Renegotiation of Long-Term Contracts as Part of an Implicit Agreement

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Abstract

Long-term relationships are often governed by a combination of contracts and implicit agreements. I show that there are welfare gains to writing long-term contracts with the intention of rewriting their terms at a later stage, despite lack of change in the underlying environment. This form of renegotiation is perfectly anticipated as part of an implicit agreement. The benefits of renegotiation are demonstrated in a principal-agent model with observable but noncontractible effort where the players sign long-term output-contingent contracts. Continuation contracts form a basis for punishing deviations from the implicit agreement, but their terms are renegotiated away on the equilibrium path. In the baseline model continuation contracts are designed to lead to unbounded punishments to the principal who is not protected by limited liability. This facilitates the implementation of first best outcomes regardless of the patience of the players and the output technology. In contrast, the first best is not attainable in equilibria without on-path renegotiation when the players are impatient. When the principal is protected by limited liability, continuation contracts can hold either player down to their outside option in equilibrium.

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1 Introduction

Long-term relationships are rarely governed by complete contracts, partly because some outcomes cannot be verified in court. It can be difficult to prove that a worker is not exerting enough effort or that a borrower is investing a loan in undesirable projects. Nevertheless, these aspects of behaviour are often (at least partially and subjectively) observable within the relationship and they can be supported by implicit agreements which use threats and promises of future behaviour. The relational contracts literature studies implicit agreements as equilibria of a repeated game where parties take noncontractible actions in the presence of contracts specifying terms for the current period only.\textsuperscript{12} This paper explores the role of implicit agreements in a setting with long-term contracts. In this richer contracting environment the act of updating the terms of a previously signed contract can become part of an implicit agreement. I show that this component of implicit agreements has important consequences for welfare. To support noncontractible actions, parties design contracts with future terms meant to punish deviations. On the equilibrium path, the terms of these continuation contracts are amended once the correct actions are observed to have been taken. From a legal standpoint the parties choose to renegotiate the continuation contracts, but this renegotiation is perfectly anticipated as part of an implicit agreement. Such agreements prescribing renegotiation of long-term contracts are necessary to achieve efficient outcomes.

The power of renegotiating long-term contracts as part of an implicit agreement is demonstrated in a stylised repeated principal-agent model. A risk-neutral principal hires a risk-averse agent with unbounded utility, whose effort stochastically determines output. The parties can write long-term contracts specifying wages contingent on the entire history of output, but not on effort. At the beginning of each period the principal can make a contract offer. The agent can accept, reject in favour of the existing contract, or take an outside option. Thus, the output contracts commit the principal to pay certain wages only insofar as they are...
not renegotiated in the future. Effort is observable and can be rewarded with a
discretionary bonus paid by the principal. Since effort is noncontractible the bonus
is enforced by punishing the principal with an unfavourable equilibrium following
noncompliance. The model builds on Pearce and Stacchetti (1998) where output
contracts specify wages for the current period only.

The main result of the paper is that forming implicit agreements to renegotiate
long-term contracts is welfare-improving, even if the underlying environment does
not change over time. This is demonstrated formally in the model as the combi-
nation of two results: Theorem 1 shows that first best equilibria are attainable
for any parameters of the model and Proposition 2 shows that when the players
are not too patient, first best outcomes cannot be attained in equilibria where
renegotiation does not occur on the equilibrium path.

To illustrate the benefits of writing contracts with the intention of renegotiat-
ing them, consider the agent’s incentives to exert effort. A complete contract is
contingent on effort so it can directly punish the agent’s deviation by specifying
low compensation in subsequent periods. However, in the setting of this paper
contracts are incomplete and the agent can claim the same future wages regardless
of his choice of effort, as his deviation does not affect the continuation contract.
Hence, the agent can only be punished through unfavourable continuation play.
Formally, the multiplicity of equilibria of the subgame at the beginning of a period
starting from the continuation contract creates rewards and punishments for the
agent’s effort choice observed in the previous period. The principal’s voluntary
bonus payment is supported in the same manner.

It follows that a desirable continuation contract supports incentives by admit-
ting harsh punishment equilibria and efficient reward equilibria for each player.
However, the terms of the continuation contract need not be realised in any of
these equilibria, as the agent may accept a new contract. Thus, despite having
access to fully contingent output contracts, planning not to replace their terms
over the course of the relationship is suboptimal – it constrains reward equilib-
ria by mandating that the agent does not accept a contract offer with different
terms. Instead, the players can benefit from signing contracts whose continuation
terms, if realised, would not be conducive to efficiency, but, as a starting point for
renegotiation, could give rise to harsh punishment equilibria. These contracts are
supplemented with the following implicit agreement: If a deviation is observed, the punishment equilibrium corresponding to the deviator is played. If no deviation is observed, efficiency is attained by altering the terms of the continuation contract through a new contract offer by the principal accepted by the agent in equilibrium.

These implicit agreements can achieve a range of first best outcomes, as shown in Theorem 1. First best equilibria use a continuation contract $c^*$ whose wages vary significantly across output levels. The key intermediate result is that the subgame at the start of a period with initial contract $c^*$ admits harsh punishment equilibria for either player. Proposition 1 characterises the agent’s worst equilibrium payoff as the utility the agent can guarantee from the terms of the contract in the absence of bonuses and renegotiation. Hence, the agent’s risk aversion implies that he can be punished severely with the unbalanced contract $c^*$. In the principal’s punishment equilibrium, the agent expects a lot of utility if he rejects in favour of the current contract $c^*$. This is possible because the agent receives a bonus supplementing the smallest contemporaneous wage in $c^*$. When $c^*$ is sufficiently unbalanced, the unboundedness of the agent’s utility implies that the marginal utility of the bonus can be made arbitrarily high. Thus, a sufficiently skewed contract $c^*$ can admit equilibria which punish either player with arbitrary severity, holding the agent down to his outside option and providing an unbounded punishment to the principal.

The existence of first best equilibria follows from the severity of the punishments facilitated by $c^*$. Each period the players sign a contract with no immediate wages and an output-invariant continuation contract $c^*$. The severity of the punishments facilitated by $c^*$ provides incentives to the agent to exert effort and to the principal to pay bonuses upon observing the equilibrium effort. On the equilibrium path, the agent is fully insured and his compensation consists entirely of bonuses. Once the equilibrium effort and bonus have been observed, the parties renegotiate $c^*$ to the contract they signed in the previous period.

The unboundedness of the principal’s punishment delivers two striking features. First best outcomes are attainable in equilibrium regardless of the patience of the players and the output technology. Moreover, two-period contracts are sufficient to attain the first best. In contrast, the first best is not generally attainable in the one-period contract setting of Pearce and Stacchetti (1998). The difference
lies in the ability to structure future salaries for the purposes of punishment, knowing that they will not be realised in equilibrium. Indeed, Proposition 2 shows that replacing contractual terms on the equilibrium path is necessary to attain first best outcomes when the players are not too patient. Hence, in a large set of environments implicit agreements to renegotiate contracts improve welfare, even though the parties have access to long-term history-dependent output contracts and self-enforcing payments.

Renegotiation is not triggered by changes in the underlying environment: The actions taken and observed by the players become payoff-irrelevant at the beginning of the next period, when they get an opportunity to amend the contract. This contrasts with other models of equilibrium renegotiation where actions taken between the signing and renegotiation of contracts affect the value of the contract to each party.³

It is important to note that first best outcomes are not supported by threats of inefficiency. Proposition 3 shows that the first best can be supported by strongly optimal equilibria in the sense of Levin (2003), which preclude inefficient continuation play commencing at the start of a subsequent period.

I also explore an extension of the model which allows the principal to shut down the firm. While the first best outcome is not achievable in general, as the punishment for the principal is bounded by his outside option, renegotiation can still be useful. Theorem 2 shows the existence of a continuation contract where both players can be held down to their outside options in their respective worst punishment equilibrium. Hence, it suffices that every period they renegotiate to a contract with the same continuation contracts, i.e. a single continuation contract provides the harshest punishments and is rolled over every period. Apart from improving upon one-period contracts, this simplifies the model as the threats are exogenously given by the outside options, compared to Pearce and Stacchetti (1998) where they are endogenously determined as the worst equilibria of the supergame.

The only other paper to consider long-term contract renegotiation as part of an implicit agreement is the independent and simultaneous work of Miller, Olsen, and Watson (2018). They study a general environment with risk neutral players who negotiate jointly over long-term contracts governing enforceable aspects

³See the literature review in Section 5 for more details.
of the game, transfers, and strategies specifying nonverifiable actions. The outcome of this bargaining is a contractual equilibrium where the players maximise their joint surplus and split it according to their exogenous bargaining power relative to a disagreement equilibrium where the contract is not renegotiated in the current period and continuation play is efficient.\footnote{Miller and Watson (2013) provide a noncooperative foundation for contractual equilibrium using various refinements of Perfect Bayesian equilibrium.} Despite these differences, the central idea behind the equilibria studied by Miller, Olsen, and Watson (2018) is the same as the force behind the equilibria constructed in this paper: players set up unfavourable continuation contracts meant to punish deviations outside the scope of the formal contract and anticipate to renegotiate them on the equilibrium path. Miller, Olsen, and Watson (2018) stress the semi-stationary structure of contractual equilibria: the same long-term contract is negotiated every period and it specifies the same terms for all periods but the initial one. These features are present in both the baseline and extended version of my model, as evidenced in Remark 1 and Remark 3. Hence, Miller, Olsen, and Watson (2018) and my paper are complementary: Despite the differences in risk attitudes and the solution concept, the act of renegotiation as part of an implicit agreement and the structure of optimal long-term contracts remain robust.

I proceed by presenting the baseline model in Section 2 and analysing it in Section 3. The results of the extended model where the principal has limited liability are reported in Section 4. I review the literature in Section 5. The paper concludes with a short discussion.
2 The model

2.1 Preferences and timing

A principal and an agent interact repeatedly at times $t = 1, 2, ...$ They sign long-term contracts on output, which takes values in $Y = \{h, l\}$ every period, where $h > l \geq 0$. A contract is a sequence $c = (c_t)_{t=1}^{\infty}$ of functions specifying salaries for the agent contingent on the output history in previous periods and the current realisation of output. Formally, $c_t : Y^t \rightarrow [0, \bar{s}]$ where $Y^t$ denotes the $t$-fold Cartesian product of $Y$ and $\bar{s}$ is an exogenous upper bound needed to rule out Ponzi schemes.\(^5\)

**Notation 1.** Given a contract $c$, let $s_y = c_1(y)$ denote the contemporaneous salary for output $y$ and $c_y$ denote the continuation contract following output $y$, i.e.

$$c_{y,t}(y_1, ..., y_t) = c_{t+1}(y, y_1, ..., y_t) \quad \text{for all } t \in \mathbb{N}, (y_1, ..., y_t) \in Y^t.$$

Each period the principal and the agent play a sequential game of perfect information. At the beginning of each period there is a residual contract $c^*$ representing the remaining terms of the last contract the parties signed. In the initial period, the residual contract is $0$, where all salaries equal $0$. The period begins with a *contract offer* $c$ by the principal. The agent’s *contract response* is to accept, reject, or take an outside option. Accepting replaces the terms of the residual contract $c^*$ with the offer $c$. Rejection maintains the terms of $c^*$ for this period. If the agent takes the outside option, the relationship is terminated – the agent works for an outside employer at a constant wage $r \geq 0$ and the principal receives $0$ in all subsequent periods.\(^6\)

If the agent does not take the outside option, the game continues with his choice of effort $e \in E = \{\text{work}, \text{shirk}\}$. The agent incurs a cost of effort $\psi(e)$ where $\psi(\text{work}) > \psi(\text{shirk}) = 0$. Effort stochastically generates output: The probability of output $y$ given effort $e$ is $p_{ye}^e > 0$ where $p_{he}^e + p_{le}^e = 1$. In addition, $p_{he}^\text{work} > p_{he}^\text{shirk}$

\(^5\)See Appendix 7.1 for details.

\(^6\)The results would remain unchanged if the players received their outside options for one period and restarted the repeated game at residual contract $0$. 


so working is more productive than shirking. The agent is paid a wage from the prevailing contract, amounting to $s_y$ if he accepted the offer $c$ or $s^*_y$ if he rejected. After observing the contract response, effort and output, the principal makes a voluntary bonus payment $b \geq 0$ to the agent. The bonus is not contractible – it can only be enforced in equilibrium as part of an implicit agreement.

The timing of the component game is depicted in Figure 1. Solid horizontal lines connecting the nodes indicate a continuous choice of contracts and bonuses. Residual contracts in the current and following period are indicated at the beginning and the end of the tree.

The expected payoffs in a period where contract $c$ is effected, the agent exerts

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7 Binary effort and output are assumed for simplicity. The same results holds with multiple levels of effort and output as long as each effort level has full support over the realisations of output.
effort $e$ and the principal pays bonus $b_y$ upon observing effort $e$ and output $y$ are

$$\text{Agent: } \sum_{y \in Y} p_y e u(s_y + b_y) - \psi(e)$$

$$\text{Principal: } \sum_{y \in Y} p_y e (y - s_y - b_y)$$

where $u : \mathbb{R}_+ \rightarrow \mathbb{R} \cup \{-\infty\}$ is concave and strictly increasing with $u(0) = -\infty$ and $\lim_{s \to \infty} u(s) = \infty$.

Players discount future periods using the same discount factor $\delta \in (0, 1)$.

2.2 Equilibrium

Formally, the principal and the agent play a stochastic game with complete and perfect information where the residual contract acts as a state variable. I consider the subgame perfect equilibria of this game.

Notation 2. It will be useful to develop notation for subgames at various stages of the component game.

- The subgame at the beginning of a period with residual contract $c^*$ will be denoted “subgame $c^*$”.

- The subgames following the agent accepting or rejecting a contract offer $c$ when the residual contract in the current period is $c^*$ will be denoted “subgame $(c^*, c, A)$” and “subgame $(c^*, c, R)$”.

The equilibrium set of a subgame following the agent’s contract response depends only on the contract that is effected, since that contract is the only payoff relevant variable for the remainder of the game. In particular, the equilibrium set of subgame $(c^*, c, A)$ is the same as the equilibrium set of subgame $(c, \hat{c}, R)$ for any $c, c^*, \hat{c}$. 

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3 Results

3.1 Agent’s worst equilibrium

Let $u(c^*)$ be the worst feasible payoff for the agent in subgame $c^*$ if he never accepts another offer by the principal. That is, in all subsequent periods the agent chooses optimally between taking his outside option and rejecting in order to collect wages from the appropriate continuation of $c^*$. If he rejects he chooses effort optimally today knowing that following output realisation $y$ he will face the same problem tomorrow with residual contract $c^*_y$. This leads to the following recursive definition of $u(c^*)$:

$$u(c^*) = \max \{ U(c^*), u(r) \}$$

where

$$U(c^*) = \max_e \sum_y p_y^e \left( (1 - \delta) u(s_y^*) + \delta u(c^*_y) \right) - (1 - \delta) \psi(e).$$

Clearly, $u(c^*)$ and $U(c^*)$ are lower bounds on the agent’s equilibrium payoff in subgames $c^*$ and $(c^*, c^*, R)$, respectively. Proposition 1 below states that these bound are attained in equilibrium.

**Proposition 1.** Given any $c, c^* \in C^{T-1}$, the agent’s worst equilibrium payoffs in subgames $c^*$ and $(c^*, c, A)$ are $u(c^*)$ and $U(c)$, respectively.

The proof of Proposition 1, deferred to Appendix 7.2, constructs an equilibrium where the principal pays no bonuses by making the strategies independent of bonus payments. The agent expects utility $U(c)$ in any subgame where he accepts a contract offer $c$. Given the agent’s strategy, the principal maximises his payoff by offering a contract $c$ which makes the agent indifferent between accepting and the payoff $u(\hat{c})$ that the agent can guarantee under the current residual contract $\hat{c}$. The assumption $u(0) = -\infty$ ensures that such a contract $c$ exists, as the principal can decrease wages without violating the agent’s incentives until $U(c) = u(\hat{c})$. Hence, the agent is held down to the payoff he can guarantee by refusing the interact with the principal.

The principal’s payoff in the equilibrium of subgame $c^*$ constructed in Proposition 1 is denoted by $f(c^*)$. Since the equilibrium depends only on the payoff the agent can guarantee, the following corollary is immediate.
Corollary 1. Let \( c^*, \hat{c} \in C^{T-1} \). If \( u(c^*) = u(\hat{c}) \), then \( f(c^*) = f(\hat{c}) \).

3.2 First best outcomes and their unattainability without renegotiation

There is a frontier of first best outcomes – one for each agent utility level \( x \geq u(r) \). I assume that the first best does not involve the agent taking his outside option. Hence, the first best outcome associated with any utility level \( x \) insures the agent fully and exhibits an optimal effort level to be exerted each period. The principal’s associated first best payoff is given by

\[
v_{FB}(x) = \max \left\{ \sum_y p_y^{\text{work}} y - u^{-1}(x + \psi(\text{work})), \sum_y p_y^{\text{shirk}} y - u^{-1}(x) \right\}.
\]

Note that, even though “work” creates more expected output than “shirk”, the curvature of the agent’s utility function may make “shirk” the first best effort at high levels of agent utility.

Naturally, when players are patient enough, a folk theorem applies and first best outcomes can be achieved through bonus payments alone. Formal contracts reduce the self-enforcing part of the payment so they can help sustain first best outcomes at lower discount factors. However, their effectiveness is limited if they are not renegotiated in equilibrium: Proposition 2 below shows that equilibria without on-path renegotiation cannot attain the first best when the players are not too patient.

Definition 1. An equilibrium without renegotiation exhibits no acceptance of contracts different from the residual contract on the equilibrium path, except in the initial period.

Proposition 2. Fix any primitives of the model except for \( \delta \). Fix a utility level for the agent \( x \in [u(r), u(\bar{s})] \) and suppose the effort in the first best outcome that delivers \( x \) to the agent is “work”. Then there exists \( \hat{\delta} > 0 \) such that the first best is not attainable in any equilibrium without renegotiation whenever \( \delta \leq \hat{\delta} \).

Proof. Consider an equilibrium without renegotiation which implements the first best outcome with utility \( x \) for the agent. Without loss of generality, a contract \( c \)
is accepted in the initial period and is not renegotiated in any subsequent period on the equilibrium path. Let \( c(y^t) \) and \( b(y^t) \) denote the salary and the bonus the agent receives after history \( y^t \in Y^{t+1} \). The agent’s compensation must be exactly \( u^{-1}(x + \psi(\text{work})) \) following any history of output, so

\[
c(y^t) + b(y^t) = u^{-1}(x + \psi(\text{work})) \quad (1)
\]

for any \( y^t \). Since the agent’s on-path payoff in any subgame is \( x \), every continuation contract \( c_{y^t} \) has \( u(c_{y^t}) \leq x \). Hence, there exists a history \( y^t \) at which the salary is at most \( u^{-1}(x) \). Thus, \( b(y^t) \geq u^{-1}(x + \psi(\text{work})) - u^{-1}(x) \) so the principal’s incentive constraint to pay the bonus necessitates

\[
u^{-1}(x + \psi(\text{work})) - u^{-1}(x) \leq \frac{\delta}{1 - \delta} (v_{FB}^{\text{FB}}(x) - v') \quad (2)
\]

where \( v_{FB}^{\text{FB}}(x) \) is the on-path continuation payoff for the principal and \( v' \) is his payoff from a punishment equilibrium that follows his reneging on the bonus. The goal is to show there is a lower bound on \( v' \) independent of \( \delta \). Then a contradiction follows from (2) when \( \delta \) is small enough.

To obtain the bound, notice that at any history, the principal can play the following strategy: always offer the contract with constant salary \( \min\{\bar{s}, u^{-1}(x + \psi(\text{work}))\} \) and do not pay any bonus. What is the worst payoff the principal can get given this strategy? If the agent takes his outside option, the principal’s payoff is 0. If the agent accepts or rejects, the principal will pay at most \( u^{-1}(x + \psi(\text{work})) \) every period and will receive at least the expected output from shirking. This follows from (1) and the nonnegativity of the bonus. Thus,

\[
v' \geq -u^{-1}(x + \psi(\text{work})) + \sum_y p_{y}^{\text{shirk}} y
\]

which gives the desired lower bound. \( \square \)
3.3 Achieving the first best

The main result of the paper, Theorem 1 below, is that a portion of the first best frontier can be attained in equilibrium, provided that salaries are sufficiently unrestricted. Combined with Proposition 2, Theorem 1 implies that on-path renegotiation is a robust feature of efficient equilibria. Proposition 2 states that there are adverse environments where using a single long-term output-contingent contract on the equilibrium path along with self-enforcing payments is not enough to attain first best outcomes. By constructing first best equilibria, Theorem 1 shows that on-path renegotiation is used to improve welfare in these environments. In first best equilibria the principal and the agent sign contracts, whose continuation contracts admit harsh punishment equilibria for either of them. These threats enforce the noncontractible actions – the agent’s effort and the principal’s bonus payment. However, achieving the first best in the continuation equilibrium is impossible if the continuation contract is not replaced, so renegotiation is needed on the equilibrium path. 8

**Theorem 1.** Fix any primitives of the model except $\bar{s}$. Then there exists $\bar{s}^{\min}$ such that any first best outcome where the principal’s payoff lies in $[f(0), v^{FB}(u(r))]$ is attainable in equilibrium, whenever $\bar{s} \geq \bar{s}^{\min}$.

**Proof.** Consider any level of agent utility $x$ such that $v^{FB}(x) \in [f(0), v^{FB}(u(r))]$. If the associated first best effort is “shirk”, the first best can be trivially achieved through a contract with constant salaries equal to $u^{-1}(x)$ without using bonuses or renegotiation.

I proceed to construct the first best equilibrium when the first best effort is “work”. On the equilibrium path the principal offers the same contract $c$ regardless of the realised output. The contemporaneous salaries in $c$ amount to 0 and the continuation contract $c^*$ is output-invariant and has $s_h^* = \bar{s}, c_h^* = c_l^* = 0$ and $s_l^*$ small enough so that

$$\sum_y p_y^{\text{work}} u(s_y^*) - \psi(\text{work}) = u(r).$$

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8That is, the subgame $(c^*, c^*, R)$ does not admit the desired first best equilibria.
Whenever $\bar{s} > u^{-1}(x)$, the existence of such a salary $s^*_h$ follows from the fact that the agent’s utility is unbounded below. If $\bar{s}$ is high enough, the agent does not have incentives to shirk in his worst equilibrium, so $u(c^*) = U(c^*) = u(r)$. Notice that the lack of upper bound on the agent’s utility implies that $s^*_h \to 0$ as $\bar{s} \to \infty$.

On the equilibrium path, the principal offers $c$, the agent accepts and works, and the principal pays bonus $b_y = u^{-1}(x + \psi(\text{work}))$. Hence, if followed, the strategies result in the first best payoffs $(x, v^{FB}(x))$. It remains to specify off-path behaviour and argue that the strategies constitute an equilibrium when $\bar{s}$ is high enough.

Consider a deviation by the agent at a history where no previous deviation has occurred. The continuation strategies are defined to follow the agent’s worst equilibrium. Hence, if he accepts the contract $c$ offered on the equilibrium path and shirks he will get no bonuses so his utility will be $U(c) = -\infty$. If he rejects in favour of residual contract $0$ (in the initial period) or $c^*$ (in all subsequent periods) he will receive at most $u(r)$. Hence, he can never get more than his on-path payoff $x$ by deviating.

Now consider a deviation by the principal at the contract offer stage. The continuation strategies following such a deviation follow the agent’s worst equilibrium. Since $u(0) = u(c^*) = u(r)$, it follows from Corollary 1 that $f(c^*) = f(0)$. Hence, the principal can get at most $f(0)$ by making a deviant contract offer whenever the residual contract is $0$ and $c^*$. Since no other residual contracts occur on the equilibrium path, the principal has no profitable contract offer deviation.

It remains to consider deviations by the principal at the bonus payment stage. In what follows, we will show the existence of an equilibrium in subgame $c^*$ with an arbitrarily low payoff $v'$ for the principal, provided that $\bar{s}$ is large enough. On the equilibrium path, the continuation contract is $c^*$ at every history up to bonus payment stage. The above continuation equilibrium in subgame $c^*$ is played following any deviation. When $v'$ is small enough so that

$$u^{-1}(x + \psi(\text{work})) \leq \frac{\delta}{1 - \delta} \left( v^{FB}(f(0)) - v' \right),$$

the principal has no incentives to deviate. His best deviation is not to pay any bonus and receive the continuation payoff $v'$ but this is no better than paying the on-path bonus $u^{-1}(x + \psi(\text{work}))$ and receiving the first best continuation payoff.
To construct an equilibrium in subgame $c^*$ with an arbitrarily low payoff for the principal, it suffices to show that there exists an equilibrium in subgame $(c^*, c^*, R)$ with an arbitrarily high payoff $x^*$ for the agent. To see this, consider any equilibrium in subgame $c^*$ such that the continuation equilibrium giving the agent $x^*$ is played whenever the agent rejects any contract offer. In this equilibrium, the agent’s payoff cannot be lower than $x^*$, so the principal’s payoff is bounded above by $v_{FB}(x^*)$. As $x^*$ becomes arbitrarily high, the principal’s payoff becomes arbitrarily low.

Therefore, it only remains to construct an equilibrium in subgame $(c^*, c^*, R)$ with an arbitrarily high payoff for the agent. In this equilibrium the agent works and the principal pays bonuses $b^*_h = 0$ and $b^*_l = \frac{\delta}{1-\delta}(v_{FB}(u(r)) - f(0))$. The continuation equilibrium following output $h$ is the agent’s worst equilibrium in subgame $c^*_h = 0$ giving the principal $f(0)$ regardless of the history of play. The continuation strategies following output $l$ and bonus $b^*_l$ are the strategies of the first best equilibrium with payoffs $(u(r), v_{FB}(u(r)))$. If a different bonus is paid, the continuation strategies follow the agent’s worst equilibrium at $0$. Hence, the incentive constraints for the bonus payments $b^*_h$ and $b^*_l$ are satisfied. Moreover, the agent has incentives to work whenever $\bar{s}$ is high enough, even in lieu of bonuses. Hence, the strategies form an equilibrium. The agent’s payoff is given by

$$x^* = \sum_y p^y_{work}(1 - \delta)u(s^*_y + b^*_y) + \delta u(r) - (1 - \delta)\psi(work)$$

$$= x + (1 - \delta)p^l_{work}[u(s^*_l + b^*_l) - u(s^*_l)].$$

Notice that there exists $\varepsilon > 0$ such that $f(0) + \varepsilon < v_{FB}(u(r))$ regardless of the value of $\bar{s}$. If the agent’s worst equilibrium in subgame 0 involves the outside options then this follows from the assumption that $v_{FB}(u(r)) > 0$. Otherwise, $f(0)$ is the principal’s payoff in an equilibrium with no bonuses. If this equilibrium replicates the first best outcome then the salaries on the equilibrium path must be constant. However, the agent expects no bonuses, so he will not have incentives to work. Hence, there is a gap between $f(0)$ and $v_{FB}(u(r))$ independent of $\bar{s}$. It follows that,

\footnote{This self-referential approach to defining equilibrium strategies can be formalised inductively.}
as $\bar{s} \to \infty$, $s_i^* \to 0$ for some $\varepsilon > 0$. Thus, the unboundedness of the agent’s marginal utility near 0 implies that $x^* \to \infty$ as $\bar{s} \to \infty$. This completes the proof.

The players achieve the first best by signing contracts whose continuation contract $c^*$ exhibits highly skewed compensation. If either player deviates on the noncontractible aspects of the relationship (the agent’s effort and the principal’s bonus payment), their respective punishment equilibrium in subgame $c^*$ follows. If the players comply with the equilibrium strategies, $c^*$ is renegotiated away to a new contract, whose continuation contracts are $c^*$ in order to create the same incentives. To an outside observer, it may appear that the parties repeatedly sign contracts which expose them to a lot of risk and renegotiate them away. However, this constant recontracting is premeditated as part of their implicit agreement. The continuation contract $c^*$ is designed to admit harsh punishment equilibria for either player, but its terms cannot support a first best outcome, so it must be renegotiated on path.

More precisely, $c^*$ facilitates punishments in the subsequent period by acting as a starting point for contract renegotiation. In the agent’s punishment equilibrium in subgame $c^*$, the agent anticipates that if he rejects in favour of $c^*$, his worst equilibrium will be played. In this equilibrium of subgame $(c^*, c^*, R)$, the agent gets no bonuses, so the skewed compensation and his risk aversion imply that his utility equals $u(r)$. Hence, the agent can be held down to his outside option in subgame $c^*$ as well. In the principal’s punishment equilibrium in subgame $c^*$, the agent anticipates a high payoff if he rejects in favour of $c^*$. This is because he anticipates an equilibrium of subgame $(c^*, c^*, R)$ where he receives a bonus on top of the small salary for low output, which brings a lot of marginal utility. A higher upper bound on salaries allows for more skewed contracts, keeping the agent’s worst punishment at his outside option while lowering the salary for low output. A lower salary increases the marginal utility of the bonus the agent receives in the principal’s punishment equilibrium, making that punishment arbitrarily severe as the upper bound on salaries grows.

Since the punishment to the principal can be made arbitrarily severe, any bonus payment can be incentivised in equilibrium. Hence, any first best outcome which
improves the principal relative to his worst equilibrium payoff can be attained. In addition, the unboundedness of the principal’s punishment implies that two-period contracts are sufficient to achieve the first best, as shown in the proof of Theorem 1. This is a stark contrast to the one-period contracts setting of Pearce and Stacchetti (1998) where the first best is generally unattainable, providing another demonstration of the power of writing contracts whose future terms can be replaced in equilibrium.

Since the punishments are based on a contract with inefficient terms, it may appear that inefficient equilibria play an important role in the proof of Theorem 1. However, this need not be the case: Proposition 3 below shows that the first best can also be attained by equilibria with efficient continuations at the beginning of each period. This is the concept of strong optimality considered by Levin (2003).

**Definition 2** (Levin (2003)). An equilibrium is strongly optimal if the continuation equilibrium payoffs at any history up to the contract offer stage are efficient.

**Proposition 3.** Suppose \( \lim_{m \to \infty} u'(m) = 0 \). Then the first best outcomes in Theorem 1 can be implemented in strongly optimal equilibria.

**Proof.** See Appendix 7.3.

The idea behind the strong optimality of equilibria is similar to Levin (2003): If a deviation by any player is followed by an inefficient equilibrium in the next stage, continuation play can be replaced with an efficient equilibrium that gives the same payoff to the deviator. In risk-neutral settings, it is easy to show that such an efficient equilibrium exists: if \( x \) is the maximal equilibrium payoff for the agent that gives the principal utility \( v \) but \( (x, v) \) is Pareto dominated by \( (x', v') \), modifying the latter equilibrium by increasing the agent’s salary can create a Pareto improvement over \( (x, v) \) where the principal’s payoff is exactly \( v \). However, increasing the agent’s salary may distort incentives in the risk averse setting I consider. To ensure an efficient equilibrium exists for every utility level of each

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10The statement of Theorem 1 is slightly simpler, as \( f(0) \) is a lower bound on the principal’s worst equilibrium payoff at the start of the repeated game but the more general statement is also true.
player, I assume that the marginal utility of the agent vanishes at high payments.\textsuperscript{11} Then, the first best effort at high agent utility levels is “shirk” and any first best outcome can be sustained when the players start with an appropriate residual contract. The strong optimality of equilibria then follows from the above argument.

Proposition 3 implies that the continuation contract $c^*$ supports first best outcomes not by threatening deviators with inefficient equilibria but by admitting a large multiplicity of equilibria. In particular, subgame $c^*$ can access a large portion of the first best frontier in equilibrium, so deviators can be threatened with an unfavourable, yet efficient equilibrium. Hence, $c^*$ can be seen as a source of strategic ambiguity, as described by Bernheim and Whinston (1998) who argue that environments where contracts are incomplete may make it optimal for parties to sign contracts that are even more incomplete. In my setting, $c^*$ does not add strategic ambiguity by leaving out contractual terms directly, but the added incompleteness is endogenous, as $c^*$ allows for a wider range of implicit agreements in equilibrium.

Remark 1. The equilibrium contracts can exhibit the semi-stationary structure described by Miller, Olsen, and Watson (2018). The equilibrium in Theorem 1 is unchanged if $c^*$ is replaced with a contract $c^{**}$ such that $c_{t+1}^{**}(y', y) = s_y^*$ for all $t \in \mathbb{N}$ and $y' \in Y^t$. Hence, the equilibrium contract $c$ effects every period involves null salaries in the current period, and prescribes the same compensation in any future period which is independent of the history of output.

\textsuperscript{11}It is difficult to establish the monotonicity of the efficient frontier of equilibrium payoffs without a detailed characterisation which is beyond the scope of the paper. Introducing public randomisation would not help, since the residual contract at each stage acts as a limited liability constraint through the worst equilibrium payoffs for each player: A Pareto improving equilibrium may require randomisation between two equilibria which are feasible under different residual contracts, but not under a common one.
4 Limited liability for the principal

In the first best equilibria constructed in Theorem 1 the principal writes continuation contracts that expose him to liability which may be orders of magnitude above the value of the firm. This section explores an extension of the model which limits the principal’s liability. In every period, the principal can shut down the firm immediately after observing the agent’s contract response. As the exercise of the agent’s outside option, this shutdown is permanent and results in payoffs of $u(r)$ to the agent and 0 to the principal. If the principal does not shut down, the rest of the period proceeds as in the baseline model. Since both parties have limited liability, there is no need to impose an exogenous bound $\bar{s}$ on contractual salaries.

The convention of Notation 2 is retained in the following analysis with the inclusion of subgames $(c^*, c, A, \text{in})$ and $(c^*, c, R, \text{in})$ where “in” denotes the principal’s decision not to shut down. Let $V(c^*)$ denote the equilibrium payoff set of subgame $c^*$.

4.1 Worst punishments

Let $\underline{u}(c^*)$ and $\underline{v}(c^*)$ denote the worst equilibrium payoffs for the agent and the principal in subgame $c^*$. Let $\underline{u}^A(c)$ and $\underline{v}^A(c)$ denote the agent and the principal’s worst equilibrium payoffs in any subgame where contract $c$ is effected, i.e. subgame $(c^*, c, A, \text{in})$ or $(c, c^*, R, \text{in})$.\(^{12}\)

The arguments from Proposition 1 can be adapted to show that

$$
\underline{u}^A(c) = U(c) = \max_e \sum_y p_y \left[ (1 - \delta)u(s_y) + \delta u(c_y) \right] - (1 - \delta)\psi(e).
$$

\(^{12}\)The equilibrium sets of both (classes of) subgames are identical because the only payoff-relevant part of the history is the effected contract.
The principal’s worst equilibrium payoff in the same subgame is given by

\[ v^A(c) = \min_{e, (b_y, u_y, v_y)} \sum_y p_y \left[ (1 - \delta)(y - s_y - b_y) + \delta v_y \right] \]

s.t. \[ \sum_y p_y \left[ (1 - \delta)u(s_y + b_y) + \delta u_y \right] - (1 - \delta)\psi(e) \geq U(c) \]

\[ b_y \leq \frac{\delta}{1 - \delta} (v_y - v(c^*)) \]

\[ (u_y, v_y) \in V(c_y) \forall y \]

Consider the agent’s worst equilibrium in subgame \( c^* \). If \( v^A(c^*) \leq 0 \) there is an equilibrium in subgame \( c^* \) that holds the agent down to his outside option. Upon rejection, the principal shuts down the firm in anticipation of his worst equilibrium giving him \( v^A(c^*) \leq 0 \). On-path play specifies that the agent accepts a contract offer and an equilibrium that gives him \( u(r) \) is played.\(^{13}\) Effectively, the principal has full bargaining power because the agent cannot force him to pay the salaries specified in the contract. Thus, the agent’s worst equilibrium payoff is \( u(r) \) when \( v^A(c^*) \leq 0 \). If \( v^A(c^*) > 0 \), the agent can secure \( U(c^*) \) by rejecting all future principal offers, as the principal will not have an incentive to shut down the firm regardless of which equilibrium is being played. Hence, the agent’s worst equilibrium payoff in subgame \( c^* \) is \( \max\{U(c^*), u(r)\} \), as in the baseline model.

4.2 Preliminary characterisation

The following lemma connects the equilibrium sets of subgames starting at different residual contracts.

**Lemma 1.** An equilibrium \( \sigma \) of any subgame \( c \) with payoffs \((u, v)\) is an equilibrium of subgame \( c^* \) iff \((u, v) \geq (u(c^*), v(c^*))\).

**Proof.** Clearly, if \((u, v) \geq (u(c^*), v(c^*))\) does not hold, some player has a profitable deviation. It remains to show that if \((u, v) \geq (u(c^*), v(c^*))\), then \( \sigma \) constitutes an equilibrium in subgame \( c^* \).

If the equilibrium calls for the outside options, then \((u, v) = (u(r), 0) = (u(c^*), v(c^*))\) since both parties can unilaterally exit. Suppose \( \sigma \) prescribes a shutdown by the principal. It is without loss to assume that the agent accepts the

\[^{13}\text{An equilibrium where the agent’s payoff is } u(r) \text{ can be constructed similarly to Proposition 1.}\]
contract offer. It remains to show that the agent does not have incentives to reject. If $v^A(c^*) \leq 0$, there is an equilibrium following rejection that gives the agent $u(r)$. If $v^A(c^*) > 0$ then the worst equilibrium for the agent following rejection gives him at most $\max\{u(r), u^A(c^*)\} = u(c^*) = u(r)$. Hence, $\sigma$ is an equilibrium in subgame $c^*$. If $\sigma$ prescribes that the agent quits, then the agent must not have incentives to accept by construction of $\sigma$ and the above argument shows he does not have incentives to reject either.

Now suppose that the outside options are not taken in $\sigma$. Then it is without loss that the principal offers some contract $c'$ and the agent accepts. The incentive constraints after $c'$ is effected are satisfied by definition of $\sigma$. The principal has no incentive to deviate at the contract offer stage since he receives more than $v(c^*)$ in equilibrium. He has no incentives to shut down since $v(c^*) \geq u(r)$ by definition. The agent has no incentives to reject or take the outside options because this gives him at most $\max\{U(c^*), u(r)\} \leq u(c^*)$. Hence, $\sigma$ is an equilibrium in subgame $c^*$.

Lemma 1 shows that efficient equilibria are described by a frontier which is independent of the residual contract. The efficient equilibria in a given subgame $c^*$ are described by the portion of this frontier which lies above $(u(c^*), v(c^*))$.

In equilibrium it is without loss of generality to use continuation contracts which maximise the punishment of one party holding the punishment of the other party fixed. That is, for any continuation contract $c^*$, there exists no alternative contract $c'$ with $u(c') \leq u(c^*)$, $v(c') \leq v(c^*)$ and at least one strict inequality. The reason is that harsher punishments expand the set of continuation payoffs by Lemma 1 and strengthen contemporaneous incentives. Hence, for the purposes of finding the equilibrium set it is sufficient to find contracts which achieve the frontier of worst punishments. An equivalent observation holds in the baseline model but finding the harshest punishments is not necessary since the principal’s punishment is unbounded, as shown in Theorem 1.
4.3 Contracts which achieve the harshest punishments

In principle, there may be tradeoffs between punishing the principal and the agent. However, Theorem 2 below shows that there exist contracts that can hold both parties down to their outside options, providing the harshest possible punishments. The result is similar to Theorem 1 where the principal’s punishment could be made arbitrarily severe while holding the agent down to his outside option. In this extended model punishments are bounded by the outside options so efficiency is not guaranteed but the uniformity of the harshest punishments has the same implications for the renegotiation dynamics. Any equilibrium can be supported with a continuation contract $c^*$ independent of output. Every period $c^*$ is renegotiated to a contract whose continuations are $c^*$. Hence, the long-term contracting problem becomes equivalent to a one-period contracting problem where a deviating party receives their outside in subsequent periods. Hence, introducing long-term contracts and renegotiation simplifies the model relative to Pearce and Stacchetti (1998) where contracts last for one period so deviators are punished with their worst equilibrium in the supergame which is an endogenous object.

**Theorem 2.** There exists a contract $c^*$ such that $u(c^*) = u(r)$ and $v(c^*) = 0$.

The proof of Theorem 2 is simple under the following assumption.

**Assumption 1.** The efficient frontier of equilibrium payoffs contains a point where the principal’s payoff is 0.\textsuperscript{1415}

**Proof of Theorem 2 under Assumption 1.** Consider a contract $c^*$ effected in an efficient equilibrium where the principal receives 0. I construct an equilibrium of subgame $c^*$ where the principal receives 0 as follows. The agent rejects all offers. Following rejection the principal does not shut down and the efficient equilibrium which gives him 0 is played. The agent has no incentive to deviate as he receives his highest equilibrium payoff in the game and the principal cannot deviate to get more than 0 as all his offers will be rejected. To construct an equilibrium of

\textsuperscript{14}Assumption 1 refers to the single efficient frontier which describes payoffs across different subgames $c^*$. See Lemma 1 and the discussion which follows it.

\textsuperscript{15}The difficulty in proving the strong optimality of equilibria, described in the discussion of Proposition 3 applies here as well: It may be that any equilibrium which gives the principal 0 is Pareto dominated. This is precisely what Assumption 1 assumes away.
subgame \( c^* \) where the agent receives his outside option, have the principal offer a contract \( c \) that achieves the efficient outcome giving the agent \( u(r) \) and shut down the firm following rejection (since he is indifferent). If the agent accepts \( c \), the aforementioned efficient equilibrium is played. The agent can do no better by deviating since he will receive his outside option. The principal also lacks a profitable deviation as the strategies mandate that he receives his highest equilibrium payoff in subgame \( c^* \). To see this, notice that if there were a better equilibrium for the principal, it must strictly improve the agent as well. The strategies of this equilibrium can be modified by decreasing the salaries of the effected contract until the agent’s payoff equals his outside option, since \( u(0) = -\infty \). The resulting modification would be a Pareto improvement on the original equilibrium. Hence, \( c^* \) achieves the required punishments.

Theorem 2 is also valid in the absence of Assumption 1 but the argument is slightly more involved so I defer it to Appendix 7.4. Only the construction of the principal’s worst equilibrium changes. It no longer gives the agent his highest equilibrium payoff but incentive compatibility is preserved because the agent anticipates his worst equilibrium following off-path offers by the principal.

The contract dynamics in Theorem 2 can be interpreted in the following way. The parties sign a contract with continuation contracts to be used as threats. These continuation contracts, if effected, could give the principal payoffs above or below his outside options. Hence, following rejection in the next period, the principal can justify shutting down the firm as well as staying in business depending on his beliefs. If the agent deviates today, the principal becomes pessimistic about the equilibrium played following rejection in the next period. He forces the agent into the Pareto optimal equilibrium that holds the agent down to his outside option by threatening him with shutdown. If, instead, the principal deviates today, the agent believes that the principal will keep the firm in business which allows him to credibly reject all relevant offers and hold the principal down to his outside option. Hence, the principal becomes more optimistic about the relationship following rejection when he has deviated relative to when the agent has deviated. In fact, when the principal deviates, his optimism is used against him, as he can no longer credibly threaten the agent with rejection. So the continuation
contracts which achieve the worst punishments are used to enable a drastic shift in (implicit) bargaining power depending on which party has deviated in the past. This is similar to the strategic ambiguity of the continuation contracts used in Theorem 1 which can support a large range of equilibrium payoffs, strengthening implicit incentives. Under Assumption 1 this feature is especially appealing since continuation equilibria are efficient on and off the equilibrium path (and hence are also strongly optimal). Punishments simply move the deviating party to their worst equilibrium on the efficient frontier.

**Remark 2.** Unlike in the baseline model, continuation contracts do not explicitly rely on the agent’s risk aversion to punish both parties. However, risk aversion plays an implicit role through the assumption $u(0) = -\infty$ which ensures that the efficient equilibrium where the agent receives his outside option is not Pareto dominated, so that the principal can be given incentives to exert the harshest possible punishment on the agent. Alternatively, $u(0) = -\infty$ means that the agent’s limited liability has no bite which rules out risk-neutrality.

**Remark 3.** It is without loss of generality that the continuation of any contract used in equilibrium following any level of output in the initial period is a contract $c^*$ as in Theorem 2. Thus, the continuation of $c^*$ can be $c^*$ itself. In other words, $c^*$ can be taken to be stationary. Hence, it is without loss of generality that the parties use semi-stationary contracts, as in the model of Miller, Olsen, and Watson (2018).

5 Literature Review

This paper contributes to the relational contracts literature popularised by Bull (1987), Thomas and Worrall (1988), MacLeod and Malcomson (1989), and Levin (2003) which studies implicit agreements as equilibria of a repeated game. This literature emphasises the role of noncontractible outcomes, so formal contracts (the ones enforceable in a court of law) typically have limited scope. A complementary literature studies the interaction of formal contracts and implicit agreements (Baker, Gibbons, and Murphy, 1994; Schmidt and Schnitzer, 1995; Bernheim and Whinston, 1998; Pearce and Stacchetti, 1998; Che and Yoo, 2001; Battigalli and
Maggi, 2008; Kvaloy and Olsen, 2009; Hermalin, Li, and Naughton, 2013; Itoh and Morita, 2015). While the existing analysis is limited to settings with one-period formal contracts, my paper considers history-dependent long-term formal contracts and shows that their renegotiation is an essential feature of optimal relational contracts. In addition, I contribute to the comparatively small set of papers which consider risk aversion in relational contracts (Thomas and Worrall, 1988; Pearce and Stacchetti, 1998; MacLeod, 2003; Hemsley, 2013; Thomas and Worrall, 2018).

This paper considers the renegotiation of contracts in equilibrium. This is distinct from the literature on renegotiation-proofness (Bernheim and Ray, 1989; Farrell and Maskin, 1989; Pearce, 1987) where the implicit agreements between the players, i.e. the equilibrium strategies themselves, are renegotiated.

The literature on renegotiation of incomplete contracts can be split broadly into two categories. The mechanism design approach posits that contracts are contingent on messages sent by the players. In the frameworks of Maskin and Moore (1999) and Segal and Whinston (2002) the players sign a contract, observe nonverifiable information, send messages and renegotiate the outcome specified in the contract to an efficient alternative. This approach was widely used in the incomplete contracts literature, originating in Hart and Moore (1988). It implies that it is without loss of generality to consider renegotiation-proof contracts, since renegotiation is assumed to be costless (Brennan and Watson, 2013). Hence, under this approach renegotiation is not an equilibrium phenomenon but a restriction on the contract space.

In my paper renegotiation is modelled strategically as part of a noncooperative game without making use of contracts contingent on messages. This approach has been used in a variety of one-period settings, including holdup (Huberman and Kahn, 1988b), leveraged buyout (Huberman and Kahn, 1988a), and a principal-agent model (Hermalin and Katz, 1991). Guriev and Kvasov (2005) consider a holdup problem in continuous time where a seller makes persistent investments that improve the value of trade with a buyer. Under parametric conditions, a con-

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16 As discussed in the introduction, the complementary work of Miller, Olsen, and Watson (2018) also fills this gap in the literature.
17 Also see Zhao (2012) for an application of renegotiation-proofness to a relational contracting model with subjective evaluations.
18 See Bolton and Dewatripont (2005) for a survey.
tract specifying trade and prices over a period of time can be used to achieve first best surplus in an equilibrium where the players renegotiate to this contract at every instant. There is no multiplicity of outcomes on and off the equilibrium path so renegotiation is not used as a tool of implicit contracting to create rewards and punishments. Instead, the seller is punished for underinvesting because the persistence of his investment lowers the utility he can guarantee under the continuation contract deteriorating his bargaining position at the next instant when the contract is renegotiated. In contrast, the actions taken by the principal and the agent in my model do not affect payoffs at the time the contract can be renegotiated. Hence, incentives are provided entirely through variation in continuation play. Continuation contracts enable equilibrium multiplicity which provides rewards and punishments, making renegotiation an an essential component of implicit contracting.

Rey and Salanie (1990) analyse a setting with moral hazard where the agent’s action is private but every observable outcome is verifiable. The only incompleteness in the contracts is their short-term (two-period) length. Rey and Salanie (1990) provide conditions under which the outcome of long-term contracts can be replicated by overlapping short-term contracts, which are renegotiated in equilibrium.

In Iossa and Spagnolo (2011) the players breach their formal contracts in equilibrium as part of an implicit agreement. Even when the contract is breached, the players are taken to court only if their deviation was not prescribed by the implicit agreement. This method of circumventing the punishments associated with formal contracts bears resemblance to the renegotiation of punishment contracts in the equilibria of this paper. Iossa and Spagnolo (2011) allow for renegotiation explicitly but it only occurs after deviations and the punishment which is renegotiated away is assumed to call for strict adherence to the formal contract for the rest of the game.
6 Conclusion

In both models analysed in this paper the players benefit from setting up threats posed by continuation contracts which will be renegotiated on the equilibrium path. In the baseline model, Theorem 1 shows that continuation contracts can punish the principal with arbitrary severity in one equilibrium, while holding the agent down to his outside option in another. Similarly, when the principal has limited liability, Theorem 2 shows that one contract can support punishment equilibria where either player receives his or her outside option. Since the harshest possible punishments can be achieved through the same continuation contract, there are no tradeoffs between the punishments to both players. This result simplifies the exposition, but the logic of improvement through renegotiation also applies in its absence. In a model with a different bargaining protocol, bounded agent utility, or imperfectly observable effort, the harshest threats cannot necessarily be achieved in equilibrium but there is no reason to suspect that setting up continuation contracts which support inefficient equilibrium payoffs with the intention of renegotiating them on the equilibrium path will not lead to a Pareto improvement. In such a richer model there may be a frontier of “punishment” continuation contracts used in equilibrium. Hence, it is possible that continuation contracts are used as partial commitment devices to favour one party over another, leading to nonstationarities that are not necessary in the risk-neutral model of Miller, Olsen, and Watson (2018).
7 Appendix

7.1 Boundedness of contracts

Contracts must be bounded to ensure existence of equilibrium by preventing Ponzi schemes. To see this, suppose wages are unrestricted. Fix any subgame perfect equilibrium with payoffs \((u^*, v^*)\) for the agent and the principal respectively. The maximum feasible expected payoff for the principal is

\[ v_{\text{max}} = \sum_y p_y y \]

since salaries and bonuses are nonnegative. If \(v^* = v_{\text{max}}\) the agent must receive zero salaries and bonuses so he is better off deviating to his outside option. Hence, \(v^* < v_{\text{max}}\). I now argue that the principal can secure a payoff arbitrarily close to \(v_{\text{max}}\) with a multi-stage deviation involving a Ponzi scheme. In the initial period the principal offers a contract \(c\) with \(s_y\) small but positive and continuation contracts \(c_y\) that promise big payments in the future. Since the agent is free to reject subsequent offers from the principal, the continuation contracts provide a guaranteed lower bound on his utility so they can be chosen appropriately to guarantee that contract \(c\) will give the agent more utility than \(u^*\) regardless of whether a bonus will be paid and regardless of the continuation equilibrium. Moreover, if \(c_h\) guarantees sufficiently higher utility than \(c_l\), the agent will work regardless of the continuation equilibria he anticipates. This is because there is a lower bound on the equilibrium payoff of the principal in a subgame starting with a given residual contract (for instance, he could offer the residual contract and refuse to pay bonuses in all subsequent periods); as a result, there is an upper bound on the agent’s equilibrium payoff. Thus, by deviating to an alternative contract offer, the principal can guarantee that the agent accepts it and works for very small salaries in anticipation of a high continuation utility. In the following period there is an upper bound on the equilibrium utility of the agent so the principal can deviate again by offering a contract that delays the promised payments to the agent even further while making sure the agent works for almost no payment. The principal can keep making an appropriately large offer to the agent in each subsequent period and secure a payoff sufficiently close to \(v_{\text{max}}\) from the resulting infinite-stage deviation. Hence, no equilibrium exists when salaries are unbounded.
7.2 Proof of Proposition 1

The proof constructs equilibrium strategies which give the agent the desired payoffs in the relevant subgames. This is sufficient since they are also lower bounds on the agent’s equilibrium payoffs.

The agent’s strategy is to accept contract \( c \in C \) in subgame \( c^* \in C_{T-1} \) if \( U(c) \geq u(c^*) \). If the converse holds, he takes his outside option provided \( U(c^*) < u(r) \) and rejects otherwise. Upon acceptance and rejection he chooses the maximising effort in the definition of \( U(c) \) and \( U(c^*) \) respectively. That is, if he accepts, he behaves as if the utility he gets from continuation contract \( c_y \) is \( u(c_y) \) and he anticipates no bonuses.

The principal never pays bonuses. To describe his contract offer strategy, let \( f(c^*) \) denote the maximum payoff the principal can achieve in subgame \( c^* \) given the agent’s strategy. It satisfies the following Bellman equation

\[
f(c^*) = \begin{cases} 
\max_e g(e, u(c^*)) & \text{if } U(c^*) \geq u(r) \\
\max_e \left\{ \max_e g(e, u(c^*)), 0 \right\} & \text{if } U(c^*) < u(r)
\end{cases}
\]

where

\[
g(e, x) = \max_{c \in C} \sum_y p_y^c \left[ (1 - \delta)(y - s_y) + \delta f(c_y) \right]
\]

s.t. \( U(c) \geq x \) \hspace{1cm} (3)

\[
U(c) = \sum_y p_y^c \left( (1 - \delta)u(s_y) + \delta u(c_y) \right) - (1 - \delta)\psi(e)
\]

Here, \( g(e, x) \) is the maximum payoff the principal can receive from offering the agent a contract that induces effort \( e \) and gives the agent at least \( x \) in equilibrium. When \( U(c^*) \geq u(r) \) the agent will not take his outside option so the principal gets \( \max_e g(e, u(c^*)) \) by choosing the contract offer that maximises his utility while providing the agent with enough utility to accept. Notice that the outcome of rejection can be replicated if the principal offers \( c^* \). When \( U(c^*) < u(r) \) the principal has the additional option of offering \( c^* \) itself, which results in the agent taking his outside option and a zero payoff for the principal.

The principal’s contract offer strategy is defined by the policy function of the above Bellman equation for \( f \). By construction, the principal is best responding
to the agent’s strategy at every history. Moreover, paying no bonus is justified by
the fact that it does not change the agent’s behaviour.

If the prescribed strategies are followed, the agent’s payoffs in all subgames
are as stated in the proposition. In subgame $c^*$, if the principal offers a contract
which will make the agent take his outside option, then $U(c^*) < u(r)$ so the
agent’s payoff is $u(r) = u(c^*)$, as required. If the agent does not take his outside
option, constraint (3) binds since $u(0) = -\infty$ implies the principal can decrease
wages without affecting incentives. Hence, the principal will offer a contract $c$ with
$U(c) = u(c^*)$ which the agent will accept. Since the agent chooses the effort in
the definition of $U(c)$, his payoff is $u(c^*)$ provided that his continuation payoff
following any output $y$ equals $u(c_y)$. Discounting and total boundedness of payoffs
imply that a successive application of the same argument to subgame $c_y$ and further
subgames implies that the agent’s payoff is within $\varepsilon$ of $u(c^*)$ where $\varepsilon$
converges to 0 as the argument is applied to further subgames. Hence, the agent’s payoff is
$u(c^*)$. Similar arguments can be used to find the agent’s equilibrium payoff in the
rest of the subgames.

To show the optimality of the agent’s strategy, consider a one-shot deviation in
subgame $c^*$. If he deviates by rejecting, he anticipates receiving $u(c^* y)$ in the next
period if output is $y$ today. Hence, by definition, a one-shot deviation through
rejection or the outside option gives the agent at most $u(c^*)$. A one-shot deviation
in effort following acceptance of $c$ is also not profitable by definition of $U(c)$, since
the agent will receive $u(c y)$ tomorrow following output $y$. Hence, the agent has no
profitable deviation.

7.3 Proof of Proposition 3

Lemma 2. Consider an equilibrium of any subgame $c^*$ with payoffs $(x^*, v^*)$. Suppose
there exists a contract $c$ and an equilibrium in subgame $(c^*, c, A)$ with payoffs
$(x, v) \geq (x^*, v^*)$. Then there exists an equilibrium in subgame $c^*$ with payoffs $(x, v)$.

Proof. Consider the following strategies in subgame $c^*$: The principal proposes $c$,
the agent accepts, and the equilibrium in subgame $(c^*, c, A)$ with payoffs $(x, v)$. Any deviation at the initial contract offer and contract response stages is followed
by the appropriate continuation of the equilibrium in subgame $c^*$ with payoffs
\((x^*, v^*)\). Since, \((x, v) \geq (x^*, v^*)\), the players have incentives to offer and accept \(c\) and the continuation play that follows forms an equilibrium by definition. \(\Box\)

In light of Lemma 2 it is sufficient to show that each first best outcome can be implemented in an equilibrium of some subgame \((c^*, c, A)\).\(^{19}\) Then the construction in Theorem 1 can be amended so that continuation equilibria at the contract offer stage following a deviation by either player can be replaced with their first best counterparts that give the same payoff to the deviating player.\(^{20}\) The same applies to the continuations of the first best equilibria played following a deviation.

By the inverse function theorem and the fundamental theorem of calculus

\[
u^{-1}(x + \psi(\text{work})) - \nu^{-1}(x) = \int_{x}^{x + \psi(\text{work})} \frac{1}{u'(\nu^{-1}(z))} dz.
\]

Since \(u(m) \rightarrow \infty\) and \(u'(m) \rightarrow 0\) as \(m \rightarrow \infty\), it follows that \(u^{-1}(x + \psi(\text{work})) - u^{-1}(x) \rightarrow \infty\) as \(x \rightarrow \infty\). Hence, there exists a utility level for the agent \(x^{\text{shirk}}\) such that the first best effort is “shirk” whenever the agent’s payoff is at least \(x^{\text{shirk}}\).

As argued in the proof of Theorem 1, any first best outcome where the optimal effort is “shirk” can be implemented in equilibrium. There is a compact set of first best payoffs where the optimal effort is “work”. Hence, there exists \(\bar{s}\) sufficiently high such that all of these first best outcomes can be supported in an equilibrium of subgame \((c^*, c, A)\) by the continuations of the strategies described in the proof of Theorem 1 after the agent’s acceptance of contract \(c\) in the initial period (where \(c\) is the contract used on the equilibrium path mirroring the notation from the proof of Theorem 1). Thus, Lemma 2 completes the proof of Proposition 3.

7.4 Proof of Theorem 2

**Lemma 3.** Let \(c\) satisfy \(v^A(c) > 0\). Then there exists a contract \(c'\) such that \(v^A(c') > 0\) and \(U(c') - U(c) \geq \eta(s_h, s_l, v^A(c))\) where \(\eta\) is a continuous, positive function defined on \(\{\hat{c} | v^A(\hat{c}) > 0\}\).

\(^{19}\)The equilibrium sets of \((c^*, c, A)\) and \((\hat{c}, c, A)\) are the same for any \(\hat{c}\) because the only payoff relevant state variable is the effected contract.

\(^{20}\)On-path continuation equilibria at the contract offer stage are efficient by construction.
Proof. Since \( v(c) > 0 \) the worst equilibrium for the principal in the subgame \((c,c,A)\) must involve the highest bonuses that are incentive compatible, that is

\[
v^A(c) = P(e, c) := \sum_y p^e_y \left[ (1 - \delta)(y - s_y) + \delta v(c_y) \right]
\]

where \( e \) is the only part of the punishment which is not fully determined by the contract \( c \). The maximum payoff the agent can receive in this punishment equilibrium is

\[
M(e, c) := \sum_y p^e_y \left[ \max_{(u_y, v_y) \in V(c_y)} (1 - \delta)u(s_y + \frac{\delta}{1 - \delta}(v_y - v(c_y))) + \delta u_y \right] - (1 - \delta)\psi(e).
\]

Any effort \( e \) can be supported in an equilibrium of \((c,c,A)\) as long as the incentive constraint of the agent is met. Thus, the problem of finding the principal’s worst punishment in \((c, c, A)\) is simply

\[
v^A(c) = \min_{e \in \{\text{shirk, work}\}} P(e, c) \quad \text{s.t.} \quad M(e, c) \geq U(c)
\]

Let \( e^* \) be the effort that achieves the principal’s worst punishment in the above minimisation problem and \( e \) be the effort taken in the agent’s worst equilibrium in \((c, c, A)\).

Consider the case of \( e = e^* = \text{shirk} \). Suppose the salary \( s_l \) is increased to \( s'_l \) creating a new contract \( c' \). The set of incentive compatible efforts to punish the principal can be no larger at \( c' \) than at \( c \): shirking is still incentive compatible and if working was not incentive compatible at \( c \), it is not incentive compatible under \( c' \). The latter holds because shirking remains optimal in the agent’s worst equilibrium, hence \( U(c') - U(c) = p^\text{shirk}_l(u(s'_l) - u(s_l)) \), whereas \( M(\text{work, } c') - M(\text{work, } c) \) is at most \( p^\text{work}_l(u(s'_l + b_l) - u(s_l + b_l)) \) where \( b_l \) is the optimal bonus under \( c' \). Thus,

\[
v^A(c') = P(e^*, c').
\]

Now choose \( s'_l \) so that \( P(e^*, c') = \frac{1}{2}v^A(c) \). Thus, the contract \( c' \) has \( v^A(c') > 0 \) and \( U(c') - U(c) \) is bounded below by \( p^\text{shirk}_l(u(s'_l) - u(s_l)) > 0 \) which is continuous in \( v^A(c) \) and \( s_l \). If \( e^* = \text{shirk} \neq e \) then both efforts are incentive compatible at \( c \) so the same increase in salary resulting in \( c' \) can only reduce the set of incentive compatible effort levels. Hence, \( v^A(c') \geq v^A(c') > 0 \) and the same bound on \( U(c') - U(c) \) applies.
The case of $e^* = \text{work}$ is tackled similarly by increasing $s_h$ leading to a lower bound on the increase in worst agent utility that depends on $v^A(c)$ and $s_h$ only. Hence, $\eta$ can be defined as the smaller of the two lower bounds, completing the proof.

To prove Theorem 2, let $\tilde{C} = \{c|v^A(c) > 0\}$ and $\tilde{U} = \sup\{U(c)|c \in \tilde{C}\}$. If $\tilde{C} = \emptyset$ then any contract $c^*$ satisfies the property: there exists an equilibrium where the principal shuts down the firm following rejection or acceptance of any contract $c$ since $v^A(c^*) \leq 0$ and $v^A(c) \leq 0$ for all $c$.

Now suppose $\tilde{C} \neq \emptyset$. Consider any sequence $(c^n)$ which attains the supremum in the definition of $\tilde{U}$. Without loss of generality $c^n \to c^*$ and $v^A(c^n) \to v^*$ for some $c^*$ and $v^*$. Suppose $v^* > 0$. Since $U(c^n) \to \tilde{U}$ and $\eta(s^n_h, s^n_l, v^A(c^n)) \to \eta(s^*_h, s^*_l, v^*) > 0$, where $\eta$ is given by Lemma 3, there exists $n$ large enough so that $\tilde{U} - U(c_n) < \eta(s^n_h, s^n_l, v^A(c^n))$. Thus, Lemma 3 implies the existence of a contract $c'$ with $v^A(c') > 0$ and $U(c') > \tilde{U}$ resulting in a contradiction. Hence, $v^*_s = 0$.

Now consider the sequence of equilibria in subgames $c^n$ which give the principal $v^A(c^n)$ and the agent some payoff $x^*_n$ which must be at least $U(c^n)$. Without loss of generality, the strategies converge to a strategy profile $\sigma$ which is an equilibrium in subgame $c^*$, gives the principal $v^*_s = 0$, and the agent at least $\tilde{U}$.

The agent’s worst equilibrium in subgame $c^*$ is constructed in the same manner as in the proof of Theorem 2. In the principal’s worst equilibrium in subgame $c^*$, the agent rejects some contract offer, the principal does not shut down the firm and the parties play $\sigma$, giving the principal a payoff of 0. Off the equilibrium path the agent rejects all offers $c$ with $U(c) \leq \tilde{U}$, since he otherwise expects either shutdown or his worst equilibrium giving him $U(c)$. The agent also rejects all offers with $U(c) > \tilde{U}$ because they satisfy $v^A(c) \leq 0$ so there is a continuation equilibrium where the principal shuts down the firm. In both cases, rejection is either followed by shutdown or by the strategy profile $\sigma$. Hence, the principal cannot improve upon his equilibrium payoff of 0 by making a deviant offer.
References


