically *inelastic*, may cause a self-immiseration for the developing nation if she engages in a free-trade with the advanced economy. However, Samuelson's contrived artifact in this paper demonstrates that a rise in the productivity of a developing economy for the other commodity may transfer some of the comparative advantages previously possessed to the advanced economy. In such a situation, the advanced economy that has not experienced any change is immiserated as the external change abroad makes the autarkic price ratios equal between trading countries, which vanishes the gains from trade. Bhagwati et al. (2004) endorsed this finding only when the innovation gap between trading countries is too tiny. Countries like China and India are growing in such a way that they remain concentrated in the production of least productive technology services exported to the advanced economy (i.e., the USA). It is too optimistic to think that developing countries can educate their huge population to achieve knowledge of skill within a short period. In the notably celebrated paper, Vernon (1966) discussed how a more skilled labor force, social environment, external economies provide advantages to the advanced economy over developing economies in the production of new products. In this way, Bhagwati et al. dispelled the fear of Samuelson as the advanced economy has a more dynamic and innovative society than a developing economy, which will push the developed country to move up the technology ladder. However, Ruffin and Jones (2005) evaluated Samuelson's finding differently by deviating from the assumption of continuous technological developments in advanced countries, envisaged by Vernon (1966). They investigated whether an advanced country, a technology leader, can gain by sharing superior technology to a developing country without any cost in which it possesses her highest comparative advantage. They came with an affirmative answer by showing that improvement in developing country's technology, achieved by the free transfer of technology, may yield the gains from trade for the advanced economy by raising their real wages and incomes.

Krugman (1979) developed a model where trade pattern is determined by a continuous process of innovating new products and technology transfer, contemplating the reasons behind higher wage in an advanced economy. A slow rate of innovation or an accelerated technology transfer hampers the welfare of advanced economy laborers by narrowing the gap between wages earned by developing economy labors. That's why the continuity of innovation is essential to maintain the real income of technology leaders.

The rest of the paper is organized as follows. The next section describes the benchmark model; Section 3 provided a numeric characterization of the results achieved in Section 2. The closed economy presented in Section 2 is extended as an open economy model in this section when two countries engage in trade. Section 4 introduces technology sharing, which determines much of the technology should be shared. The final section comes with some concluding remarks.

2 The Baseline Model

This section restates the Acemoglu and Restrepo (2018b) model, which introduces our baseline model in the simplest way and characterizes the technology cutoff to determine factor market clearing conditions along with factor prices. We carry out the theoretical inquiry using a two-country world where countries are identical in terms of technology endowment and productivity, but different in technology access. The main focus will be given on the international sharing of exclusive technologies. Theoretical results derived below, which is a closed economy model as in Acemoglu and Restrepo (2018b), will drive us to determine the optimal amount of technology sharing that will ensure the highest level of utility. We will first focus on the autarky model, and then extend our analysis introducing trade and technology sharing.

2.1 Environment

The world economy consists of two countries. We refer to the two countries as Home and Foreign, denoted by h and f, respectively. Countries are identical in terms of factor endowment and productivity. As this section describes the model for autarky case, denote a country by j where $j \in \{h, f\}$. Country j produces a unique final good by combining a continuum of intermediate goods $i \in [\underline{i}, \bar{i}]$, with an elasticity of substitution $\sigma \in (0, \infty)$:

$$C_j = (\int_{\underline{i}}^{\bar{i}} c_{i,j}^{\frac{\sigma-1}{\sigma}} di)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

where i denotes the index of intermediates. However, index of intermediates are identical across the countries. Market for intermediates and final good are perfectly competitive.

Each good i is produced by a linear production function as follows:

$$y_{i,j} = \eta_{i,j} k_{i,j} + \gamma_{i,j} l_{i,j}$$

where $\eta_{i,j}$ and $\gamma_{i,j}$ denote the productivity of capital and labor in intermediate good i, respectively. So there is perfect substitutability between capital and labor in the production of intermediate good i. However, all intermediates can be produced with labor. Technology constraint in this environment is imposed by *automation* that there exists $I_j \in [\underline{i}, \bar{i}]$ such that goods $i \leq I_j$ are technologically automated. It means all the goods below I_j should be produced by capital. On the other hand, goods iI_j are produced with manual technology where labor is factor of production. Consider the following assumption:

Assumption 1. *$\eta_{i,j}$, $\gamma_{i,j}$ and $\frac{\gamma_{i,j}}{\eta_{i,j}}$ are strictly increasing in i.*

It implies labor productivity is rising at a faster rate compared to the capital which means labor has a strict comparative advantage in the production of higher indexed intermediates. Following figure ensures that goods with lower index will be automated, higher indexed goods will be produced with labor.

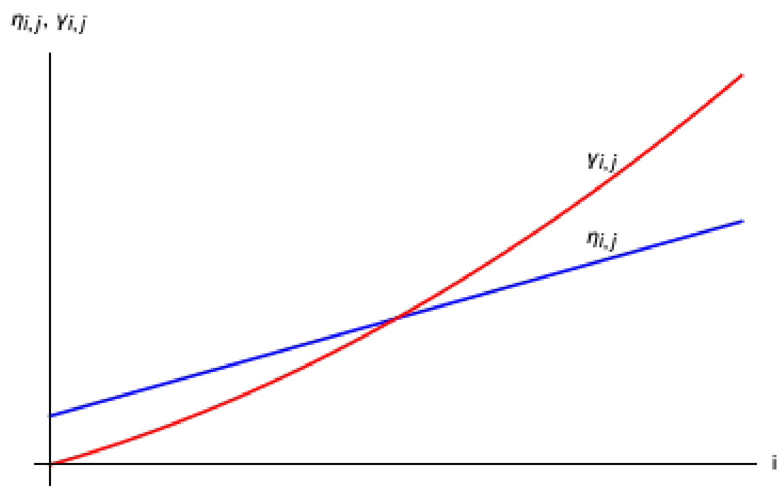

Figure 1: Factor productivity for the production of intermediate goods $i \in [\underline{i}, \bar{i}]$

$U(C_j, L_j)$ is the preference of a representative household which illustrates the demand side of the economy. C_j is consumption and L_j denotes the labor supply of representative household of country j. $U(C_j, L_j)$ is concave. Finally, as this is a static model, capital stock K_j is exogenous. There is no innovation of new goods. As a result, continuum of intermediate goods $i \in [\underline{i}, \bar{i}]$ is given (and unchanged). Next subsection characterizes the equilibrium price of intermediates, factor prices, endogenous labor supply, and technology cutoff I_j^*.

2.2 Equilibrium under Autarky

All the intermediates are supplied competitively which says the price of good i, p_i is equal to the marginal cost of production:

$$p_{j,i} = \begin{cases} \min\{\frac{R_j}{\eta_{i,j}}, \frac{W_j}{\gamma_{i,j}}\} & \text{if} \quad i \leq I_j \\ \frac{W_j}{\gamma_{i,j}} & \text{if} \quad i > I_j \end{cases}$$

Here, W_j denotes the wage rate and R_j is the return to capital. It is obvious that all goods $i > I_j$ are produced at a cost of $\frac{W_j}{\gamma_{i,j}}$. But the production cost for the goods $i \leq I_j$, $\min\{\frac{R_j}{\eta_{i,j}}, \frac{W_j}{\gamma_{i,j}}\}$ ensures the fact that capital and labor are perfect substitute. In such a situation, an intermediate good producer will pick a cheaper option. Assumption 1 ensures that there should a commodity $\tilde{I}_j$ for which the production costs using capital and labor are same.

$$\frac{R_j}{\eta_{\tilde{I}_j}} = \frac{W_j}{\gamma_{\tilde{I}_j}} \tag{2}$$

Consider the following figure:

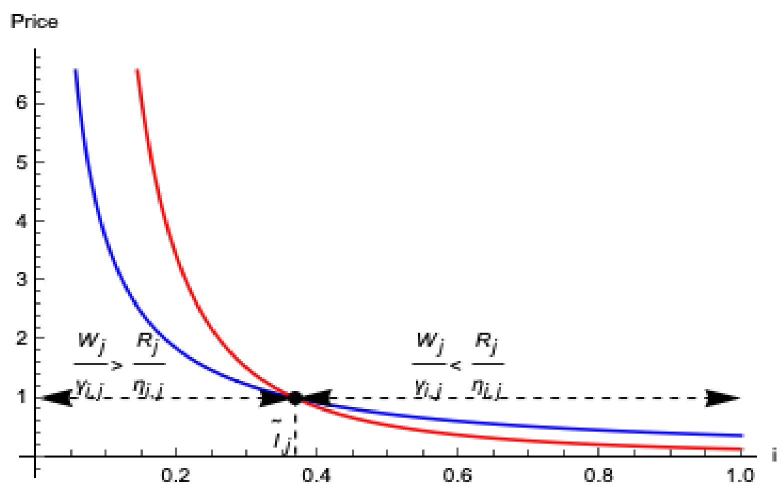

Figure 2: $\tilde{I}_j$ is the technology cutoff

The figure says all the goods $i < \tilde{I}_j$ should be produced with capital as $\frac{R_j}{\eta_{i,j}} < \frac{W_j}{\gamma_{i,j}}$, while the remaining goods will be produced by using manual technology. So, $\tilde{I}_j$ is the good which determines which intermediates should be produced with capital and which of them should be produced with labor. If $\tilde{I}_j < I_j$, it implies country j has the access to automated technology for producing goods i_j. But $\tilde{I}_j$ is such a commodity for which $\frac{R_j}{\eta_{i,j}} < \frac{W_j}{\gamma_{i,j}}$, country j will stop using automated technology after reaching $\tilde{I}_j$. Goods between the range of $(\tilde{I}_j, I_j]$ will be produced using manual technology; nonetheless, the presence of automated technology. But what will happen if country j is constrained by technology ($\tilde{I}_j > I_j$)? In such a situation, firms in country j cannot use automated technology beyond I_j because of the constraint on technology. That's why the equilibrium technology cutoff can be written as

$$I_j^* = \min\{\tilde{I}_j, I_j\} \tag{3}$$

If the capital stock is too high, then there is a chance that all of the goods should be produced using automated technology. To ensure the interiority of the solution, we are imposing an additional assumption:

Assumption 2. *Country j is endowed with capital stock $K_j < \bar{K}_j$, where $\bar{K}_j$ is such that $\frac{R_j}{\eta_{\bar{i}}} = \frac{W_j}{\gamma_{\bar{i}}}$.*

This assumptions implies that some of the goods are produced with capital, while some are using labor.

Throughout the analysis, final good is the numeraire. Demand for intermediate i from equation (1) can be written as

$$c_{i,j} = (p_{i,j})^{-\sigma} C_j$$

Existence of I_j^* ensures that the goods in the range of $[\underline{i}, I_j^*]$ are produced by capital only (i.e., $l_i = 0$). Market clearing condition for goods $i \leq I_j^*$ provides the demand for capital. Opposite thing will occur for the goods $i > I_j^*$

$$k_{i,j} = \begin{cases} (R_j)^{-\sigma}(\dfrac{1}{\eta_{i,j}})^{1-\sigma} C_j & \text{if} \quad i \leq I_j^* \\ 0 & \text{if} \quad i > I_j^* \end{cases} \quad \text{and} \quad l_{i,j} = \begin{cases} 0 & \text{if} \quad i \leq I_j^* \\ (W_j)^{-\sigma}(\dfrac{1}{\gamma_{i,j}})^{1-\sigma} C_j & \text{if} \quad i \leq I_j^* \end{cases}$$

Endogenous technology cutoff I_j^* can be determined by equation (2) and (3). Once it is determined, factor market clearing condition can be written as

$$K_j = \int_{\underline{i}}^{I_j^*} k_{i,j} di = \int_{\underline{i}}^{I_j^*} (R_j)^{-\sigma} (\frac{1}{\eta_{i,j}})^{1-\sigma} C_j di \tag{4}$$

$$L_j = \int_{I_j^*}^{\bar{i}} l_{i,j} di = \int_{I_j^*}^{\bar{i}} (W_j)^{-\sigma} (\frac{1}{\gamma_{i,j}})^{1-\sigma} C_j di \tag{5}$$

Equilibrium condition says that $-\frac{U_{C_j}}{U_{L_j}} = \frac{1}{W_j}$. Since the budget constraint of country j is $C_j = W_j L_j + R_j K_j$, this can be rearranged along with the equilibrium condition to yield the following endogenous labor supply:

$$L_j = L_j(K_j, I_j^*) \tag{6}$$

All the intermediates are assembled together to produce the final composite good. Determination I_j^* ensures that goods in the range of $[\underline{i}, I_j^*]$ are produced with capital at a cost of $\frac{R_j}{\eta_{i,j}}$, while manual technology is used for producing remaining good at a unit cost of $\frac{W_j}{\gamma_{i,j}}$. The choice of final good as the numeraire yield the ideal price condition as follows:

$$1 = [\int_{\underline{i}}^{I_j^*} (\frac{R_j}{\eta_{i,j}})^{1-\sigma} di + \int_{I_j^*}^{\bar{i}} (\frac{W_j}{\gamma_{i,j}})^{1-\sigma} di]^{\frac{1}{1-\sigma}} \tag{7}$$

We can define the competitive equilibrium as follows. Given the continuum of intermediate goods $[\underline{i}, \bar{i}]$, technology constraint I_j, and capital stock K_j, the system has **5** unknowns $\{I_j^*, R_j, W_j, L_j, C_j\}$ with **5** equations (3), (4), (5), (6) and (7). Equation (3) says that minimum value between $\tilde{I}_j$ and I_j will be considered as I_j^*. The technology constraint, I_j is exogenous, where $\tilde{I}_j$ is endogenous.

Proposition 1. *Suppose that Assumptions 1 and 2 hold. Then there exists a unique technology cutoff* I_j^* *such that*

$$\frac{W_j}{R_j} = (\frac{K_j}{L_j(K_j, I_j^*)})^{\frac{1}{\sigma}} [\frac{\int_{I_j^*}^{\bar{i}} (\frac{1}{\gamma_{i,j}})^{1-\sigma} di}{\int_{\underline{i}}^{I_j^*} (\frac{1}{\eta_{i,j}})^{1-\sigma} di}]^{\frac{1}{\sigma}}$$

Equations (4) and (5) provide the preceding equation. If $I_j^* = I_j < \tilde{I}_j$, deployment of factors is constrained by technology. But if $I_j^* = \tilde{I}_j < I_j$, an intermediate good producer will go for cost-minimizing factor allocation, which is determined by factor prices. Equation presented in the proposition along with equation (2) yield a solution for $\tilde{I}_j$ in terms of capital stock K_j.

As there is no functional form of factor productivity, all the endogenous variables are expressed in reduced form. In spirit of Acemoglu and Restrepo (2018b), we propose an economy model where two identical countries engage in trade without any cost. The only difference is that the Home country is using cost-minimizing technology, while Foreign is facing technology constraint. Difference in technology access works as a driving force of trade between these countries. This paper explores how the technology cutoff is determined in such a scenario. What will happen if the technology leader shares some of her exclusive technology with other countries? Previous literature (i.e., Krugman (1979)) takes this sharing technology as exogenous. A novel contribution of this paper is to determine the endogenous technology sharing level which will ensure the highest level of utility for technology leader, Home after trade. Following sections illustrate these issues **numerically**.

3 Numerical Exercise

We study trade and technology transfer in a world economy comprising two countries. The countries- which we refer to as Home and Foreign- are distinguished by their respective technology constraint. Apart from the technology constraints, both countries have identical factor endowments. Each countries are endowed with K units of capital and 1 unit of labor. Both countries are producing an identical continuum of intermediate goods, $i \in [0, 1]$. Factor productivity in those goods which are produced in both countries will be assumed to be same in Home and Foreign, so that the technology constraint is the mere source of trade in this model.

Demand side of the representative household, which is identical across the world, is expressed by an additively separable utility function:

$$U(C_j, L_j) = \log C_j + \log(1 - L_j) \quad \text{where} \quad j \in \{h, f\} \tag{8}$$

where C_j denotes consumption, L_j is the labor supply of the representative household at country j. The utility functions are concave and twice differentiable which positively depends on own consumption while $\log(1 - L_j)$ expresses the utility cost of labor supply.

Both of the countries are producing a unique final good C_j by combining a continuum of intermediate goods $i \in [0, 1]$, with an elasticity of substitution $\sigma = 1$. It means all of the intermediate goods will be used for the production of final good as follows:

$$C_j = \int_0^1 \log c_{i,j} di$$

Each of the intermediate goods are produced by a linear production function. It is possible to produce all goods by using labor only. However, assumption 1 says that labor productivity is growing at a faster rate compared to capital productivity. Keeping this assumption in mind, we can write, labor productivity $\gamma_i = i^2$ and $\eta_i = i$. So there should a good $\tilde{I}_j$ such that

$$\frac{W_j}{R_j} = \tilde{I}_j \tag{9}$$

It says the cost of producing with either capital or labor is equal. For all goods $i < \tilde{I}_j$, we have $\frac{R_j}{i} < \frac{W_j}{i^2}$. So it is cost saving for a country to produce these goods by using capital. However, if $\tilde{I}_j > I_j$, intermediate goods producers cannot use capital technology even if it is profitable to use capital technology. It happens due to the technology constraint. It is mentioned earlier that Foreign is constrained by technology ($\tilde{I}_j > I_j$) while Home is not. It means some goods are produced with capital in Home while labor is used for the production of those goods in Foreign even it is cost saving for Foreign to use capital technology for the production process. In this general setup, equilibrium threshold good can be defined as $I_j^* = \min\{\tilde{I}_j, I_j\}$. All of the capital will be deployed below I_j^* while labor will be used for the rest of the goods. As $\sigma = 1$, the factor market clearing condition can be written as

$$K = R_j^{-1} C_j I_j^*$$

$$L_j = W_j^{-1} C_j [1 - I_j^*]$$

Think about the supply of labor in country j. It has to be determined endogenously as there is a labor leisure trade-off. The marginal rate of substitution between consumption and labor should be equal to the price ratio which says

$$\frac{1}{1 - L_j} = \frac{C_j}{W_j}$$

3.1 World Without any Trade

Using this condition along with the budget constraint and equation (9) will provide the endogenous labor supply equation. For Home, it can be written as follows

$$L_h = \frac{1}{2} - \frac{K}{2\tilde{I}_h} \tag{10}$$

Home endogenous labor supply is expressed as a function fo K and $\tilde{I}_h$. However, the story will be different for Foreign labor. As Foreign constrained by using technology, she has no business of determining $\tilde{I}_f$. Rather she has to work with I_f. Factor market clearing conditions for Foreign provides a relationship between R_f and W_f as follows:

$$\frac{W_f}{R_f} = \frac{K}{L_f}\frac{1-I_f}{I_f} \tag{11}$$

Plugging this relation in Foreign budget constraint will provide the endogenous labor supply as a function I_f which is exogenously given.

$$L_f = \frac{1-I_f}{2-I_f} \tag{12}$$

Value of I_f determines how much labor will be supplied by Foreign. As $\tilde{I}_h < I_h$, Home has to endogenously determine the threshold $\tilde{I}_h$ which demarcates the use of production technology. Factor market clearing conditions along with endogenous labor supply condition provide the following equation

$$\tilde{I}_h^2 + K\tilde{I}_h - 2K = 0$$

K is exogenously given in this case. So the closed-form solution for $\tilde{I}_h$ will come out as an explicit function K as follows

$$\tilde{I}_h = \frac{-K + \sqrt{K^2 + 8K}}{2}$$

Final good is the numeraire in this analysis. As goods $i \in [0, I_j^*]$ are produced with capital, and goods $i \in (I_j^*, 1]$ are produced with labor, the price index can be written as

$$\log 1 = \int_0^{I_j^*} \log(\frac{R_j}{i})di + \int_{I_j^*}^1 \log(\frac{W_j}{i^2})di \tag{13}$$

As the countries are different by technology possession, the area of deploying factors is separate as well. It will cause a different factor returns across the borders. As it is the Home country business to determine $\tilde{I}_h$, equation (9) will be applicable only for Home. Using this condition, factor returns for Home can be expressed as

$$W_h = e^{\tilde{I}_h - 2}$$

$$R_h = \frac{e^{\tilde{I}_h - 2}}{\tilde{I}_h}$$

Plugging equation (11) in the price index will give the Foreign factor return as a function of K and I_f.

$$W_f = e^{I_f - 2}[\frac{K(2 - I_f)}{I_f^2}]^{I_f}$$

$$R_f = e^{I_f - 2}[K(2 - I_f)]^{I_f - 1} I_f^{1 - 2I_f}$$

Home capital will enjoy a higher return compared to the Foreign capital as Home capital is deployed at a larger territory. As a result demand for Home capital will higher than Foreign capital which causes a higher price. On the other hand, labor is cheaper in Foreign. Once we have factor returns and endogenous labor supply conditions, it is straightforward to find the consumption of each countries utilizing the respective budget constraint. Obviously, the level of consumption will be different across countries due to the different factor returns, so will be the level of welfare. It completes the description of this model for the autarky case.

So far in the analysis, we are saying that none of the economies are open for trade. Even after having identical factor endowments, due to the difference in technological constraints, results like factor returns, labor supply, consumption etc. are different across countries. Following sections will investigate the impact of cost-less trade between these two countries. The trade scenario will be discussed under two special cases. First, even though the countries are different in technology possession, assume that the technology difference between these two countries are tiny (i.e., $\tilde{I}_h \approx I_f$). Second scenario is comparatively complicated. In this case, we will study what will happen if the technology difference between these countries are high. We will answer the question of technology sharing in this scenario. Should a technologically advanced country share her technology with the competitor? If the answer is yes, then what type of technology should be shared?

3.2 What will Happen if $I_f \approx \tilde{I}_h$?

Assume that both countries allocate their respective capital uniformly for the production of intermediate goods. Mobility of capital across the border is strictly denied. Nonetheless, technology constraint limits the access of Foreign to capital technology, trade without any cost makes these two countries to use identical technology for the production of intermediate goods i. As the capital is immobile, Foreign cannot deploy her capital beyond I_f. We have to determine the Home threshold at first. Suppose, after allowing trade, there will be a new threshold $\tilde{I}_{h,\text{trade}}$ which demarcates the range of technology used in the production of intermediate goods $i \in [0, 1]$. Cost-free trade will make the price of intermediate goods equal between the countries. Additionally, both countries are assumed to be almost identical technologically. As a result, after trade consumption pattern will be same as well. Consider the following figure. The figure is saying that due to technology

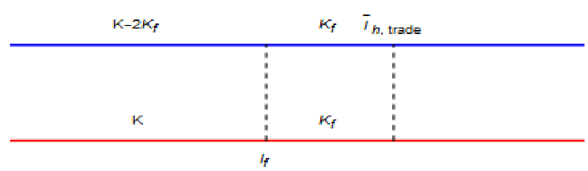

Figure 3: Trade between two countries when $I_f \approx \tilde{I}_h$

constraint, Foreign cannot invest her capital K beyond I_f. Rather, she invests her total capital in the range $[0, I_f]$. On the other hand, Home deploys $K - 2K_f$ units of capital for producing in the range of $[0, I_f]$ while $2K_f$ units of capital are used in the production of goods $(I_f, \tilde{I}_{h,\text{trade}}]$ for Home and Foreign as well. So, the total of amount of capital invested by both countries for producing goods $i \in [0, I_f]$ is $K - 2K_f + K = 2K - 2K_f$. These units of capital are used to satisfy the demand for goods $i \in [0, I_f]$ in both countries while $2K_f$ units of capital are used by Home only for satisfying the demand for goods $i \in (I_f, \tilde{I}_{h,\text{trade}}]$ herself and Foreign as well. Capital market clearing conditions after trade can be stated as follows:

$$2K - 2K_f = 2R_{\text{trade}}^{-1} C_{\text{trade}} I_f \qquad \text{and} \qquad 2K_f = 2R_{\text{trade}}^{-1} C_{\text{trade}} [\tilde{I}_{h,\text{trade}} - I_f]$$

Those two equations jointly provide the world capital market clearing condition. However, above two equations will provide the amount of capital deployed by Home in the range $[0, I_f]$ as follows:

$$K - 2K_f = R_{\text{trade}}^{-1} C_{\text{trade}} [2I_f - \tilde{I}_{h,\text{trade}}] \tag{14}$$

It has to be ensured that $2I_f - \tilde{I}_{h,\text{trade}} > 0$. Otherwise, amount of capital deployed by Home in the range $[0, I_f]$ will be destroyed. As it is assumed that $\tilde{I}_{h,\text{trade}} > I_f$, algebraic manipulation provides $2I_f - \tilde{I}_{h,\text{trade}} > 0$. In the autarky case, Foreign has higher concentration of capital for producing goods $i \in [0, I_f]$ which makes those goods cheaper compared to Home price. As there is no cost for trade, it is cost-saving for Home to import some of the goods from Foreign. Equation (14) says goods $i \in [\frac{\tilde{I}_{h,\text{trade}}}{2}, I_f]$ are produced by Home herself. Remaining goods in that range is imported from Foreign as follows:

$$\begin{aligned} \text{Home Import} &= R_{\text{trade}}^{-1} C_{\text{trade}} I_f - R_{\text{trade}}^{-1} C_{\text{trade}} [2I_f - \tilde{I}_{h,\text{trade}}] \\ &= R_{\text{trade}}^{-1} C_{\text{trade}} [\tilde{I}_{h,\text{trade}} - I_f] \end{aligned}$$

Foreign produces $[0, I_f]$ for herself and exports a fraction of the goods in this range to Home by deploying K units of capital.

$$K = \text{Own Demand} + \text{Home Import} = R_{\text{trade}}^{-1} C_{\text{trade}} \tilde{I}_{h,\text{trade}} \tag{15}$$

Remaining $2K_f$ units of capital owned by Home are employed in the production of $(I_f, \tilde{I}_{h,\text{trade}}]$ to satisfy her own demand and Foreign as well.

$$2K_f = 2R_{\text{trade}}^{-1} C_{\text{trade}} [\tilde{I}_{h,\text{trade}} - I_f] \tag{16}$$

Home will split this production into equal two parts. One part will go for for her own use while the other part will exported to Foreign which is same as the Home import. So far the trade is balanced for the zone $[0, \tilde{I}_{h,\text{trade}}]$. Adding (14), (15) and (16), we will get the world demand for capital

$$2K = 2R_{\text{trade}}^{-1} C_{\text{trade}} \tilde{I}_{h,\text{trade}} \tag{17}$$

Labor technology is common in both countries. As the labor productivity is identical and trade is cost-free, both countries will produce the goods beyond $\tilde{I}_{h,\text{trade}}$. However, the pattern of trade is indeterminate in this case. It may happen that there is no trade in that region. As the price of goods beyond $\tilde{I}_{h,\text{trade}}$ will be same after trade, each countries can produce goods $i \in (\tilde{I}_{h,\text{trade}}, 1]$ by herself. On the other hand, it may happen that Home will only produce a fraction of those goods while the remaining portion will be produced by Foreign. Amount of Home (Foreign) export (import) will be equal to Home (Foreign) import (export). World demand for labor can be written as

$$2L_{\text{trade}} = 2W_{\text{trade}}^{-1} C_{\text{trade}} [1 - \tilde{I}_{h,\text{trade}}] \tag{18}$$

Dividing (17) by (18) and algebraic manipulation will provide an equation for determining $\tilde{I}_{h,\text{trade}}$ which comes with a solution

$$\tilde{I}_{h,\text{trade}} = \frac{-K + \sqrt{K^2 + 8K}}{2} = \tilde{I}_h$$

After trade Home threshold is same as the autarky threshold. To sum up, it can be said that if $I_f \approx \tilde{I}_h$, Home will invest capital in the range $[0, I_f]$, i.e., $K - 2K_f > 0$. Trade without cost will provide an equilibrium threshold in Home $\tilde{I}_{h,\text{trade}}$ which is same as the autarky Home threshold $\tilde{I}_h$ which was solved endogenously by utilizing Home capital and Home labor. It means equilibrium post-trade factor price, labor supply and consumption in Home country will be same as pre-trade equilibrium value. Gains from trade in this case is zero for Home country while Foreign will be benefited from this trade as consumption goes up after trade due to the higher return to factors.

Things are kind of trivial when $I_f \approx \tilde{I}_h$. Interesting scenario will come up when two countries who are identical by factor endowment but totally different in technology possession engage in trade. For the convenience of the analysis, assume that even after having K units of capital, Foreign has no access to any capital-technology. On the other hand, Home is producing at her frontier. Following section will shed light on this issue.

3.3 Trade Between Two Countries when Foreign has no Access to Capital Technology

Start to think about a situation when Foreign cannot utilize her capital even after having same capital endowment as Home. It is possible only when Foreign has no access to the usage of capital technology. In such a situation, Foreign will produce all of her intermediate goods, $i \in [0,1]$ using labor only if there is no scope of trade. On the other hand, due to having capital technology, Home is utilizing their capital at the fullest. Now the question is what will happen if these two countries engage in trade under this scenario. Will the Home country gain anything from this kind of trade? To answer this question, at first we have to determine the threshold commodity $\tilde{I}_{h,\text{trade}}$. Goods $i \in [0, \tilde{I}_{h,\text{trade}}]$ will be produced by capital technology while the remaining goods will use the labor as production factor. Our key assumption states that the labor productivity is growing at a faster rate than the capital productivity. As the trade is free of cost, price of all intermediates will be same across the borders. Capital technology will be used until the production cost of using it is less than that of labor technology. That's why our key assumption defines a good $\tilde{I}_{h,\text{trade}}$ such that

$$\frac{R_h}{\tilde{I}_{h,\text{trade}}} = \frac{W_{\text{trade}}}{\tilde{I}^2_{h,\text{trade}}} \Rightarrow \frac{W_{\text{trade}}}{R_h} = \tilde{I}_{h,\text{trade}} \tag{19}$$

So it is profitable for Foreign to import the goods below $\tilde{I}_{h,\text{trade}}$ as she has no access to the capital technology. In this environment, only Home has the comparative advantage to produce those goods using capital technology. So Home capital, K is used to satisfy her own demand for goods $i \in [0, \tilde{I}_{h,\text{trade}}]$ and Foreign as well. Foreign capital is wasted in this case because of her inability to use any capital technology. The capital market clearing condition is

$$K = R_h^{-1}[C_h + C_f]\tilde{I}_{h,\text{trade}} \tag{20}$$

Both of the countries are endowed with 1 unit of labor, but the demand for labor is not inelastic. They can produce all the goods using labor technology while the labor productivity across the intermediate goods are assumed to be identical. Considering Foreign is bound to use only labor technology, the demand for labor will be different across countries in the autarky. It happens due to the technology constraints. Return to labor will also be different in this case. But when trade is allowed without any cost, it is already found that goods, $i \in [0, \tilde{I}_{h,\text{trade}}]$ will be produced by capital technology. Intermediate goods price equalization due to free trade will force the price of goods equal beyond $\tilde{I}_{h,\text{trade}}$. As the labor productivity is similar, the return to labor in both countries will be same. It means two countries will jointly produce goods $i \in (\tilde{I}_{h,\text{trade}}, 1]$. However, the direction of trade is indeterminate in this case. Home will produce a fraction of the goods, $i \in (\tilde{I}_{h,\text{trade}}, 1]$ by using labor while Foreign labor will be used to produce the remaining fraction. It will happen for balancing the trade. Complete specialization by any country for the production of this goods is strictly prohibited as it will make the labor of one country to be wasted fully. World labor market clearing condition is

$$L_h + L_f = W_{\text{trade}}^{-1}[C_h + C_f][1 - \tilde{I}_{h,\text{trade}}] \tag{21}$$

which says world demand for goods $i \in (\tilde{I}_{h,\text{trade}}, 1]$ will be met by the labor of both countries. But even after having the equal labor return, the labor supply decision is different across the countries. It happens due to the difference of technology constraint. The additively separable utility function says that the marginal rate of substitution between consumption and labor is equal to the price ratio as follows

$$1 - L_j = \frac{C_j}{W_{\text{trade}}} \qquad \text{where} \quad j \in \{h, f\}$$

Combining the above mentioned equilibrium condition with respective countries budget constraint, the endogenous labor supply function can be stated as follows

$$L_h = \frac{1}{2} - \frac{K}{2\tilde{I}_{h,\text{trade}}} \tag{22}$$

$$L_f = \frac{1}{2} \tag{23}$$

Home labor supply, L_h is a function of K and $\tilde{I}_{h,\text{trade}}$. On the other hand, Foreign labor supply, L_f is constant due to having access to capital technology. Once we have the condition for determining technology threshold, factor market clearing conditions and endogenous labor supply conditions, we can easily find out the closed form solution for $\tilde{I}_{h,\text{trade}}$. Diving equation (20) by (21) and plugging the endogenous labor supply conditions of the respective countries we can get an equation

$$2\tilde{I}_{h,\text{trade}}^2 + K\tilde{I}_{h,\text{trade}} - 2K = 0$$

K is exogenous variable in this case. The solution for $\tilde{I}_{h,\text{trade}}$ will come as function of K as follows

$$\tilde{I}_{h,\text{trade}} = \frac{-K + \sqrt{K^2 + 16K}}{4}$$

Previous section shows that when $\tilde{I}_h \approx I_f$, equilibrium technology threshold is same as the Home autarky threshold which force both economy to be identical after trade. Home country cannot gain anything from this kind of trade. But when two countries are "far" different from the point of view of technology constraint, new technology threshold is not same as the autarky one, i.e., $\tilde{I}_{h,\text{trade}} \neq \tilde{I}_h$. It means factor returns, consumption and welfare decisions will be different compared to the autarky case in this environment.

It is mentioned earlier that the final good is the numeraire. After trade, the price of intermediates are same across the countries while the final goods are produced by blending all of these intermediates. So the price of the final good will also be same after trade. The price index for final good after trade can be written as

$$\log 1 = \int_0^{\tilde{I}_{h,\text{trade}}} \log(\frac{R_h}{i})di + \int_{\tilde{I}_{h,\text{trade}}}^1 \log(\frac{W_{\text{trade}}}{i^2})di$$

It says goods produced by Home capital, $i \in [0, \tilde{I}_{h,\text{trade}}]$ are sold at the price $\frac{R_h}{i}$. On the other hand, price of the goods beyond $\tilde{I}_{h,\text{trade}}$ is $\frac{W_{\text{trade}}}{i^2}$. Using equation (19), factor returns can be written as follows

$$W_{\text{trade}} = e^{\tilde{I}_{h,\text{trade}}-2}$$

$$R_h = \frac{e^{\tilde{I}_{h,\text{trade}}-2}}{\tilde{I}_{h,\text{trade}}}$$

Once we have the equilibrium factor returns and labor supply conditions, it is a piece of cake to find out the equilibrium consumption. Utilizing the budget constraints of the respective countries, equilibrium consumption can be written as

$$C_h = R_hK + W_{\text{trade}}L_h = e^{\tilde{I}_{h,\text{trade}}-2}[\frac{1}{2} + \frac{K}{2\tilde{I}_{h,\text{trade}}}] \tag{24}$$

$$C_f = W_{\text{trade}} L_f = e^{\tilde{I}_{h,\text{trade}} - 2}[\frac{1}{2}] \tag{25}$$

We have to determine seven endogenous variables $\{\tilde{I}_{h,\text{trade}}, R_h, W_{\text{trade}}, L_h, L_f, C_h, C_f\}$ with seven equations (19), (20), (21), (22) (23), (24) and (25). An equilibrium in this open economy environment is technology threshold $\tilde{I}_{h,\text{trade}}$, a collection of sequence of factor prices $\{W_{\text{trade}}, R_h\}$ and prices for intermediate goods $\{\frac{R_h}{i}, \frac{W_{\text{trade}}}{i^2}\}$, endogenous labor supply decision $\{L_h, L_f\}$, and aggregate consumption decision $\{C_h, C_f\}$ such that each country maximize their utility subject to their respective budget constraints, intermediate goods producers in each country maximize their profits, and all of the factor market clearing conditions are satisfied.

Now it is the time to analyze whether the countries go for trade. To do so, we illustrate the

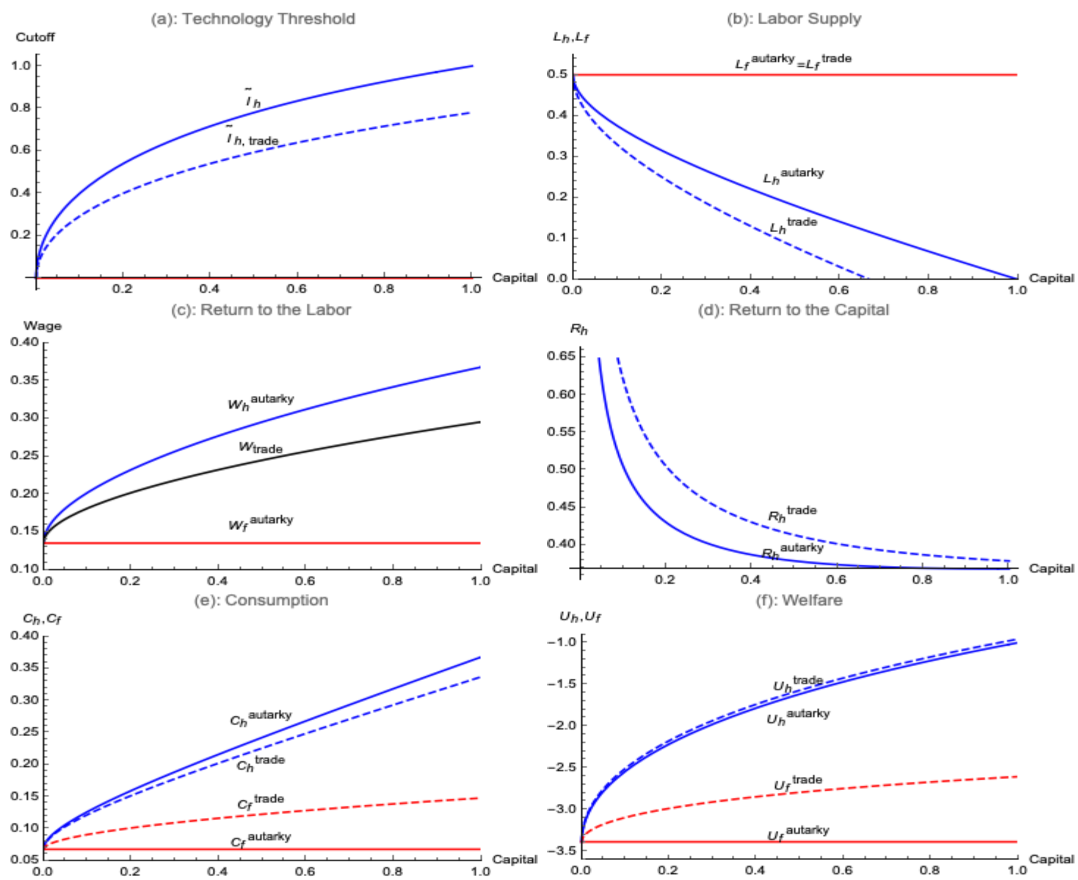

Figure 4: Trade when Foreign has no Access to Capital Technology

equations for determining threshold, labor supply, factor returns, consumption and welfare in the following figure. Panel (a) of figure 4 says that the level of technology threshold will go up with an increase in Home capital stock K. In the autarky, all the goods solid blue line will be produced by capital in the Home. The red line lies on the horizontal axis which means Foreign is producing everything by using labor only, no matter how much capital she has. After allowing trade, the new technology threshold is denoted by the dashed blue line which lies below the solid blue line for every amount of capital stock. It means trade shrinks the range of goods which are produced by capital. To investigate the reason, we have to look at panel (d) of the figure. After allowing at free of cost, Home capital is used to satisfy the demand of goods $i \in [0, \tilde{I}_{h,\text{trade}}]$ for herself and Foreign as well. As the amount of capital is fixed, the demand for Home capital will go up which raise the price of capital. Panel (d) illustrates that the after trade capital return is higher than that of pre-trade scenario. It means the price of those goods will go up. So it will be rational for Home to shrink the range of goods produced by capital technology. On the other hand, post-trade labor wage is same for both countries. Panel (c) shows that $W_h^{\text{autarky}} > W_{\text{trade}} > W_f^{\text{autarky}}$. As there is a fall in Home wage after trade, Home will reduce her supply of labor which is depicted in panel (b). It means Home produces a smaller fraction of goods in the range $(\tilde{I}_{h,\text{trade}}, 1]$. Majority part will be produced by Foreign which will be imported by the Home country. Due to a fall in labor supply and wage, Home consumption will go down after allowing trade even after there is a rise in capital return. On the other hand, Foreign consumption will go up as there is a rise in labor wage after trade. Panel (f) shows both countries are benefited after doing trade. Home gets the benefit as she can enjoy more of the leisure while Foreign is happier because of having more consumption. In a nutshell, both countries are better off due to this type of trade.

So far it is assumed that Foreign cannot use any of the capital technology even after having the same endowment of capital as Foreign. Next important thing is to investigate how to bring the Foreign capital in the production channel. Capital is immobile across the borders. So Foreign cannot transfer her capital to Home. One possible solution could be technology sharing. Let Home decides to share her some of the capital technology to Foreign. Is this kind of technology sharing by

Home well worth? If the answer is yes, how much of the technology should be shared? Following section will investigate this issue.

4 Trade with Technology Sharing

For ensuring the best use of Foreign capital, let Home decides to provide logistic support to Foreign so that Foreign can make use of their unused capital. Assumption 1 says that capital productivity is rising in i. Suppose Home decides to share her less productive capital technology μ to Foreign country without any cost where $\mu \in (0, \tilde{I}_h)$. Foreign will deploy her K units of capital for producing goods $i \in [0, \mu]$. Rest of the goods produced by capital technology is manufactured by Home. Like the previous case, $\tilde{I}_{h,\text{trade}}$ is the threshold commodity which is determined by the following equation:

$$\frac{W_{\text{trade}}}{R_h} = \tilde{I}_{h,\text{trade}} \tag{26}$$

Goods beyond $\tilde{I}_{h,\text{trade}}$ will be produced by labor technology as usual. In the previous section, it was assumed that even after having K units of capital, Foreign was unable to make any use because of the having no capital technology. After having μ units of capital technology from Home, Foreign can use her capital to produce goods $i \in [0, \mu]$ for herself and Home as well. Home capital will take care of the goods $i \in (\mu, \tilde{I}_{h,\text{trade}}]$. Capital market clearing condition in this case is

$$\text{Home:} \qquad K = R_h^{-1}[C_h + C_f][\tilde{I}_{h,\text{trade}} - \mu] \tag{27}$$

$$\text{Foreign:} \qquad K = R_f^{-1}[C_h + C_f]\mu \tag{28}$$

The question is why Home should share technology with Foreign. One possible explanation may be by sharing technology for the goods $i \in [0, \mu]$, Home can import them at a cheaper rate. On the other hand, she can enjoy higher return on capital by producing goods $i \in (\mu, \tilde{I}_{h,\text{trade}}]$. How much of the technology should be shared? As the endowment of K is same in both countries, equation (27) and (28) say

$$\frac{R_h}{R_f} = \frac{\tilde{I}_{h,\text{trade}} - \mu}{\mu} \tag{29}$$

There should be a threshold for sharing technology beyond which transferring any technology will say that the return to capital will be equalized in the both countries (like $K > 2K_f$ case). In that case, $R_h = R_f$ which will make the above equation as follows

$$1 = \frac{\tilde{I}_{h,\text{trade}} - \mu}{\mu} \qquad \Rightarrow \qquad \mu = \frac{\tilde{I}_{h,\text{trade}}}{2}$$

For having a higher capital return in Home country after trade, Home has to share technology below $\frac{\tilde{I}_{h,\text{trade}}}{2}$. However, like the previous case, goods beyond $\tilde{I}_{h,\text{trade}}$ will be produced by the labor of two countries. Due to having same labor productivity and trade at free of cost, wage in the two countries will be equal as usual. The labor market clearing condition in this case is

$$L_h + L_f = W_{\text{trade}}^{-1}[C_h + C_f][1 - \tilde{I}_{h,\text{trade}}] \tag{30}$$

This equation looks alike equation (21). However, technology threshold $\tilde{I}_{h,\text{trade}}$ in this case is different than that of no-sharing case. As the marginal rate of substitution between consumption and labor is equal to price ratio, endogenous labor supply condition after sharing technology can be found by exploiting the budget constraints of respective countries:

$$L_h = \frac{1}{2} - \frac{K}{2\tilde{I}_{h,\text{trade}}} \tag{31}$$

$$L_f = \frac{1}{2} - \frac{\mu K}{2\tilde{I}_{h,\text{trade}}(\tilde{I}_{h,\text{trade}} - \mu)} \tag{32}$$

Foreign labor supply decision now depends on the level of capital stock, technology threshold and shared technology as she can use her capital stock for the production of goods $i \in [0, \mu]$. Determination of $\tilde{I}_{h,\text{trade}}$ depends on world capital-labor ratio after trade. Algebraic manipulation using equations (26), (29), (31) and (32) provides the equation for determining $\tilde{I}_{h,\text{trade}}$ as follows:

$$2\tilde{I}_{h,\text{trade}}^2 + (K - 2\mu)\tilde{I}_{h,\text{trade}} - 2K = 0 \tag{33}$$

The closed-form solution for $\tilde{I}_{h,\text{trade}}$ can be written as

$$\tilde{I}_{h,\text{trade}} = \frac{-(K - 2\mu) + \sqrt{(K - 2\mu)^2 + 16K}}{4}$$

It is mentioned earlier that Home will share technology below $\frac{\tilde{I}_{h,\text{trade}}}{2}$. What will happen if $\mu \geq \frac{\tilde{I}_{h,\text{trade}}}{2}$? To answer this question, lets consider the case $\mu = \frac{\tilde{I}_{h,\text{trade}}}{2}$. Now plugging this in equation (33), algebraic computation will provide $\tilde{I}_{h,\text{trade}} = \tilde{I}_h$. All the results like factor returns, consumption, welfare for Home will be same as autarky condition in this case. Gains from trade after sharing technology will be zero. That's why for ensuring positive gains from trade, Home has to control the amount of technology sharing. Home has to make sure that $\mu < \frac{\tilde{I}_{h,\text{trade}}}{2}$. In other words, μ should be restricted as $\mu \in (0, \frac{\tilde{I}_{h,\text{trade}}}{2})$.

For the production of numeraire i.e., non-tradable final good, intermediate goods $i \in [0, \mu]$ are bought from Foreign at a price $\frac{R_f}{i}$, goods $i \in (\mu, \tilde{I}_{h,\text{trade}}]$ are bought from Home at a price $\frac{R_h}{i}$ and goods $i \in (\tilde{I}_{h,\text{trade}}, 1]$ are bought at a price $\frac{W_{\text{trade}}}{i^2}$. The price index for the final good can be written as

$$\log 1 = \int_0^{\mu} \log(\frac{R_f}{i})di + \int_{\mu}^{\tilde{I}_{h,\text{trade}}} \log(\frac{R_h}{i})di + \int_{\tilde{I}_{h,\text{trade}}}^{1} \log(\frac{W_{\text{trade}}}{i^2})di$$

Equations (26) and (29) provide the relationship between factor returns in terms of $\tilde{I}_{h,\text{trade}}$ and μ. Plugging these relations in the price index, we can find the returns for this technology sharing environment:

$$W_{\text{trade}} = \frac{e^{\tilde{I}_{h,\text{trade}}-2}[\tilde{I}_{h,\text{trade}} - \mu]^{\mu}}{\mu^{\mu}}$$

$$R_h = \frac{e^{\tilde{I}_{h,\text{trade}}-2}[\tilde{I}_{h,\text{trade}} - \mu]^{\mu}}{\mu^{\mu}\tilde{I}_{h,\text{trade}}}$$

$$R_f = \frac{e^{\tilde{I}_{h,\text{trade}}-2}[\tilde{I}_{h,\text{trade}} - \mu]^{\mu-1}}{\mu^{\mu-1}\tilde{I}_{h,\text{trade}}}$$

Now it is pretty straight forward to get the equations for consumption. Both countries are utilizing their capital and labor in this case, so consumption should come out as a function of $K, \tilde{I}_{h,\text{trade}}, \mu$.

$$C_h = e^{\tilde{I}_{h,\text{trade}}-2}[\frac{\tilde{I}_{h,\text{trade}} - \mu}{\mu}]^{\mu}[\frac{1}{2} + \frac{K}{2\tilde{I}_{h,\text{trade}}}] \tag{34}$$

$$C_f = e^{\tilde{I}_{h,\text{trade}}-2}[\frac{\tilde{I}_{h,\text{trade}} - \mu}{\mu}]^{\mu}[\frac{1}{2} + \frac{\mu K}{2\tilde{I}_{h,\text{trade}}(\tilde{I}_{h,\text{trade}} - \mu)}] \tag{35}$$

The total system has **8** unknowns $\{\tilde{I}_{h,\text{trade}}, R_h, R_f, W_{\text{trade}}, L_h, L_f, C_h, C_f\}$ with **8** equations (26), (27), (28), (30), (31), (32), (34) and (35). Plugging the maximized consumption and labor in the utility function will provide the indirect utility function a.k.a. welfare for respective countries.

Figure 5 illustrates the scenario when Home provides technical support to Foreign. All the graphs are drawn in terms of technology sharing to capture the impact of sharing on relevant variables. It

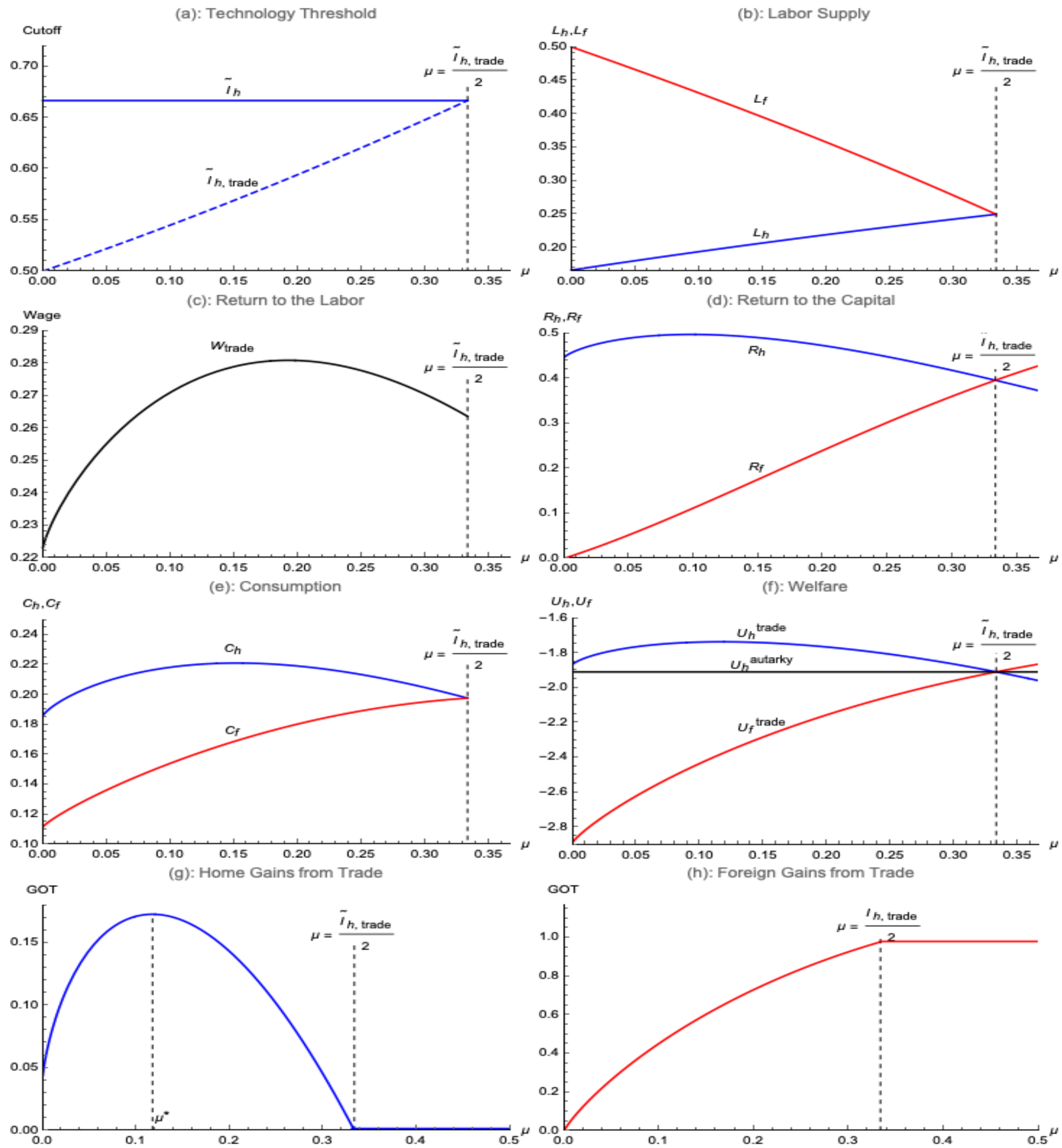

Figure 5: Trade when Home Shares Technology with Foreign

will help us to answer the question of how much of the technology should be shared and what will happen if Home continues to share onwards. As none of the technology is shared in the autarky, the decision of determining threshold $\tilde{I}_h$ will be independent of μ which is shown by the horizontal line in panel (a). There is a positive correlation between after trade technology threshold $\tilde{I}_{h,\text{trade}}$ and μ.

The higher the value of μ, the range of Foreign capital deployment will expand. As this is a static model, capital accumulation is not allowed. Due to the expansion of the range of operations, demand for Foreign capital will go up which, will raise the Foreign capital return. This thing is shown by the red line in panel (d). After transferring μ amount of technology, which were previously produced by Home, K units of Home capital will be deployed beyond μ for producing goods. This will push up the threshold $\tilde{I}_{h,\text{trade}}$ as due to the relative abundance of capital, now it will be profitable for Home to produce some of the goods using capital which were previously produced by labor. It is evident from the figure that sharing technology will provide higher factor returns and higher consumption to both countries. In fact, it will increase the welfare of both countries. Now the question is how much of the technology should be allowed to transfer. Consider panel (f). It is showing that if Home shares technology more than $\frac{\tilde{I}_{h,\text{trade}}}{2}$ (i.e., $\mu \geq \frac{\tilde{I}_{h,\text{trade}}}{2}$), Home country will start to loose. The blue line falls below the black line (which denotes the Home autarky welfare) beyond $\frac{\tilde{I}_{h,\text{trade}}}{2}$. So there is no reason for Home country to share capital technology more than $\frac{\tilde{I}_{h,\text{trade}}}{2}$. Additionally, panel (d) shows that if Home shares capital technology more than $\frac{\tilde{I}_{h,\text{trade}}}{2}$, her return to capital will fall below Foreign. The same thing will happen to consumption. However, Foreign will always be better off by getting the technology from Home and this will continue even after $\frac{\tilde{I}_{h,\text{trade}}}{2}$. Panel (f) says Home will get positive gains from trade only if she shares capital technology with Foreign until $\mu < \frac{\tilde{I}_{h,\text{trade}}}{2}$.

The most important task is to find out the level of shared technology which will provide the highest level of gains from trade. Consider panel (g) and (h) which show the Home and Foreign gains from trade respectively. As there is no incentive for Home to share technology beyond $\frac{\tilde{I}_{h,\text{trade}}}{2}$, the gains from trade for Home after $\frac{\tilde{I}_{h,\text{trade}}}{2}$ will be zero. This is shown by the blue line on the horizontal axis of panel (g). The concave shape of the blue line in panel (g) ensures the interiority of shared technology. It is showing that Home gains from trade will go up until she shares technology up to μ^*. After μ^*, it will start to fall, and will be vanished after crossing $\frac{\tilde{I}_{h,\text{trade}}}{2}$. Panel (h) depicts that Foreign gains from trade will continue to go up until $\frac{\tilde{I}_{h,\text{trade}}}{2}$. After $\frac{\tilde{I}_{h,\text{trade}}}{2}$, Home will stop to share technology with Foreign. As a result, gains from trade for Foreign beyond $\frac{\tilde{I}_{h,\text{trade}}}{2}$ will remain same (parallel to the horizontal axis). Following proposition characterizes the unique interior equilibrium.

Proposition 2. *For* $K \in (0,1)$, $\lim_{\mu \to 0} \frac{\delta U_h}{\delta \mu} = +\infty$*; (ii)* $\lim_{\mu \to \frac{\tilde{I}_h}{2}} \frac{\delta U_h}{\delta \mu} < 0$*; and (iii)* $\frac{\delta^2 U_h}{\delta \mu^2} < 0$. *These three conditions ensure the interiority of* μ.

Proof. See Appendix A. □

The remarkable result in this proposition is the uniqueness of μ^*. The reason behind the interiority of the equilibrium is one of the main economic forces of our model. It describes the strategic behavior of a technology leader. In a two-country setup, when countries engage in trade-war, this model says the technology leader should provide how much of the technology to the opponent to get the optimal gains from trade. Figure 6 ensures the existence of the unique interior equilibrium μ^*. In a nutshell, the amount of technology sharing should be controlled by the technology leader, Home, to ensure the positive gains from trade. Otherwise, it will bring immiseration for Home.

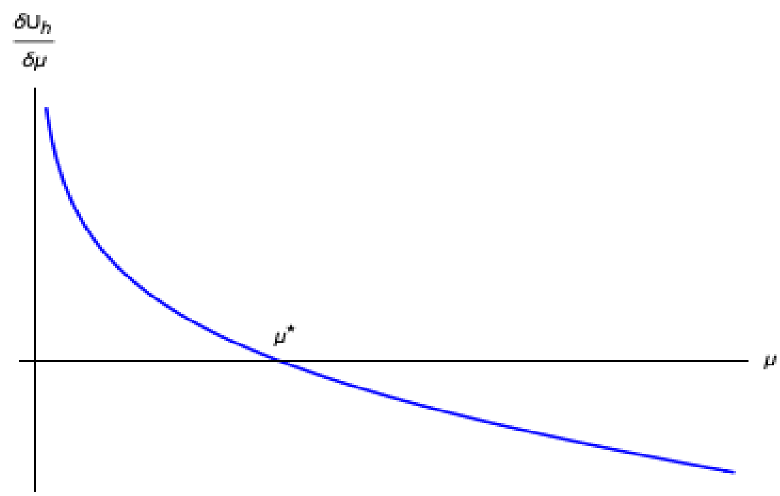

Figure 6: Unique Interior Equilibrium

Less productive technologies are assumed to be shared by Home so far. Consider a case of sharing most productive capital technology. Let Home will produce goods $i \in [0, \mu]$ while technologies for $(\mu, \tilde{I}_{h,\text{trade}}]$ will be given to Foreign. All goods at Home must be produced utilizing cheaper technology. As a result, the marginal good produced with capital at Home, when Foreign is given the technology at the top, must equal

$$\frac{W_{\text{trade}}}{R_h} = \mu \tag{36}$$

In this environment, commodity threshold $\tilde{I}_{h,\text{trade}}$ is determined by the following equation

$$\frac{W_{\text{trade}}}{R_f} = \tilde{I}_{h,\text{trade}} \tag{37}$$

Above equations will hold only if $\frac{R_f}{R_h} = \frac{\mu}{\tilde{I}_{h,\text{trade}}}$. Story for the factor market clearing condition is same as before. Home capital will be used to produce goods $i \in [0, \mu]$ while goods in the range of $(\mu, \tilde{I}_{h,\text{trade}}]$ will be produced by Foreign capital. Labors of both countries will jointly produce the goods beyond $\tilde{I}_{h,\text{trade}}$. Factor market clearing conditions in this case can be written as

$$K = R_h^{-1}[C_h + C_f]\mu \tag{38}$$

$$K = R_f^{-1}[C_h + C_f][\tilde{I}_{h,\text{trade}} - \mu] \tag{39}$$

$$L_h + L_f = W_{\text{trade}}^{-1}[C_h + C_f][1 - \tilde{I}_{h,\text{trade}}] \tag{40}$$

Capital market clearing condition says that $\frac{R_f}{R_h} = \frac{\tilde{I}_{h,\text{trade}} - \mu}{\mu}$. Using all of these conditions, it is possible to get an equation which show the relationship between μ and $\tilde{I}_{h,\text{trade}}$ as follows

$$\mu = \frac{(\sqrt{5} - 1)\tilde{I}_{h,\text{trade}}}{2} \tag{41}$$

It says determination of μ actually depends on the optimal choice of $\tilde{I}_{h,\text{trade}}$. Obviously, now μ is not an optimal choice.

Budget constraint of the respective country will provide the endogenous labor supply condition. However, this time the equations for labor supply will different from the previous case as Home is sharing her best capital technology this time.

$$L_h = \frac{1}{2} - \frac{1}{\sqrt{5} - 1} \frac{K}{\tilde{I}_{h,\text{trade}}} \tag{42}$$

$$L_f = \frac{1}{2} - \frac{K}{2\tilde{I}_{h,\text{trade}}} \tag{43}$$

Algebraic manipulations using equation (36)-(43) provides a closed-form solution for $\tilde{I}_{h,\text{trade}}$ as a function of K:

$$\tilde{I}_{h,\text{trade}} = \frac{-K + \sqrt{(K^2 + 8(3-\sqrt{5})K}}{2(3-\sqrt{5})}$$

K is exogenous in the static model. Determination of $\tilde{I}_{h,\text{trade}}$ depends solely on the value of K. Once $\tilde{I}_{h,\text{trade}}$ is determined, it is straightforward to find out μ. So we can see there is a basic difference between technology sharing from the bottom and the top. If Home decides to share her less productive capital technology (sharing from the bottom), Home has to decide on μ at first. The optimal decision of μ will tell us about how much of the goods should be produced with capital technology and how much with labor technology. The story will be different when Home decides to share her more productive capital technology (sharing from the top). μ is not an optimal choice anymore. Determination of optimal $\tilde{I}_{h,\text{trade}}$ will say how much of the technology should be shared.

There will be a slight change in the price index. Now Home will sell goods $[0, \mu]$ at price $\frac{R_h}{i}$, Foreign will sell $(\mu, \tilde{I}_{h,\text{trade}}]$ at a price $\frac{R_f}{i}$. Goods in the range $i \in (\frac{R_f}{i}, 1]$ will be sold at $\frac{W_{\text{trade}}}{i^2}$ in both countries. Price index for the numeraire final good is

$$\log 1 = \int_0^{\mu} \log(\frac{R_h}{\eta_i})di + \int_{\mu}^{\tilde{I}_{h,\text{trade}}} \log(\frac{R_f}{\eta_i})di + \int_{\tilde{I}_{h,\text{trade}}}^{1} \log(\frac{W_{\text{trade}}}{\gamma_i})di$$

Using the relationship between W_{trade}, R_h and R_f, factor returns after sharing technology from the top can be written as

$$W_{\text{trade}} = e^{\tilde{I}_{h,\text{trade}}-2}[\frac{\sqrt{5}-1}{2}]^{\frac{(\sqrt{5}-1)\tilde{I}_{h,\text{trade}}}{2}}$$

$$R_h = \frac{e^{\tilde{I}_{h,\text{trade}}-2}}{\tilde{I}_{h,\text{trade}}}[\frac{\sqrt{5}-1}{2}]^{\frac{(\sqrt{5}-1)\tilde{I}_{h,\text{trade}}}{2}-1}$$

$$R_f = \frac{e^{\tilde{I}_{h,\text{trade}}-2}}{\tilde{I}_{h,\text{trade}}}[\frac{\sqrt{5}-1}{2}]^{\frac{(\sqrt{5}-1)\tilde{I}_{h,\text{trade}}}{2}}$$

Plugging the factor returns and labor supply in the respective budget constraint will provide the consumption. It will complete the description of competitive equilibrium model in this environment. Now the crucial question is what will happen if Home decides to control the technology sharing from the top. In this case, Home has to decide on μ at first. Determination of optimal μ will provide the new automation cutoff. Home will transfer automated technology to Foreign only if Home achieves a higher capital return compared to Foreign. Capital market clearing conditions, equation (38) and (39), ensures that Home will get a higher return on capital compared to Foreign if $\mu > \frac{\tilde{I}_{h,\text{trade}}}{2}$. As Home will provide capital goods, $[0, \mu]$ to both Home and Foreign, she will try to set μ at the nearest possible location of $\tilde{I}_{h,\text{trade}}$. Home will deploy K units of capital for satisfying her own demand for the goods $i \in [0, \mu]$ and Foreign as well. It will provide a higher return to the Home capital owner(s). Remaining automated goods $(\mu, \tilde{I}_{h,\text{trade}}]$ will be produced by using Foreign capital. Comparing to the autarky situation, this type of transfer will provide higher return to capital owners of both countries. Meanwhile, a rise in μ pushes $\tilde{I}_{h,\text{trade}}$ to the right as $\frac{\delta \tilde{I}_{h,\text{trade}}}{\delta \mu} > 0$. Additionally, $\mu > \frac{\tilde{I}_{h,\text{trade}}}{2}$ ensures the after trade automation cutoff to exceed the Home autarky automation cutoff ($\tilde{I}_{h,\text{trade}} > \tilde{I}_h$). It means a higher value of μ narrows the range of the goods produced by labor. After trade, labor wage will be same on both countries. Shrinking the range of goods produced by labor will lower the labor wage. Lets compare the Home factor returns after trade:

$$\log W_{\text{trade}} - \log R_h = \log[\frac{\tilde{I}_{h,\text{trade}} - \mu}{\mu}]$$

It is mentioned earlier that Home will transfer automated technology only if $\mu > \frac{\tilde{I}_{h,\text{trade}}}{2}$. This condition ensures that $\log W_{\text{trade}} < \log R_h \Rightarrow W_{\text{trade}} < R_h$. Our model is based on the assumption that labor productivity is growing at a faster rate than that of capital. As a result, labor can produce some of the goods $i \in [0, \mu]$ cheaper at both Home and Foreign that what is being produced by Home capital (i.e.,$\frac{W_{\text{trade}}}{\gamma_i} < \frac{R_h}{\eta_i}$). Consider the following graph:

Home is supposed to produce goods $i \in [0, \mu]$ using capital for both Home and Foreign country. Due to controlled technology sharing, a subdomain of goods that should be produced by Home capital technology is instead produced by cheaper labor. It is clear from the figure that goods in the lower tail of $[0, \mu]$ should be produced by labor. And it creates the problem by violating the assumption of faster labor productivity. According to the assumption, labor has the comparative advantage in the production of higher index goods. It ensures the existence of a cutoff good which demarcates the range of goods according to the mode of production. After controlling the share of technology provision, even though the goods in the lower tail of $[0, \mu]$ are produced by labor,

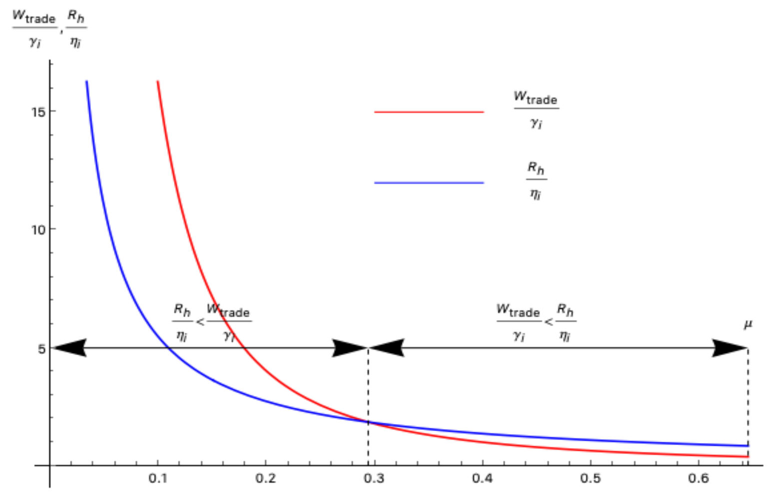

Figure 7: When controlled technology sharing occurs from the top

goods $i \in (\mu, \tilde{I}_{h,\text{trade}}]$ are produced by Foreign capital as $\frac{W_{\text{trade}}}{R_f} = \tilde{I}_{h,\text{trade}}$, which means goods beyond $\tilde{I}_{h,\text{trade}}$ are produced by labor again. So there is a reversal in the mode of production in the range $(\mu, \tilde{I}_{h,\text{trade}}]$ which is the violation of key assumption. Because of the infringement of this assumption, technology sharing variable, μ should not be determined endogenously. Instead of being an optimal choice, it is up to $\tilde{I}_{h,\text{trade}}$ (which is a function of K only) to use the discretion in setting how much of the technology should be shared from the top (μ).

5 Concluding Remarks

In this paper, we offered a two-country model where countries are symmetric in terms of factor endowment and productivity. Technology possession transpires the difference between these countries. This difference in technology is the driving force of trade between the two countries. However, sharing exclusive techniques by technology leader, who uses her available technology at the frontier, with the opponent makes the scenario more interesting. If she decides to provide her *less productive* automated technology to the follower, there should be an optimal level of sharing, which will ensure the highest level of utility to the technology provider. In such a situation, the determination of the optimal level of sharing dictates the technology cutoff- the cutoff which demarcates the automated technology from manual. On the other hand, the situation will be different if the technology leader decides to share her *more productive* exclusive automated technology. In this case, the leader must determine how much of the goods should be produced by automated technology at first. Once the technology cutoff is discovered, she can use her discretionary power to decide how much of the *more productive* technologies can be shared. Otherwise, it will end up with the result that violates the complete analysis's fundamental assumption that manual technology has a strict comparative advantage over automated technology in the production of higher indexed goods.

The whole analysis is based on the assumption that there is no innovation of new goods, and the stock of capital is fixed. Endogenizing capital formation and factor productivity will add some exciting features to the current analysis. A large body of the literature in this field conducts its analysis, assuming the rate of technology sharing is exogenous. A crucial result of this research is the endogenous determination of optimal sharing. The decision of optimal dynamic sharing is left for future research.

Appendix A

Proof of Unique Interior Equilibrium, μ^*

Proof. Home country utility function can be written as

$$\begin{aligned} U_h &= \log C_h + \log(1 - L_h) \\ &= \log(R_h K + W_{\text{trade}} L_h) + \log(1 - L_h) \end{aligned}$$

Differentiating with respect to μ

$$\frac{\delta U_h}{\delta \mu} = \frac{\frac{\delta R_h}{\delta \mu} K + \frac{\delta W_{\text{trade}}}{\delta \mu} L_h + W_{\text{trade}} \frac{\delta L_h}{\delta \mu}}{C_h} - \frac{\frac{\delta L_h}{\delta \mu}}{1 - L_h}$$

Taking the differentiation of R_h, W_{trade} and L_h

$$\begin{aligned}
\frac{\delta R_h}{\delta\mu} =& \frac{\delta}{\delta\mu}[\frac{e^{\tilde{I}_{h,\text{trade}}-2}[\tilde{I}_{h,\text{trade}}-\mu]^{\mu}}{\mu^{\mu}\tilde{I}_{h,\text{trade}}}] \\
=& \frac{\delta}{\delta\mu}[\mu^{-\mu}]\tilde{I}^{-1}_{h,\text{trade}}e^{\tilde{I}_{h,\text{trade}}-2}[\tilde{I}_{h,\text{trade}}-\mu]^{\mu} + \mu^{-\mu}\frac{\delta}{\delta\mu}[\tilde{I}^{-1}_{h,\text{trade}}]e^{\tilde{I}_{h,\text{trade}}-2}[\tilde{I}_{h,\text{trade}}-\mu]^{\mu} \\
&+ \mu^{-\mu}\tilde{I}^{-1}_{h,\text{trade}}\frac{\delta}{\delta\mu}[e^{\tilde{I}_{h,\text{trade}}-2}][\tilde{I}_{h,\text{trade}}-\mu]^{\mu} + \mu^{-\mu}\tilde{I}^{-1}_{h,\text{trade}}e^{\tilde{I}_{h,\text{trade}}-2}\frac{\delta}{\delta\mu}[\tilde{I}_{h,\text{trade}}-\mu]^{\mu} \\
\frac{\delta W_{\text{trade}}}{\delta\mu} =& \frac{\delta}{\delta\mu}[\frac{e^{\tilde{I}_{h,\text{trade}}-2}[\tilde{I}_{h,\text{trade}}-\mu]^{\mu}}{\mu^{\mu}}] \\
=& \frac{\delta}{\delta\mu}[\mu^{-\mu}]e^{\tilde{I}_{h,\text{trade}}-2}[\tilde{I}_{h,\text{trade}}-\mu]^{\mu} + \mu^{-\mu}\frac{\delta}{\delta\mu}[e^{\tilde{I}_{h,\text{trade}}-2}][\tilde{I}_{h,\text{trade}}-\mu]^{\mu} + \mu^{-\mu}e^{\tilde{I}_{h,\text{trade}}-2}\frac{\delta}{\delta\mu}[\tilde{I}_{h,\text{trade}}-\mu]^{\mu} \\
\frac{\delta L_h}{\delta\mu} =& \frac{\delta}{\delta\mu}[\frac{1}{2}-\frac{K}{2\tilde{I}_{h,\text{trade}}}] = \frac{K}{2\tilde{I}^2_{h,\text{trade}}}\frac{\delta}{\delta\mu}[\tilde{I}_{h,\text{trade}}]
\end{aligned}$$

i. Consider the term $\mu^{-\mu}$ in R_h or W_{trade} which can be written as

$$\mu^{-\mu} = e^{-\mu\log\mu}$$

Taking the derivative

$$\frac{\delta}{\delta\mu}[\mu^{-\mu}] = \frac{\delta}{\delta\mu}[e^{-\mu\log\mu}] = e^{-\mu\log\mu}[-1-\log\mu] = \mu^{-\mu}[-1-\log\mu]$$

Consider the case when $\mu \to 0$.

$$\lim_{\mu\to 0}\frac{\delta}{\delta\mu}[\mu^{-\mu}] = \lim_{\mu\to 0}\mu^{-\mu}[-1-\log\mu] = \lim_{\mu\to 0}\mu^{-\mu}\lim_{\mu\to 0}[-1-\log\mu] = -1-\lim_{\mu\to 0}\log\mu = +\infty$$

which actually says that $\lim_{\mu\to 0}\frac{\delta R_h}{\delta\mu} = +\infty$ and $\lim_{\mu\to 0}\frac{\delta W_{\text{trade}}}{\delta\mu} = +\infty$ as $\lim_{\mu\to 0}\frac{\delta}{\delta\mu}[\mu^{-\mu}] = +\infty$. All of these translates that

$$\lim_{\mu\to 0}\frac{\delta U_h}{\delta\mu} = +\infty$$

ii. Taking the following derivatives

$$\begin{aligned}
\frac{\delta}{\delta\mu}[\tilde{I}^{-1}_{h,\text{trade}}] &= -\frac{1}{\tilde{I}^2_{h,\text{trade}}}\frac{\delta}{\delta\mu}[\tilde{I}_{h,\text{trade}}] \\
\frac{\delta}{\delta\mu}[e^{\tilde{I}_{h,\text{trade}}-2}] &= e^{\tilde{I}_{h,\text{trade}}-2}\frac{\delta}{\delta\mu}[\tilde{I}_{h,\text{trade}}]
\end{aligned}$$

$[\tilde{I}_{h,\text{trade}} - \mu]^\mu$ can be written as $e^{\mu \log[\tilde{I}_{h,\text{trade}} - \mu]}$. Taking the derivative

$$\begin{aligned}\frac{\delta}{\delta\mu}[\tilde{I}_{h,\text{trade}} - \mu]^\mu &= e^{\mu \log[\tilde{I}_{h,\text{trade}} - \mu]}[\frac{\mu}{\tilde{I}_{h,\text{trade}} - \mu}(\frac{\delta}{\delta\mu}[\tilde{I}_{h,\text{trade}}] - 1) + \log[\tilde{I}_{h,\text{trade}} - \mu]] \\ &= [\tilde{I}_{h,\text{trade}} - \mu]^\mu[\frac{\mu}{\tilde{I}_{h,\text{trade}} - \mu}(\frac{\delta}{\delta\mu}[\tilde{I}_{h,\text{trade}}] - 1) + \log[\tilde{I}_{h,\text{trade}} - \mu]]\end{aligned}$$

Consider the case when $\mu \to \frac{\tilde{I}_h}{2}$.

$$\lim_{\mu \to \frac{\tilde{I}_h}{2}} \frac{\delta}{\delta\mu}[\mu^{-\mu}] = \underbrace{\lim_{\mu \to \frac{\tilde{I}_h}{2}} \mu^{-\mu}}_{>0} \underbrace{\lim_{\mu \to \frac{\tilde{I}_h}{2}} [-1 - \log \mu]}_{<0} < 0$$

As $\mu \in (0, 1)$, we should always have $[-1 - \log \mu] < 0$.

It is already proved that when $\mu \geq \frac{\tilde{I}_h}{2}$, the equilibrium trade cutoff is always equal to the Home autarky cutoff, $\tilde{I}_{h,\text{trade}} = \tilde{I}_h$. Taking the derivative of $\tilde{I}_h$

$$\frac{\delta \tilde{I}_h}{\delta\mu} = \frac{\delta}{\delta\mu}[\frac{-K + \sqrt{K^2 + 8K}}{2}] = 0$$

It actually says

$$\lim_{\mu \to \frac{\tilde{I}_h}{2}} \frac{\delta}{\delta\mu}[\tilde{I}_{h,\text{trade}}] = \lim_{\mu \to \frac{\tilde{I}_h}{2}} \frac{\delta \tilde{I}_h}{\delta\mu} = 0$$

Using this relation

$$\lim_{\mu \to \frac{\tilde{I}_h}{2}} \frac{\delta}{\delta\mu}[\tilde{I}^{-1}_{h,\text{trade}}] = -\underbrace{\lim_{\mu \to \frac{\tilde{I}_h}{2}} \frac{1}{\tilde{I}^2_{h,\text{trade}}}}_{>0} \underbrace{\lim_{\mu \to \frac{\tilde{I}_h}{2}} \frac{\delta}{\delta\mu}[\tilde{I}_{h,\text{trade}}]}_{=0} = 0$$

$$\lim_{\mu \to \frac{\tilde{I}_h}{2}} \frac{\delta}{\delta\mu}[e^{\tilde{I}_{h,\text{trade}} - 2}] = \underbrace{\lim_{\mu \to \frac{\tilde{I}_h}{2}} e^{\tilde{I}_{h,\text{trade}} - 2}}_{>0} \underbrace{\lim_{\mu \to \frac{\tilde{I}_h}{2}} \frac{\delta}{\delta\mu}[\tilde{I}_{h,\text{trade}}]}_{=0} = 0$$

$$\lim_{\mu \to \frac{\tilde{I}_h}{2}} \frac{\delta}{\delta\mu}[\tilde{I}_{h,\text{trade}} - \mu]^\mu = \underbrace{\lim_{\mu \to \frac{\tilde{I}_h}{2}} [\tilde{I}_{h,\text{trade}} - \mu]^\mu}_{>0}[\underbrace{\lim_{\mu \to \frac{\tilde{I}_h}{2}} \frac{\mu}{\tilde{I}_{h,\text{trade}} - \mu}}_{>0}(\underbrace{\lim_{\mu \to \frac{\tilde{I}_h}{2}} \frac{\delta}{\delta\mu}[\tilde{I}_{h,\text{trade}}] - 1)}_{<0} + \underbrace{\lim_{\mu \to \frac{\tilde{I}_h}{2}} \log[\tilde{I}_{h,\text{trade}} - \mu]]}_{<0}$$

$$\Rightarrow \lim_{\mu \to \frac{\tilde{I}_h}{2}} \frac{\delta}{\delta\mu}[\tilde{I}_{h,\text{trade}} - \mu]^\mu < 0$$

We want to see what will happen to $\frac{\delta R_h}{\delta\mu}$, $\frac{\delta W_{\text{trade}}}{\delta\mu}$ and $\frac{\delta L_h}{\delta\mu}$ if $\lim_{\mu \to \frac{\tilde{I}_h}{2}}$.

$$\lim_{\mu\to\frac{\tilde{I}_h}{2}} \frac{\delta R_h}{\delta\mu} = \underbrace{\lim_{\mu\to\frac{\tilde{I}_h}{2}} \frac{\delta}{\delta\mu}[\mu^{-\mu}]}_{<0} \underbrace{\lim_{\mu\to\frac{\tilde{I}_h}{2}} \tilde{I}^{-1}_{h,\text{trade}}}_{>0} \underbrace{\lim_{\mu\to\frac{\tilde{I}_h}{2}} e^{\tilde{I}_{h,\text{trade}}-2}}_{>0} \underbrace{\lim_{\mu\to\frac{\tilde{I}_h}{2}} [\tilde{I}_{h,\text{trade}}-\mu]^{\mu}}_{>0}$$
$$+ \underbrace{\lim_{\mu\to\frac{\tilde{I}_h}{2}} \mu^{-\mu} \lim_{\mu\to\frac{\tilde{I}_h}{2}} \frac{\delta}{\delta\mu}[\tilde{I}^{-1}_{h,\text{trade}}] \lim_{\mu\to\frac{\tilde{I}_h}{2}} e^{\tilde{I}_{h,\text{trade}}-2} \lim_{\mu\to\frac{\tilde{I}_h}{2}} [\tilde{I}_{h,\text{trade}}-\mu]^{\mu}}_{=0}$$
$$+ \underbrace{\lim_{\mu\to\frac{\tilde{I}_h}{2}} \mu^{-\mu} \lim_{\mu\to\frac{\tilde{I}_h}{2}} \tilde{I}^{-1}_{h,\text{trade}} \lim_{\mu\to\frac{\tilde{I}_h}{2}} \frac{\delta}{\delta\mu}[e^{\tilde{I}_{h,\text{trade}}-2}] \lim_{\mu\to\frac{\tilde{I}_h}{2}} [\tilde{I}_{h,\text{trade}}-\mu]^{\mu}}_{=0}$$
$$+ \underbrace{\lim_{\mu\to\frac{\tilde{I}_h}{2}} \mu^{-\mu}}_{>0} \underbrace{\lim_{\mu\to\frac{\tilde{I}_h}{2}} \tilde{I}^{-1}_{h,\text{trade}}}_{>0} \underbrace{\lim_{\mu\to\frac{\tilde{I}_h}{2}} e^{\tilde{I}_{h,\text{trade}}-2}}_{>0} \underbrace{\lim_{\mu\to\frac{\tilde{I}_h}{2}} \frac{\delta}{\delta\mu}[\tilde{I}_{h,\text{trade}}-\mu]^{\mu}}_{<0}$$
$$\Rightarrow \lim_{\mu\to\frac{\tilde{I}_h}{2}} \frac{\delta R_h}{\delta\mu} < 0$$

$$\lim_{\mu\to\frac{\tilde{I}_h}{2}} \frac{\delta W_{\text{trade}}}{\delta\mu} = \underbrace{\lim_{\mu\to\frac{\tilde{I}_h}{2}} \frac{\delta}{\delta\mu}[\mu^{-\mu}]}_{<0} \underbrace{\lim_{\mu\to\frac{\tilde{I}_h}{2}} e^{\tilde{I}_{h,\text{trade}}-2}}_{>0} \underbrace{\lim_{\mu\to\frac{\tilde{I}_h}{2}} [\tilde{I}_{h,\text{trade}}-\mu]^{\mu}}_{>0}$$
$$+ \underbrace{\lim_{\mu\to\frac{\tilde{I}_h}{2}} \mu^{-\mu} \lim_{\mu\to\frac{\tilde{I}_h}{2}} \frac{\delta}{\delta\mu}[e^{\tilde{I}_{h,\text{trade}}-2}] \lim_{\mu\to\frac{\tilde{I}_h}{2}} [\tilde{I}_{h,\text{trade}}-\mu]^{\mu}}_{=0}$$
$$+ \underbrace{\lim_{\mu\to\frac{\tilde{I}_h}{2}} \mu^{-\mu}}_{>0} \underbrace{\lim_{\mu\to\frac{\tilde{I}_h}{2}} e^{\tilde{I}_{h,\text{trade}}-2}}_{>0} \underbrace{\lim_{\mu\to\frac{\tilde{I}_h}{2}} \frac{\delta}{\delta\mu}[\tilde{I}_{h,\text{trade}}-\mu]^{\mu}}_{<0}$$

which says $\lim_{\mu\to\frac{\tilde{I}_h}{2}} \frac{\delta W_{\text{trade}}}{\delta\mu} < 0$.

Moreover, $\lim_{\mu\to\frac{\tilde{I}_h}{2}} \frac{\delta L_h}{\delta\mu} = \underbrace{\lim_{\mu\to\frac{\tilde{I}_h}{2}} \frac{K}{2\tilde{I}^2_{h,\text{trade}}}}_{>0} \underbrace{\lim_{\mu\to\frac{\tilde{I}_h}{2}} \frac{\delta}{\delta\mu}[\tilde{I}_{h,\text{trade}}]}_{=0} = 0.$

Plugging all these relations in $\frac{\delta U_h}{\delta\mu}$

$$\lim_{\mu\to\frac{\tilde{I}_h}{2}} \frac{\delta U_h}{\delta\mu} < 0$$

iii. The utility function is a log function which ensures the strict concavity of the utility function.

$$\frac{\delta^2 U_h}{\delta\mu^2} < 0$$

It means the slope of utility function is decreasing at a decreasing rate with an increase in μ.

Combining the above three conditions, it can be said that there exists an interior solution where $\frac{\delta U_h}{\delta \mu} = 0$ which will provide us $\mu^* = 0$. $\square$

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