The effect of networks on cooperation in the finitely repeated prisoner's dilemma

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January 19, 2024

Abstract

I discuss the effect of networks on cooperation in the finitely repeated prisoner's dilemma using three approaches: the game-theoretical model approach, the belief-based learning model approach, and the experimental approach. (1) The game-theoretical model consists of two games: the simple repeated PD (prisoner's dilemma) game for finite rounds and the repeated PD game on a network after the simple repeated PD game. In the repeated PD game on a network, players choose their game partners who can be neighbors on a network based on mutual consent. I find the subgame perfect pairwise-Nash equilibria in which all players defect and are completely connected or isolated. (2) The belief-based learning model allows updating players' strategies by learning opponents' strategies so that it explains the cooperative actions not explained by the game-theoretical model but observed in the experiments. My learning model shows that the strategies based on excluding defectors in network formation drive cooperation. (3) I examine the effect of networks on cooperative actions in human groups in the finitely repeated PD using three game experiments: the simple repeated game, the cheap talk repeated game, and the networked repeated game. First, the average cooperation rate in the networked repeated game is the highest among the three games. Second, the network formation in the networked repeated game is associated with increased cooperation. Finally, cooperators (defectors) are connected with other cooperators (defectors) in the network, and cooperation can be achieved by connections among cooperators who exclude defectors from the network.

Keywords: evolution of cooperation, the networked repeated prisoner's dilemma game, excluding defectors in network formation, reputation building

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1 Introduction

Why do we make connections with others? One reason to make connections with others is to find cooperators. We can get valuable information from cooperators and increase our economic welfare or total utilities through cooperators who give mutual benefits. For instance, in the labor market, unemployed persons can get valuable information to be hired through their social networks. Financial institutions can reduce the risk in the financial market through others connected to them. Thus, connections (or interactions) and cooperation are closely tied, and understanding interactions in the economic system is important to understand cooperation.

Several researchers have tried to improve economic thinking to understand cooperation. Also, several new theories and methodologies have been suggested to explain the evolution of cooperation. Behavioral and experimental economists have identified many examples of cooperation, especially in repeated games (see Bó (2005); Dal Bó and Fréchette (2011, 2018)), but we still need a unifying economic theory that explains interaction behaviors in economics because cooperation is closely related to interaction. By developing network theory, we can now understand the human interaction structure in the socio-economic system (see Goyal (2009); Jackson (2014); Carvalho and Tahbaz-Salehi (2019)).

This paper asks the question: What is the impact of human interaction on cooperative actions in human groups in the finitely repeated prisoner's dilemma game? To answer this question, I do three studies: the game-theoretical model study, the belief-based learning model study, and the experimental study.

In the game-theoretical model study, I analyze the networked repeated prisoner's dilemma game using equilibrium concepts. The game consists of n players and T rounds $(n, T \gg 2)$. From the 1st to round t_s (1 < $t_s \ll$ T), all players play the simple PD (prisoner's dilemma) with their partners randomly re-matched in each round. After round t_s , all players play the repeated PD game on a network based on a non-random partner selection rule using network formation. All players can see all histories of others' actions. Using that information,

all players propose links to others they want to play with. If the player who receives a proposal accepts the link, the proposer and the acceptor will be linked in the network. After link formation, all players play the PD game simultaneously with their neighbors, who are connected to them in the network. As a result of the equilibrium analysis, pairwise-stable networks exist in which cooperators and defectors are fully separated. There exist two pairwise-Nash equilibrium networks. Suppose the cost of network formation is less than a player's payoff when all players defect. In that case, the pairwise-Nash equilibrium network is the complete network in which all players defect and are completely connected. However, if the cost of network formation is greater than a player's payoff when all players defect, then the pairwise-Nash equilibrium network is an empty network in which all players defect and are isolated. These pairwise-Nash equilibrium networks are also subgame perfect equilibria. Thus, the game-theoretic model predicts that all players defect.

In the belief-based learning model study, I outline a novel belief-based learning model, which explains players' cooperative actions in the finitely repeated PD game. The model consists of two games: the simple repeated PD and the repeated PD on a network. Two players are randomly re-matched each round from the 1st to the 5th round, playing the PD game. From the 6th to the 20th round, all players play the PD game on a network. In the PD game on a network, players invite and select their game partners using the histories of all players' actions and form networks to play. In the PD game on a network, all players play the PD game simultaneously with their neighbors. There may be many strategies playing in the game, but this study focuses on three strategies that help us understand the relationship between cooperation and network formation: an m-threshold strategy, an excluding trigger strategy, and a network tit-for-tat (TFT) strategy. An m-threshold strategy player cooperates from the 1st round to the mth round and defects from the (m + 1)th round. An m-threshold strategy player would like to select all others as game partners. An excluding trigger strategy player always cooperates but never plays the game with defectors in the networked game. An excluding trigger strategy player never selects players who

previously defected as partners in the networked game. A network TFT strategy player always cooperates but does not play the game with the players who defected in the previous round. A network TFT strategy player does not select players who defected in the previous round as game partners in the network game. A player updates his or her beliefs by learning from the game opponents based on imitating the best strategy of his or her partners. A player in the game selects his or her best strategy to maximize the expected total payoff of the game at each round.

Hundreds of computer simulations show robust results in various parameter sets. First, the high cooperation rate at the first round and the decreasing pattern of cooperation as rounds increase in the simple repeated PD game and repeated PD game on a network are observed. Second, defectors are excluded by cooperators in the networked game. Third, in the simple repeated PD game, which corresponds to the game from the 1st round to the 5th round, a short threshold strategy, the m-threshold strategy for $0 \le m \le 5$, is the most popular. On the other hand, in the repeated PD game on a network, from the 6th to the 20th round, the most popular strategy is a long threshold strategy, the m-threshold strategy for $6 \le m \le 20$. Finally, the excluding trigger strategy and network TFT strategies lead to increases in cooperation for most parameter sets considered. The belief-based learning model shows that, in general, excluding defectors in network formation can promote players' cooperative actions in the finitely repeated PD game.

In the experimental study, I show the results of human behavioral experiments to measure the impact of networks on cooperative actions in human groups in the finitely repeated PD game using the students' subjects recruited from Iowa State University. Three kinds of games in experiments were conducted: the simple repeated game, the cheap talk repeated game, and the networked repeated game. In the simple repeated game, two players randomly selected played the simple repeated PD game for 20 rounds. In the cheap talk repeated game, two players randomly selected played a simple repeated PD game for 20 rounds, but from the 6th to the 20th round, each player could communicate with the game partner before submitting

his or her action. The networked repeated game consisted of two types of repeated PD games: the simple repeated PD game from the 1st to the 5th round and the repeated PD game on a network from the 6th to the 20th round. All players played the simple PD game with randomly re-matched partners each round from the 1st to the 5th round. From the 6th to the 20th round, players could select their partners using all players' reputation scores and histories of their actions. If two players agreed to be partners for a round, they would be linked and form links in the network for that round. After finishing the partner selection, the network was formed, and players played with their partners or neighbors on the network.

I recruited 216 unique human subjects and conducted 29 experiments with 2 players in the simple repeated games, 23 with 2 players in the cheap talk repeated games, and 8 with 14 players in the networked repeated games. First, the average cooperation rate in the networked repeated games is the highest among the three games. I hypothesize this is due to the reputation-building mechanism from Round 1 to 5 and the network formation from Round 6 to 20. Second, the networked repeated games show a significant increase in the fraction of cooperative actions in human groups after the $5^{\rm th}$ round. Moreover, the regression analysis confirms the positive effect of network formation on cooperative actions in human groups. In the cheap talk repeated games, the positive effect of cheap talk on cooperative actions in human groups is observed but not statistically significant. The cooperation rate in the simple repeated games shows a decreasing pattern as the round increase. Third, cooperators are linked to other cooperators, and defectors are linked to other defectors in the experiments. This result is measured by a significant positive similarity index. Finally, I conclude that cooperation can be achieved by connecting cooperators with excluding defectors from the network. Thus, the experimental results support the predictions of the belief-based learning model over the game-theoretic model.

The remainder of the paper is organized as follows. Section 2 presents the literature review on the evolution of cooperation and networked experiments. Section 3 considers the game-theoretical model study. Section 4 shows the belief-based learning model study.

Section 5 presents the experimental study. Finally, I conclude and discuss future research in Section 6.

2 Literature Review

Nowak (2006) suggests five rules for the evolution of cooperation: kin selection, direct reciprocity, indirect reciprocity, network reciprocity, and group selection. The last two rules are related to the network structure of human groups, and they are critical rules for improving cooperative actions in human groups. My research is related to the last two rules.

My game-theoretical study is related to the previous research to understand the equilibrium state in the networked game and the effect of network formation on cooperative actions in the equilibrium state. Ule (2008) finds that cooperation due to the threat of exclusion is achieved in the subgame perfect Nash equilibrium of the finitely repeated PD game when players endogenously form the network under the assumption that the number of rounds is sufficiently large. Mengel (2009) studies the bilateral PD game under local interaction with neighbors. She introduces the imitation process through their neighbors in the model. She finds that full cooperation emerges when there is a sufficiently strong conformist bias in the imitation process. Fosco and Mengel (2011) suggest the PD model that allows local interaction with their neighbors in the network. They find "full separation" of defectors and cooperators in the network. Also, they observe the "marginalization" of defectors. That is, connected cooperators are in the core of the network, and defectors are in the network's periphery. The result of Wolitzky (2013) shows that network structure matters in sustaining the level of cooperation in the public game. He finds that players with higher centrality¹ are more cooperative than others with lower centrality. Cho (2014) finds that a sequential equilibrium, which supports cooperation, is constructed by local communication on a

¹A network consists of nodes and links. Nodes represent agents in the network, and links represent the relationship between nodes. For example, in a friendship network, nodes represent persons, and links depict the friendship relationship between persons. Ch. 2 in Jackson et al. (2008) explains the representation of networks. Centrality shows the degree of importance of a node in the network. For example, the node with a high degree centrality – the number of the node's links – will be important in the network.

network in the repeated PD game. Wolitzky (2015) shows that only private information by cheap talk on a 2-connected network replicates public information in repeated games². Mihm and Toth (2020) investigate the relationship between cooperative network formation and information asymmetries in a dynamic network game. They find the equilibrium regardless of players' beliefs under private monitoring when the network structures satisfy a triadic closure property. It implies that players could be strongly connected in the cooperative networks under private monitoring. Sugaya and Wolitzky (2021) show that cooperation is reinforced by communication within the community. The results of Mihm and Toth (2020) and Sugaya and Wolitzky (2021) indicate that interaction or community structure could be critical to promoting cooperation in the repeated game in terms of information gathering and dissemination.

My game-theoretical model and belief-based learning model are also related to the previous research about social learning on a network because we need to understand social learning on networks to understand the effect of network formation on cooperative actions in human groups. Much previous research on social learning in networks has focused on the relationship between opinion formation or information diffusion and network structure.

Bala and Goyal (1998) suggest a network model that explains the relationship between neighborhoods' structures and the social learning process. They show that in a connected society, all agents get the same payoff in the long run due to learning from their neighbors.

Acemoglu et al. (2011) study the equilibrium of sequential social learning on networks, particularly a perfect Bayesian equilibrium. They show that agents' actions converge to the right action, which is the action to maximize the payoff, by learning from their neighbors as the size of the social network increases.

Mueller-Frank (2013) studies the process of rational learning in social networks, which is the process of making all possible inferences based on the history of neighbors they observed.

²This means that there are two independent paths between any two nodes in a network. That is, the removal of any single node in a 2-connected network does not prevent information flow in a network (see Wolitzky (2015)).

He shows that rational learning in social networks leads to a consensus of players' opinions when there is common knowledge of strategies. He shows that the duration to consensus, which is the time taken to reach consensus, is a function of the network's diameter³: the larger the diameter is, the longer the duration to consensus is.

Lobel and Sadler (2015) study how information in networks diffuses through social learning using the Bayesian learning approach. They show that information can diffuse when neighbors are independent if a minimal connection holds but can fail to diffuse when neighbors are correlated in a well-connected network. It implies that weak connections based on heterogeneous agents would be more efficient in diffusing information if a minimal connection holds than strong connections based on homogeneous agents.

Dasaratha and He (2020) study the naïve sequential learning model on networks. In the naïve sequential learning setting, an agent follows aggregating actions of predecessors linked to him or her in the network. They show that the naïve sequential learning model in networks can lead to incorrect actions or mislearning, which are not payoff-relevant actions. They also characterize the networks that lead to correct actions or payoff-relevant actions. They show that network structure does matter in leading to correct actions, which is the action to maximize the payoff.

Board and Meyer-ter Vehn (2021) study the dynamics of learning in social networks based on Bayesian learning on large random networks. They focus on the relationship between network structure and agents' decisions on whether they adopt an innovation. They show that in directed networks⁴, the characteristics of links in the network, including direction, have a significant effect on agents' learning, but in undirected networks (see the definition

³The diameter is measured by the largest distance between two nodes in the network (see Ch.2 in Jackson et al. (2008)). Thus, a network that has a large diameter means a larger network.

⁴ There are generally two types of networks: an undirected network and a directed network. In an undirected network, links have no direction, but links have a direction in a directed network. Suppose that persons A and B are linked on Twitter. A follows B, but B does not follow A. A and B are linked regardless of link direction if the network is undirected. However, if the network is directed, we should consider the direction of links. In this network, the direction is defined by the direction from a node following to a node followed. For example, there is a link from A to B in the directed network because A follows B. However, there is no link from B to A because B does not follow A.

of undirected networks in Footnote 4), learning and social welfare are lower than in directed networks.

Chandrasekhar, Larreguy and Xandri (2020) test the social learning model on networks using experiments. In experiments, all participants should guess the binary state of the world. The world could have one of two states, zero or one, in the experiment. They test a model that consists of sophisticated (Bayesian) and naïve learners (DeGroot). Bayesian agents are facing incomplete information about other's types. DeGroot agents follow the majority of their neighbors' previous guesses. They conducted two lab experiments at different locations: 665 subjects in an Indian village with 665 subjects and ITAM in Mexico with 350 students. They estimate the mixing parameter for two experiments, which show the ratio between sophisticated and naïve learners in the experiment. The share of Bayesian agents in the Indian-villager is estimated at 10%, and that in Mexican-student samples is estimated at 50%.

My belief-based learning is related to the previous research about individual learning based on individual belief because my model is also based on belief-based learning. In particular, a player updates his or her beliefs about strategies from game opponents' strategies. Many previous works have focused on learning about rational decisions in games, where a rational strategy is defined as a strategy that maximizes players' utilities.

Fudenberg and Levine (1995) study the relationship between players' decisions achieved by learning and the best decisions that maximize their utilities. In particular, they focus on a variation of a fictitious play. Fictitious play is based on belief-based learning that gives more weight to the actions frequently chosen by opponents. Thus, fictitious play switches off 'infrequent' action in the game, which is rarely chosen by players, and can lead to a rational decision of a player. They show that the utility achieved by an agent using the fictitious play rule is nearly the best, meaning play that maximizes utility.

Roth and Erev (1995) suggest the reinforcement learning model, which allows reinforcing actions experienced in the past to explain the behavior observed in experiments consisting of

three different two-state sequential games: a public goods provision game, a market game, and an "ultimatum" bargaining game. Their model shows good predictions of players' actions for the periods between short-term and long-term predictions for those three games.

Fudenberg et al. (1998) show that learning models are useful for understanding the equilibrium state which arises from less fully rational players in the long run and suggest ways to use learning models to evaluate and modify traditional equilibrium concepts in economics.

Camerer and Hua Ho (1999) suggest the 'experience-weighted attraction' (EWA) learning model, which includes the characteristics of the belief-based learning model and the reinforcement learning model. They calibrate three parameters of their model using experimental data: the weight of the strength of reinforcement of not chosen strategies relative to chosen strategies, the discount factor of previous attractions, and an experienced weight. They show that reinforcement and belief-based learning, which are special cases in an EWA learning model, are rejected in most results of calibrations, but belief-based learning is better in some constant-sum games.

Camerer, Ho and Chong (2002) discuss the generalized EWA learning model that includes adaptive and sophisticated learners, rationally best responding to all others' predicted behaviors. They estimate the model using experimental data obtained from the p-beauty contest and repeated games with incomplete information. Their generalized model performs better than their EWA learning model in explaining and predicting human behaviors in experiments.

Learning models are useful for describing the adaptive behaviors of players in games. Thus, they have been used to explain experimental data. Feltovich (2000) explains the data obtained from asymmetric-information games using reinforcement-based and belief-based learning models. The belief-based learning models have been used to explain the experimental data from the infinitely and finitely repeated PD games (see Dal Bó and Fréchette (2011); Embrey, Fréchette and Yuksel (2018)).

My experimental study is related to the previous experimental research about network

games. Cassar (2007) finds, using lab experiments, that players in the local network⁵ are more likely to be coordinated than in the small-world network⁶ in the finitely repeated PD games. Gracia-Lázaro et al. (2012) conducted lab experiments to investigate the relationship between the heterogeneity of networks and cooperation in the finitely repeated PD game. However, they do not find any evidence that network heterogeneity promotes cooperation. Wang, Suri and Watts (2012) conducted online experiments on MTurk to analyze the effect of dynamic network formation on cooperative actions in human groups in the finitely repeated PD game. They find that when players can change their partners in the dynamic network formation setting, the fraction of cooperative actions in human groups increases compared to static network formation. Cuesta et al. (2015) conducted lab experiments to investigate the effect of a player's reputation on cooperative behaviors and network formation in human groups under the finitely repeated PD game in the lab. They find that reputation drives cooperative behavior and makes the links in human groups more robust. Gallo and Yan (2015) also show that reputational and social knowledge promote cooperation in the dynamic networked repeated PD game using online experiments on MTurk⁷. Recently, large-sized networked experiments have used hundreds of human subjects to hundreds of millions of people simultaneously, using developed online platforms such as Facebook app adoption, instant messaging networks of Yahoo, and other online social network platforms (see Aral (2016)).

In experimental economics, the relationship between information gathering and cooperation has also been one of the most interesting topics (see the literature survey of Dal Bó and Fréchette (2018)). According to Dal Bó and Fréchette (2018), many subjects cooperate

⁵In the local network, the number of an agent's interaction with others or the number of an agent's neighbors is fixed (see Cassar (2007)).

⁶In the small-world network, most agents are closely connected to their neighbors and can reach others who are not their neighbors by a small number of steps in the network (see Ch. 10 in Newman (2018)).

⁷Many researchers have conducted the reputation effect on cooperation in the finitely simple repeated prisoner's dilemma game without network formation (see Andreoni and Miller (1993); Cooper et al. (1996); Gong and Yang (2010); Cox et al. (2015); Kamei (2017); Kamei and Putterman (2017); Honhon and Hyndman (2020)). Contrary to the literature, reputation is used as one of the pieces of information to form networks in my experiments.

in the first round even with imperfect public information about their partner's behavior in repeated PD games. Additionally, they report that cooperation is possible despite imperfect private monitoring. In my experiments, all players can do perfect public monitoring using network information. Thus, it implies that network information can induce players to cooperate with others by the logic of Dal Bó and Fréchette (2018).

My paper contributes to the literature related to the study of the impact of dynamic network formation on cooperation (Ule (2008); Wang, Suri and Watts (2012); Cuesta et al. (2015); Gallo and Yan (2015)) and the evolution of networks in the finitely repeated PD game (Mengel (2009); Fosco and Mengel (2011)). I consider the repeated game, which consists of the simple repeated PD game and the repeated PD game on a network, to be more general than the previous experiments (see Wang, Suri and Watts (2012); Cuesta et al. (2015); Gallo and Yan (2015)). Thus, my game captures the transition from the simple PD game to the PD game on a network when network information flows. This transition was not captured in the previous experiments. In particular, my research discusses the process of building reputations about human behavior before forming networks and is different from the previous experiments (see Fosco and Mengel (2011); Wang, Suri and Watts (2012); Cuesta et al. (2015); Cho (2014))⁸. More specifically, the game in this paper consists of the simple repeated PD game from the 1st to the 5th round and the repeated PD game on a network from the 6th to the final round. This study shows a high cooperation rate related to building a reputation in the simple repeated PD game. Previous research only considered the networked PD game without the simple repeated PD game (see Fosco and Mengel (2011); Wang, Suri and Watts (2012); Cuesta et al. (2015); Cho (2014)). Thus, they did not see human behavior related to forming networks in the future. The strength of my research is showing the value of reputation formation in the simple repeated PD game before network

⁸This reputation-building process is also related to the threat of exclusion in network formation, and players in my experiments can set strategies based on the threat of exclusion. Thus, my study contributes to the literature about the effects of threatening strategy or punishment strategy, such as the tit-for-tat (TFT) strategy (see Axelrod (1984); Dal Bó and Fréchette (2011); Fréchette and Yuksel (2017); Embrey, Fréchette and Yuksel (2018); Dal Bó and Fréchette (2019)). I discuss the relationship between TFT and network formation in Section 5.4.

formation. Additionally, I conduct simple repeated PD and cheap talk repeated PD games to determine how much network formation for promoting cooperation is more efficient than cheap talk and simple repeated games.

3 The game-theoretical model study

3.1 Model description

Let $N = \{1, ..., n\}$ be the set of players $(n \gg 2)$. All players play the repeated prisoner's dilemma game for T rounds $(T \gg 2, T)$ is an integer). In every round $t \in \{1, ..., T\}$, every player i $(i \in N)$ simultaneously chooses an action $x_i^t \in \{C, D\}$, where C and D denote cooperate and defect, respectively. If player i selects C (D) at round t, then player i is a cooperator (defector) at round t. All players know all players' history of actions. Thus, the information set in the game at round t (I^t) is as follows: $I^t = \{x_1^1, ..., x_1^{t-1}, x_2^1, ..., x_n^1, ..., x_n^{t-1}\}$.

Assumption 1 The finitely repeated game consists of two games: (1) the simple repeated PD game; (2) the repeated PD game on a network.

All players are doing the simple repeated PD game from the 1st round to round t_s (1 < $t_s \ll T$). The game partner of a player is randomly selected at each round for the simple repeated PD game. Then, a player plays with his or her partner under the payoff structure as follows:

		Player i	
		C	D
Player j	C	(c,c)	(e,f)
	D	(f,e)	(d,d)

Table 1: The payoff table of the game.

where f > c > d > e, f + e < 2c, e < 0. If both players cooperate, they each get c. If both players defect, they each get d. If player i (j) cooperates and player j (i) defects, player i

(j) gets e, and player j (i) gets f.

Assumption 2 The repeated PD game on a network consists of two stages in each round: (1) the partner selection; (2) the choice of action. At the beginning of each round in the repeated PD game on a network, all players must choose their partners. After finishing the partner selection, all players play the PD game with only their partners under the same payoffs as the simple repeated PD game. A player submits the same action to all partners of a player. In general, the number of a player's partners is between zero and n-1 (inclusive).

Assumption 3 Players who are not selected by others as partners get nothing, regardless of their actions. Thus, all players should be selected by others to get payoffs in the PD game on a network.

The details of the partner selection process are as follows:

- 1. At the beginning of the PD game on a network, players have to propose links to others with whom they want to play the game simultaneously. Players can see other players' histories of actions. There is no constraint on the number of links to propose. Thus, a player can propose n-1 links in each round. However, there exists a network formation cost per link c_{net} ($c_{net} < f$).
- 2. Players who receive proposals from others should decide whether they play the PD game with proposers. If the player accepts the proposal, both players are linked to each other in the network and play the PD game. However, if the player does not accept, both players are not linked to each other in the network and do not play the PD game. If they do not play the PD game, they get nothing. It means that a player not selected as a partner by others gets zero payoffs.

3. The partner selection process is repeated at the beginning of the repeated PD game on a network after round t_s . All players can change their partners at every round after round t_s . Also, all players can unilaterally sever links with others in the partner selection process.

The partner selection process captures the general making-a-friend process. In general, we make friends based on mutual consent using our experiences. Also, we can unilaterally sever friendship relations with others whom we do not like.

The partner selections of player i can be captured by a binary vector

$$b_i = (b_{ij})_{j \in N} \in \{0, 1\}^N$$
, such that $b_{ii} = 0$ and $\sum_{j \in N} b_{ij} \le n - 1$. (1)

(1) $b_{ij} = 1$ if player i proposes a link to player j; (2) $b_{ij} = 0$ otherwise.

Let g(b) denote the network of the established link given the profile of partner selections $b=(b_1,...,b_n)$. In the partner selection process, a link between two players is formed when two players agree to be linked to each other. By the mutual link formation process, $g_{ij}=\min\{b_{ij},b_{ji}\}$, where g_{ij} is a binary variable that denotes the link between players i and j: (1) $g_{ij}=1$ if players i and j are linked; (2) $g_{ij}=0$ otherwise. Let $L_i(g)=\{j|g_{ij}=1\}$ denote the set of neighbors of player i and let $l_i(g)=|L_i(g)|$ be the number of neighbors of player i: (1) $l_i(g)=1$ in the simple repeated prisoner's dilemma $(t \leq t_s)$; (2) $0 \leq l_i(g) \leq n-1$ in the repeated PD game on a network $(t_s < t \leq T)$. Let $G=\{g|0 \leq l_i(g) \leq n-1$, for all $i \in N\}$ be the set of feasible networks. Thus, $g(b) \in G$.

After finishing partner selections, all players choose their actions. Then, the payoff of player i in the PD game on a network given the network of established links and the profile of actions at round t ($\pi_i(x^t, b^t)$) is as follows:

$$\pi_i(x^t, b^t) = \sum_{j \in L_i(g^t)} u_i(x_i^t, x_j^t) - \sum_{j \in L_i(g^t)} c_{net} = \sum_{j \in N} (u_i(x_i^t, x_j^t) - c_{net}) g_{ij}(b^t). \tag{2}$$

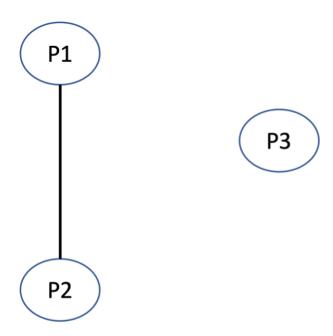


Figure 1: An example of a network formed by three players.

where $u_i(x_i^t, x_j^t)$ denotes the payoff of player i if players i and j choose x_i and x_j at round t, respectively. b_i^t denotes the partner selections of player i at round t. c_{net} is the cost of a link formation. $(x^t, b^t) = ((x_1^t, b_1^t), ..., (x_n^t, b_n^t))$.

Example 1: The payoffs of three players in the PD game on a network.

Suppose three players (P1, P2, and P3) form a network at round t (see Figure 1).P1 is connected with P2, and P3 is isolated. Then, $g_{12}(b^t) = g_{21}(b^t) = 1$, $g_{13}(b^t) = g_{31}(b^t) = g_{23}(b^t) = 0$.

Assume that $x_1^t = C, x_2^t = D$, and $x_3^t = D$. x_i^t denotes the action of Pi at round t. Then, $\pi_1(x^t, b^t) = e - c_{net}, \pi_2(x^2, b^t) = f - c_{net}$, and $\pi_3(x^t, b^t) = 0$. $\pi_i(x^t, b^t)$ denotes the payoff of Pi at round t.

The action and partner selection of player i $(x_i^*, b_i^*) \in S_i$ is a best response of player i to the profile of player i's opponents' $(N \setminus \{i\})$ actions and partner selections (x_{-i}, b_{-i}) if

$$\pi_i(x_i^*, b_i^*, x_{-i}, b_{-i}) \ge \pi_i(x_i', b_i', x_{-i}, b_{-i}), \tag{3}$$

for any $(x_i', b_i') \in S_i$, where $S_i = X_i \times B_i$, X_i and B_i denote the set of actions of player i and the set of partner selections of player i, respectively. A Nash equilibrium of the PD game on a network $\Gamma = \langle N, S, \pi, I \rangle$ is a strategy profile $(x^*, b^*) \in S$ (where $S = X \times B$. X and B denote the set of profiles of actions and partner selections) is as follows:

$$\pi_i(x^*, b^*) \ge \pi_i(x_i, b_i, x_{-i}^*, b_{-i}^*),$$
(4)

for any $(x_i, b_i) \in S_i$. $\pi = (\pi_1, ..., \pi_n)$, I is the information set. A network g is an equilibrium network if there exists a Nash equilibrium (x^*, b^*) such that $g = g(b^*)$.

 $\Gamma^T = \langle N, S, \pi, I, t_s, T \rangle$ presents the repeated PD game for T rounds. All players select their actions and their sets of partner selections at each round to maximize their total payoffs in the game. The total payoff of player i in the repeated PD game $\Pi_i(h^T)$ under the given sequence of actions and partner selections for T rounds h^T ($h^T = ((x^1, b^1), ..., (x^T, b^T))$) is as follows:

$$\Pi_i(h^T) = \sum_{t=1}^T \pi_i(x^t, b^t).$$
 (5)

Calvó-Armengol and Ilkılıç (2009) argue that the Nash equilibrium is weak to capture the equilibrium of a network based on mutual link formation because there are multiple Nash equilibrium networks, including the empty network in the network formation game. In a network formation game, the empty network is always a Nash equilibrium network. Thus, we need another equilibrium concept to understand the game on a network based on mutual link formation and unilateral link severance.

Jackson and Wolinsky (1996) suggest pairwise stability that captures stability based on mutual link formation and unilaterally removing a link. I use pairwise stability to analyze the equilibrium network in the networked game.

Definition 1 (Pairwise stability [Jackson and Wolinsky (1996)]). A network g is

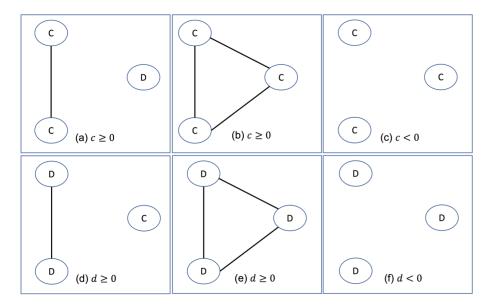


Figure 2: Pairwise stable networks in the networked prisoner's dilemma game with three players if $c_{net} = 0$ (see **Definition 1** and **Example 2**). C denotes a cooperator, whose action is cooperation, and D denotes a defector, whose action is defection. (e) and (f) are pairwise-Nash equilibrium networks (see **Definition 2** and **Example 3**).

pairwise stable if the following two conditions are satisfied:

- (i) For all $i, j \in N$ where $g_{ij} = 1$, $u_i(g) \ge u_i(g \ominus ij)$ and $u_j(g) \ge u_j(g \ominus ij)$,
- (ii) For all $i, j \in N$ where $g_{ij} = 0$, if $u_i(g \oplus ij) > u_i(g)$, then $u_j(g \oplus ij) < u_j(g)$.

 where $u_i(g)$ is the utility or the payoff to player i under the network g. $g \oplus ij$ denotes removing a link between players i and j in the network g. $g \oplus ij$ denotes forming a link between players i and j in the network g.

A pairwise network allows a link unless a link between two players makes their payoffs worse off. However, a player does not create a link with the other, making his or her payoff worse in a pairwise network.

Example 2: Pairwise stable networks in the PD game on a network with three players.

Suppose three players form a network. Assume that $c_{net} = 0$. Figure 2 shows the possible pairwise stable networks. Cooperators whose action is cooperation do not want to connect with defectors whose action is defection since cooperators' payoffs are decreased by e per

link with a defector. If $c \geq 0$, cooperators want to make links with other cooperators since cooperators can increase their payoffs by c per link with a cooperator (see Figure 2 (a) and (b)). However, if c < 0, cooperators do not want to connect with other cooperators since cooperators' payoffs are decreased by c per link with a cooperator (see Figure 2 (c)). Thus, if $c \geq 0$, cooperators are only connected with cooperators in pairwise stable networks, but cooperators are isolated if c < 0. Similarly, if $d \geq 0$, defectors are only connected with other defectors in pairwise stable networks since cooperators do not want defectors (see Figure 2 (d) and (e)), but defectors are isolated if d < 0 (see Figure 2 (f)).

Then, pairwise stability and Nash equilibrium can be combined. I define a *pairwise-Nash* equilibrium network that is robust to one-link creation based on mutual consent and unilateral multi-link severance.

Definition 2 (Pairwise-Nash equilibrium). A network g is a pairwise-Nash equilibrium network if and only if there exists a pure strategy Nash equilibrium x^* that satisfies $g = g(x^*)$, and for all $i, j \in N$, if $g_{ij} = 0$, then $u_i(g \oplus ij) > u_i(g)$ implies $u_j(g \oplus ij) < u_j(g)$.

Example 3: Pairwise-Nash equilibrium in the PD game on a network with three players.

Suppose three players form a network. Assume that $c_{net} = 0$. The best response of a player is defection because u(D,C) = f > u(C,C) = c and u(D,D) = d > u(C,D) = e. Thus, all players defect in Nash equilibrium. There are two pairwise stable networks when all players defect (see Figures 2 (e) and (f)). Therefore, pairwise-Nash equilibrium networks are as follows: the complete network in which all players are connected (see Figure 2 (e)) if $d \ge 0$ and the empty network in which all players are isolated if d < 0 (see Figure 2 (f)).

3.2 Equilibrium analysis

In **Examples 2** and **3**, I analyze the equilibrium state of the PD game on a network with three players. This subsection extends the equilibrium analysis to the repeated PD game on a network with n players.

Proposition 1 Consider the repeated PD game $\Gamma^T = \langle N, S, \pi, I, t_s, T \rangle$. Then, there exist pairwise stable networks in Γ^T : (i) all cooperators are only completely connected to other cooperators if $c \geq c_{net}$ or isolated if $c < c_{net}$; (ii) all defectors are only completely connected to other defectors if $d \geq c_{net}$ or isolated if $d < c_{net}$.

Proof. Suppose all players are in the network g during the game at round t ($t_s < t \le T$). To prove **Proposition** 1, I have to consider the following three cases of link formations in pairwise stable networks: (1) between two cooperators; (2) between a defector and a cooperator; (3) between two defectors.

Let us see the first case. Suppose two cooperators i and j are linked in $g(g_{ij}=1)$. If cooperator i severs a link from cooperator j, then $u_i(g) - u_i(g \ominus ij) = (c - c_{net})$ and $u_j(g) - u_j(g \ominus ij) = (c - c_{net})$. Thus, if $c \ge c_{net}$, then $u_i(g) \ge u_i(g \ominus ij)$ and $u_j(g) \ge u_j(g \ominus ij)$. The first case satisfies condition (i) for pairwise stability, and cooperators are linked if $c \ge c_{net}$ in pairwise stable networks. However, if $c < c_{net}$, then $u_i(g) < u_i(g \ominus ij)$ and $u_j(g) < u_j(g \ominus ij)$. It implies that cooperators lose their payoffs by connecting if $c < c_{net}$ and cooperators are not connected if $c < c_{net}$. Thus, all cooperators are linked to each other if $c \ge c_{net}$ but are isolated if $c < c_{net}$ in pairwise networks.

Let us see the second case. Suppose player i is a cooperator, and player j is a defector in the network g. If they form a link between them, $u_i(g \oplus ij) - u_i(g) = e - c_{net} < 0$ and $u_j(g \oplus ij) - u_j(g) = f - c_{net} > 0$. Thus, $g_{ij} = 0$ in pairwise stable networks. It means that the payoff of cooperator i is reduced, but the payoff of defector j increases, and it satisfies condition (ii) for pairwise stability. Thus, all cooperators do not add links with all defectors in pairwise stable networks.

Let us see the third case. Suppose players i and j are defectors in the network g. If they form a link between them, player i gets $d - c_{net}$, and player j gets $d - c_{net}$. Thus, if $d \geq c_{net}$, then $u_i(g \oplus ij) - u_i(g) = d - c_{net} \geq 0$ and $u_j(g \oplus ij) - u_j(g) = d - c_{net} \geq 0$. Defectors i and j get mutual benefits by linking each other if $d \geq c_{net}$. It implies that $g_{ij} = 1$ in pairwise stable networks if $d \geq c_{net}$, and condition (i) for pairwise stability is satisfied. However, if $d < c_{net}$, they lose their payoffs by connecting and are not connected. Thus, all defectors are connected if $d \geq c_{net}$ but are isolated if $d < c_{net}$ in pairwise stable networks. Q.E.D.

Proposition 1 implies "full separation" of cooperators and defectors in the network is consistent with the result in Fosco and Mengel (2011). Fosco and Mengel (2011) show that cooperators and defectors are fully separated using exclusion and imitation strategies in the repeated prisoner's dilemma game. **Proposition** 1 shows that mutual link formation and unilateral multi-link severance prohibit link formation between a cooperator and a defector like the exclusion strategy in Fosco and Mengel (2011).

Proposition 1 characterizes the equilibrium states of networks. However, it does not consider the equilibrium states, including players' actions. Thus, we need to find pairwise-Nash equilibrium networks considering the equilibrium state of network formation and players' actions.

Proposition 2 Consider the repeated PD game $\Gamma^T = \langle N, S, \pi, I, t_s, T \rangle$. Then, there exist two unique pairwise-Nash equilibrium networks in Γ^T : (i) all players defect and are completely connected if $d \geq c_{net}$; (2) all players defect and are isolated if $d < c_{net}$. These pairwise-Nash equilibrium networks are also subgame perfect equilibrium networks.

Proof. The best response of all players at each round in the game Γ_T is defection because u(D,C)=f>u(C,C)=c and u(D,D)=d>u(C,D)=e. Thus, by backward induction, all players defect for all rounds, and this is the unique subgame perfect Nash equilibrium. By **Proposition** 1, there exist pairwise stable networks: All defectors are only completely connected to other defectors if $d \geq c_{net}$ or isolated if $d < c_{net}$. Therefore, there exist two

unique pairwise-Nash equilibrium networks in Γ_T : (i) all players defect and are completely connected if $d \geq c_{net}$; (ii) all players defect and are isolated if $d < c_{net}$. These pairwise-Nash equilibrium networks are also subgame perfect equilibrium networks since the equilibrium state in which all players defect is the subgame perfect equilibrium. Q.E.D.

Proposition 2 characterizes the pairwise-Nash equilibrium networks in the finitely repeated PD game on a network. Wang, Suri and Watts (2012) find that it is more likely that the networks in which all players defect and are completely connected in the finitely repeated networked prisoner's dilemma game are observed as a round goes to the final round in experiments. **Proposition** 2 is consistent with the result in Wang, Suri and Watts (2012).

3.3 Discussion: What do the equilibrium networks tell us?

As a result of the equilibrium analysis, all players defect and are completely connected or isolated in pairwise-Nash equilibrium networks. The result cannot explain the cooperative actions in the experiments. However, my equilibrium analysis provides an important understanding of forming networks in human decisions under the repeated prisoner's dilemma to create an advanced model that explains cooperative actions.

Network formation can be the strategy to drive cooperation by excluding defectors if players can learn and imitate others' actions. By **Propositions** 1 and 2, cooperators and defectors never be connected in pairwise stable networks, including pairwise-Nash equilibrium networks. Also, if we assume that the model allows cooperative actions of players, the pairwise stable network in which cooperators are completely connected exists if the cost of a link formation is lower than the mutual benefit between two cooperators. It implies that cooperators can benefit from playing games with other cooperators by excluding defectors in network formation. The strategy of excluding defectors through network formation can drive cooperation. This idea is incorporated into the belief-based learning model in the following section.

4 The belief-based learning model study

In the game-theoretical model study, I characterize the equilibrium state of the networked game. In the pairwise-Nash equilibrium, all players defect, and the complete or empty network is formed. However, pairwise stable networks in which cooperators are connected with other cooperators can be possible if some players cooperate in the game (see **Proposition** 1). In the previous experiments of networked PD games, the pairwise stable networks with cooperators connecting with other cooperators were observed (see Wang, Suri and Watts (2012); Cuesta et al. (2015)). As I discussed in Section 3.3, players can use the network formation rule to exclude or isolate defectors in the network as a strategy in the game. To add the characteristics of strategies based on network formation in the model, I need another approach with more flexible assumptions than the game-theoretical model.

To explain cooperative actions in the networked PD games not explained by the gametheoretical model, I suggest a belief-based learning model. The belief-based learning model is a structural form model based on learning from game opponents and easy to add strategies. Also, we can do computer or agent-based simulations using the belief-based learning model, and it provides a deep understanding of the relationship between the cooperation actions of players and network formation in the game and the dynamics of cooperation rate in the game.

4.1 Model description

4.1.1 The structure of the game

The belief-based learning model is based on the model in Embrey, Fréchette and Yuksel (2018). I extend their model to a novel belief-based learning model, including network formation strategies to understand the effects of networks in the finitely repeated prisoner's dilemma game.

Let $N = \{1, 2, ..., n\}, n \geq 3$ be the set of players in the belief-based learning formation

model. Let $A_{it} = \{C, D\}$ denote the set of actions of player i at round t. C denotes cooperate, D denotes defect. Let a_{it} denote the action of player i at round t. A player plays the finitely repeated prisoner's dilemma game for T rounds (T > 2). Let $u_{it}(a_{it}, a_{jt})$ denote the payoff of player i when player i plays with player j at round t. Table 1 shows the payoff of players i and j for a one-shot game. If both players cooperate, then each gets c. If both players defect, then each gets d. If one defects and the other cooperates, the cooperator (defector) gets e(f).

The game consists of two types: the simple repeated PD game from the 1st round to round t_s and the repeated PD game on a network from round $t_s + 1$ to round T. I assume that there is no cost of link formation. The structures of the simple repeated game and the repeated PD game on a network are identical to the game-theoretical model with zero cost of link formation. Please see **Assumptions** 1, 2 and 3 in the game-theoretical model.

Let me give an example of the PD game on a network. For example, players A, B, and C are linked and play in the networked game at round t. Assume that $a_{At} = C$, $a_{Bt} = D$, and $a_{Ct} = C$. Then, the payoffs of players A,B, and C are as follows: $\pi_{At} = c + e$, $\pi_{Bt} = 2f$, and $\pi_{At} = c + e$. π_{it} denotes the payoff of player i at round t. A strategy in this game includes a network formation strategy. I explain strategies adopted by players in the game in Section 4.1.2 and describe network formation strategies.

The total payoff of player i from the round t to T denoted by U_{tT}^i is as follows:

$$U_{tT}^{i} = \sum_{k=t}^{t_{s}} u_{ik}(a_{ik}, a_{j(k)k}) + \sum_{k=t_{s}+1}^{T} \sum_{j \in N_{k}(i), i \in N_{k}(j)} u_{ik}(a_{ij}, a_{jk}), \text{ if } 1 \le t \le t_{s}$$

$$= \sum_{k=t}^{T} \sum_{j \in N_{k}(i), i \in N_{k}(j)} u_{ik}(a_{ij}, a_{jk}), \qquad \text{if } t_{s} < t \le T$$

$$(6)$$

where j(k) denotes the partner of player i at round k in the simple repeated PD game. $N_k(i)$ is the set of player i's neighbors at round k in the repeated PD game on a network. I assume

that player i at round t selects his or her action to maximize U_{tT}^i .

4.1.2 Strategies of players

This game consists of the simple repeated PD game and the repeated PD game on a network. Thus, a player considers not only his or her actions but also his or her network formation to select and be selected by neighbors in the networked game.

In general, we can think of many strategies in the repeated game. Thus, I focus on important strategies to describe players' network formation and cooperative actions: *m*-threshold strategies, excluding trigger strategy, and network tit-for-tat (TFT). I assume that all players choose one among these strategies. Future research will introduce other and more complicated strategies.

An m-threshold strategy is based on threshold strategies suggested by Embrey, Fréchette and Yuksel (2018). The action of player i following an m-threshold strategy at round t is as follows: (1) $a_{it} = C$ if $t \leq m$; (2) $a_{it} = D$ otherwise. The set of neighbors of an m-threshold strategy player i in the network game at round t is as follows: $N_t(i) = \{1, ..., n\} \setminus \{i\}$, if $t_s < t \le T$. If m = 0, then player i defects for all rounds. If m = T, then player i cooperates for all rounds. Thus, an m-threshold strategy includes short-term cooperators and longterm cooperators. Threshold strategies used in Embrey, Fréchette and Yuksel (2018) do not consider network formation because their model is based on the simple repeated PD game. They could explain the high cooperation rate at the first round and a decreasing pattern of cooperation rate as a round increases in the finitely repeated PD game. Wang, Suri and Watts (2012) find a high cooperation rate and a decreasing pattern of cooperation as rounds increase in the networked finitely repeated PD game. It is a similar pattern of cooperation rate as the simple repeated games. Their findings imply that it may be possible to exist short-term cooperators and long-term cooperators in the networked finitely repeated PD game. In addition, I assume that an m-threshold strategy player makes as many links as possible because an m-threshold strategy player does not consider others' past actions. In this case, maintaining as many links as possible with others gives the highest payoff under zero cost of link formation.

An excluding trigger strategy is a strategy that excludes defectors forever in the network formation phase. The action and the set of neighbors of player i following an excluding trigger strategy at round t are as follows: $a_{it} = C$, $N_t(i) = \{j | R_{jt} = 1\}$. R_{jt} denotes the reputation score of player j at round t. The definition of R_{jt} is as follows: $R_{jt} = \sum_{k=1}^{t-1} 1_{a_{jk}=C}/(t-1)$. $1_{a_{jk}=C}$ denotes the indicator function: (1) $1_{a_{jk}=C} = 1$ if $a_{jk} = C$; (2) $1_{a_{jk}=C} = 0$ otherwise. R_{jt} is between 0 and 1 ($0 \le R_{jt} \le 1$). If player j did defect at least once from the 1st round to a round t, R_{jt} is less than 1. An excluding trigger strategy excludes all js whose R_{jt} is less than 1. Thus, an excluding trigger strategy player excludes all defectors in the network formation phase and only plays the game with the players who have never defected. An excluding trigger strategy is so harsh because it never allows defection. However, it can make players start to cooperate in the game in the initial rounds due to threats of exclusion in the network formation phase.

A network tit-for-tat (TFT) is a strategy that only excludes players who defected in the previous round. A network TFT player i's action and set of neighbors at round t are as follows: $a_{it} = C, N_t(i) = \{j | a_{jt-1} = C\}$. A network TFT is more generous than an excluding trigger strategy because it allows connections with players with reputation scores that are less than one if they cooperated at the previous round. Like an excluding trigger strategy, a network TFT also drives players to cooperate due to threats of exclusion in the network formation phase. A network TFT can reinforce connections between cooperators in the PD game on a network, and this connection can sustain a high cooperation rate.

4.1.3 The rule for selecting the best strategy

The core of the belief-based learning model is that, at each round, a player chooses his or her best strategy, which maximizes the expected total payoff at that round. Players can alter their best strategies at every round according to the game situation. In particular, the belief-based learning model allows altering strategies by learning opponents' best strategies.

Each player updates his or her beliefs by learning the strategy adopted by his or her opponent. Let $\overrightarrow{\beta_{it}}$ be the belief vector of player i to strategies at round t. The dimension of $\overrightarrow{\beta_{it}}$ is $N_s \times 1$. N_s is the number of strategies. β_{it}^k represents the weight that player i assigns to a strategy k to be selected by his or her opponent at round t. $\overrightarrow{\beta_{it}}$ is updated as follows:

$$\overrightarrow{\beta_{it+1}} = \theta_i \overrightarrow{\beta_{it}} + \overrightarrow{L_{it}}, \tag{7}$$

where θ_i denotes how player i discounts past beliefs ((1) $\theta_i = 0$: Cournot dynamics; (2) $\theta_i = 1$: fictitious play). $\overrightarrow{L_{it}}$ is the update vector dimensions of $N_x \times 1$ given plays of player i and i's opponent at round t: (1) $L_{it}^k = 1$ if strategy k is selected as the best strategy by player i's opponent; (2) $L_{it}^k = 0$ otherwise. Thus, the belief in the opponent's best strategy is higher than other strategies by updating belief.

Let $\overrightarrow{\mu_{it}}$ be the utility vector of player i at round t for all strategies dimension of $N_s \times 1$.

$$\overrightarrow{\mu_{it+1}} = \overrightarrow{u_{it}} + \lambda_i \overrightarrow{\epsilon_{it}}, \tag{8}$$

where $\overrightarrow{u_{it}} = \boldsymbol{U(t)}\overrightarrow{\beta_{it}}$. $\overrightarrow{u_{it}}$ is the weighted expected payoff vector of player i for all strategies at round t dimensions of $N_s \times 1$. $\boldsymbol{U(t)}$ is the payoff matrix of the strategies dimension of $N_s \times N_s$. Thus, $U_{ij}(t)$ represents the expected total payoff of playing strategy i against playing a strategy j at round t. Note that $\boldsymbol{U(t)}$ contains the expected total payoff of each strategy against every other strategy from round t to round t. Thus, $\boldsymbol{U(t)}$ is a function of t - t + 1 and the stage-game payoffs. t = t + 1 and the stage-game payoffs. t = t + 1 and it is a scaling parameter that measures how well each player best responds to his or her beliefs. t = t + 1 is an idiosyncratic error vector size of t = t + 1 and t = t + 1 is an idiosyncratic error vector size of t = t + 1 and t = t + 1 is an idiosyncratic error vector size of t = t + 1 and t = t + 1 is an idiosyncratic error vector size of t = t + 1 and the stage-game payoffs are ideal to the probability that player t = t + 1 and the stage-game payoffs. t = t + 1 is the probability that player t = t + 1 and the stage-game payoffs are ideal to the payoff of playing strategy t = t + 1 and t = t

Let p_{it}^k denote the probability of choosing a strategy k of player i at round t, and it can be written as follows:

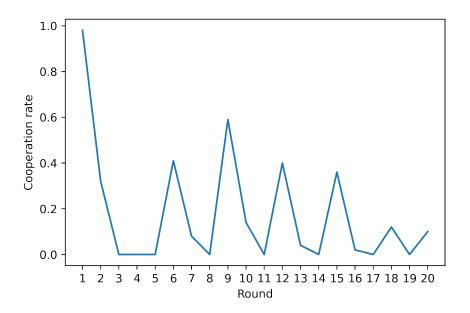


Figure 3: The dynamics of the cooperation rate.

$$p_{it}^k = \frac{\exp\left(\frac{u_{it}^k}{\lambda_i}\right)}{\sum_{j=1}^{N_s} \exp\left(\frac{u_{it}^j}{\lambda_i}\right)},\tag{9}$$

where u_{it}^k is u_{it} for strategy k in $\overrightarrow{u_{it}}$. The higher u_{it}^k is, the higher p_{it}^k is. Thus, a player in the game chooses the strategy that gives him or her the highest expected total payoff with the highest probability.

4.2 The simulation results

4.2.1 Simulations

The parameters used in the baseline model are as follows: $n = 100, \beta_{i0}^k = 4.45, c = 4, d = 0, e = -1, f = 7, t_s = 5, T = 20, \lambda_i = 0.83, \sigma_i = 0.16, \theta_i = 0.83, \kappa_i = 1.0, 0 \le m \le 20$ for every i and k. These parameter values are based on the parameters of the belief-based learning model estimated from experiments based on the finitely repeated PD game (see Embrey, Fréchette and Yuksel (2018)). I run a hundred Monte Carlo simulations given the parameters to get statistically robust results.

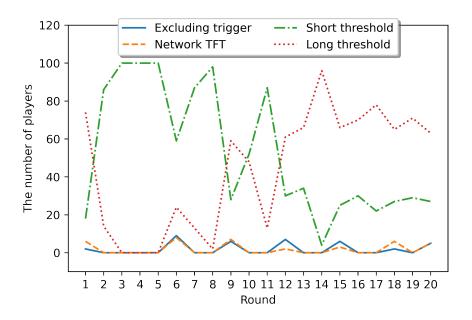


Figure 4: The dynamics of populations of strategies: Short (Long) threshold strategy corresponds to the m-threshold strategies for $0 \le m \le 5$ ($6 \le m \le 20$).

4.2.2 The result of the baseline model

Figure 3 shows the dynamics of the cooperation rate for one simulation. A high cooperation rate in the initial period is observed. The cooperation rate decreases fast after the 1st round and is close to zero at the 5th round, the final round of the simple repeated PD game. However, at the 6th round, the first round of the PD game on a network, the cooperation rate significantly increases and decreases with fluctuations after the 6th round. This pattern of the cooperation rate is observed robustly for most simulations (see Figure 7). The high cooperation rate in initial periods and the decreasing pattern of it have been observed in the simple and networked finitely repeated PD game (see Wang, Suri and Watts (2012); Cuesta et al. (2015); Embrey, Fréchette and Yuksel (2018)). In particular, the simple and networked games are combined in my belief-based learning model, and my model explains the experimental results of both games. The high cooperation rate in the initial round of the finitely repeated PD game is relevant because all agents are incentivized to increase their reputation scores to be selected as partners in the networked game. The expected total

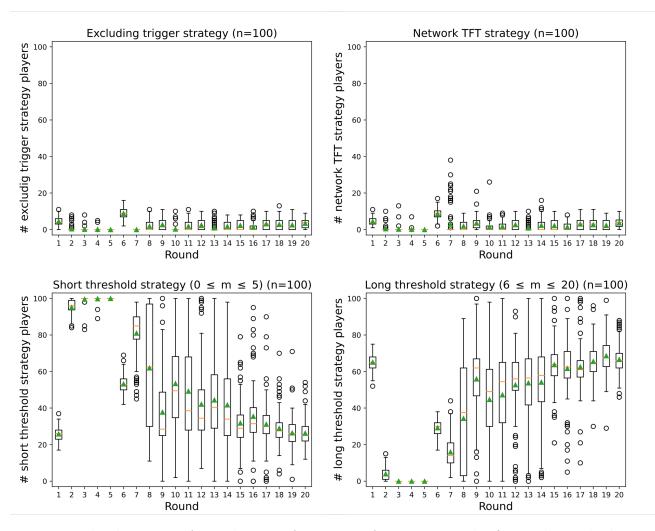


Figure 5: The dynamics of populations of strategies for n = 100: The figure shows the box plot of the cooperation rate for each round. Green triangles are the means of cooperation rates for a hundred Monte Carlo samples at each round.

payoff of a player considers a strategy based on a reputation score, such as an excluding trigger strategy. Thus, cooperation in the first round is rational based on expected total payoff maximization in the belief-based learning model. In addition, the effect of reputation building on cooperation is close to zero as a round is close to the final round. Thus, the cooperation rate decreases as a round increases.

Figure 4 depicts the dynamics of populations of strategies. In Figure 4, the short (long) threshold strategy corresponds to the m-threshold strategies for $0 \le m \le 5$ ($6 \le m \le 20$). The short threshold strategy is the most dominant in the simple repeated PD game from the

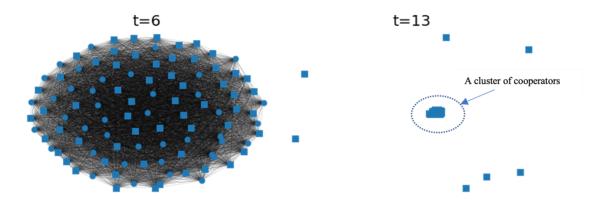


Figure 6: Networks formed in different rounds: Circles indicate cooperators, and squares indicate defectors. The left (right) panel shows the network formed in the 6th (13th) round.

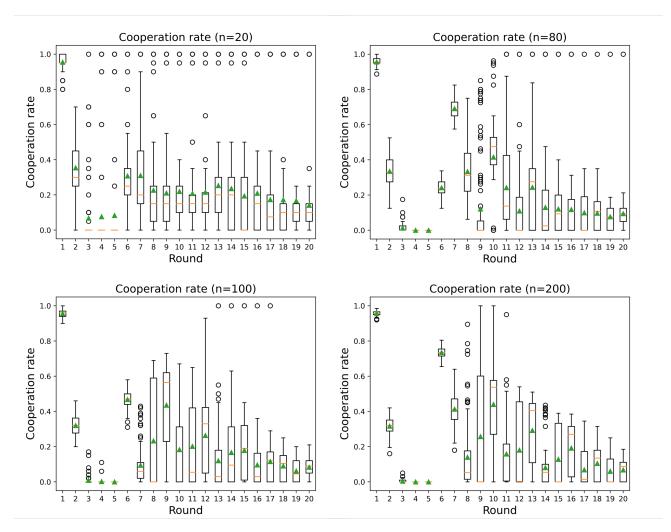


Figure 7: The dynamics of the cooperation rate for n = 20, 80, 100, 200: The figure shows the box plot of the cooperation rate for each round. Green triangles are the means of cooperation rates for a hundred Monte Carlo samples at each round.

1st to the 5th round, and the long threshold strategy is the most dominant in the repeated PD game on a network from the 6th to the final round. It shows that short-term cooperators can survive in the simple repeated PD game from the 1st to the 5th round but cannot survive in the repeated PD game on a network from the 6th to the final round. On the other hand, long-term cooperators cannot survive from the 1st round to the 5th round but can survive from the 6th round to the final round. These results show that long-term cooperation is more advantageous than defection in the repeated PD game on a network. The populations of excluding trigger strategy and network TFT players increase slightly in the 6th round, the first round in the repeated PD game on a network, but they are small compared to short and long threshold strategies (they are 4.3% of populations of 23 strategies for all rounds: 21 m-threshold (m = 0, 1, ..., 20), excluding trigger and network TFT strategies). Also, when the cooperation rate increases, the populations of excluding trigger strategy and network TFT players increase (see the 9th, 12th, 15th, 18th, and 20th rounds in Figure 4). Even though their populations are small, they significantly contribute to an increase in the cooperation rate. This is explained in Section 4.2.4.

Figure 5 shows the dynamics of populations of strategies in a hundred Monte Carlo data. It shows the same characteristics as the dynamics of populations of strategies as in Figure 4: Large fraction of short (long) threshold strategy players in the simple (networked) repeated game and small fractions of excluding trigger and network TFT strategies players.

Figure 6 shows how networks form in different rounds. Circles and squares in Figure 4 denote cooperators and defectors, respectively. The left (right) panel in Figure 6 depicts the network formed in the 6th (13th) round. In the 6th round, cooperators and defectors are connected, but defectors are isolated in the 13th round. The isolation of defectors is due to both the excluding trigger strategy and the network TFT strategy. This threat of exclusion in the network can sustain a positive cooperation rate. This is explained in Section 4.2.4.

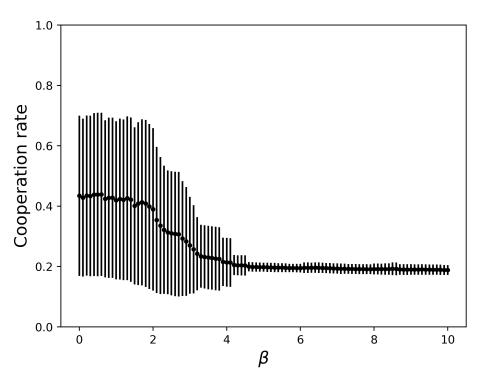


Figure 8: The average cooperation rate of the game as a function of $\beta (= \beta_{i0}^k, 0 \le \beta_{i0} \le 10)$: The error bar shows the standard deviation of a hundred Monte Carlo data.

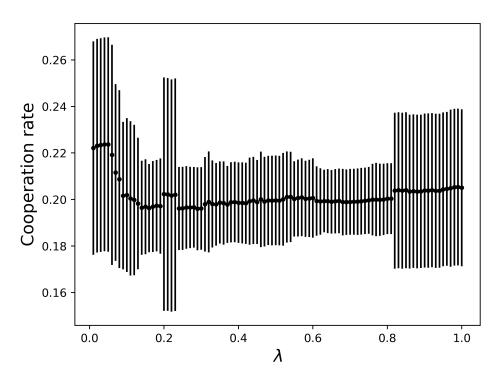


Figure 9: The average cooperation rate of the game as a function of $\lambda (= \lambda_i, 0.01 \le \lambda_i \le 1)$: The error bar shows the standard deviation of a hundred Monte Carlo data.

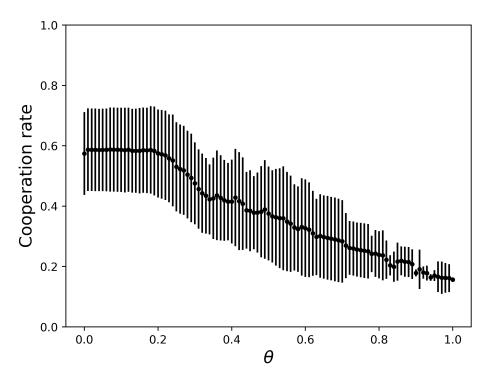


Figure 10: The average cooperation rate of the game as a function of $\theta (= \theta_i, 0 \le \theta_i \le 1)$: The error bar shows the standard deviation of a hundred Monte Carlo data.

4.2.3 Robustness checks of the model

I test the robustness of the model using various parameter values for n, β_{i0}^k , λ_i , and θ_i . For each parameter set, I run a hundred Monte Carlo simulations and get a positive average cooperation rate at a 1% significance level.

Figure 7 shows the dynamics of the cooperation rate for n = 20, 80, 100, 200 with other parameters whose values are the same as the baseline model. High cooperation rates in the 1st round, which is the initial period in the simple repeated PD game, and in the 6th round, which is the initial period in the repeated PD game on a network, are observed for the different numbers of players. Also, the decreasing patterns of cooperation rate in the simple repeated PD game and the repeated PD game on a network are observed robustly.

Figure 8 shows the average cooperation rates of the game for different values for β : $0 \le \beta_{i0}^k \le 10$ for every i and k. As $\beta (= \beta_{i0}^k)$ increases, the cooperation rate decreases and

converges to some value, about 0.2. Also, a decrease in the fluctuation of the cooperation rate is observed. If $\beta = 0$, the best strategy of a player is chosen randomly. However, when β increases, the difference between the probability of selecting the strategy that gives a player the highest utility and the probabilities of other strategies will be larger. Thus, a player can distinguish his or her best strategy among all strategies more easily as β becomes higher. This leads to a decrease in the fluctuation of the cooperation rate as β increases. Also, in the final round of the game, the cooperation rate is almost zero. By updating $\overrightarrow{\beta}_{it}$, the β_{iT} for defection in the final round will be the greatest. Thus, the larger β is, the lower the average cooperation rate of the game is due to a higher belief of selecting defection. This is why the pattern of the average cooperation rate of the game decreases as β increases.

Figure 9 shows the average cooperation rate of the game as a function of $\lambda (= \lambda_{i0}, 0.01 \le \lambda_{i0} \le 1)$. As λ increases, the cooperation rate decreases slightly. The average cooperation rate for all λ s is about 0.21, and the fluctuations are not significant.

Figure 10 shows the average cooperation rate of the game as a function of $\theta(=\theta_i, 0 \le \theta_i \le 1)$. As θ increases, the average cooperation rate of the game decreases. The fluctuation of the cooperation rate also decreases as θ increases. The effect of an increase in θ is almost the same as that of an increase in β since β_{it}^k increases as θ_i increases by the updating rule of β_{it}^k . This is why the pattern of the cooperation rate as a function of θ shows a similar pattern as that of the cooperation rate as a function of β .

4.2.4 The effects of an excluding trigger strategy and a network TFT strategy

To measure the effects of an excluding trigger strategy and a network TFT strategy, I run simulations of the learning model without an excluding trigger strategy and a network TFT strategy.

Figure 11 shows the comparison between the average cooperation rate of the game with an excluding trigger strategy and a network TFT strategy and that of the game without them. The average cooperation rate of the game with an excluding trigger strategy and a

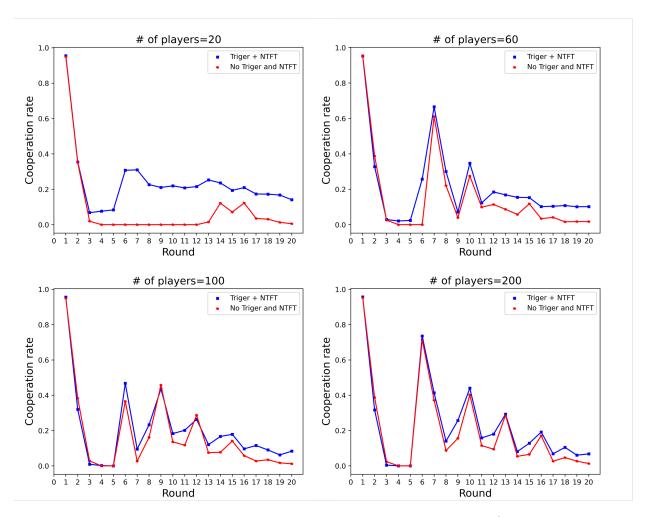


Figure 11: The comparison between the average cooperation rate of the game with an excluding trigger strategy and a network TFT and without them for n = 20, 60, 100, 200: Blue squares (Red circles) show the average cooperation rate in each round of the game with (without) an excluding trigger strategy and a network TFT for a hundred Monte Carlo data.

network TFT strategy is higher than the game without them.

In particular, the effect of these two strategies on the cooperation rate is more significant when the number of players is smaller. In Figure 11, the difference between the game's average cooperation rate with and without these two strategies is higher with 20 players than with 60. It implies that the smaller the number of players is, the greater the effect of threats of exclusion in the network on the cooperation rate is. Also, the cooperation rate in the simple repeated PD game with an excluding trigger strategy and network TFT strategy is higher than without them. This positive effect of excluding trigger and network TFT

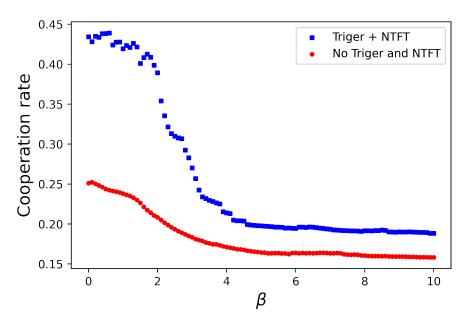


Figure 12: The comparison between the average cooperation rate of the game with an excluding trigger strategy and a network TFT and without them for $0 \le \beta \le 10$: Blue squares (Red circles) show the average cooperation rate of the game with (without) an excluding trigger strategy and a network TFT for a hundred Monte Carlo data.

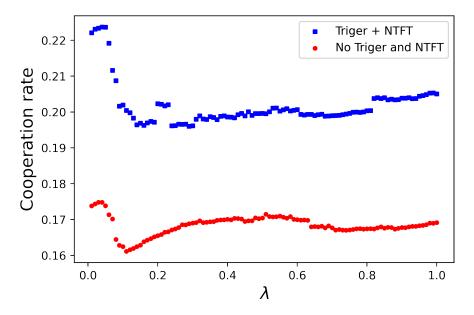


Figure 13: The comparison between the average cooperation rate of the game with an excluding trigger strategy and a network TFT and without them for $0.01 \le \lambda \le 1$: Blue squares (Red circles) show the average cooperation rate of the game with (without) an excluding trigger strategy and a network TFT for a hundred Monte Carlo data.

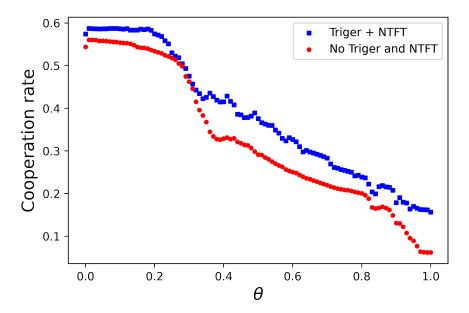


Figure 14: The comparison between the average cooperation rate of the game with an excluding trigger strategy and a network TFT and without them for $0 \le \theta \le 1$: Blue squares (Red circles) show the average cooperation rate of the game with (without) an excluding trigger strategy and a network TFT for a hundred Monte Carlo data.

strategies on the cooperation rate in the simple game becomes clearer when the number of players is smaller.

The higher cooperation in the simple repeated PD game is, the higher the reputation score is. Thus, it implies that the threat of exclusion in the network formation leads to an increase in reputation score, resulting in an increase in the cooperation rate in the simple repeated PD game.

Figure 12 shows the comparison between the average cooperation rate of the game with an excluding trigger strategy and a network TFT and without them for $0 \le \beta \le 10$. The average cooperation rate of the game with these two strategies is significantly higher than the game without them for all β s.

The average cooperation rate of the game with an excluding trigger strategy and a network TFT strategy is significantly higher than the game without these two strategies for different values of other parameters such as λ and θ (see Figures 13 and 14).

4.3 Discussion: How can we observe excluding trigger and Network TFT strategies in the experiments?

The belief-based learning model explains that cooperation can be driven by excluding trigger and network TFT strategies based on excluding defectors in network formation. The excluding and network TFT strategies can be used by players in the experiments. Now I discuss how the evidence of these strategies can be measured in the experiments.

First, reputation building in the initial rounds and an increased cooperation rate during network formation will be measured in the experiments. Reputation building by network formation will be stronger than the simple repeated PD game. Many researchers have observed the reputation effect in the finitely simple repeated PD game (see Andreoni and Miller (1993); Cooper et al. (1996); Gong and Yang (2010); Cox et al. (2015); Kamei (2017); Kamei and Putterman (2017); Honhon and Hyndman (2020)). Most research about the reputation effect in the finitely repeated PD is related to avoiding punishment in the next rounds. Similarly, to be selected as game partners in the network games, players can increase their reputation scores by selecting more cooperative actions in the initial rounds before starting network games. Avoiding exclusion in network formation will be stronger than avoiding punishment by defection since excluded players cannot play the game and make any profit. Also, all players can unilaterally exclude others with low reputation scores during network games. Thus, all players are incentivized to submit more cooperative actions during the network games.

In addition, connections among players who have the same actions will be measured in the experiments. By excluding defectors who prefer defection to cooperation in network formation among cooperators, cooperators who prefer cooperation to defection will be connected with other cooperators. Defectors will also be connected to other defectors to play the games due to exclusion by cooperators.

The following section represents the experimental study, and the pieces of evidence discussed above are investigated in the experimental study.

5 The experimental study

5.1 Experimental Design

The experiments were conducted with 216 unique student subjects recruited from Iowa State University. I performed 29 experiments with 2 players in the simple repeated games, 23 with 2 players in the cheap talk repeated games, and 8 with 14 players in the network repeated games from Oct. 2022 to Mar. 2023.

The simple repeated games. In each round, two players randomly selected play the prisoner's dilemma game under the given payoff structure, and they repeat the game for 20 rounds. The payoff of both players in the game is as follows:

		Player i	
		A	B
Player j	A	(4,4)	(-1,7)
	B	(7, -1)	(0,0)

Table 2: The payoff table of the game.

where A and B denote cooperate and defect, respectively. If both players choose A, then both players get 4 points. If a player i (j) chooses B and a player j (i) chooses A, then a player i (j) gets 7 points, and a player j (i) loses 1 point. If both choose B, both get nothing. This payoff structure is the same as the simple repeated game in the game-theoretical model and the belief-based learning model (c = 4, d = 0, e = -1, f = 7 in Table 1). To avoid framing, C and D in Table 1 are replaced with A and B since players can guess cooperation and defection using C and D. In addition, the name 'Prisoner's Dilemma Game' is never mentioned in all my experiments for the same reason. You can see the details of the simple repeated game experiment in the online Appendix.

The cheap talk repeated games. All players play the repeated PD game for 20 rounds under the given payoff structure (see Table 2). Two players are randomly selected, and they play the PD game in each round. From the 6th to the 20th round, all players can communicate with their partners using free-form texting before submitting their decisions. You can see the detail of the cheap talk repeated game experiment in the online Appendix.

The networked repeated games. All players play the simple repeated PD game from the 1st to the 5th round under the given payoff structure (see Table 2). From the 6th to the 20^{th} round, all players play the repeated PD game on a network based on partner selection. In the repeated PD game on a network, all players can select partners using the players' reputation scores and histories of players' actions for the past five rounds. The reputation score is measured by the average number of cooperative actions by a player during the past rounds (see R_{jt} in Section 4.1.2 on Page 25). All players must send proposals to others with whom they want to play the game from the 6th round. If the player who receives a proposal accepts (rejects) it, the proposer and acceptor do (do not) play the game. Players also see the network formed by players after network formation from the 6th round. The network shows the interaction structure of players. All players can see who is connected with them and who is the most connected in the network. After seeing the network formed, a player submits the same action to all neighbors of a player. You can see the details of the network repeated game experiment in the online Appendix.

5.2 Data Set

I collect data by conducting experiments using students' subjects at Iowa State University from Oct. 2022 to Mar. 2023. 216 unique students (111 male students, 102 female students, and 3 non-binary students) participate in the experiments: (1) the simple repeated games with 58 students – 29 experiments with 2 players in the simple repeated games; (2) the cheap talk repeated games with 46 students – 23 experiments with 2 players in the cheap

talk repeated games; (3) the networked repeated games with 112 students – 8 with 14 players ⁹ in the network repeated games. All students are paid 10 dollars as a participation reward and an additional bonus based on payoffs earned in the game (1 payoff point is rescaled to 3 cents). You can see the details of the demographics and compensations of participants in the online Appendix.

5.3 Data Analysis

The overall aim of this study is to understand the relationship between human cooperative behavior and human interaction behavior in the repeated PD game. In particular, I focus on measuring the effect of networks on cooperative actions in human groups in the repeated PD. Additionally, I analyze the characteristics of network structure to promote cooperative actions in human groups in the networked repeated game.

The first hypothesis (H1) that I want to test is as follows:

H1. The cooperation rate in the networked repeated game is higher than in the cheap talk repeated game and the simple repeated game.

Crawford and Sobel (1982) show that cheap talk should play no role in strategic interaction in theory. However, in experimental economic studies, cheap talk increases cooperation in repeated PD games (see Kagel and McGee (2016); Arechar et al. (2017); Kagel (2018); Cason and Mui (2019)). Network formation can exclude players more likely to defect in the future using reputation score and histories of actions of players, but cheap talk cannot. Thus, the networks would provide more useful information to promote cooperation than cheap talk. The fraction of cooperative actions in human groups in the networked repeated game should be higher than in the cheap talk repeated game and the simple repeated game.

⁹According to power analysis using the belief-based learning model, the effect of excluding trigger and network TFT strategies on cooperative actions in human groups in the repeated PD is maximized when the number of players is 14. Thus, I conduct the networked repeated games with 14 players.

Types of repeated games	Rounds 1 - 20	Rounds 1 - 5	Rounds 6 - 20
The simple repeated games The cheap talk repeated games The networked repeated games	0.466***	0.525***	0.447***
	0.538***	0.522***	0.543***
	0.644***	0.614***	0.654***

Table 3: The average cooperation rate in the games. *** p < 0.01, ** p < 0.05, * p < 0.1

To test **H1**, I compare the fraction of cooperative actions in human groups in the networked repeated games with the cheap talk repeated games and the simple repeated games. Table 3 shows the average cooperation rate in the games: the simple repeated games, the cheap talk repeated games, and the networked repeated games. The average cooperation rates in the networked repeated games for all rounds (Rounds 1-20), from the 1^{st} to the 5^{th} round (Rounds 1-5) and from the 6^{th} to the 20^{th} round (Rounds 6-20), are the highest among the three games. The highest cooperation rate in the networked repeated games for Rounds 1-5 can be due to reputation-building since players should have a high reputation score to be selected as game partners for Rounds 6-20. Thus, **H1** is strongly supported.

Additionally, contrary to the network repeated games, the cooperation rate in the cheap talk games for Rounds 1-5 is 0.522, lower than the simple repeated games. It implies that the communication from the 6^{th} to the 20^{th} round does not affect cooperative actions in human groups before the 6^{th} round, unlike network formation.

The second hypothesis (H2) that I want to test is as follows:

H2. Network formation promotes cooperative actions in human groups in the finitely repeated PD game.

To test **H2**, I compare the fraction of cooperative actions in human groups from the 1st to the 5th round with that from the 6th to the 20th round in the networked repeated games. Figure 15 shows the dynamics of the fraction of cooperative actions in human groups in the networked repeated games. In the networked repeated games, an increase in the cooperation

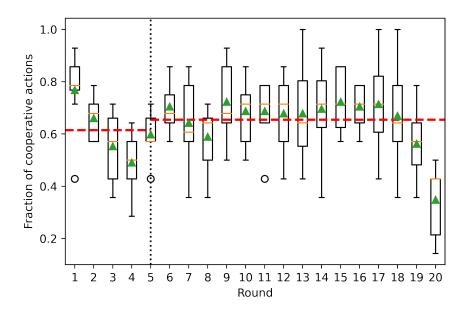


Figure 15: The figure shows the box plot of the fraction of cooperative actions in human groups in the networked repeated games at each round. Green triangles are the means of cooperation rates at each round. The black dotted vertical line indicates the 5th round. The red dashed lines show the average values in the average fraction of cooperative actions in human groups from the 1st to the 5th round and from the 6th to the 20th round, respectively.

rate by network formation from the 6th to the 20th round is observed. Thus, the result shows the evidence that supports **H2**. The result is also consistent with the result of the belief-based learning model: (1) the high cooperation rate in the first round; (2) the decreasing pattern of the cooperation rate in the dynamics of cooperation rate (see the dynamics of cooperation rate allowing excluding trigger and network TFT strategies in Figure 11). In the cheap talk repeated games, an increase in the cooperation rate by communication from the 6th to the 20th round is also observed (see Figure 16). However, the cooperation rate in the simple repeated games shows a decreasing pattern as a function of a round (see Figure 17).

To confirm the result observed in Figure 15, I additionally run two regressions in the

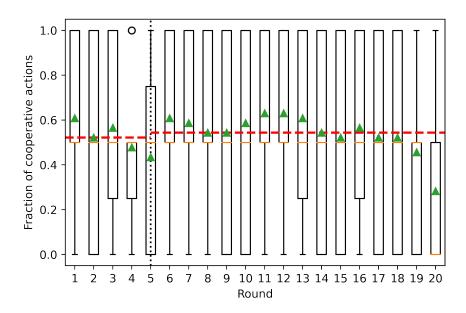


Figure 16: The figure shows the box plot of the fraction of cooperative actions in human groups in the cheap talk repeated games at each round. Green triangles are the means of cooperation rates at each round. The black dotted vertical line indicates the 5th round. The red dashed lines show the average values in the average fraction of cooperative actions in human groups from the 1st to the 5th round and from the 6th to the 20th round, respectively.

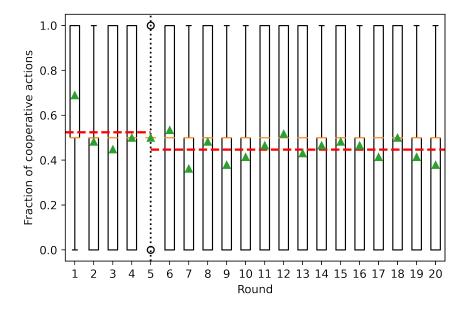


Figure 17: The figure shows the box plot of the fraction of cooperative actions in human groups in the simple repeated games at each round. Green triangles are the means of cooperation rates at each round. The black dotted vertical line indicates the 5th round. The red dashed lines show the average values in the average fraction of cooperative actions in human groups from the 1st to the 5th round and from the 6th to the 20th round, respectively.

Dependent variable: Cooperative action of a player (y_{it})			
Panel A: REG1			
Independent variable	Coefficient (β)	Standard Error	t-Statistic
Reputation score: Rep_{it}	0.810***	0.026	29.544
Quiz: $Quiz_i$	0.028	0.018	1.537
Network effect: NE_t	0.132^{***}	0.024	5.413
Total payoff: $TotPayoff_{it-1}$	-0.001***	0.0001	-4.389
Intercept	0.049***	0.026	1.906
	Panel B: REG2		
Independent variable	Coefficient (β)	Standard Error	z-Statistic
Reputation score: Rep_{it}	4.245***	0.199	21.379
Quiz: $Quiz_i$	0.192*	0.113	1.703
Network effect: NE_t	0.833***	0.161	5.185
Total payoff: $TotPayoff_{it-1}$	-0.004***	0.0001	-4.525
Intercept	-2.424***	0.177	-13.704
Number of observations: 2,128			

Table 4: The regression results using **REG1** and **REG2** in the network repeated games.

*** p < 0.01, ** p < 0.05, * p < 0.1

networked repeated games as follows:

REG1:
$$y_{it} = \beta_0 + \beta_1 Rep_{it} + \beta_2 Quiz_i + \beta_3 NE_t + \beta_4 Tot Payof f_{it-1} + \epsilon_{it},$$
 (10)

$$\mathbf{REG2}: P(y_{it} = 1|X_{it}) = \frac{\exp(\beta^T X_{it})}{1 + \exp(\beta^T X_{it})},\tag{11}$$

where y_{it} is a binary variable that denotes a cooperative action of a player: (i) $y_{it} = 1$, if player i's decision at round t is A; (ii) $y_{it} = 0$, otherwise. Rep_{it} denotes the reputation score of player i at round t (see the definition of the reputation score (R_{jt}) in Section 4.1.2 on Page 25). $Quiz_i$ is a binary variable that denotes whether player i passed the quiz before starting the game: (i) $Quiz_i = 1$, if player i passed the quiz; (ii) $Quiz_i = 0$, otherwise. NE_t denotes a binary variable that denotes the network effect on cooperation at round t: (i) $NE_t = 1$, if t > 5; (ii) $NE_t = 0$, otherwise. From the 6^{th} round, players can form their networks. Thus, NE_t quantifies the network effect on cooperation in the experiments. $TotPayoff_{it-1}$ denotes the total payoff of player i at round i at round i and i are residual term. i and i are residual term. i and i are residual term. i and i are residual term.

			`
Dependent variable: Cooperative action of a player (y_{it})			
Panel A: REG1			
Independent variable	Coefficient (β)	Standard Error	t-Statistic
Reputation score: Rep_{it}	0.657***	0.045	14.472
Quiz: $Quiz_i$	0.003	0.046	0.074
Cheap talk effect: NE_t	0.005	0.039	0.124
Total payoff: $TotPayoff_{it-1}$	0.001	0.001	0.894
Intercept	0.145^{***}	0.055	2.665
Panel B: REG2			
Independent variable	Coefficient (β)	Standard Error	z-Statistic
Reputation score: Rep_{it}	2.996***	0.247	12.145
Quiz: $Quiz_i$	-0.011	0.229	-0.049
Cheap talk effect: NE_t	0.037	0.202	0.814
Total payoff: $TotPayoff_{it-1}$	0.003	0.004	0.779
Intercept	-1.606***	0.280	-5.734
Number of observations: 874			

Table 5: The regression results using **REG1** and **REG2** in the cheap talk repeated games. In these regressions, NE_t quantifies the effect of cheap talk on cooperation.

*** p < 0.01, ** p < 0.05, * p < 0.1

denotes the probability that y_{it} is equal to 1 given X_{it} . X_{it} denotes a vector of independent variables: $X_{it} = (1, Rep_{it}, Quiz_i, NE_t, TotPayoff_{it-1})$. β^T denotes a vector of the logistic regression coefficients $(\beta = (\beta_0, \beta_1, \beta_2, \beta_3, \beta_4))$.

Table 4 depicts the regression result using **REG1** (Panel A) and **REG2** (Panel B) in the networked repeated games. The coefficients of Rep_{it} for **REG1** and **REG2** are positive $(\beta_1 > 0)$, and the values are statistically significant at the 1 percent level. It shows that a higher reputation drives more cooperative action. This is consistent with the previous results (see Cuesta et al. (2015); Gallo and Yan (2015)). The coefficient of $Quiz_i$ is positive for **REG1** $(\beta_2 > 0)$. It implies that players who passed the quiz are more likely to cooperate than others who did not. However, this result is not statistically significant. The coefficients of NE_t for **REG1** and **REG2** are positive $(\beta_3 > 0)$, and the values are statistically significant at the 1 percent level. It implies that players are more likely to cooperate due to network formation. The coefficient of NE_t is 0.132 in **REG1**. It implies that the probability of a player's cooperative action increases by 13.2% due to network formation. Thus, **H2** is

Dependent variable: Cooperative action of a player (y_{it})				
Panel A: REG1				
Independent variable	Coefficient (β)	Standard Error	t-Statistic	
Reputation score: Rep_{it}	0.803***	0.036	22.123	
Quiz: $Quiz_i$	0.040	0.028	1.435	
# of rounds effect (> 5 rounds): NE_t	0.007	0.031	0.226	
Total payoff: $TotPayoff_{it-1}$	0.001	0.001	1.063	
Intercept	0.001	0.037	0.028	
Panel B: REG2				
Independent variable	Coefficient (β)	Standard Error	z-Statistic	
Reputation score: Rep_{it}	4.092***	0.253	16.163	
Quiz: $Quiz_i$	0.173	0.160	1.083	
# of rounds effect (> 5 rounds): NE_t	0.104	0.184	0.566	
Total payoff: $TotPayoff_{it-1}$	0.003	0.003	0.834	
Intercept	-2.593***	0.237	-10.949	
Number of observations: 1,102				

Table 6: The regression results using **REG1** and **REG2** in the simple repeated games. In these regressions, NE_t quantifies the effect of the number of rounds bigger than 5 on cooperation.

$$p < 0.01$$
, ** $p < 0.05$, * $p < 0.1$

supported again using **REG1** and **REG2**. This result is also consistent with previous results (see Cuesta et al. (2015); Wang, Suri and Watts (2012)). The coefficients of $TotPayoff_{it-1}$ for **REG1** and **REG2** are negative but very close to zero ($\beta_4 < 0$). It implies that a high payoff drives defection. However, this effect is nearly zero.

Additionally, I run the regressions in the cheap talk games to identify the effect of communication on cooperation in the cheap talk repeated games using **REG1** and **REG2**. From the 6th round, players can communicate with their game opponents using cheap talk. Thus, NE_t quantifies the effect of communication on cooperation in the cheap talk repeated games.

Table 5 shows the results of **REG1** (Panel A) and **REG2** (Panel B) in the cheap talk repeated games. The coefficients of NE_t are positive for **REG1** and **REG2** ($\beta_3 > 0$) but not statistically significant. The coefficient of NE_t in **REG1** is 0.005. It implies that the probability of a player's cooperative action increases by 0.5% due to the communication, but this change is very small as well as not statistically significant.

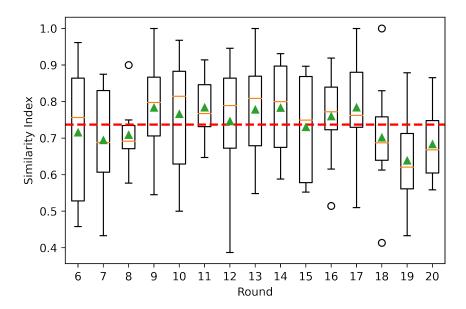


Figure 18: The similarity index as a function of a round in the networked repeated games. Green triangles represent the means of the similarity indices of each round. Error bars represent the 95% confidence interval of the similarity indices of each round. The red horizontal dashed line shows the mean of similarity indices of all rounds in the networked repeated games. The value indicated by the red line is 0.737.

Lastly, I run the regressions in the simple games to identify the effect of the number of rounds on cooperation in the simple repeated games using **REG1** and **REG2**. In the regressions, NE_t quantifies the effect of the number of rounds bigger than five on cooperation.

Table 6 shows the results of **REG1** (Panel A) and **REG1** (Panel B) in the simple repeated games. The coefficients of NE_t are positive ($\beta_3 > 0$) but not statistically significant. The coefficient of NE_t in **REG1** is 0.007. It implies that the probability of a player's cooperative action increases by 0.7% after the 5th round, but this change is very small as well as not statistically significant.

In summing up, the effect of network formation on cooperative actions is positive in the networked repeated games ($\beta_3 > 0$ in **REG1** and **REG2**). It indicates that **H2** is strongly supported by the regression analysis. The effect in the networked repeated games is the greatest among the three games. That is, **H1** is also strongly supported again.

The third hypothesis (**H3**) that I want to test is as follows:

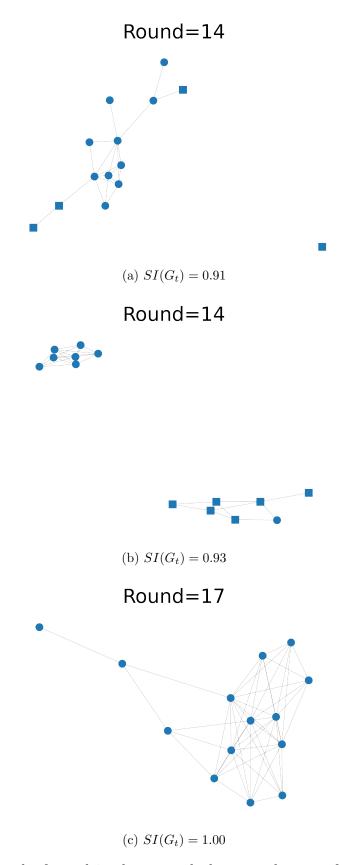


Figure 19: The networks formed in the networked repeated games for different similarity indices $(SI(G_t)s)$. Circles and squares indicate cooperators and defectors, respectively.

H3. Cooperators are more likely to be connected to other cooperators, and defectors are more likely to be connected to other defectors.

To test **H3**, I need a network measure to provide the similarity of nodes or players tied in the same link. I suggest the *similarity index* $SI(G_t)$, which measures the similarity of nodes linked in the same link as follows:

$$SI(G_t) = \frac{\sum_{i} \sum_{j \in N_{it}} \delta(a_{it}, a_{jt})}{2 \cdot S(G_t)},$$
(12)

where G_t denotes the network at round t. N_{it} denotes the set of neighbors of node or player i at round t. a_{it} is the action of a node i at round t. $\delta(a_{it}, a_{jt}) = 1$ if $a_{it} = a_{jt}$, and $\delta(a_{it}, a_{jt}) = 0$ otherwise. $S(G_t)$ is the size of network G_t , and is measured by the number of links in G_t . $SI(G_t)$ is between 0 and 1 ($0 \le SI(G_t) \le 1$). If there is no link between two nodes whose actions are the same, $SI(G_t)$ is zero. If nodes tie all links with the same action, $SI(G_t)$ is one. Thus, if a positive $SI(G_t)$ is measured with a high statistical significance, there exist nodes linked with the same action in network G_t .

Figure 18 depicts the dynamics of the similarity indices of each round in the networked repeated game. The mean of the similarity indices of all rounds is 0.737 (see the red horizontal dashed line in Figure 18). It implies that the probability that players tied with the same link have the same action is about 0.737. The mean for each round (see the green triangles in Figure 18) is greater than 0, with significance at the 1 percent level. It means that cooperators are more likely to be linked to other cooperators, and defectors are more likely to be linked to other defectors.

Figure 19 shows the networks formed in the networked repeated games for different similarity indices. We can see the higher clustering of cooperators (defectors) with a higher similarity index in Figure 19. Thus, **H3** is strongly supported.

The final hypothesis $(\mathbf{H4})$ that I want to test is as follows:

H4. The connections among cooperators with excluding defectors in the networked repeated games promote cooperative actions of players.

H4 is easily tested by testing H2 and H3. H2 and H3 are both strongly supported. The effect of network formation on cooperation is positive (H2), and cooperators are linked to other cooperators in the networked repeated games (H3). Thus, H4 is strongly supported.

5.4 Discussion: The use of the excluding trigger and network TFT strategies in human society

My experimental study shows that excluding defectors by network formation, which can be evidence of the excluding trigger and network TFT strategies, drives cooperative actions in human groups in the finitely repeated prisoner's dilemma game. I discuss the usefulness of excluding trigger and network TFT strategies to drive cooperation in human society.

First, the excluding trigger and network TFT strategies can be very efficient in solving conflicts in social dilemma situations and increasing social capital. There are diverse groups in society, and they might be in a social dilemma. In most cases, cooperation among groups based on common sense is required to solve a social dilemma and to increase diversity without too much conflict in society, but only a few extreme groups might be antisocial and disagree with cooperation. They might not be good for making society resilient and increasing diversity. Communication with these groups can be useful for consensus or cooperation, but it might be challenging since communication cannot punish them when they do not cooperate. They are more likely to cooperate if they are threatened by social exclusion. My experimental study supports this by showing that the cooperation rate by the threat of exclusion through network formation is higher than by communication (see **H1** in Section 5.3). Also, social connections among cooperators can increase social capital. The increased social capital decreases inequality in society. Thus, excluding trigger and network TFT strategies

might be helpful in increasing social capital and decreasing societal inequality.

In addition, the network TFT strategy is more realistic than TFT in human society. TFT has been one of the best strategies that give the highest profits in the finitely repeated prisoner's dilemma and observed in the experiments as a strategy to drive cooperation (see Axelrod (1984); Dal Bó and Fréchette (2011); Fréchette and Yuksel (2017); Embrey, Fréchette and Yuksel (2018); Dal Bó and Fréchette (2019)). However, TFT is based on a simple PD game that differs from reality since reality is more similar to the PD game on a network with more than one game partner. Also, TFT players cannot avoid playing with defectors. However, network TFT players can avoid playing with defectors by excluding defectors in network formation. Thus, the network TFT strategy can be more widely used in human society than TFT and can drive cooperation in human society.

6 Conclusion and discussion

In this paper, I analyze the effect of networks on cooperative actions in human groups in the finitely repeated PD game using three approaches: the game-theoretical model approach, the belief-based model approach, and the experimental approach.

In the game-theoretical model approach, I suggest the game-theoretical model, which explains the equilibrium state by network formation in the finitely repeated PD game. The model consists of two repeated PD games: the simple repeated PD game and the repeated PD game on a network. From the 1st to the round t_s , players play the simple repeated PD game. From the round t_s+1 to the final round T_s , players play the repeated PD game on a network. In the simple repeated PD game, players play the PD game with their partners randomly re-matched in each round. In the repeated game on a network, players can select their partners using histories of others' actions for the past rounds before playing the game in each round. There exist pairwise stable networks in which cooperators (defectors) are linked to other cooperators (defectors) and pairwise-Nash equilibrium networks: all players

defect and are completely connected or isolated.

In the belief-based learning model study, I suggest the belief-based learning model, which explains the cooperative actions of players and connections among cooperators observed in the experiment not explained in the game-theoretical model. The belief-based learning model also consists of the simple repeated PD and the repeated PD game on a network. The partner selection in the network game is the same as in the game-theoretical model.

Additionally, I assume that players select the best strategy that maximizes the expected total payoff for each round in the game. Three strategies are considered in the model: m-threshold strategy, excluding trigger strategy, and network TFT. An m-threshold strategy player cooperates from the 1^{st} round to the m^{th} round and defects from the $(m+1)^{th}$ round to the final round. An m-threshold strategy player wants to make connections with all others in the networked game. An excluding strategy player cooperates at any round and does not allow connections with the players who defected in past rounds. A network TFT strategy player cooperates at every round and does not allow connections with the players who defected in each previous round.

I generate one hundred Monte Carlo data sets using simulations of the model with different random seeds and analyze the data. First, I find high cooperation rates in the initial rounds of the simple repeated PD game and repeated PD game on a network and decreasing patterns of the cooperation rate as a round increases in the simple repeated PD game and repeated PD game on a network. Second, I find the behavior of excluding defectors in the repeated PD game on a network. Third, I find the short threshold strategy $(0 \le m \le 5)$ is the most popular strategy among all strategies in the simple game, and the long threshold strategy $(6 \le m \le 20)$ is the most popular in the repeated PD game on a network. Finally, both an excluding trigger strategy and a network TFT have significant positive effects on the cooperation rate. In particular, the smaller the number of players, the higher the positive effect of both strategies on the cooperation rate. Furthermore, the cooperation rate in the simple repeated PD game with excluding trigger and network TFT strategies is higher

than without them. It implies that the threat of exclusion in networks results in a high cooperation rate before forming networks, called "reputation-building."

Finally, I experimentally examine the effect of networks on cooperative actions in human groups in the finitely repeated PD game. I conduct 60 experiments with 216 unique student subjects recruited from Iowa State University: (1) the simple repeated games with 58 students – 29 experiments with 2 players in the simple repeated games; (2) the cheap talk repeated games with 46 students – 23 experiments with 2 players in the cheap talk repeated games; (3) the networked repeated games with 112 students – 8 with 14 players in the network repeated games from Oct. 2022 to Mar. 2023. In the simple repeated games, two players randomly matched play the simple PD game for 20 rounds. In the cheap talk repeated games, two players randomly matched play the simple PD games from the $1^{\rm st}$ to the 5th round and can communicate with their game opponents before submitting their decisions from the 6th to the 20th round. In the networked repeated game, players play the simple repeated PD game from the 1st to the 5th round. From the 6th to the 20th round, players play the repeated PD game on a network. From the 6th to the 20th round, all players can form their networks using the partner selection based on the reputation score, which is measured by the average number of cooperative actions by a player, and the histories of players' actions for the past five rounds before submitting their decisions.

First, the average cooperation rate in networked repeated games is higher than in simple and cheap talk games. This finding implies that the reputation-building mechanism from the 1st to the 5th round and network formation from the 6th to the 20th round encourage cooperative actions in human groups. Second, a significant increase in the cooperation rate after the 5th round is observed. As a result of the regression analysis, the positive effect of network formation on cooperative actions in human groups is confirmed. In the cheap talk repeated games, the positive effect of cheap talk on cooperative actions in human groups is observed but not statistically significant. The cooperation rate in the simple repeated games decreases as a round increases. Third, cooperators are linked to other cooperators, and defectors are

linked to other defectors in the experiments. This result is measured by a significant positive similarity index. Finally, I conclude that cooperation can be achieved by connections among cooperators with excluding defectors in the network. Thus, the experimental results support the predictions of the belief-based learning model over the game-theoretic model.

My study also has room for improvement. In the game-theoretical model approach, I need to study the effect of network structure on cooperation in the repeated PD game. The network structure is endogenously determined by partner selection. After network formation, players' actions are impacted by the network structure. Several models have been suggested to explain the relationship between cooperation and network structure in the repeated PD game. I can extend my game-theoretical model to a new model that considers network structure based on previous models.

The belief-based learning model can be applied to the analysis of the experiments, and the parameters can be estimated using the experimental data. Embrey, Fréchette and Yuksel (2018) estimate the parameters in their belief-based learning model using the experimental data of the finitely repeated PD game. Thus, in the future study, I will estimate the parameters in my belief-based learning model and the fractions of the strategies used in my model using a similar approach of Embrey, Fréchette and Yuksel (2018). On top of that, other new strategies can be added to my belief-based learning model. There are many kinds of strategies studied in the repeated PD game¹¹, and we can make new strategies by combing preexisting strategies and network formation.

In my experimental study, I can measure the fraction of players who uses strategies based on excluding defectors in networked repeated games. The observation of cooperation by excluding defectors is confirmed by testing hypotheses ($\mathbf{H1} - \mathbf{H4}$). However, to understand the relationship between cooperation and network formation by excluding defectors more

¹⁰There are several theoretical studies on the relationship between cooperation and network structures in the repeated PD (see Lieberman, Hauert and Nowak (2005); Ohtsuki et al. (2006); Allen, Lippner and Nowak (2019); Fotouhi et al. (2019); Alvarez-Rodriguez et al. (2021)). I cite these literatures without detailed explanations. My next model will add similar approaches used in these literatures.

¹¹See https://plato.stanford.edu/entries/prisoner-dilemma/strategy-table.html.

deeply, I need to measure the fractions of excluding trigger strategy players and network TFT players using my belief-based learning model and the network repeated games data.

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