The effect of network formation on cooperation in the finitely repeated prisoner's dilemma

Kyubin Yim
Department of Economics, Iowa State University, Ames, IA 50011, USA
kyubin48@iastate.edu

Elizabeth Hoffman
Department of Economics, Iowa State University, Ames, IA 50011, USA
bhoffman@iastate.edu

Abstract

We study the effect of network formation on cooperation in the finitely repeated prisoner's dilemma based on the game-theoretical model approach. We suggest the model explaining the effect of endogenous network formation on cooperation. We find a subgame perfect strongly pairwise-Nash equilibrium in which cooperation is achieved by the trigger strategy based on non-random partner selection. Also, in the subgame perfect strongly pairwise-Nash equilibrium, cooperators are completely connected with other cooperators, and defectors are isolated in the network. These results imply "full separation" of cooperators and defectors in the network and "marginalization" of defectors in the network.

Keywords: The evolution of cooperation, Network formation, Strongly pairwise-Nash equilibrium, Repeated prisoner's dilemma, Excluding trigger strategy

1. Introduction

We often conflict between our individual optimal decisions and socially optimal decisions. The prisoner's dilemma describes the social dilemma, which shows the conflicts between us and our society. In the standard economic theory based on individual rationality, there exists the unique subgame-perfect Nash equilibrium in the finitely repeated two-person prisoner's dilemma game in which all players choose to defect even if cooperation is a socially optimal decision.

Cooperation or defection is a result of interaction among players in the finitely repeated prisoner's dilemma. It implies that cooperative outcome in the finitely repeated prisoner's dilemma can be possible if we assign specific rules in the interaction structure among players. Nowak (2006) suggests network reciprocity as one of five rules to promote cooperation in the repeated prisoner's dilemma. It means that we can find the conditions based on interaction structure that make cooperative outcomes possible in the finitely repeated prisoner's dilemma.

We can deal with interaction behavior in the socio-economic system due to the development of network theory. A network consists of nodes and links. Nodes indicate players or agents in the system. In economic systems, nodes can be firms, players, households, governments, etc. Links indicate relationships or interactions among nodes. In a friendship network, a link denotes the friendship relationship between two persons. In an interbank network, a link denotes the credit relationship between two banks. We can analyze interaction structures using nodes and links. Network theory has been applied to many areas in economics, such as game theory, finance, macroeconomics, etc. (see Goyal 2012; Jackson 2014; Carvalho and Tahbaz-Salehi 2019).

This paper applies network theory to solving social dilemmas in the finitely repeated prisoner's dilemma game. We suggest the finitely repeated prisoner's dilemma game based on network formation. The game consists of n players and T rounds ($n, T \gg 2$). From the 1st to round $t_c (1 < t_c \ll T)$, all players play the simple two-person prisoner's dilemma game with their partner randomly matched in each round. After round t_c, all players play the repeated n-person prisoner's dilemma game based on the non-random partner selection rule using network formation. All players can see all players' reputation scores, histories of others' actions during the past t_c rounds, and histories of network information. The reputation score is calculated by the average times of a player's cooperative actions during the past rounds. Using the information, all players propose links to others with whom they want to play. If the player who received proposals accepts the link, the proposer and the acceptor will be linked in the network and play the game. As a result of the equilibrium analysis, there exists the strongly pairwise-Nash equilibrium network in which cooperators are completely connected, and defectors are marginalized after round t_c by an excluding trigger strategy that allows exclusion of defectors. Thus, by the network formation, cooperation can be achieved in the subgame perfect strongly pairwise-Nash equilibrium in the repeated game.

Related literature. The attempt to find a way to reach cooperative outcomes in the repeated game using networks has been tried. In particular, many studies have focused on non-random partner matching using networks. The paper is related to the literature about the non-random matching based on networks in the repeated game. Ule (2008) focuses on the trigger strategy based on network formation to promote cooperation in the game and shows that the subgame-perfect equilibrium exists in which all players cooperate in initial periods in the finitely repeated prisoner's dilemma game by the trigger strategy. Mengel (2009) introduces the bilateral prisoner's dilemma game that allows local interactions with neighbors in the network and shows that cooperation can be achieved by imitating neighbors' strategies. Fosco and Mengel (2011) focus on the effect of imitation and exclusion on cooperative outcomes in the repeated prisoner's dilemma allowing local interactions with neighbors in the network. They show that cooperation can be achieved by imitating neighbors' strategies and excluding defectors in neighbors. They also find that cooperators and defectors are fully separated in the network. Wolizky (2013) studies the effect of the grim trigger strategy based on network monitoring on cooperative outcomes in the repeated public good game using the Bayesian approach and shows that cooperation can be achieved by the grim trigger strategy. Cho (2014) focuses on the effect of local interaction in the network on cooperation in the repeated prisoner's dilemma and shows that there exists a sequential equilibrium in which cooperation is supported in the game.

This paper contributes to the previous research about network formation in the repeated prisoner's dilemma (see Ule 2008; Fosco and Mengel 2011). Also, we develop the equilibrium concept in network formation based on mutual link formation and unilateral multi-link severance. Ule (2008) shows that there exists the subgame perfect Nash equilibrium in which cooperation is achieved by network formation but does not explain interaction structure among players, such as full separation of cooperators and defectors in the network. Fosco and Megel (2011) explain the full separation of cooperators and defectors in the network, but they do not explain the network structure using the equilibrium concept in the repeated game. This paper introduces the strong network equilibrium concept in the repeated game to explain the full connections among cooperators in the network.

The remainder of the paper is organized as follows. We introduce the model in Section 2. We provide the equilibrium analysis to show the possibility of the cooperative outcome by an excluding trigger strategy in Section 3. The paper is concluded by summarizing all results in the paper, and we discuss the future work in Section 4.

2. Model

Let $N = \{1, .., n\}$ be the set of players ($n \gg 2$). All players play the repeated prisoner's dilemma game for T rounds ($T \gg 2, T$ is an integer). In every round $t \in \{1, \ldots, T\}$, every player i ($i \in N$) simultaneously chooses an action $x_i^t \in \{C, D\}$, where C and D denote *cooperation* and *defection*, respectively. We define the type of player i at round t using x_i^t: (1) *cooperator* if $x_i^{t'} = C$, for all $t' \leq t$; (2) *defector* otherwise[1].

Assumption 1. The finitely repeated game consists of two games: (1) the simple repeated two-person prisoner's dilemma game; (2) the networked repeated n-person prisoners' dilemma game.

From the first round to round $t_c (1 < t_c \ll T)$, all players are doing the simple repeated two-person prisoner's dilemma game. The game partner of a player is randomly selected at each round during the simple repeated game. Then, a player plays with her partner under the payoff structure as follows:

		Player *j*	
		C	*D*
Player i	*C*	(*c*,*c*)	(*e*,*f*)
	D	(*f*,*e*)	(*d*,*d*)

Table 1. The payoff table of the game

where $f > c > d > e, f + e < 2c, e < 0, d = 0$. If both players cooperate, they each get *c*. If both players defect, they each get *d*. If player $i(j)$ cooperates and player $j(i)$ defects, player $i(j)$ gets *e,* and player $j(i)$ gets *f.* In the simple repeated two-person prisoner's dilemma game, all players do not know others' information.

Assumption 2. There are some players inclined to choose to cooperate in the first round.

[1] The definition of a cooperator is strict because any defection in the past is not allowed to be a cooperator. However, this definition is useful to see the effect of the trigger strategy based on threats of excluding defectors on cooperation.

Assumption 2 is supported by several in the previous experimental studies about the finitely repeated prisoner's dilemma game (see Wang, Suri, and Watts 2012; Cuesta et al. 2015; Galo and Yan 2015; Embrey, Fréchette, and Yuksel 2018). Embrey, Fréchette, and Yuksel (2008) show that the cooperation rate at the first round is positively correlated with the length of rounds in the finitely repeated prisoner's dilemma game. Also, it is possible to apply the grim trigger strategy not to deviate from the cooperative outcome path at the first round when the length of rounds is sufficiently long in the finitely repeated game (see Friedman 1985; Ule 2008). In the networked game, there exist threats of excluding defectors in the partner selection process. Thus, some players have an incentive to choose to cooperate in the first round.

From round $t_c + 1$ to the final round T, players do the networked repeated n-person prisoners' dilemma game based on the non-random partner selection.

Assumption 3. The networked repeated n-person prisoner's dilemma game consists of two stages in each round: (1) the partner selection; (2) the choice of action. At the beginning of the round in the networked repeated n-person prisoner's dilemma game, all players have to choose their partners. After finishing the partner selection, all players play the prisoner's dilemma game with only their partners under the same payoff as the simple repeated game.

Assumption 4. Players who are not selected by others as a partner will get nothing regardless of their actions. Thus, all players should be selected by others to get a payoff in the networked game.

The details of the partner selection process are as follows:

1. At the beginning of the networked prisoner's dilemma game, players have to propose links to others with whom they want to play the game simultaneously. Players can see other players' reputation scores and histories of actions during the past t_c rounds. The reputation score of a player is calculated by the average times of a player's cooperative actions during the past rounds as follows: $Rep_i^t = \frac{\sum_{k=1}^{t-1} C_i^k}{t-1}$[2], where Rep_i^t denotes the reputation score of player i at round t. C_i^k is the cooperation score of player i at round k: (1) $C_i^k = 1$, if player i chooses to cooperate at round k; (2) $C_i^k = 0$, if player i chooses to defect at round k. All players use others' reputation scores, histories of others' actions, histories of network information while proposing links to others. There is no constraint on the number of links to propose. Thus, a player can propose $n - 1$ links in each round.

[2] In general, reputation in game theory is referred to as a posterior belief on the likelihood of choosing cooperation in the next period, given the history on the equilibrium path. Mailath and Samuelson (2006) provide a rigorous analysis of the effect of reputation in the repeated game using the Bayesian approach. However, the reputation score in our game is one of the information to choose an action. In the networked game, all players' reputation scores are publicly available to all players. In general, the more the player cooperated in the previous rounds, the more the player cooperates in the current period. Using this intuition, we define the reputation score calculated by the average of cooperative actions in the past rounds. Cuesta et al. (2015) measure the positive effect of reputation score, which is the same as our game, on cooperative actions in human groups in the finitely repeated prisoner's dilemma game.

2. Players who received proposals from other players should decide whether they play the prisoner's dilemma game with proposers simultaneously. If the player accepts the proposal, both will be linked in the network and play the game. However, if the player does not accept, both will not be linked in the network, and they will not play the game. If they do not play the game, they will get nothing. It means that a player who is not selected as a partner from others gets zero payoffs.

3. The partner selection process is repeated at the beginning of the networked repeated game after the round t_c. Thus, all players can change their partners in every round after the round t_c. Also, all players can unilaterally sever links between others in the partner selection process.

The partner selection process captures the general making a friend process. In general, we make friends based on mutual consent using our experiences. We can sever friendship relations with others whom we do not like.

Also, the partner selection process can explain the mechanism for making social norms or social rules that teach us the importance of cooperation. Most social knowledge is formed by mutual consent among members of society based on their experiences. Due to the partner selection process, players can experience making as many connections with cooperators as possible is the best strategy to get the best outcome. By repeating the partner selections for multiple rounds, players can see that they must be a cooperator to connect with others since players do not select defectors as their partners using the given information, such as histories of all players' actions and all players' reputation scores. Okada (1993) shows that cooperation can be achieved in the n-person prisoner's dilemma game with institutional arrangements that enforce an agreement of cooperation. The partner selection process is such a strong way of enforcing cooperation agreements by excluding defectors based on network formation. Thus, the partner selection process may promote more cooperation than other games not based on network formation.

This partner selection process based on a non-random matching process using the information that allows predicting players' actions will promote cooperation. The previous experimental studies show that the information that allows guessing others' actions and non-random partner matching promote cooperation in the game. In the ultimatum and dictator games, players are more likely to cooperate when the information that allows predicting their actions by others is given (see Hoffman et al. 1994; Hoffman, McCabe, and Smith 1996a, 1996b). Also, the previous networked experiments show the network formation process promotes cooperation (Wang, Suri, and Watts 2012; Cuesta et al. 2015; Galo and Yan 2015).

The partner selection of player i can be captured by a binary vector

$$b_i = \left(b_{ij}\right)_{j\in N} \in \{0,1\}^N, \text{ such that } b_{ii} = 0 \text{ and } \sum_{j\in N} b_{ij} \leq n-1. \tag{1}$$

$$b_{ij} = 1 \text{ if player } i \text{ proposes a link to player } j, \\ = 0 \text{ otherwise.}$$

Let $g(b)$ denote the network of the established link given the profile of partner selections $b = (b_1, \dots, b_n)$. In the partner selection, the link between two players is formed when two players agree with being linked to each other. By the mutual link formation process, $g_{ij} = \min\{b_{ij}, b_{ji}\}$, where g_{ij} is a binary variable that denotes the link between players i and j: (1) $g_{ij} = 1$ if players i and j are linked; (2) $g_{ij} = 0$ otherwise. Let $L_i(g) = \{j | g_{ij} = 1\}$ denote the set of neighbors of player i and let $l_i(g) = |L_i(g)|$ be the number of neighbors of player i: (1) $l_i(g) = 1$ in the simple repeated prisoner's dilemma repeated game $(t \leq t_c)$; (2) $0 \leq l_i(g) \leq n - 1$ in the network repeated prisoner's dilemma game ($t_c < t \leq T$). Let $G = \{g | 0 \leq l_i(g) \leq n - 1, \text{for all } i \in N\}$ be the set of feasible networks. Thus, $g(b) \in G$.

All players use the reputation scores of all players, histories of all players' actions for the past t_c rounds, and network information in the partner selection. Thus, I define the information set of the network game at round t (I^t) as follows: $I^t = \{IRep^t, IX^t, g(b)^t\}$, where $IRep^t = \{Rep_1^t, \dots, Rep_n^t\}$, $IX^t = \{\{x_1^{t-t_c}, \dots, x_n^{t-t_c}\}, \dots, \{x_1^{t-1}, \dots, x_n^{t-1}\}\}$, $g(b)^t$ denotes the network information from round $t_c + 1$ to round $t - 1$. I^t is publicly accessible to all players at round I^t. Thus, I^t is common knowledge at round t. Players select their partners using the reputation scores of players, histories of players' actions, and histories of the network information, which shows all players' connections. Additionally, we set the assumption about network information.

Assumption 5. Network information allows all players to discriminate who are not selected by others in the partner selection stage.

Network information depicts the structure which shows all interconnections among players by the partner selection. Thus, if all players can access network information during the network game, they can discriminate who are not selected by others in the partner selection stage.

By **Assumptions 1** to **5**, players have an incentive to maintain cooperation for all rounds. From the 1st round to round t_c, players have an incentive to cooperate to be selected as a partner in games after round t_c by increasing their reputation scores. From round $t_c + 1$ to the final round, all players have an incentive to maintain cooperation not to be excluded in the partner selection. We can define the trigger strategy based on this intuition. We describe the trigger strategy in **Definition 3**.

After finishing the partner selection, all players choose their actions. Then, the payoff of player i in the network game given the network of established link and the profile of actions at round t ($\pi_i(x^t, b^t)$) is as follows:

$$\begin{aligned} \pi_i(x^t, b^t) &= \textstyle\sum_{j \in L_i(g^t)} u_i(x_i^t, x_j^t) \\ &= \textstyle\sum_{j \in N} u_i(x_i^t, x_j^t) g_{ij}(b^t), \end{aligned} \tag{2}$$

where $u_i(x_i^t, x_j^t)$ denotes the payoff of player i if players i and j choose x_i and x_j at round t, respectively. $(x^t, b^t) = ((x_1^t, b_1^t), \dots, (x_n^t, b_n^t))$.

The action and partner selection of player i $(x_i^*, b_i^*) \in S_i$ is a *best response of player* i to the profile of player i's opponents' $(N\backslash\{i\})$ actions and partner selections (x_{-i}, b_{-i}) if

$$\pi_i(x_i^*, b_i^*, x_{-i}, b_{-i}) \geq \pi_i(x_i', b_i', x_{-i}, b_{-i}), \tag{3}$$

for any $(x_i', b_i') \in S_i$, where $S_i = X_i \times B_i$, X_i and B_i denote the set of action of player i and the set of partner selection of player i. A *Nash equilibrium* of the networked prisoner's dilemma game $\Gamma = < N, S, \pi, I >$ is a strategy profile $(x^*, b^*) \in S$ (where $S = X \times B$. X and B denote the set of profiles of actions and partner selections) is as follows:

$$\pi_i(x^*, b^*) \geq \pi_i(x_i, b_i, x_{-i}^*, b_{-i}^*), \tag{4}$$

for any $(x_i, b_i) \in S_i$. $\pi = (\pi_1, \dots, \pi_n), I$ is the information set. A network g is an *equilibrium network* if there exists a Nash equilibrium (x^*, b^*) such that $g = g(b^*)$.

The repeated prisoner's dilemma game for T rounds can be represented by $\Gamma^T =< N, S, \pi, I, T >$. The total payoff of player i in the repeated prisoner's dilemma game is as follows:

$$\Pi_i(h^T) = \sum_{t=1}^{T} \pi_i(x^t, b^t). \tag{5}$$

where $\Pi_i(h^T)$ denotes the total payoff of player i. h^T is the sequence of actions and partner selections for T rounds ($h^T = ((x^1, b^1), \dots, (x^T, b^T))$).

We can easily see that there exist multiple Nash equilibrium networks in which all players are defectors in the networked repeated prisoner's dilemma game $\Gamma^T =< N, S, \pi, I, T >$ [3]. However, in the previous experimental studies based on mutual link formation and unilateral like severance, cooperation is achieved in non-empty networks (see Wang, Suri, and Watts 2012; Cuesta et al. 2015; Galo and Yan 2015). Calvó-Armengol and Ilkılıç (2009) also argue that Nash equilibrium is weak to capture the equilibrium of network based on mutual link formation because there are multiple Nash equilibrium networks including the empty network in the network formation game. The empty network is always a Nash equilibrium network in network formation game. Thus, we need another equilibrium concept to explain cooperative actions in the game on a network based on mutual link formation and unilateral link severance.

Jackson and Wolinsky (1996) suggest pairwise stability that captures stability based on mutual link formation and unilaterally removing a link. We apply the strong version of pairwise stability in Jackson and Wolinsky (1996) to explain the cooperative actions of players in the networked repeated prisoner's dilemma game.

[3] The best response for a player is a defection regardless of any given network structure. Defectors get nothing from connections with other defectors because $u_i(D, D) = d = 0$, for all $i \in N$. It implies that defectors add links with other defectors or not. Thus, any feasible network $g \in G$ in which all players are defectors is a Nash equilibrium network in Γ^T.

Definition 1 (Strongly pairwise stability[4]). A network g is *strongly pairwise stable* if the following two conditions are satisfied:

(i) For all $i, j \in N$ where $g_{ij} = 1$, $u_i(g) > u_i(g \ominus ij)$ and $u_j(g) > u_j(g \ominus ij)$,
(ii) For all $i, j \in N$ where $g_{ij} = 0$, if $u_i(g \oplus ij) \geq u_i(g)$, then $u_j(g \oplus ij) \leq u_j(g)$.

where $u_i(g)$ is the utility or the payoff of player i under the network g. $g \ominus ij$ denotes removing a link between players i and j in the network g. $g \oplus ij$ denotes forming a link between players i and j in the network g.

A strongly pairwise network only allows a link that gives a mutual benefit. This strongly pairwise network concept is useful to explain the clustering of the same type of agents or coordination in the repeated game based on mutual consent.

Then, we define a *strongly pairwise-Nash equilibrium network* that are robust to one-link creation based on mutual consent, and to unilateral multi-link severance.

Definition 2 (Strongly pairwise-Nash equilibrium). A network g is a *strongly pairwise-Nash equilibrium* network if and only if there exists a pure strategy Nash equilibrium x^* that satisfies $g = g(x^*)$, and for all $i, j \in N$, if $g_{ij} = 0$, then $u_i(g \oplus ij) \geq u_i(g)$ implies $u_j(g \oplus ij) \leq u_j(g)$.

We can easily prove that there exists a strongly pairwise stable network in which cooperators are completely connected with other cooperators in the networked repeated prisoner's dilemma game, and defectors are isolated (see **Theorem 1** in equilibrium analysis).

We can see that there exists the trigger strategy based on unilateral multi-link severance. Several studies have supported the existence of trigger strategies in the network game (see Ule 2008; Wolitzky 2013; Cho 2014). Let us see an example in my game. Suppose that player i defected at the round t^*. Then, player i is removed from the subnetwork g^* in which cooperators are completely connected from the round $t^* + 2$ to the round T by a unilateral exclusion. Other players cooperate with others in the subnetwork g^i in which all links between player i are removed. Player i earns zero ($(T - t^* - 1) \cdot 0 = 0$) payoffs in the subnetwork g^i for the last $T - t^* - 1$ rounds due to the exclusion from g^*. Threats of unilateral exclusion are credible. Thus, by threats of unilateral exclusion of defectors, players do not want to deviate from cooperation. We define an *excluding trigger strategy* based on threats of unilateral exclusion of defectors.

---

[4] The definition of original pairwise stability in Jackson and Wolinsky (1996) is as follows: (i) For all $i, j \in N$ where $g_{ij} = 1$, $u_i(g) \geq u_i(g \ominus ij)$ and $u_j(g) \geq u_j(g \ominus ij)$; (ii) For all $i, j \in N$ where $g_{ij} = 0$, if $u_i(g \oplus ij) > u_i(g)$, then $u_j(g \oplus ij) < u_j(g)$. I use the strong version of pairwise stability to remove the meaningless links among defectors, which do not give defectors mutual benefit in the pairwise stable network.

Definition 3 (Excluding trigger strategy). A strategy profile σ is an *excluding trigger strategy profile* for player i if there exist $(x^t, g(b^t)) = \left((x_1^t, g(b_1^t)), \ldots, (x_n^t, g(b_n^t))\right)$ such that

$$\rho\left(g, g^*, \{g^i\}_{i \in N}, g^0, T, t^*\right) \equiv \sigma\left((\boldsymbol{C}, g), (\boldsymbol{C}', g^*), \{(\boldsymbol{C}'', g^i)\}_{i \in N}, (\boldsymbol{D}, g^0), T, t^*\right), \quad (6)$$

where $g^i = g^* \ominus 1i \ominus \ldots \ominus ni$, and $t_c < t^* < T$. g and g^* are the subnetwork in which all cooperators are completely connected. g^0 is an empty network. (1) $(x^t, g(b^t)) = (\boldsymbol{C}, g)$ for $t \leq t^*$; (2) $(x^t, g(b^t)) = (\boldsymbol{C}', g^*)$ for $t = t^* + 1$; (3) $(x^t, g(b^t)) = \{(\boldsymbol{C}'', g^i)\}_{i \in N}$ for $t^* + 1 < t < T$; (4) $(x^t, g(b^t)) = (\boldsymbol{D}, g^0)$ for $t = T$, $g^0, g, g^*, g^1, \ldots, g^n \in G$, where G is the set of feasible networks in the networked repeated prisoner's dilemma game $\Gamma^T = < N, S, \pi, I, T >$.

(i) $x_i^t = C$, for $t \leq t^*$, $x_i^t = D$ for $t = t^* + 1$, and $x_i^t = C$ or D for $t^* + 1 < t \leq T$.

(ii) $\boldsymbol{C}$ is a strategy profile such that $x_j = C$ for all $j \in N$. $\boldsymbol{C}'$ is a strategy profile such that $x_j = C$ for $j \in N \backslash \{i\}$ and $x_i = D$. $\boldsymbol{C}''$ is a strategy profile such that $x_j = C$ for $j \in N \backslash \{i\}$ and $x_i = C$ or $x_i = D$. $\boldsymbol{D}$ is a strategy profile such that $x_j = D$ for all $j \in N$

Also, other strategies weaker than the trigger strategy are possible using network formation in the game. However, other strategies may also be based on threats of unilateral exclusion of defectors. For example, some players can change their previous defective actions to cooperative actions by the exclusion in the partner selection, and they might be selected as partners in the next partner selection stage. This strategy is similar to tit-for-tat using the partner selection, but it is also based on threats of exclusion of defectors. Thus, we focus on the trigger strategy which is less complicated than other strategies to see the effect of threats of excluding defectors on cooperation.

We define a subgame perfect strongly pairwise-Nash equilibrium in the finitely repeated prisoner's dilemma game on a network[5] using the similar way in Friedman (1985): (i) all players cooperate, and they are completely connected in the network during all initial rounds; (ii) some players sequentially or simultaneously start to defect from early rounds to the final rounds, and defectors are being excluded in the network; (iii) all players defect for final rounds, and all players are isolated in the network. All networks are strongly pairwise stable in the subgame. In the following, we do

[5] Friedman (1985) introduces a *discriminating trigger strategy* based on player-specific punishment, not considering the network structure among players in the finitely repeated game. Friedman (1985) shows that there exists the subgame perfect non-cooperative equilibrium in which cooperative outcome can be achieved by discriminating defectors using the discriminating trigger strategy. An excluding trigger strategy is also player-specific based on the partner selection process in the finitely repeated prisoner's dilemma game. Several studies show that cooperative outcome can be achieved in the initial periods in the subgame in the finitely repeated game due to trigger strategies or ostracism (see Friedman 1985; Hirshleifer and Rasmusen 1989; Ule 2008). Thus, we define the subgame perfect strongly pairwise-Nash equilibrium using the similar way as the subgame perfect equilibrium in Friedman (1985) by achieving cooperative outcome in the initial periods.

equilibrium analysis based on strongly pairwise stability and strongly pairwise-Nash equilibrium.

3. Equilibrium analysis

Theorem 1. A strongly pairwise stable network exists in which all cooperators are completely connected to other cooperators, and defectors are isolated in the repeated prisoner's dilemma game $\Gamma^T = < N, S, \pi, I, T >$.

Proof. Suppose all cooperators are completely connected in the network g, and defectors are isolated. To prove **Proposition 1**, we have to see that the following three cases of link formations satisfy strongly pairwise stability: (1) between two cooperators; (2) between a defector and a cooperator; (3) between two defectors.

Let us see the first case. If the number of cooperators in the network g is $N_c (N_c \leq n)$, then the number of neighbors of a cooperator in the network g is $(N_c - 1)$. Thus, each cooperator in the network g will get $(N_c - 1)c$. If cooperator i in the network g severs a link from cooperator j in the network g, then the payoff of player i or j will be $(N_c - 2)c$. Thus, $u_i(g) = (N_c - 1)c > (N_c - 2)c = u_i(g - ij)$ and $u_j(g) = (N_c - 1)c > (N_c - 2)c = u_j(g - ij)$ for all ij where $g_{ij} = 1$.

Let us see the second case. Suppose player j is a cooperator, and player i is a defector in the network g. Thus, $g_{ij} = 0$. If they form a link between them, the payoff of player j is $(N_c - 1)c + e$, and the payoff of player i is f. The payoff of player j is reduced from $(N_c - 1)c$ to $(N_c - 1)c + e$ because e is negative. However, the payoff of player i increases from 0 to f. Thus, player j in the network g will not add a link with player i outside the network g. It implies that for all ij where $g_{ij} = 0$, if $u_i(g + ij) = f \geq 0 = u_i(g)$, then $u_j(g + ij) = (N_c - 1)c \leq (N_c - 1)c + e = u_j(g)$, if player j is a cooperator, and player i is a defector in the network g.

Let us see the third case. Suppose player j is a defector in the network and player i is a defector in the network g. Thus, $g_{ij} = 0$. If they form a link between them, the payoff of player j is d, and the payoff of player i is d. Thus, $u_i(g + ij) = d \geq 0 = u_i(g)$, $u_j(g + ij) = d \leq 0 = u_j(g)$ ($\because d = 0$). It implies that for all ij where $g_{ij} = 0$, if $u_i(g + ij) = d \geq 0 = u_i(g)$ then $u_j(g + ij) = d \leq 0 = u_j(g)$, if players i and j are defectors in the network g.

Therefore, the network g in which all cooperators are connected to other cooperators, and defectors are isolated satisfies (i) and (ii) in **Definition 1** to be strongly pairwise stable. *Q.E.D.*

Theorem 1 implies "full separation" of cooperators and defectors in the network and "marginalization" of defectors in the network. These are consistent with the results in Fosco and Mengel (2011). Fosco and Mengel (2011) focus on the imitation strategy of neighbors and the exclusion of defectors in the networked prisoner's dilemma game. In our game, by the exclusion trigger strategy, there exists the subgame perfect strongly pairwise-Nash equilibrium in which cooperators are completely connected, and defectors are isolated for finite times.

Theorem 2 Consider the repeated prisoner's dilemma game $\Gamma^T =< N, S, \pi, I, T >$ such that there exists a $g^* \in G$. G is the set of feasible networks. For sufficiently large T, there exists a positive integer $t^* \in (t_c, T-\gamma)$, $0 < \gamma < T - t_c$, such that the excluding trigger strategy profile $\rho\left(g^*, g^*, \{g^i\}_{i\in N}, g^0, T, t^*\right)$ for Γ^T that creates the subgame perfect strongly pairwise-Nash equilibrium subnetwork g^* in which cooperators are completely connected: (i) all players cooperate, and they are completely connected in the subnetwork g^* in the early rounds $t_c + 1, \dots, t^*$; (ii) some players in the subnetwork g^* sequentially or simultaneously start to defect from t^* to the final round, and defectors are excluded in the subnetwork g^*; (iii) all players defect in the final round.

Proof. Suppose all players are taking the excluding trigger strategy. Then, all players are cooperators in the subnetwork g^*. Then, g^* is a strongly pairwise stable network in which all players are cooperators, and cooperators are completely connected with other cooperators. I assume player i cooperates in the early rounds $t_c + 1, \dots, t^*$ and defect at the round $t^* + 1$. From the round $t^* + 2$ to the round T, player i will be excluded in the subnetwork g^* due to her defection. Thus, the subnetwork g^* is changed to g^i from the round $t^* + 2$. In the final round T, all players defect, and there exists the empty network g^0, which is also a strongly pairwise-Nash equilibrium network. Let Π_i^c be the sum of the payoff of player i if player i does not deviate from cooperation to round $T - 1$ and let Π_i^d be the sum of payoff of player i if player i deviates from cooperation at the round $t^* + 1$. Then, $\Pi_i^c = (T - t_c)(n-1)c + t_c \cdot c$ and $\Pi_i^d = (t^* - t_c)(n - 1)c + t_c \cdot c + (n-1)f + (T - t^* - 1) \cdot 0 = (t^* - t_c)(n-1)c + t_c \cdot c + (n-1)f$.

Additionally, $\Pi_i^c > \Pi_i^d$ if $t_c < t^* < T - \frac{f}{c} < T$ ($\because c < f$). It implies that there exists $t^* \in (t_c, T - \gamma)$, $\gamma = \frac{f}{c}$ such that players do not deviate from cooperation. Thus, all players in the subnetwork g^* cooperate, and g^* is a strongly pairwise stable network in which cooperators are completely connected for $t^* \in (t_c, T - \frac{f}{c})$ rounds by the excluding trigger strategy $\rho\left(g^*, g^*, \{g^i\}_{i\in N}, g^0, T, t^*\right)$.

It is straightforward to prove that the excluding trigger strategy equilibrium is the subgame perfect strongly pairwise-Nash equilibrium. For the subgame from round t' to round $T'(t' > t_c, T' < T)$, there exist $t'^* \in (t', T' - \gamma)$ such that players do not deviate from cooperation using the same approach in the above. Also, all players defect in the final rounds T' of all subgames, and there exist empty strongly pairwise-Nash equilibrium networks. It implies that the excluding trigger strategy applied from any $t' \in (t_c, T - 1)$ onward forms a strongly pairwise-Nash equilibrium point for the subgame beginning at round t' with the available information set. *Q.E.D.*

Theorem 2 implies that cooperation can be achieved in the subgame perfect strongly pairwise stable network in which cooperators are completely connected for the finite time t^*. **Theorem 2** is also consistent with the result of Friedman (1985). Friedman (1985) shows that γ depends on the payoff structure in the finitely repeated game. In general, γ depends on the number of links as well as the payoff structure in the networked game. In our game, the network formation is costless. Thus, all players can propose as many links as possible, and it results in only dependence on the payoff structure in γ.

4. Conclusion and discussion

In this paper, we suggest the game-theoretical model explaining the relationship between cooperation and network formation in the finitely repeated prisoner's dilemma. The model consists of two repeated prisoner's dilemma games: the simple repeated two-person prisoner's dilemma game and the repeated n-person game on a network. From the 1st to round t_c, players play the simple repeated prisoner's dilemma game. From round t_c+1 to the final round T, players play the repeated prisoner's dilemma game on a network. In the simple repeated game, players play the prisoner's dilemma game with their partner randomly matched in each round. In the repeated network game, players can select their partners using others' reputation scores, histories of others' actions for the past t_c rounds, and histories of network information before playing the prisoner's dilemma game in each round. Players can exclude defectors unilaterally using the partner selection, and cooperation is achieved in the repeated network game by an excluding trigger strategy. By threats of unilateral exclusion of defectors in the network, there exists the subgame-perfect strongly pairwise-Nash equilibrium in which cooperation is achieved. In the subgame-perfect strongly pairwise-Nash equilibrium network, cooperators are completely connected with other cooperators and defectors are marginalized.

Our study has room for development. The effect of network structure on cooperation in the repeated prisoner's dilemma game can be considered in the next model. The network structure is endogenously determined by the partner selection in the model. After network formation, players' actions are impacted by the network structure. In particular, to measure the effect of network structure in a dynamic network setting, we need to consider the stochastic process of network formation. Several models have been suggested to explain the relationship between cooperation and network structure in the repeated prisoner's dilemma game based on stochastic process (see Lieberman, Hauert, and Nowak 2005; Ohtsuki et al. 2006; Allen, Lippner, and Nowak 2019; Fotouhi et al. 2019; Alvarez-Rodriguez et al. 2021). We will consider the stochastic process of network formation using the previous models in the next model.

Acknowledgments

We thank Dr. Li (Department of Economics, Iowa State University, Ames) for her valuable comments. We have improved our paper due to her comments.

References

Allen, B., Lippner, G., and Nowak, M. A., 2019. Evolutionary games on isothermal graphs. *Nature communications*, *10*(1), 1-9.

Alvarez-Rodriguez, U., Battiston, F., de Arruda, G. F., Moreno, Y., Perc, M., and Latora, V., 2021. Evolutionary dynamics of higher-order interactions in social networks. *Nature Human Behaviour*, 1-10.

Calvó-Armengol, A., and Ilkılıç, R., 2009. Pairwise-stability and Nash equilibria in network formation. *International Journal of Game Theory*, *38*(1), 51-79.

Carvalho, V. M., and Tahbaz-Salehi, A., 2019. Production networks: A primer. *Annual Review of Economics*, *11*, 635-663.

Cho, M., 2014. Cooperation in the repeated prisoner's dilemma game with local interaction and local communication. *International Journal of Economic Theory*, *10*(3), 235-262.

Cuesta, J.A., Gracia-Lázaro, C., Ferrer, A., Moreno, Y. and Sánchez, A., 2015. Reputation drives cooperative behaviour and network formation in human groups. *Scientific reports*, *5*(1), pp.1-6.

Embrey, M., Fréchette, G. R., and Yuksel, S., 2018. Cooperation in the finitely repeated prisoner's dilemma. *The Quarterly Journal of Economics*, *133*(1), 509-551.

Fosco, C., and Mengel, F., 2011. Cooperation through imitation and exclusion in networks. *Journal of Economic Dynamics and Control*, *35*(5), 641-658.

Fotouhi, B., Momeni, N., Allen, B., and Nowak, M. A., 2019. Evolution of cooperation on large networks with community structure. *Journal of the Royal Society Interface*, *16*(152), 20180677.

Friedman, J. W., 1985. Cooperative equilibria in finite horizon noncooperative supergames. *Journal of Economic Theory*, *35*(2), 390-398.

Gallo, E., and Yan, C., 2015. The effects of reputational and social knowledge on cooperation. *Proceedings of the National Academy of Sciences*, *112*(12), 3647-3652.

Goyal, S., 2012. *Connections: an introduction to the economics of networks*. Princeton University Press.

Hirshleifer, D., and Rasmusen, E.,1989. Cooperation in a repeated prisoners' dilemma with ostracism. *Journal of Economic Behavior & Organization*, *12*(1), 87-106.

Hoffman, E., McCabe, K., Shachat, K., and Smith, V., 1994. Preferences, property rights, and anonymity in bargaining games. *Games and Economic behavior*, *7*(3), 346-380.

Hoffman, E., McCabe, K., and Smith, V. L., 1996a. Social distance and other-regarding behavior in dictator games. *The American economic review*, *86*(3), 653-660.

Hoffman, E., McCabe, K. A., and Smith, V. L., 1996b. On expectations and the monetary stakes in ultimatum games. *International Journal of Game Theory*, *25*(3), 289-301.

Jackson, M. O., 2014. Networks in the understanding of economic behaviors. *Journal of Economic Perspectives*, *28*(4), 3-22.

Jackson, M. O., and Wolinsky, A., 1996. A strategic model of social and economic networks. *Journal of economic theory*, *71*(1), 44-74.

Lieberman, E., Hauert, C., and Nowak, M. A., 2005. Evolutionary dynamics on graphs. *Nature*, *433*(7023), 312-316.

Mailath, G. J., and Samuelson, L., 2006. *Repeated games and reputations: long-run relationships*. Oxford university press.

Mengel, F., 2009. Conformism and cooperation in a local interaction model. *Journal of Evolutionary Economics*, *19*(3), 397-415.

Nowak, M.A., 2006. Five rules for the evolution of cooperation. *Science*, *314*(5805), pp.1560-1563.

Ohtsuki, H., Hauert, C., Lieberman, E., and Nowak, M. A., 2006. A simple rule for the evolution of cooperation on graphs and social networks. *Nature*, *441*(7092), 502-505.

Okada, A., 1993. The possibility of cooperation in an n-person prisoners' dilemma with institutional arrangements. *Public Choice*, *77*(3), 629-656.

Suri, S. and Watts, D.J., 2011. Cooperation and contagion in web-based, networked public goods experiments. *ACM SIGecom Exchanges*, *10*(2), pp.3-8.

Ule, A., 2008. *Partner choice and cooperation in networks: Theory and experimental evidence* (Vol. 598). Springer Science & Business Media.

Wang, J., Suri, S. and Watts, D.J., 2012. Cooperation and assortativity with dynamic partner updating. *Proceedings of the National Academy of Sciences*, *109*(36), pp.14363-14368.

Wolitzky, A., 2013. Cooperation with network monitoring. *Review of Economic Studies*, *80*(1), 395-427.

Wolitzky, A., 2015. Communication with tokens in repeated games on networks. *Theoretical Economics*, *10*(1), 67-101.