

Wealth Accumulation, On-the-Job Search and Inequality

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Abstract

We study a directed search equilibrium with risk-averse workers who can search on the job and accumulate non-contingent assets under a borrowing limit. Search outcomes affect earnings and wealth accumulation. In turn, wealth and earnings affect search decisions by changing the optimal trade-off between the wage and the matching probability. The calibrated model yields sizable inequality in wages and wealth among homogeneous workers. Wealth significantly reduces a worker's transition rates from unemployment to employment and from one job to another. The interaction between search and wealth provides important self-insurance as it reduces the pass-through of earnings inequality into consumption by more than 60%. Moreover, we analyze the dynamic welfare effects of changes in unemployment insurance and find that unemployment benefits should be smaller than they currently are.

Keywords: Wealth accumulation; On-the-job search; Inequality; Directed search.
JEL classifications: E21; E24; J60.

1 Introduction

For a typical individual, labor income is the chief source of income. This income fluctuates as a worker's job or labor market status changes. A worker is unable to insure against such risks perfectly, because assets contingent on a worker's labor market outcome typically do not exist. However, a worker can partially insure against earnings risks by accumulating non-contingent assets and modifying job search strategies. For the latter, in particular, a worker can search for jobs that are relatively easy to obtain. In the equilibrium, how do job search and wealth accumulation interact? What are the implications of these interactions

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on inequalities in income, wealth and consumption? We address these questions with a model where workers can accumulate assets under a borrowing limit and search in the labor market both off and on the job. In addition, we compute dynamic welfare effects of changing the borrowing limit and the unemployment benefit.

It is well understood that labor income risks affect workers' consumption and wealth because of incomplete asset markets. Recent literature also documents how wealth affects job search. Specifically, higher wealth implies a lower job-finding probability for an unemployed worker, a lower transition rate between jobs for an employed worker, and higher future wages. For example, Chetty (2008) reports that individuals who receive larger severance payments stay unemployed for longer periods of time. Herkenhoff et al. (2015) show that increasing individuals' ability to borrow in the U.S. increases unemployment duration significantly. This positive effect of wealth on unemployment duration also exists in France (Algan et al., 2003), Germany (Bloemen and Stancenelli, 2001), and Denmark (Lentz and Tranbaes, 2005). For employed workers, Lise (2013) shows that higher wealth implies a lower transition rate from one job to the next job. Hartung et al. (2018) document that changes in unemployment insurance affect employed workers' behavior substantially in Germany. Griffy (2017) finds that relaxing liquidity constraints of unemployed workers results in higher wages in their next job. Together, these different sources of evidence show convincingly that job search and wealth accumulation interact with each other and that this interaction affects an individual's earnings process endogenously.

The interaction between wealth accumulation and job search should be of first-order importance in macroeconomics. It naturally connects two major markets – the intertemporal goods market with borrowing constraints and the labor market with search frictions. A voluminous literature has studied each market separately. A few exceptions that study the two markets jointly either assume an exogenous distribution of wage offers or exclude on-the-job search. As reviewed later in this Introduction, those studies severely distort the interaction between wealth accumulation and job search. Another motivation for studying this interaction is to understand how it affects economic inequality and its implications for unemployment insurance policy.

We integrate an equilibrium model of labor market search into an intertemporal model of consumption and savings. Individuals are risk averse. They can save and borrow with non-contingent assets but face a borrowing limit. Both employed and unemployed workers can search. Search is directed. Firms create vacancies competitively in submarkets that differ in the wage offer and the matching probability. When unemployed, workers have access to unemployment insurance that expires with some probability. The possibilities of failing to match and losing unemployment insurance generate labor income risks. Over time, workers differ in the history of search outcomes. This endogenous heterogeneity induces dispersion in earnings, wealth and consumption. To focus on such heterogeneity induced endogenously by search frictions, we assume that all workers have the same productivity and all jobs use the same production technology.

On the worker side, wealth accumulation and labor market search interact as follows. A worker with high wealth optimally chooses to apply for a high wage and face a low matching

probability. If the application fails to yield a match, the worker can still maintain high consumption by decumulating wealth. This stretches the high end of the wage ladder. In contrast, a worker whose wealth is close to the borrowing limit has only a limited scope of using wealth to smooth consumption. To prevent consumption from falling sharply over time, the worker optimally chooses to apply for a low wage in order to obtain the job quickly. This stretches the low end of the wage ladder. Thus, the effect of wealth on search, in turn, increases heterogeneity in the paths of earnings and wealth.

Firms cater to endogenously heterogeneous workers by offering wages conditional on the applicants' wealth. For any given wage offer, firms prefer to hire workers with higher wealth. This preference does not arise from discrimination or any direct contribution of workers' wealth to output. Rather, it arises from firms' rational expectation that an employee with higher wealth will search for another job that has a relatively high wage. Because such a job has a low matching probability for the worker, the employee will separate from the current job with a low probability, which increases the present value of the job to the current employer. Thus, each submarket is indexed by the wage offer and the applicants' wealth. In section 3.2, we will discuss the empirical evidence that firms' hiring can depend on the applicants' wealth. Moreover, we will show that the role of wealth in hiring can be replaced by observables such as the applicants' current wage and labor market status.

We calibrate the baseline model and compare the results with two benchmarks. One benchmark is a "no-search" model similar to Aiyagari (1994), where the labor market is Walrasian but workers face employment risks and a borrowing limit. The probability of becoming employed and the probability of losing employment are set to be equal to the average counterparts in the baseline model. The other benchmark is a "no-wealth" search model where workers are hand-to-mouth. This benchmark differs from the canonical search model (e.g., Diamond, 1981, Mortensen, 1982, and Pissarides, 2000) in that search is directed, workers can search on the job and workers are risk averse.

Our analysis yields four main results. First, the equilibrium interaction between wealth accumulation and on-the-job search is quantitatively important for the aforementioned empirical patterns of labor market dynamics. The policy functions show that high wealth levels or current earnings significantly reduce the transition rates of workers from unemployment to employment and from one job to another. With the data generated by the model, regressions show that wealth and current earnings both have significantly negative effects on the transition rates.

Second, wealth accumulation and directed search both provide important self-insurance against earnings risks. The model generates a Gini coefficient in wealth equal to 0.621. This is large given that the model abstracts from all exogenous heterogeneity among workers. The Gini coefficient in consumption is significantly smaller. In the no-search model, the Gini coefficient in consumption is 45.8% of that in earnings. That is, 54.2% of the earnings risk is self-insured by accumulating wealth. Allowing for directed search reduces the Gini coefficient in consumption further to 38.4% of that in earnings. In other words, workers' job search behavior has a quantitatively important role as a self-insurance mechanism. As discussed in section 5.3, the result is robust to incorporating alternative sources of earnings

such as home production.

Third, the model generates large frictional wage inequality, i.e., wage inequality among homogeneous workers. Hornstein et al. (2011) show that the *mean-min ratio* in wages among homogeneous workers is about 2 in the U.S data but is about 1.04 in a variety of search models. In contrast, the mean-min ratio is 1.728 in our model. It is the interaction between wealth accumulation and on-the-job search that generates large frictional wage dispersion. Shutting down on-the-job search reduces the mean-min wage ratio to 1.171. On the other hand, shutting down wealth accumulation reduces the mean-min wage ratio to 1.368. Relative to hand-to-mouth, the ability to accumulate wealth stretches out both ends of the wage ladder, as explained above.

Fourth, there is a limited role for unemployment insurance. Using the calibrated model, we study the welfare effects of changes in the replacement rate and the duration of unemployment insurance. To finance the insurance scheme, the government balances the budget intertemporally by adjusting an ad-valorem wage tax. We measure the welfare effect of a policy change by the percentage change in lifetime consumption, taking into account transitional dynamics. Keeping the expected duration of the benefit at the baseline value of 26 weeks, the optimal replacement rate is around 38%. If the replacement rate remains at the baseline value of 50%, the optimal duration of unemployment insurance is 15 weeks. For each of these two policies on unemployment insurance, moving from the baseline to the optimal level increases welfare by 0.02% to 0.04%. Welfare gains from the three policies are relatively small because (i) the average unemployment duration is relatively short in the data, and (ii) the long-run benefits of reducing wage taxes are in a large part compensated by the transition costs of increasing self-insurance.

Comparing welfare across steady states, the optimal replacement rate is 5%, and moving from the replacement rate of 50% to 5% increases welfare by 0.9%. Individuals prefer to pay a lower wage tax, receive lower unemployment benefits when unemployed, and use wealth and search to smooth consumption. Dynamically, this large reduction in unemployment insurance is not optimal, because workers need to have low wages and consumption for a relatively long time before getting close to the new steady state. We believe that this is an important contribution to the debate on the optimal size of unemployment insurance programs. Our findings suggest that, at the current size of the unemployment insurance program, the risk-sharing benefits of unemployment insurance do not compensate for the cost generated by distortionary taxes. The reason is that workers could instead modify their job search and savings decision to insure against earnings risk.

Our work builds on several strands of the literature.¹ To our knowledge, Krusell et

¹Models of business cycles have incorporated capital accumulation and undirected search (e.g., Andolfatto, 1996, Merz, 1995). With both directed and undirected search, Shi and Wen (1999) have studied taxes and subsidies. In these models, idiosyncratic risks generated by labor search are completely smoothed either within large households or in a perfect insurance market. Moreover, individuals do not face a borrowing limit. Lentz and Tranaes (2005) and Lentz (2009) analyze the optimal unemployment insurance when wealth affects unemployed workers' search effort. Since these models do not allow for on-the-job search, the equilibrium has a single wage rate. There are also papers that analyze how hidden savings affect unemployment insurance when there is moral hazard, e.g. Werning (2001) and Kocherlakota (2004).

al. (2010) and Lise (2013) are the pioneers to integrate labor search into models of precautionary savings in the style of Bewley (1977), Huggett (1993) and Aiyagari (1994). In contrast to our paper, Krusell et al. (2010) do not allow for on-the-job search, and Lise (2013) assumes the distribution of wage offers to be exogenous. Moreover, these papers assume undirected search, where workers are randomly matched with wage offers instead of choosing the wage to search for. Our paper have all three features: an equilibrium model of wages, on-the-job search, and directed search. We explain below why these features of our model are necessary for understanding wealth accumulation and job search.

Why is it necessary to endogenize the distribution of wage offers? An important contribution of our analysis is the finding that, taking into account optimal search behavior, there is limited scope for unemployment insurance. The literature with an exogenous distribution of wage offers is subject to the usual critique for policy analysis, because it fails to incorporate how a policy affects this distribution. In addition, by determining the expected return to search, the distribution of wage offers is a central factor affecting how job search interacts with wealth accumulation. With an endogenous distribution of wage offers, an equilibrium generates the mapping from a model’s mechanism to the data. In contrast, a large literature ignores firms’ responses and takes the offer distribution as exogenous. In this literature, the assumed distribution of wage offers is generically inconsistent with an equilibrium, and the typical claim of success of such a model in “matching” the wage distribution in the data is not warranted, especially after a policy changes (see the end of section 4.3).

Why is it important to incorporate on-the-job search and directed search? On-the-job search is necessary for explaining the large job-to-job transition in the data (e.g., Fallick and Fleischman, 2004). Obtaining the first job is only part of a worker’s transition in the labor market. The ability to search on the job in the future affects a worker’s wealth accumulation and current search. Shutting down on-the-job search, as in Krusell et al. (2010), significantly weakens the power of job search as an insurance mechanism and might misleadingly increase the role of unemployment insurance. A model without on-the-job also fails to explain the empirical evidence that increasing wealth reduces a worker’s job-to-job transition rate. Directed search is both intuitive and realistic.² In particular, it enables workers to adjust their search target according to their wealth. In addition, directed search simplifies the analysis and the computation by making the equilibrium block recursive (see Shi, 2009). Namely, individuals’ decisions and the market tightness do not depend on the large dimensional object – the distribution of workers across individual states.

There is a view that models of on-the-job search, such as Burdett and Mortensen (1998), have already generated sufficiently large frictional wage dispersion. They have not. As Hornstein et al. (2011) demonstrate, those models calibrate or estimate the value of home production to be unrealistically low. When this value is set to be realistic, those models generate a mean-min ratio in wages between 1.16 and 1.27, which is still much

²For example, in a survey data in the U.S., Hall and Krueger (2008) document that 84 percent of white, male, non-college workers either “know exactly” or “had a pretty good idea” about how much their current job would pay before the job interviews.

smaller than in the data. Even this modest ratio relies on the unrealistic assumption of undirected search, which implies that an unemployed worker accepts any wage offer as long as it is not lower than the value of home production. If search is directed, as in reality, the wage searched for by an unemployed worker is a significant jump above the value of home production, which reduces frictional wage dispersion (see Figure 4 later).

Our paper is closely related to Herkenhoff (2019) who uses a directed search model to study how increasing the access to credit card debt impacts unemployment duration and job recoveries after recessions. The main parts of his analysis assume that only unemployed workers can search. Although he discusses in an appendix how to extend the analysis to allow for on-the-job search, he does not characterize or describe the equilibrium. In addition to providing such an analytical characterization, our model generates the following features that Herkenhoff (2019) either did not examine or was unable to produce: the relationship between wealth and job transition among employed workers, the high wealth Gini coefficient, and the high mean-min wage ratio. Braxton et al. (2018) examine the optimal unemployment insurance when unemployed workers have private information about their own type and have access to credit. Corbae and Glover (2017) analyze the effects of banning employers' use of workers' credit record in the hiring process. In contrast, we emphasize the accumulation of wealth and on-the-job search.

Our model abstract from ex ante heterogeneity in worker and firm characteristics. This is done for two purposes: (i) to examine frictional wage inequality (Hornstein et al., 2011); and (ii) to focus on the ex post heterogeneity in the employment status, earnings, and wealth accumulation.

2 Model of Consumption, Savings and Search

2.1 Environment of the model

Time is discrete and lasts forever. There is a unit measure of risk averse workers whose utility in each period is $u(c)$, where c is consumption. The function u satisfies $u'(c) \in (0, \infty)$ and $u''(c) \in (-\infty, 0), \forall c \in \mathbb{R}_+$, with $u'(0) = \infty$. A worker has a discount factor, $\beta \in (0, 1)$, and can accumulate non-contingent assets denoted as $a \in A = [\underline{a}, \bar{a}]$. The lower bound on asset holdings, \underline{a} , can be negative, in which case it is a borrowing limit. The upper bound \bar{a} can be chosen so that it is not binding. The net rate of return on assets is r , which is determined in the world market and taken as given in this model. We assume $\beta(1+r) < 1$ to ensure that workers' asset levels are bounded in the equilibrium.³ In this model, ex ante identical workers become endogenously heterogeneous ex post in three aspects: the labor market status, current earnings, and wealth.

A worker's labor market status is $\varepsilon \in \{e, u\}$, where e indicates being employed and u unemployed.⁴ Denote a worker's earnings as ω . An employed worker's earnings are the

³As shown by Aiyagari (1994), when individuals face income risks and a borrowing limit, the precautionary saving motive would induce them to accumulate infinite wealth if $\beta(1+r) < 1$ were violated.

⁴We have extended the model to allow a worker to choose whether to participate in the market. Since

wage $w \in W \equiv [\underline{w}, \bar{w}]$. An unemployed worker’s “earnings” are unemployment insurance b , which can change over the unemployment spell. In the first period after a worker becomes unemployed, the worker receives unemployment insurance ρw_{-1} , where w_{-1} is the worker’s wage prior to unemployment and $\rho \in (0, 1)$ is the replacement rate. Thereafter, the insurance benefit expires with the probability χ at the end of each period, which we will choose to match the average duration of unemployment insurance in the data. The two parameters (ρ, χ) approximate unemployment insurance in reality. The stochastic expiration reduces the state space relative to a deterministic expiration schedule, since the latter will introduce the time till expiration as a state variable.⁵

Competitive entry determines the measure of firms. Firms are separate identities instead of being owned by workers. They are risk neutral but have the *same* discount factor β as workers. The production technology has constant returns to scale in jobs, and a firm treats the jobs independently. We refer to a job as a firm, as in most search models. All jobs yield a constant stream of output, y , and the production cost is normalized to 0. The vacancy cost per period is $k > 0$. Firms commit to offers, but workers can quit at anytime. In addition, as in Burdett and Mortensen (1998), firms cannot respond to the employee’s outside offers, which is realistic for most jobs. If an employed worker receives a better offer, the worker quits the current job.⁶ In addition to endogenous separation, exogenous separation destroys an existing match with the probability $\delta \in (0, 1)$. Exogenous separation is independent across matches and time.

In each period, an unemployed worker is able to search with probability one, and an employed worker with the probability λ_e .⁷ The labor market consists of a continuum of submarkets indexed by the wage offer w and the applicants’ wealth a . Each submarket (w, a) has a tightness $\theta(w, a)$, which is the ratio of vacancies to applicants. Taking the tightness function as given, recruiting firms and searching workers choose the submarket to enter. The tightness function is independent of the distribution of workers because the equilibrium is block recursive, as explained in section 3. Moreover, workers who search in submarket (w, a) must have the wealth level a . Conditioning a submarket on applicants’ wealth simplifies the analysis of the equilibrium. In section 3.2, we will explore alternative ways to describe the submarkets when wealth is not observable.

Matching is random in each submarket. The matching technology has constant returns to scale in the measure of applicants and vacancies. In any submarket with tightness θ , the matching probability is $p(\theta)$ for an applicant and $q(\theta)$ for a vacancy, where $p(\theta) = \theta q(\theta)$ because of constant returns to scale. Also, the matching probabilities satisfy the standard assumptions: $p(\theta), q(\theta) \in [0, 1]$, $p'(\theta) \in (0, \infty)$, $q'(\theta) \in (-\infty, 0)$ and $p''(\theta) < 0$ for all θ . Although $p(\theta)$ and $q(\theta)$ are exogenous functions of θ , the tightness function $\theta(w, a)$ is

the extension yields quantitatively similar results to the model presented here, we omit the extension.

⁵To simplify the analysis, we take the unemployment insurance as given in the baseline model. In section 5 on welfare analysis, we will introduce a wage tax to finance the unemployment insurance.

⁶See Postel-Vinay and Robin (2002) for an undirected search model where firms can match an employee’s outside offers.

⁷In the baseline exercise, we will set λ_e for the model to match the average rate of job-to-job transition. In a counter-factual experiment, we will set $\lambda_e = 0$ to examine the importance of on-the-job search.

endogenously determined by firms' entry and workers' search decisions. Thus, the matching probabilities are endogenous functions of (w, a) .

Each period is divided into five stages: (i) production, (ii) consumption and savings, (iii) search and matching, (iv) exogenous separation, and (v) the expiration of unemployment insurance. In the production stage, all existing matches produce, and workers obtain earnings (including unemployment insurance). In the second stage, workers choose consumption and savings. In the third stage, the opportunity to search is realized according to the probability λ_e for each employed worker and to probability 1 for each unemployed worker. Searchers choose the submarket to search, where new matches are formed. In stage (iv), a separation shock destroys a fraction $\delta \in (0, 1)$ of old matches and throws the workers into unemployment. To reduce repetitive accounting, we assume that a newly formed match is not subject to exogenous separation until the next period. Finally, in stage (v), unemployment insurance expires with the probability χ .

We define some objects. A worker's individual state is $s \equiv (\varepsilon, \omega, a)$, which consists of the labor market status ε , current earnings ω , and wealth a . The labor market status lies in the set $E \equiv \{e, u\}$. Earnings lie in the set $W \equiv [\underline{w}, \bar{w}]$ for an employed worker, and in the set $\rho W \cup \{0\}$ for an unemployed worker. Denote $\bar{W} \equiv W \cup \rho W \cup \{0\}$ so that $\bar{W} \ni \omega$. Wealth lies in the set $A \equiv [\underline{a}, \bar{a}]$. Denote the Borel sets as \mathcal{E} for E , \mathcal{W} for \bar{W} , and \mathcal{A} for A . The set of individual states is $S \equiv E \times \bar{W} \times A$, whose Borel sets are $\mathcal{S} = \mathcal{E} \times \mathcal{W} \times \mathcal{A}$.

The aggregate state of the economy is $\psi : \mathcal{S} \rightarrow [0, 1]$, a distribution of workers over individual states. Let $\Psi(S, \mathcal{S})$ be the space of distribution functions on the measurable space (S, \mathcal{S}) . Let $\mathcal{T} : \Psi(S, \mathcal{S}) \rightarrow \Psi(S, \mathcal{S})$ be the law of motion of the state of the economy. As explained later, the equilibrium in this model is block recursive in the sense that individuals' optimal decisions, value functions and the market tightness are independent of ψ . In particular, the market tightness function is $\theta : W \times A \rightarrow \mathbb{R}_+$, as introduced earlier.

2.2 Decisions on consumption and savings

Consider a worker in the state $s = (\varepsilon, \omega, a)$. When choosing consumption and savings in a period, the worker's value function is $V_\varepsilon(\omega, a)$. Let \hat{a} be the choice of savings, i.e., future wealth. The continuation value immediately after the choice is $\beta R_\varepsilon(\omega, \hat{a})$, which is computed later. The optimal choices of consumption and savings solve:

$$V_\varepsilon(\omega, a) = \max_{(c, \hat{a})} [u(c) + \beta R_\varepsilon(\omega, \hat{a})] \quad (1)$$

$$\text{s.t. } c + \frac{\hat{a}}{1+r} = y_{\min} + \omega + a \quad \text{and} \quad \hat{a} \geq \underline{a}.$$

The first constraint is the budget constraint and the second constraint the borrowing limit. The amount $y_{\min} > 0$ is a small amount of home production that prevents $c \leq 0$ in the case $\omega + a \leq 0$.⁸ Denote the policy function as $c_\varepsilon(\omega, a)$ for optimal consumption and $\hat{a}_\varepsilon(\omega, a)$ for optimal savings.

⁸If $\underline{a} > 0$, then $y_{\min} > 0$ is not needed for $c > 0$. If $\underline{a} < 0$, however, an unemployed worker's income can reach 0 with positive probability when the unemployment insurance expires.

2.3 Unemployed workers' search

Consider an unemployed worker who receives unemployment insurance b in the period and whose wealth after consumption is \hat{a} . The worker chooses a submarket (\hat{w}, \hat{a}) to search and receives a match with the probability $p(\theta(\hat{w}, \hat{a}))$. If the worker gets a match, the worker will start the next period employed with the value $V_e(\hat{w}, \hat{a})$. If the worker fails to match, unemployment insurance (if any) will expire with the probability χ . Let $R_u(b, \hat{a})$ denote the worker value generated by the optimal choice of search. Then,

$$R_u(b, \hat{a}) \equiv \max_{\hat{w}} \left\{ p(\theta(\hat{w}, \hat{a})) V_e(\hat{w}, \hat{a}) + [1 - p(\theta(\hat{w}, \hat{a}))] [\chi V_u(0, \hat{a}) + (1 - \chi) V_u(b, \hat{a})] \right\}. \quad (2)$$

Denote the policy function as $\tilde{w}_u(b, \hat{a})$ for the optimal search target. It is useful to rewrite this optimal choice as a function of the worker's wealth at the beginning of the period, a , instead of wealth at the end of the period, \hat{a} . Substituting $\hat{a} = \hat{a}_u(b, a)$ from the optimal choice of savings, we have $\hat{w}_u(b, a) \equiv \tilde{w}_u(b, \hat{a}_u(b, a))$. Similarly, we express the tightness of the submarket searched by the unemployed worker as $\hat{\theta}_u(b, a) \equiv \theta(\hat{w}_u(b, a), \hat{a}_u(b, a))$.

The above formulation reveals that search serves as a mechanism of partial self-insurance. A worker adjusts the search target according to the wealth level and earnings to smooth consumption. Consider an unemployed worker with low wealth. The worker expects to find a job in the future with some probability, in which case the income will increase. The risk averse worker would like to smooth consumption by decumulating wealth to transfer some of the higher future income to the present. However, the borrowing limit constrains the worker's ability to do so. When the borrowing constraint is binding, the worker is forced to consume a relatively low amount. If the worker fails to find a job, consumption will likely be even lower in the next period. To reduce the likelihood of such falling consumption, the worker will choose to search for jobs that have relatively high job-finding probabilities. In equilibrium, those jobs are the ones paying relatively low wages. Similarly, for any given wealth, unemployed workers who receive a relatively low unemployment insurance have incentive to apply for lower wages. This income effect on search exists even if unemployment insurance does not expire, but it is strengthened by the possibility that unemployment insurance can expire.

2.4 Employed workers' search

The search decision of an employed worker is formulated similarly to an unemployed worker's. Consider a worker employed at wage w whose wealth at the end of the period is \hat{a} . If the worker receives an opportunity to search, which occurs with the probability λ_e , the worker chooses the submarket (\hat{w}, \hat{a}) to search. In this submarket, the worker gets a match with the probability $p(\theta(\hat{w}, \hat{a}))$, in which case the continuation value will be $V_e(\hat{w}, \hat{a})$. If the worker fails to find a match, the worker will be hit by exogenous separation with the probability δ . In this case, the worker will receive unemployment insurance ρw , and the continuation value will be $V_u(\rho w, \hat{a})$. If the worker escapes exogenous separation,

the continuation value will be $V_e(w, \hat{a})$. Thus, the optimal search decision generates the value $R_e(w, \hat{a})$ to the worker and solves:

$$R_e(w, \hat{a}) \equiv \max_{\hat{w}} \left\{ \lambda_e p(\theta(\hat{w}, \hat{a})) V_e(\hat{w}, \hat{a}) + [1 - \lambda_e p(\theta(\hat{w}, \hat{a}))] [\delta V_u(\rho w, \hat{a}) + (1 - \delta) V_e(w, \hat{a})] \right\} \quad (3)$$

Denote the optimal target wage of search by the policy function $\tilde{w}_e(w, \hat{a})$. Recall that worker's wealth at the beginning of the period is a . We express the optimal search target as $\hat{w}_e(w, a) \equiv \tilde{w}_e(w, \hat{a}_e(w, a))$. The tightness of the submarket searched by the employed worker is $\hat{\theta}_e(w, a) \equiv \theta(\hat{w}_e(w, a), \hat{a}_e(w, a))$.

Similar to an unemployed worker, search serves as a mechanism for an employed worker to partially self-insure, which is reflected by the effects of wealth and earnings on the search decision. An employed worker with relatively low wealth or a relatively low wage has incentive to obtain a wage increase quickly in order to build up wealth for self insurance. To do so, the worker searches for a job that yields a relatively small wage gain but has a higher matching probability for the worker.⁹

2.5 Firms and market tightness

Consider a job that pays wage w and is filled by a worker with wealth a . Current profit of the job is $(y - w)$. The worker will quit for another job with the probability $\lambda_e p(\hat{\theta}_e(w, a))$, where $\hat{\theta}_e(w, a)$ is the tightness of the submarket searched by the worker. In addition to the quit for another job, the worker separates exogenously into unemployment with the probability δ . When either type of separation occurs, the future value of the job to the firm will be zero. If no separation occurs, the future value of the job to the firm will be $\beta J(w, \hat{a}_e(w, a))$, where $\hat{a}_e(w, a)$ is the worker's future wealth. Note that the firm discounts future profits with the same discount factor β as workers. The value of the filled job for the firm satisfies:

$$J(w, a) = y - w + (1 - \delta) [1 - \lambda_e p(\hat{\theta}_e(w, a))] \beta J(w, \hat{a}_e(w, a)). \quad (4)$$

Competitive entry of vacancies determines the tightness in each submarket. Consider submarket (\hat{w}, \hat{a}) . The matching probability for a vacancy is $q(\theta(\hat{w}, \hat{a}))$. If the firm is matched, production will start in the next period, and the present value of the firm is $\beta J(\hat{w}, \hat{a})$. The flow cost of a vacancy is $k > 0$. For all (\hat{w}, \hat{a}) , if $\beta J(\hat{w}, \hat{a}) \geq k$, competitive entry of vacancies will equate the expected value of a vacancy to the cost k . If $\beta J(\hat{w}, \hat{a}) < k$, no vacancy will enter the submarket. Thus, competitive entry of vacancies yields

$$q(\theta(\hat{w}, \hat{a})) \beta J(\hat{w}, \hat{a}) \leq k \text{ and } \theta(\hat{w}, \hat{a}) \geq 0, \forall (\hat{w}, \hat{a}),$$

⁹Although the effect of the current wage on search exists even when workers are risk neutral, risk aversion strengthens the effect for workers with low wealth because the marginal utility of consumption is higher for such workers.

where the two inequalities hold with complementary slackness. We solve:

$$\theta(\hat{w}, \hat{a}) = \begin{cases} q^{-1}\left(\frac{k}{\beta J(\hat{w}, \hat{a})}\right) & \text{if } \beta J(\hat{w}, \hat{a}) \geq k \\ 0 & \text{otherwise.} \end{cases} \quad (5)$$

Note that $q(\theta)$ is a decreasing function and $p(\theta)$ an increasing function. A submarket with relatively high job-finding probability for an applicant must have a relatively large tightness. Because such a submarket has a relatively low matching probability for a vacancy, firms enter such a submarket only if the submarket yields a relatively high firm value J and, hence, has a relatively low wage offer.¹⁰

3 Equilibrium and Submarkets

3.1 Equilibrium definition and block recursivity

The aggregate state of the economy is a distribution of workers over individual states. Given the distribution of workers at the beginning of a period, ψ , individuals' optimal decisions and matching outcomes induce the distribution of workers at the beginning of the next period, $\hat{\psi}$. We omit the characterization of this transition of the aggregate state because it is cumbersome counting of the flows of workers between states.

Definition 1 An *equilibrium* consists of the value function of workers, V_ε (for $\varepsilon = e, u$), the firm value function J , policy functions, $(c_\varepsilon, \hat{a}_\varepsilon)$ and (\hat{w}_u, \hat{w}_e) , and the transition function of the aggregate state, \mathcal{T} , that satisfy requirements (i)-(iv) below:

- (i) The value function of workers, $V_\varepsilon : \bar{W} \times A \rightarrow \mathbb{R}$, satisfies (1). The corresponding optimal decisions of participation, consumption and savings yield the policy functions $(c_\varepsilon, \hat{a}_\varepsilon)$, and optimal search decisions yield the policy functions (\hat{w}_u, \hat{w}_e) .
- (ii) The firm's value function, $J : W \times A \rightarrow \mathbb{R}$, satisfies (4).
- (iii) The tightness function θ satisfies (5) for all $(\hat{w}, \hat{a}) \in W \times A$.
- (iv) The aggregate state transition, $\mathcal{T} : \Psi(S, \mathcal{S}) \rightarrow \Psi(S, \mathcal{S})$, is consistent with the policy functions and induces the aggregate state in the next period as $\hat{\psi} = \mathcal{T}(\psi)$.

The equilibrium defined above is a *block recursive equilibrium* (BRE), as defined and analyzed by Shi (2009) and Menzio and Shi (2010, 2011). Namely, value functions, policy functions and the market tightness function in the equilibrium are determined by (i)-(iii) independently of the distribution of workers. We can verify block recursivity as follows. Start with the hypothesis that the value function of a filled job, $J(w, a)$, is independent of ψ . Competitive entry of vacancies into submarkets requires the expected value of a filled job to be equal to the vacancy cost. This requirement determines the market tightness function, $\theta(w, a)$, independently of ψ (see (5)). Because matching probabilities in a submarket are

¹⁰Features such as an applicant's labor market status, earnings and past employment history can affect the applicants' wealth and the target wage of search. However, given (\hat{w}, \hat{a}) , these features do not add information to the determination of the market tightness.

only a function of the market tightness, they are also independent of ψ . Given these matching probabilities, individuals can calculate the present value of a job and make their decisions. The resulting value functions and optimal choices are independent of ψ . In particular, the value function of a filled job to a firm, $J(w, a)$, is independent of ψ , which supports the initial hypothesis.

Directed search and competitive entry of vacancies are critical for block recursivity. Because search is directed, a worker chooses to search in the submarket that features the optimal trade-off between the gain in value and the matching probability. For this decision, the worker does not need to know the distribution of workers, provided that the matching probabilities are independent of the distribution. Because competitive entry of vacancies drives down expected profit of a vacancy to zero in every viable submarket, the tightness and the matching probabilities are indeed independent of the distribution.

Block recursivity reduces the complexity of the equilibrium substantially. If the equilibrium is not block recursive, then the distribution of workers is a state variable relevant for individuals' decisions. This aggregate state has infinite dimension because earnings and wealth lie in intervals. Moreover, the law of motion of this aggregate state is endogenous because it must be consistent with the flows of workers induced by individuals' optimal decisions. Exactly computing such a non-block recursive equilibrium is not feasible. The approximation in the literature assumes that only a small number of moments of the distribution matter (e.g., Krusell and Smith, 1998, Krusell et al., 2010). In contrast, when the equilibrium is block recursive, individuals' optimal decisions and the market tightness function depend only on the individual state that has a small dimension.

We take advantage of block recursivity to study the welfare implications of policy changes and counterfactual analyses (see section 5). Block recursivity allows us to compute welfare effects, including the cost and benefits of transitional dynamics between an initial and a final steady states. All without the need to keep track of the distribution of workers' individual states nor its evolution in between steady states.

3.2 The role of wealth in the description of submarkets

In the modeling above, submarkets are described by the wage offer and applicants' wealth. Why does a firm care about an applicant's wealth in addition to the offer? An employee's wealth affects the survival probability of a match, because the employee's probability of quitting for another job is $\lambda_e p(\hat{\theta}_e(w, a))$. For any given current wage w , a worker with high wealth has incentive to search for a high wage. Since a high wage offer comes with a relatively low matching probability for the worker, the worker will succeed in getting the outside offer with a low probability. Thus, the employee's wealth increases the survival probability of the match and, hence, increases the expected value of the match to the firm. For any wage offer, a recruiting firm prefers to hire a worker with a high level of wealth.

If wealth is not observable, how can the equilibrium be implemented? One way is to assume that firms can elicit information about applicants' wealth. An applicant is not able to show more wealth than what he has. If an applicant chooses to hide some wealth, the

worker does not gain once the worker gets the applied job, but the worker increases the risk of getting the job if other applicants to the job report wealth truthfully. Thus, it is incentive compatible for the applicant to show wealth truthfully. In the majority of states in the U.S., employers are allowed to check job applicants' credit records before hiring, and they do. In a survey, the Society for Human Resource Management (2012) found that 60 percent of the managers in 2010 checked job applicants' credit records before hiring.¹¹ Although a credit record may not precisely identify a worker's wealth, it can be highly informative of a worker's wealth.

Another way to implement the equilibrium without assuming observable wealth is to include an applicant's *current job*, defined as $\eta \equiv (\varepsilon, \omega)$, in the description of a submarket. The term "current job" is used for both an employed worker with $\eta = (e, w)$, and for an unemployed worker with $\eta = (u, \rho w_{-1})$. The additional information η may be available from an applicant's resume and can be used as a qualification for a job. The following lemma provides the conditions under which the offer wage and the applicants' current job together are sufficient for describing submarkets (see Appendix A for a proof):

Lemma 1 *Suppose that a worker's search choice is not observable to the worker's current employer unless search results in a match and that a worker's wealth is not observable. However, the information about a worker's current job η is publicly observable and contractible. Assume that (i) for any given (ε, ω) , the policy function $\tilde{w}_\varepsilon(\omega, \hat{a})$ is one-to-one of \hat{a} , and (ii) the policy functions $\tilde{w}_\varepsilon(\omega, \hat{a})$ and $\hat{a}_\varepsilon(\omega, a)$ are single-valued. Then, the equilibrium where submarkets are described by (\hat{w}, η) is the same as the equilibrium where wealth \hat{a} is observable and submarkets are described by (\hat{w}, \hat{a}) .*

Under the assumption that $\tilde{w}_\varepsilon(\omega, \hat{a})$ is one-to-one of \hat{a} for any given (ε, ω) , a wage \hat{w} is searched only by applicants with a particular wealth level \hat{a} for any given current job $\eta = (\varepsilon, \omega)$. Such self-selection in the search process implies that recruiting firms are able to infer the applicant's wealth from the wage offer and an applicant's current job. Given this inference, a firm that succeeds in recruiting a worker can forecast perfectly the worker's future search choices and, hence, the separation rate in the future. Thus, a recruiting firm's expected value in a submarket is a function of only the wage offer \hat{w} and the applicants' current job η . Then, competitive entry of vacancies determines the tightness in each submarket as a function of only (\hat{w}, η) . Once an applicant's wealth is inferred from (\hat{w}, η) , the applicant's past jobs and the duration at the current job do not add any relevant information to the description of the submarket, since they affect only the path by which the worker has reached the particular wealth level.

The above exposition explains why the two elements (\hat{w}, η) are sufficient for describing a submarket. When wealth is not observable, the two elements are also necessary for separating the applicants into different submarkets. It is possible that two employed workers can differ in both wealth and current earnings (or the labor market status) in a particular way that induces them to search for the same wage. If both succeed in matching, they

¹¹The question asked in the survey is: "Does your organization, or an agency hired by your organization, conduct credit background checks for any job candidates by reviewing the candidates' consumer reports?"

will have the same wage. But because they differ in wealth, they will search for different wages in the future. This implies that the two firms matched with the two workers will face different separation rates in the future and, hence, different firm values. If submarkets were described only by the offer wage, the two workers could not be separated.

Despite Lemma 1, we will continue to describe submarkets by the offer wage and applicants' wealth. It is difficult to analytically verify the assumptions (i) and (ii) in the lemma, although they hold in the quantitative analysis later. Also, the dimension of submarkets (\hat{w}, \hat{a}) is lower than (\hat{w}, η) by one.

4 Quantitative Analysis

4.1 Calibration

The utility function and the matching probabilities have the following forms:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \quad p(\theta) = [1 + \theta^{-\gamma}]^{-\frac{1}{\gamma}}, \quad q(\theta) = \frac{p(\theta)}{\theta}.$$

The matching probability function, $p(\theta)$, is a Dagum (1975) function, with $\gamma > 0$. If a submarket has a measure N of applicants and tightness θ , the function $p(\theta)$ implies that the measure of matches is:

$$\mathcal{M}(N, \theta N) = p(\theta)N = \frac{N \times (\theta N)}{[(N)^\gamma + (\theta N)^\gamma]^{\frac{1}{\gamma}}}.$$

We calibrate the model to the monthly frequency. Normalize $y = 1$. Other parameters and calibration targets are listed in Table 1. The interest rate is $r = 0.327\%$ per month or 4% annually. The difference between r and the discount rate $\frac{1}{\beta} - 1$ is critical for the wealth distribution. We set β to match the median of monthly income to liquid wealth ratio of 23% in the Survey of Consumer Finance (SCF) of 2007.^{12,13} With the borrowing limit $\underline{a} = -5.90$, the model generates a fraction of the population with negative liquid wealth of 20%, as in the SCF of 2007. The curvature of the utility function is set to $\sigma = 2$ as in most macro calibration.

On the labor market, the exogenous separation rate $\delta = 0.026$ matches the average transition rate from employment to unemployment in the Current Population Survey (CPS). For an employed worker, the probability of having the search opportunity in a period, λ_e , is chosen to yield a monthly job-to-job transition rate 2.9%, which lies in the range [2.2%, 3.2%] in recent empirical evidence (e.g., Fallick and Fleischman, 2004, Hornstein et al., 2011). We adjust γ to target the average elasticity of the job-finding probability of unemployed workers with respect to average market tightness for unemployed workers. This

¹²This method is similar to that in Kaplan (2012), who identifies β by targeting a worker's asset level. However, because he focuses on young workers, he obtains a lower value of β than we do.

¹³We define liquid wealth as total financial assets net of retirement savings and credit card balance.

Table 1. Baseline calibration parameters

Parameter	Baseline	Target
β	0.996315	median of income to liquid wealth ratio = 23%
r	0.327%	annual interest rate = 4%
\underline{a}	-5.90	fraction of population with $a < 0$ = 20%
δ	2.6%	separation rate into unemployment in CPS
σ	2.0	standard in macro calibration
γ	0.65	elasticity of p_u to θ_u = 0.32
λ_e	0.23	EE transition rate = 2.9%
k	0.314	unemployment rate = 6.5%
ρ	0.5	replacement rate = 50%
χ	0.197	expected UI duration = 26 weeks

elasticity is 0.32 in the calibration, which lies in the range $[0.27, 0.5]$ in the literature (see Shi, 2018). Home production in the absence of the unemployed benefit is set to $y_{\min} = 0.05$ to prevent workers from reaching $c = 0$.¹⁴ The replacement ratio of the unemployment benefit is set to a realistic value $\rho = 0.5$. In our model, the expected duration of the unemployment benefit is $1 + \frac{1}{\chi}$, since χ is the expiration probability of the benefit in each period after receiving the benefit in the first period of unemployment. Calibrating this expected duration to a common value in reality, 26 weeks, yields $\chi = 0.197$. The vacancy cost is set to $k = 0.314$ to match an unemployment rate of 6.5%.

The model is able to exactly match all the targeted moments. With the identified parameters, we solve the equilibrium and simulate the model (for the procedure, see Appendix C in the supplementary appendix). We will analyze the computed policy functions in the next subsection. Subsection 4.3 will analyze the distribution and inequality.

4.2 Policy functions, value functions and market tightness

A critical feature of the equilibrium is that an individual's decision on consumption and savings interacts with search decision in the labor market. We analyze this interaction using the computed policy functions.

Figure 1 depicts workers' optimal search decision (panel (a)), the difference between next period's wealth and current wealth (panel (b)), and consumption (panel (c)), all as functions of current wealth. In each panel, the green dashed line is for an unemployed worker. The other three lines are for employed workers at three levels of current earnings: low $w \in (0, \hat{w}_u(0, a))$ (the black solid line), medium $w \in (\hat{w}_u(0, a), \bar{w})$ (the red dashed

¹⁴This small amount of unearned income guarantees that a borrowing-constrained unemployed worker who has exhausted unemployment insurance is able to consume about 3% of the average consumption level across all workers. In section 5 where we study optimal unemployment insurance, we conduct robustness analysis to consider a larger amount of unearned income and conclude that our main findings do not rely on a low level of y_{\min} .

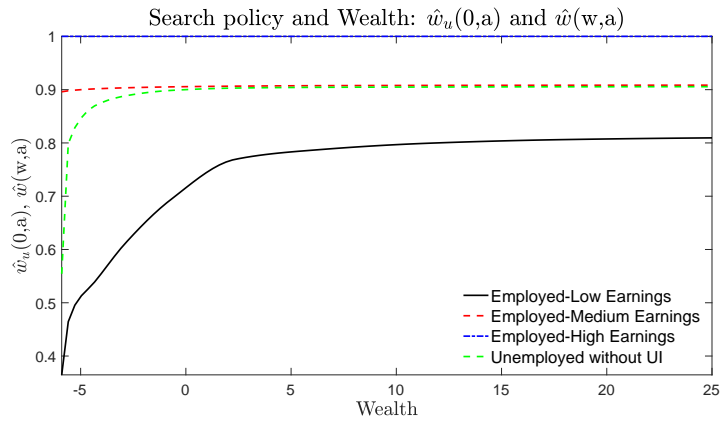
line), and high $w = \bar{w}$ (the blue line).¹⁵ For earnings at the maximum, the target wage for search stays constant at this maximum over all wealth levels, as depicted by the blue line in the panel (a). For all earnings lower than the maximum, the target wage for search is increasing in wealth if wealth is moderate or low. This policy function for the search target becomes flat when wealth is high. Thus, the effect of wealth on job search is strong at low wealth levels and dissipates as wealth increases.

These features of the optimal search target are intuitive. For a worker with low wealth, the borrowing limit is either binding or will be binding if the wage does not increase soon. In both cases, future consumption is expected to fall. To partially insure against this outcome, the worker tries to obtain a wage increase quickly by search. The optimal target wage for search is low because only low-wage jobs have high job-finding probabilities. An increase in wealth reduces the likelihood that the borrowing limit will be binding soon. This enables the worker to tolerate a lower job-finding probability and, hence, to search for higher wages. That is, the target wage as a function of wealth is positively sloped at low wealth levels. As wealth keeps increasing, the effect diminishes, and so the target wage policy function becomes less steep. When wealth is sufficiently high, the worker is perfectly self-insured against income risks, in which case further increases in wealth do not affect the optimal target wage for search.

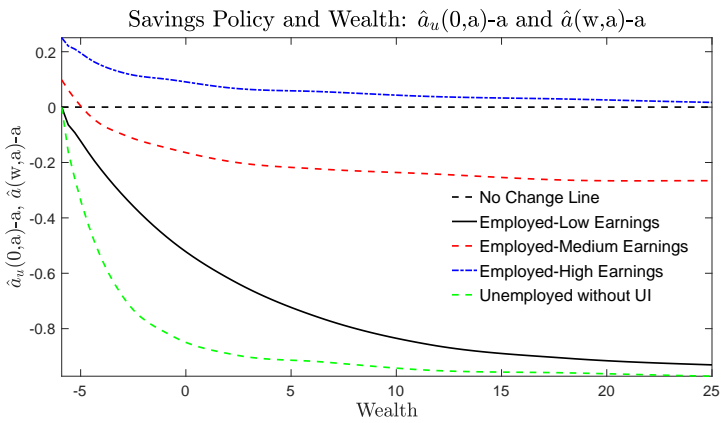
Panel (b) in Figure 1 shows that, as wealth increases, the difference between future and current wealth decreases in current wealth. The decreasing difference implies different dynamics of wealth for workers with different earnings, because the sign of the difference depends on earnings. For a worker with high earnings, the difference between future and current wealth is above zero. The worker is accumulating wealth as a precaution for exogenous job separation into unemployment. As wealth increases toward the level of perfect self-insurance, the difference between future and current wealth decreases toward zero. In contrast, for a worker with low earnings, the difference between future and current wealth is negative. The worker is decumulating wealth in order to maintain smooth consumption. As wealth decreases, the difference between future and current wealth increases but remains negative. The absolute value of this difference declines. For a worker with medium earnings, the dynamics of wealth depend on current wealth. If current wealth is lower than a threshold, the worker accumulates wealth over time; if current wealth is higher than the threshold, the worker decumulates wealth over time.

Panel (c) in Figure 1 shows consumption as an increasing function of wealth. The slope of the consumption function represents the marginal propensity to consume out of wealth, which decreases as wealth increases to improve the ability to self-insure. When wealth approaches the level of perfect self-insurance, consumption becomes constant, and so the marginal propensity to consume approaches zero. Moreover, for any given wealth,

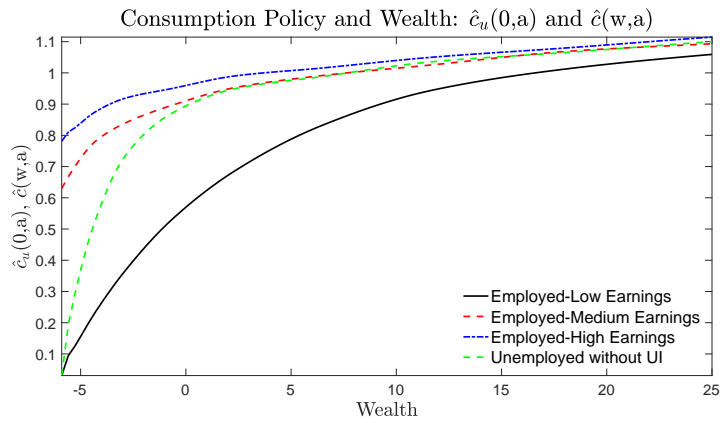
¹⁵The low wage used in Figure 1 is lower than $\hat{w}_u(0, a)$ and, hence, is out of the (steady-state) equilibrium. Nevertheless, it is useful to include this low wage in Figure 1 since it may occur during the transition to the steady state equilibrium. This is analogous to the common practice of plotting the value function over the entire domain of the state variables even though only a particular sequence of the state variables will be observed in equilibrium.



(a) Job Search and Wealth



(b) Savings and Wealth



(c) Consumption and Wealth

Figure 1. Optimal search target, future wealth and current consumption as functions of wealth for earnings fixed at low $w \in (b, \hat{w}_u(a))$, medium $w \in (\hat{w}_u(a), \bar{w})$, and high $w = \bar{w}$.

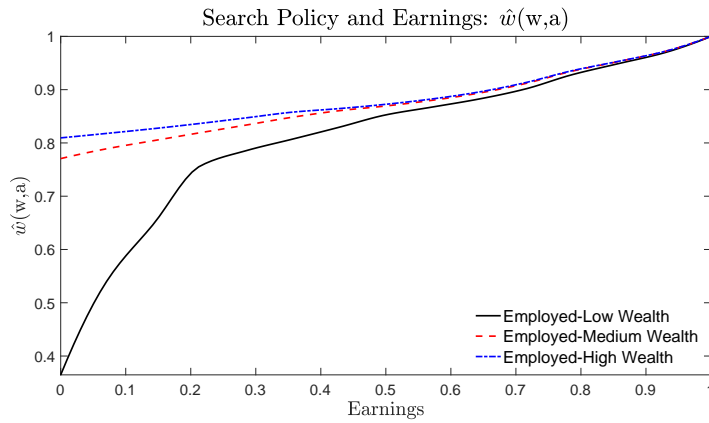
workers with low earnings have lower consumption and, to smooth consumption, these workers consume more out of their wealth proportionally. Reflecting this effect of current earnings, the consumption function is lower but steeper for workers with low earnings than for workers with high earnings.

Figure 2 depicts employed workers' target wage for search (panel (a)), the difference between future and current wealth (panel (b)), and consumption (panel (c)), all as functions of current earnings. In each panel, the three lines correspond to three wealth levels: low (the black solid line), medium (the red dashed line) and high (the blue dot-dashed line). These panels confirm the above analysis. Relative to workers with high earnings, workers with low earnings search for lower wages, decumulate wealth less to smooth consumption, and have higher propensities to consume out of wealth. These differences narrow as wealth increases. When wealth is high, consumption change little with earnings and the difference between future and current wealth becomes positive for all wealth levels because precautionary savings motives are stronger.

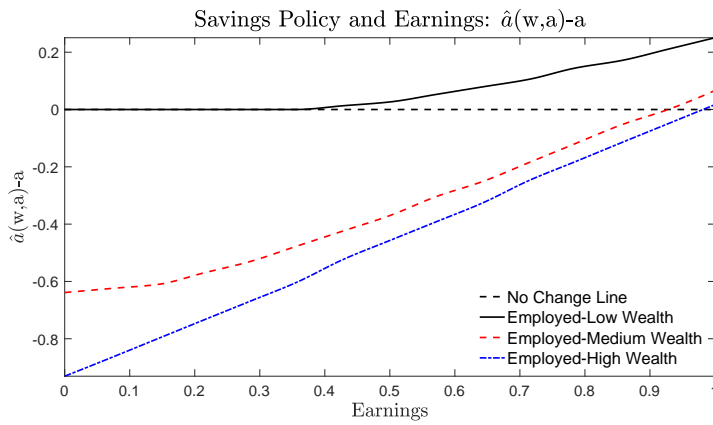
Now we examine how the firm value of a filled job and the market tightness depend on the wage and the applicant's wealth. In Figure 3, the left panels depict the dependence on an applicant's wealth, where the applicant's current earnings are set to three levels with the same legends and colors as in Figure 1. The right panels depict the dependence on the wage offer, where the applicant's wealth is set to be three levels with the same legends and colors as in Figure 2. Not surprisingly, the firm value of a filled job is a decreasing function of the wage offer for any given wealth level of the applicant. Anticipating the low value of a filled job, not many firms enter the submarket to offer the high wage. Thus, the market tightness is also a decreasing function of the wage offer.

For any given wage offer, the firm value of a filled job and the tightness of the submarket offering the wage are increasing functions of an applicant's wealth, provided that the applicant's current wage is lower than the maximum. That is, for any given wage offer, a recruiting firm prefers an applicant with high wealth to an applicant with low wealth. This preference arises from the effect of a worker's wealth on search decisions. When a worker's wealth is high, it is not urgent for the worker to obtain a wage increase for self insurance. As a result, the worker will search for higher wages that are less likely to be obtained. This reduces the endogenous separability probability of the worker from the current job and, hence, increases the firm value of the job filled by the worker. Anticipating this higher value of a job filled by a wealthier worker, more vacancies enter the submarket to attract such workers. The tightness increases in this submarket. These effects of a worker's wealth weaken as the wage offer increases. When the wage offer is at the maximum, a worker employed at such a wage is not expected to move to another job. In this limit, the firm value of a filled job and the tightness of the submarket offering the maximum wage are independent of the worker's wealth.¹⁶

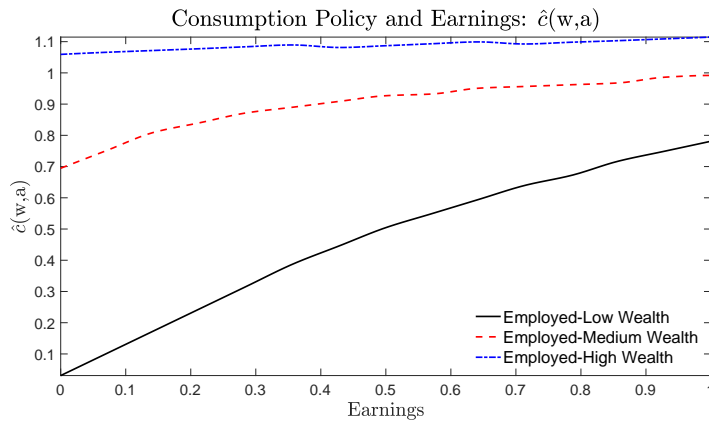
¹⁶At very low wage and wealth levels, the benefit to a firm of increasing the wage to retain a worker may be even higher than the direct cost of the wage increase. In this case, an increase in the wage offer improves the payoff to both a recruiting firm and an applicant. Thus, submarkets with such low wages are not active in the equilibrium.



(a) Job Search and Earnings



(b) Savings and Earnings



(c) Consumption and Earnings

Figure 2. Optimal search target, future wealth and current consumption as functions of earnings for wealth fixed at low, medium and high levels.

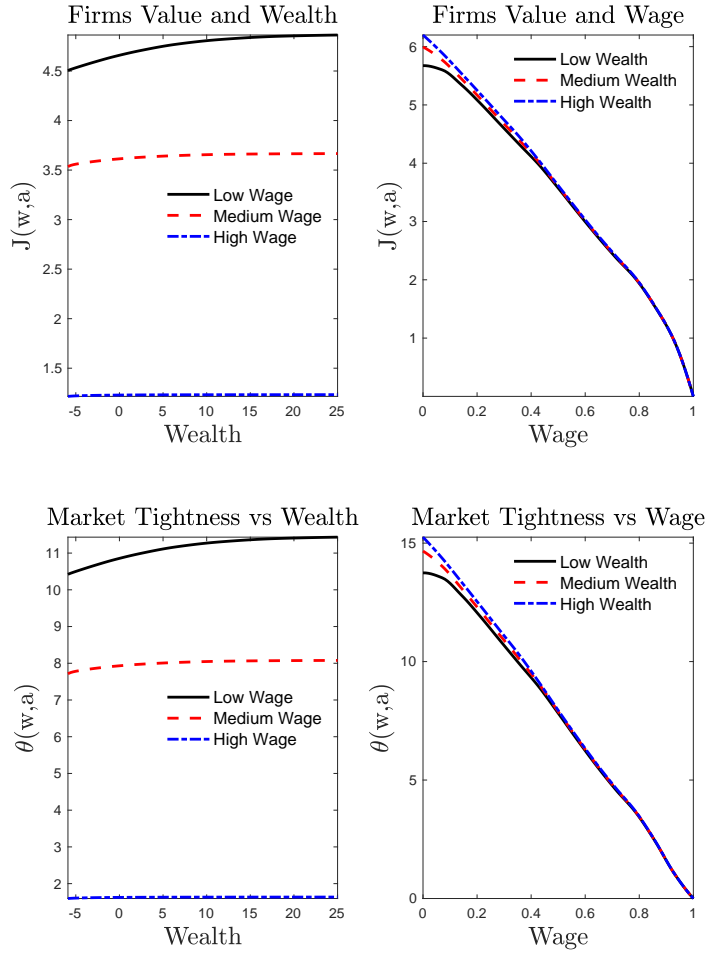


Figure 3. Firm value and market tightness as functions of worker’s wealth and the wage offer

4.3 Distribution of workers and inequality

Table 2 reports measures of inequality in earnings, income, wealth and consumption. Income is equal to earnings plus interest income on assets. The results in the baseline model, reported in the first column, are compared with those in two benchmarks:

(i) The “no-search” model (the second column in Table 2): This is similar to Aiyagari (1994), where the labor market is frictionless but workers face employment risks and a borrowing limit.¹⁷ In each period, exogenous separation destroys a job with the probability δ and makes the worker unemployed. An unemployed worker becomes employed with the

¹⁷This benchmark differs from the model in Aiyagari (1994) primarily in that the interest rate is exogenous rather than endogenous.

probability p_u , which is set to the average job-finding probability of an unemployed worker in the baseline model. The wage is competitive that sets the profit of a vacancy to 0.

(ii) The “no-wealth” (the third column in Table 2): This is a model where workers are hand-to-mouth. As in the baseline model, employed and unemployed workers can search, search is directed, and workers are risk averse.

Appendix D in the supplementary appendix describes these benchmarks in more detail. In the baseline and the no-wealth model, all wage inequality is frictional in the sense that it is caused by search frictions. Since all workers have the same ability and preferences, and all jobs produce the same amount of output, all workers would have the same wage if search frictions were absent in the labor market as in the no-search model.

The mean-min wage ratio, proposed by Hornstein et al. (2011) as a measure of wage inequality, is the ratio of the average wage earned by an employed worker to the lowest wage in the equilibrium. Our model generates a mean-min wage ratio of 1.728. To put this number in perspective, it is useful to know that the mean-min wage ratio is no more than 1.04 in a variety of search models as demonstrated by Hornstein et al. (2011).¹⁸

Wage inequality in our model is higher for two reasons: (i) unemployed workers without unemployment insurance are willing to take low wages; and (ii) the search decision interacts with wealth. Ignoring wealth accumulation, as in the no-wealth model, results in lower frictional wage dispersion. As explained in section 4.2, on the one hand, the desire for self insurance motivates unemployed workers with low wealth to take jobs that pay low wages. This expands the left tail of the equilibrium wage distribution. On the other hand, as wealth increases, a worker becomes better insured and can take the chance of searching for higher wages. This expands the right tail of the wage distribution. It is worth noting that Krusell et al. (2010) also analyze search with wealth accumulation but they do not allow for on-the-job search. Their model generates a mean-min ratio of 1.02 when it is calibrated as in Shimer (2005).¹⁹ The presence of on-the-job search in our model is important for the higher dispersion, as examined further in Appendix B. Not allowing for on-the-job search reduces the mean-min ratio from 1.728 to 1.171.

Table 2 also reports Gini coefficients in wealth, earnings, income, and consumption.²⁰ The baseline model shows more earnings and income inequality than the no-search model. With no search, there is no dispersion in wages, and so earnings inequality is only driven by the uncertainty in whether workers are employed. Despite having higher inequality in earnings and income, the baseline model generates a lower ratio of the Gini in consumption to the Gini in earnings than the no-search model. This ratio can be interpreted as the proportion of earnings risks that are passed through to consumption. The pass-through

¹⁸Shi (2018) constructs a directed model of on-the-job search that can generate a mean-min ratio of 2.6. In that model, the cost of posting a vacancy is increasing and convex in the capital stock of the vacancy. In the equilibrium, firms create jobs with a low capital stock for unemployed workers and invest to increase the capital stock later.

¹⁹They get a mean-min ratio of 1.0002 if home production is calibrated as in Hagedorn and Manovskii (2008).

²⁰Since some households hold negative wealth we adjust the Gini coefficient in wealth following Chen, Tsaur and Rhai (1982).

rate is 19% higher in the no-search model. Thus, workers in the baseline model are able to shield consumption better from fluctuations in earnings than in the no-search model. This better insurance comes from workers' ability to choose which jobs to apply for as well as the amount of savings. Moreover, if workers are hand-to-mouth, as in the no-wealth model, then all earnings risks are passed through to consumption. Comparing this ratio in the no-search and the no-wealth models, we see that savings provide significant self insurance against the earnings risk. In the no-search model, the Gini coefficient in consumption is 45.8% of that in earnings. In other words, 54.2% of the earnings risk is self insured by accumulating wealth. The last row of table 2 shows the welfare loss of moving a population of individuals with a given initial distribution of employment status and wealth to each of the two benchmark economies. We measure welfare by the percentage change in lifetime consumption as described in equation (7) of section 5. Moving workers from our baseline economy to an economy where savings cannot be used as self-insurance reduces aggregate welfare by 33%. In addition, moving workers from our baseline economy to the "no-search", where individuals cannot use search as self-insurance, reduces welfare by 1%. Although the insurance benefits of job search decisions are significantly smaller than those of savings, they are still quantitatively very large. As a reference, the welfare gains of relaxing the borrowing limit by 10 months of the average monthly salary are just above 0.9%.^{21,22}

Table 2. Inequality measures (not targeted)

	Baseline	No-search	No-wealth	Wage-Tax
Mean-min wage ratio	1.728	1.000	1.368	1.701
Gini: wealth	0.621	0.513	—	0.620
Gini: earnings	0.053	0.038	0.107	0.052
Gini: income	0.055	0.043	0.107	0.054
Gini: consumption	0.021	0.019	0.107	0.021
$\frac{\text{Gini: consumption}}{\text{Gini: earnings}}$	0.384	0.458	1.000	0.387
Welfare comparison	0.000	-1.070	-33.09	—

We go beyond the statistics of inequality to examine the entire distributions of wages and wealth. Figure 4 shows the equilibrium density of wage earnings. The support of the distribution is endogenous. This support excludes the income y_{\min} and the interest income on assets. The red bar at zero earnings shows the mass of workers who are unemployed without unemployment insurance. The orange bars show the density of unemployed workers receiving unemployment insurance which is dispersed because it is proportional to the wage prior to unemployment. Each blue bar shows the mass of workers at an equilibrium wage, which ranges from approximately 0.55 to almost 1.²³ Although the optimal search target is unique given a worker's wealth and income, there is dispersion among each group

²¹See section 5.2.

²²The insurance benefits of job search are robust to increases in the amount of unearned income such as additional transfers from the government or family members. In particular, we consider an exogenous income $y_{min} = 0.3$ and find a significant welfare cost of eliminating job search as self-insurance.

²³The horizontal axis in figure 4 is not plotted to scale. It has two breaks at the horizontal dashed lines

of the blue bars because the searchers are originated in the dispersed group of workers situated immediately below. Start with unemployed workers depicted by the orange bars and the red bar. The differences in unemployment duration and the expiration of the unemployment benefit cause dispersion in income and wealth in this group. The ones with the highest wealth and income apply to the highest wage in the lowest group of the blue bars immediately above. As wealth and income fall, the target wage of search falls, since these workers attempt to increase the employment probability. Next, employed workers in the lowest group of the blue bars apply to the next group of the blue bars immediately above, each selecting a unique target wage according to wealth and income. In contrast to undirected search on the job, e.g., Burdett and Mortensen (1998), directed search implies that the search target is strictly higher than a worker’s current wage or income. This generates the gap between any two adjacent groups of bars. Moreover, most workers are employed at a few values of wages.

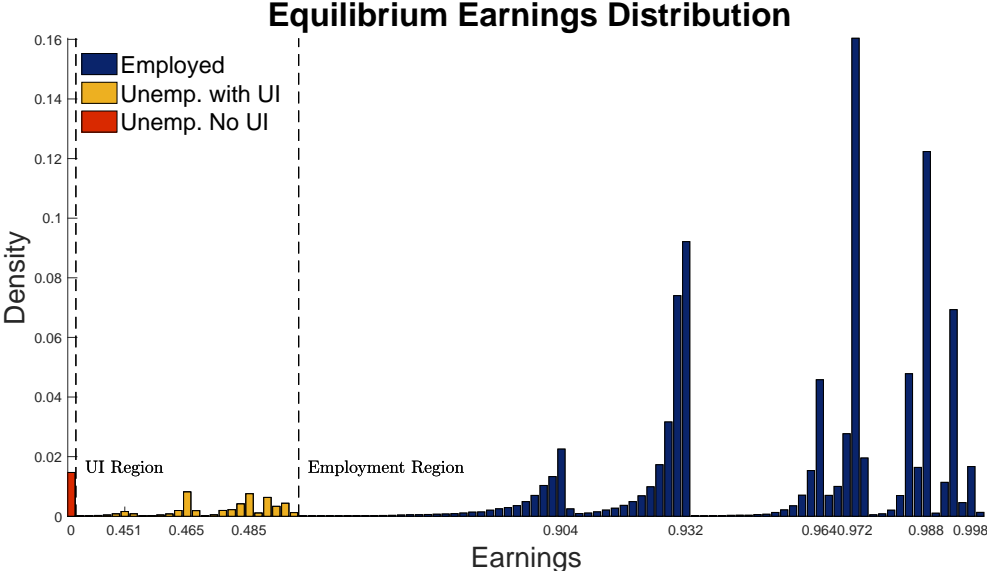


Figure 4. Equilibrium earnings distribution

Table 2 shows the fraction of workers that are accumulating and decumulating wealth according to their position with respect to the average asset holdings in equilibrium ($\bar{a} = 2.8$). The elements on the main diagonal of the table show the fraction of the population that push the wealth distribution towards the mean. The two elements off the diagonal show the fraction of the workers that are spreading the wealth distribution away from the mean. In total, 57.6% of the population is spreading wealth away from the mean, with about $\frac{3}{5}$ of them pushing the right tail of the distribution. In addition, 42.4% of the

because some levels of earnings are not in the support of the distribution. The dashed lines separate the region of workers’ earnings into three parts: (i) unemployment without insurance; (ii) unemployment with insurance (UI Region); and, (iii) employment (Employment Region).

population is contracting the distribution towards the mean.

Table 3. Fraction of workers accumulating and decumulating wealth

current wealth \ future wealth	$\hat{a} \leq a$	$\hat{a} > a$
$a > \bar{a}$	10.6%	36.0%
$a \leq \bar{a}$	21.6%	31.9%

The above behavior of savings affects the wealth distribution in equilibrium. Figure 5 plots the density function of wealth, in the baseline model and in the model with no search. Relative to the baseline, the no-search model generates higher average wealth and a wealth distribution less skewed to the left than the baseline. In particular, the fraction of people with negative wealth is smaller in the no-search model. These contrasts indicate that the option to modify job search is quantitatively important for wealth accumulation. Because workers do not have this option in the no-search model, they need to rely more on wealth as self-insurance. This increases the average wealth level and reduces the mass of workers whose wealth is close to the borrowing limit.

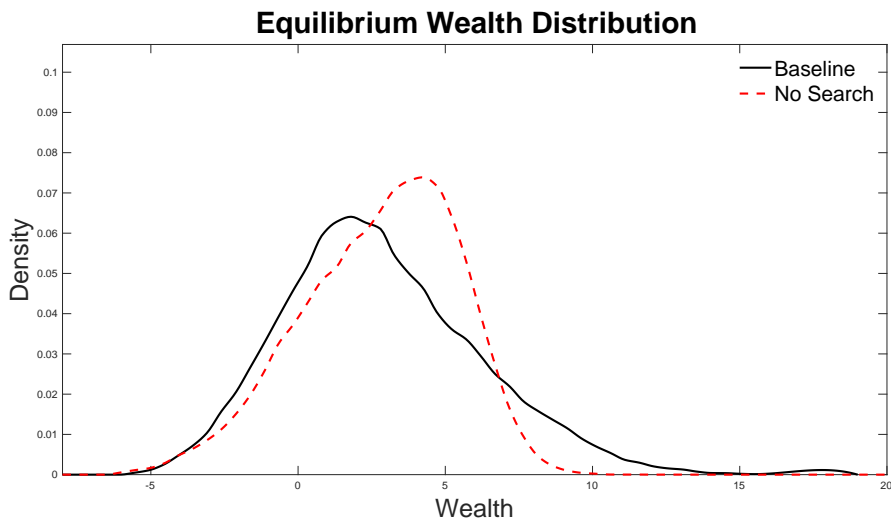


Figure 5. Equilibrium earnings distribution

In Appendix B we conduct counter-factual exercises. The results can be summarized as follows. First, shutting down on-the-job search (OJS) reduces the mean-min wage ratio from 1.728 to 1.171. OJS is important for wage dispersion by reducing the minimum wage in the equilibrium.²⁴ Second, without OJS, average wealth is higher than in the baseline

²⁴Allowing for wealth accumulation but not for on-the-job search, Krusell et al. (2010) find that the mean-min wage ratio is only 1.023, which is even smaller than 1.04 that Hornstein et al. (2011) find in models without wealth.

model. As job search decisions are less useful for consumption smoothing they need to stock more wealth for insurance. However, average wealth without OJS (but with unemployed search) is still lower and the wealth distribution is more skewed to the left than in the no-search benchmark (see Figure 9 in Appendix B).

To conclude this section, we echo the discussion in the Introduction on the importance of using an equilibrium model for the analysis, which determines the offer distribution of wages endogenously. To be specific, we discuss the paper by Lise (2013) who incorporates OJS to examine the effect of wealth on job search under an exogenously given offer distribution of wages. He estimates the offer distribution and other parameters by matching the model and the data on the observables such as the distribution of employed workers over wages and workers' transition rates. Consistent with the findings in Hornstein et al. (2011) we find that it is quantitatively challenging to generate the wage distribution observed in the data when it arises from firms' optimal decisions. In order to generate as much wage dispersion as in the data, Lise (2013) leaves the value of an unemployed worker's home production, b , to be estimated rather than calibrated. The estimated value is $b = 0$, in contrast to $b = 0.5w_{-1}$ in our calibration. Thus, as other partial-equilibrium models, Lise's (2013) model generates large dispersion in employed wages by forcing the value of home production to be unrealistically low.²⁵ Moreover, since search is undirected in Lise (2013), the value $b = 0$ implies that unemployed workers who differ in wealth have the same transition rate into employment and accept the same distribution of wages. These implications are inconsistent with the empirical evidence discussed in the Introduction on how wealth affects unemployment duration. Furthermore, a partial-equilibrium model is not suitable for policy analysis, to which we turn in the next section.

5 Welfare Analysis

Our baseline model allow workers to use a number of tools to insure against earnings risk. Namely, workers have access to an unemployment benefit scheme, access to savings and credit, and they can modify their job search. To the best of our knowledge this is the first paper in the literature to allow jointly for all these source of insurance together with on-the-job and allowing the distribution of wages to be determined in equilibrium. It is important then, to quantitatively assess the relevance of credit frictions and unemployment insurance in the presence of all these ingredients.

In this section, we quantify the welfare effects of varying the extent to which these tools can be used to smooth consumption. In particular, we study the effects of changing the borrowing limit, the replacement rate and the expected duration of the unemployment benefit. We take advantage of block recursivity to compute the dynamic effect of policy changes. That is, including the transition dynamics between an initial and a final steady states. In addition we compute the steady-state effect, which provide a north for the ideal policy if transition costs were not to be paid.

²⁵Hornstein et al. (2011) make the same observation.

5.1 Welfare measure and the computational method

We measure the welfare effect in terms of the percentage change in consumption in the lifetime, as in Lucas (1987). In all experiments, case 0 refers to the benchmark and case 1 to the case after the change. Let $[c_{i,t}^j]_{t=0}^{\infty}$ be the consumption path of individual i in case $j \in \{0, 1\}$. When the economy moves from case 0 to case 1, the welfare effect on individual i , denoted μ_i , is given by

$$\sum_{t=0}^{\infty} \beta^t u(c_{i,t}^0 (1 + \mu_i)) = \sum_{t=0}^{\infty} \beta^t u(c_{i,t}^1). \quad (6)$$

The welfare measure μ_i is the percentage increase in lifetime consumption that individual i obtains when the economy moves from case 0 to case 1. Similarly, the aggregate welfare effect, denoted as $\bar{\mu}$, is defined as follows:

$$\int_0^1 \sum_{t=0}^{\infty} \beta^t u(c_{i,t}^0 (1 + \bar{\mu})) \, di = \int_0^1 \sum_{t=0}^{\infty} \beta^t u(c_{i,t}^1) \, di. \quad (7)$$

A policy can change the sum of unemployment insurance. To compare welfare in a meaningful way, it is necessary to require unemployment insurance to be financed. We assume that the government uses a proportional wage income tax rate, τ_w , to finance the unemployment benefit in the present value. Given the tax rate, the tax revenue needed to finance the unemployment benefit depends on the distribution of workers over wages and the unemployment rate. This dependence makes the equilibrium fail to be block recursive, because the tax rate affects households' decisions on consumption, savings and search. Despite this failure, we compute equilibrium dynamics exactly rather than using the typical method of approximation in Krusell and Smith (1998). It is important to compute equilibrium dynamics instead of only the steady state. As shown later, the welfare effect of a policy along the transitional path differs significantly in the magnitude, and sometime in the sign, from the steady-state effect.

Given this importance of the dynamic equilibrium, it is helpful to explain how we compute such an equilibrium without encountering the typical problem of dimensionality of the state space. The tractability comes from the property that the equilibrium is block recursive for any *given* tax rate. Exploring this property, we iterate on the tax rate. Starting with an arbitrarily given tax rate, we compute the dynamic equilibrium without imposing the requirement of a balanced budget for the government. Since the tax rate is treated as a parameter in this step, the equilibrium is block recursive as defined in section 3.1 and can be computed relatively quickly. Then, we calculate the present value of the government surplus. If this surplus is not zero, we adjust the tax rate and recompute the equilibrium until the government's intertemporal surplus becomes zero.

5.2 Welfare effect of changing the borrowing limit

The presence of the borrowing limit reduces the extent of consumption smoothing. How much better off would workers be if their borrowing limit were relaxed? To answer this

question we take the steady state in the baseline model with $\underline{a} = 0$ as case 0. Then, we relax the borrowing limit to a new level $\underline{a} < 0$, which is case 1.²⁶ This change is treated as a permanent shock. We compute the transitional dynamics and assess the welfare effect. The tax rate τ_w is adjusted to balance the government budget intertemporally.

Figure 6 shows the welfare effects measured by the aggregate welfare change as defined in (7), $\bar{\mu}$. The horizontal axis shows the relaxed borrowing limit. The vertical axis shows welfare changes in percentage points. The vertical dashed line signals the borrowing limit in our baseline calibration of the model. Relaxing the borrowing limit increases welfare monotonically. In terms of magnitudes, moving from an economy to with $\underline{a} = 0$ to one with $\underline{a} = -5.90$ (51% of the average annual income), as in the baseline calibration, increases welfare by approximately 0.8%.

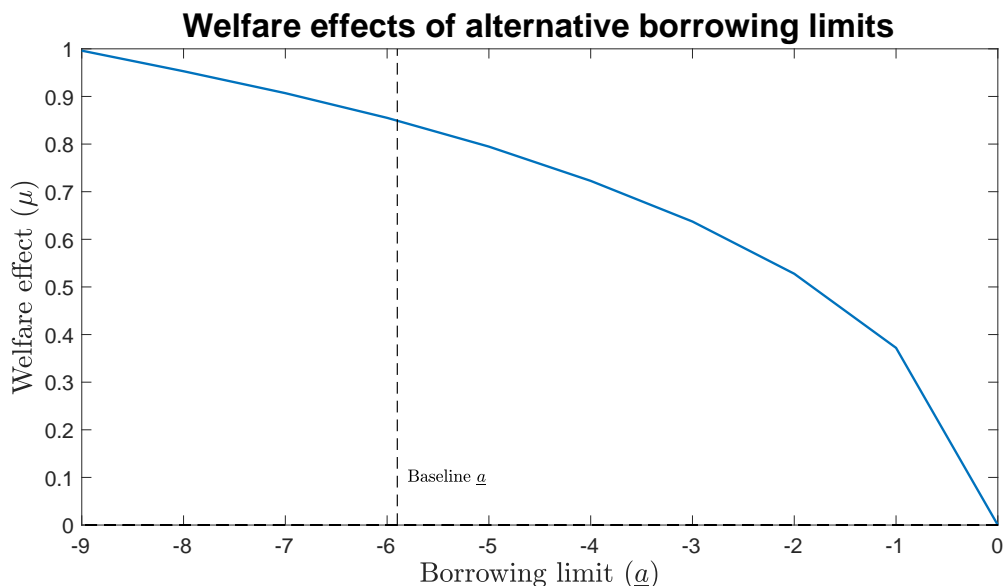


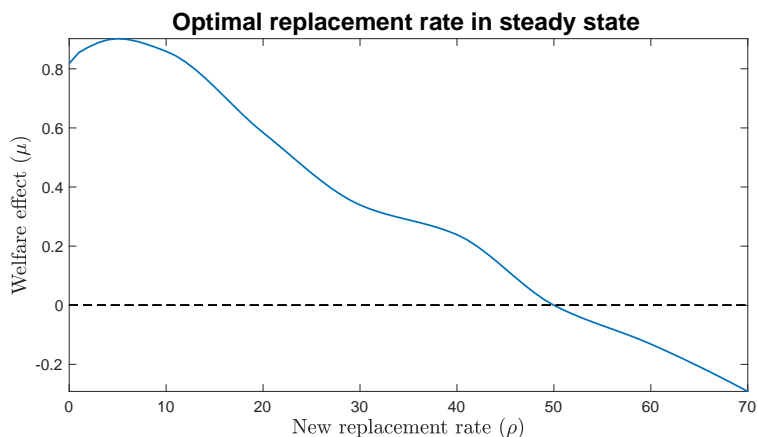
Figure 6. Welfare consequences of changes in the borrowing limit.

All welfare gains from relaxing the borrowing limit occur during the transition as the extra borrowing capacity enables workers to maintain higher consumption in bad times. Once the economy reaches the new steady state, workers have reduced their average asset holdings and exhausted the benefits of moving to the economy with a larger borrowing limit. In fact, the steady-state welfare effect of relaxing the borrowing limit is negative. Thus, correctly assessing welfare gains of relaxing borrowing constraints requires precise computation of the transition to the new steady state, as we do here.

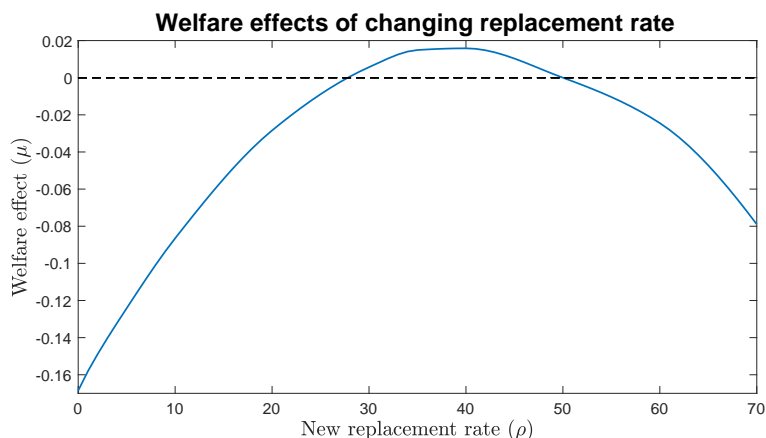
²⁶It is also meaningful to conduct the experiment in the opposite direction by treating the steady state of an economy with $\underline{a} < 0$ as case 0 and tightening the borrowing limit. However, such tightening of the limit imposes negative consumption on individuals who are constrained by the borrowing limit in the initial steady state.

5.3 Optimal Unemployment Insurance

We examine the welfare effects of changing the generosity and the expected length of unemployment insurance. The generosity of the benefit is parameterized by the replacement rate ρ and the expected length of the benefit by $1/\chi$, where χ is the probability that the benefit expires. As in the previous exercise, the tax rate τ_w adjusts to balance the government budget intertemporally.



(a) Without Transitions



(b) Including Transitions

Figure 7. Welfare effects of changes in the replacement rate.

Figure 7 shows the effects of changing the replacement rate ρ from the baseline value, 0.5. Panel (a) depicts the welfare effect comparing across steady states, and panel (b) depicts the welfare effect when transitional dynamics are taken into account. In both panels the horizontal axis is the value of ρ to which the policy changes from 0.5, while the vertical axis is the welfare change in percentage points.

Steady state comparisons (panel (a)) show that the optimal replacement rate is $\rho \approx$

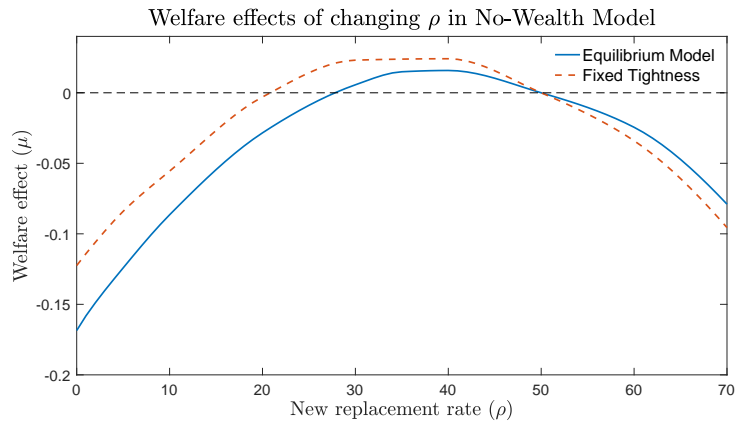
0.05, with $\tau_w = 0.25\%$. Moving from $\rho = 0.5$ to $\rho = 0.05$, aggregate steady-state welfare increases by approximately 0.9%. In the steady state, workers prefer to receive a very small amount in unemployment insurance in order to pay a low tax rate on wages. Instead of unemployment insurance, they prefer to rely on wealth accumulation and job search decisions as self-insurance mechanisms. Relative to the baseline with $\rho = 0.5$, the new steady state with $\rho = 0.05$ has more job creation since firms can pay lower gross wages and yet deliver higher after-tax wages.

The dynamic effect of ρ differs significantly from the steady-state effect. Taking transitional dynamics into account, panel (b) in Figure 7 shows that aggregate welfare is maximized at $\rho \approx 0.38$ with an associated wage tax of $\tau_w = 1.98\%$. Moving from the baseline calibration to this optimal replacement rate increases aggregate welfare by approximately 0.02%. Although this optimal replacement rate is lower than in the baseline calibration, it is significantly positive. Why do not workers want to move all the way to an economy with $\rho = 0.05$ which is optimal if only the steady state is concerned? The reason is that to build enough insurance to compensate for the zero replacement rate, workers have to accumulate wealth and apply to lower wages over the transition path. Although employed workers pay lower wage taxes if $\rho = 0.05$, consumption is lower for a sufficiently long time while workers accumulate enough wealth to self-insure against fluctuations in labor earnings. In the steady state with $\rho = 0.38$, the aggregate level of wealth is approximately 23% larger than in the steady state under the baseline calibration. In contrast, in steady state with $\rho = 0.05$, aggregate wealth is approximately two times as much as in the steady state in the baseline calibration. Taking into account the transition cost of accumulating assets significantly reduces the welfare gain from lowering the replacement rate.

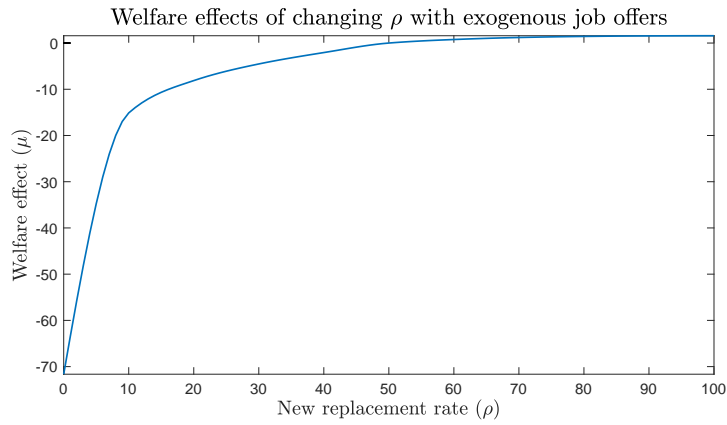
In addition, we consider changes in the expected length of unemployment insurance by changing the expiration probability χ and adjusting the wage tax rate τ_w to keep the government budget balanced intertemporally. The main message of this exercise is similar to the previous one. Steady-state comparisons show that the optimal probability of losing unemployment insurance in each period is $\chi = 1$. In the steady state, workers prefer to pay as low taxes as possible and accumulate wealth to self-insure against unemployment risks. In contrast, taking into account transitional dynamics, the optimal probability of losing unemployment insurance in each period is $\chi \approx 0.4$, which is higher than in the baseline calibration ($\chi = 0.197$) but less than one. The value $\chi = 0.4$ implies that the optimal expected duration of the unemployment benefit, $1 + \frac{1}{\chi}$, is 15 weeks instead of 26 weeks as in the baseline calibration. Moving from $\chi = 0.197$ to $\chi = 0.4$, aggregate welfare increases by approximately 0.04%.

We conclude this section by comparing our welfare results in the baseline model to what would be obtained in alternative scenarios. Figure 8 summarizes the results. In all calculations we include the effects of transitions and the government budget is balanced intertemporally.

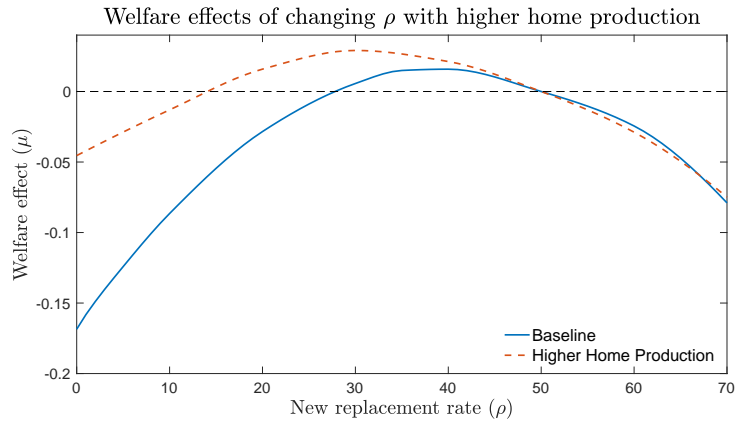
To illustrate the importance of using an equilibrium model, panel (a) compares our results for the baseline model (blue solid line) to a model where the distribution of wages is determined exogenously (red dashed line). More precisely, we first compute the market



(a) Exogenous distribution of wages



(b) No-Wealth model



(c) High Home Production

Figure 8. Optimal replacement rate in alternative scenarios.

tightness for each submarket in our baseline model under the baseline calibration. Then, we calculate the welfare effects of changes in ρ but keeping the tightness function fixed as in the baseline calibration. That is, we do not let firms adjust their policy functions to respond to changes in the behavior of workers. In other words, we simulate a partial equilibrium model where the demand side for workers is exogenously given. The red-dashed line shows that the model with exogenous wage distribution over-estimates the value of reducing the replacement rate. This is because after the reduction in ρ workers accumulate more wealth and are willing to apply for relatively lower wages than with $\rho = 0.5$. Therefore, firms should offer more vacancies tailored to workers with larger savings and aiming for lower wages but they are not allowed to do so. As a result, there is an excess of positions in some submarkets and a lack of openings in some others.

To highlight the importance of allowing for wealth accumulation, in panel (b) of figure 8, we plot the welfare effects of changes in the replacement rate in the "No-Wealth" model. In this case, workers can still use job search decisions as self-insurance but they cannot manage their asset holdings. As a result, welfare increases monotonically from a $\rho = 0$ to a $\rho = 1$. That is, it would be optimal to have a 100% replacement rate.²⁷ Absent wealth as an insurance mechanism, the welfare effect of increasing the size of the unemployment insurance program are opposite to those in the baseline model. Moreover, welfare losses of eliminating unemployment insurance are as large as 33%.

Finally, in panel (c) of figure 8, we consider the possibility of allowing for a larger value of unearned income y_{min} . We set $y_{min} = 0.3$, re-calibrate the model to match the same targets as in the baseline model, and then consider the effects of modifying ρ . The larger value of unearned income could be interpreted as the value of expenditure savings due to home production or other sources of earnings and transfers unrelated to workers' wage earnings and capital income.²⁸ The blue solid line repeats the baseline results while the red-dashed line shows the welfare effects of changes in ρ under $y_{min} = 0.3$. A larger value y_{min} reduces the earnings risk face by the possibility of unemployment as workers have an additional source of income. As a result, the optimal replacement rate would be 30%, lower than in our baseline case.²⁹ In that sense, our results can be understood as an upper bound for the optimal replacement rate if we took into account additional sources of income not considered in our analysis.

All in all, our analysis suggests that when we take into account job search and wealth accumulation as self-insurance, there is a limited role for unemployment insurance programs. The main reason to keep the size of the replacement rate at around 38% is not its long run benefit in terms of consumption smoothing but the transition costs that a reduction in ρ induces. Considering additional sources of unearned income and potential moral hazard costs of unemployment benefits would result in a lower optimal level for the

²⁷Notice that since unemployment benefits expire with positive probability the optimal replacement rate could be higher than 100%.

²⁸Ganong and Noel (2019) find that on average unemployed workers who exhausted unemployment insurance, receive deposits in their checking accounts for a value of about 30% of their previous wage earnings, which motivates the choice of $y_{min} = 0.3$ in this scenario.

²⁹The steady state that delivers the highest welfare in this case is achieved when $\rho = 0$.

replacement rate.

6 Conclusion

We have studied a search equilibrium with risk-averse workers who can accumulate non-contingent assets under a borrowing limit. Workers can search both on and off the job, and search is directed. Workers with high wealth optimally choose to search for high wages and, in the case of failing to match, they decumulate wealth to smooth consumption. Workers with low wealth optimally choose to search for low wages in an attempt to get a job quickly to build up wealth. Firms create vacancies and offer different wages to cater to these differential needs of workers. In the other direction, search frictions affect wealth accumulation by generating heterogeneity in search outcomes and earnings. After calibrating the model, we have found that wealth significantly reduces worker transitions in the labor market. The baseline equilibrium generates significantly higher inequalities in earnings and wealth than both the model without search and the model without wealth. However, the interaction between wealth and search provides important self-insurance against earnings risk as it reduces the pass-through of earnings inequality into consumption by more than 60%. Moreover, we have analyzed welfare effects of the borrowing limit and unemployment insurance. Steady state comparisons call for very large reductions in unemployment insurance programs. However, the transitional costs of accumulating wealth and searching for lower wages offset part of the benefits of such reductions. We obtain that the optimal replacement rate, keeping the expected duration of unemployment benefit in 26 weeks, should be around 38%. Finally, we showed that if workers have access to some source of unearned income the optimal replacement rate would be significantly lower.

At least two extensions of this model are worth pursuing. First, firms can post dynamic contracts instead of a fixed wage. With dynamic contracts, firms have incentive to back-load wages to increase retention, as analyzed by Burdett and Coles (2003) and Shi (2009). This force can stretch the upper tail of the wage distribution. Also, an unemployed with low wealth may be willing to accept even lower wages than in the baseline model in the expectation of wage increases in a contract. This force can stretch the lower tail of the wage distribution. Although both forces can widen frictional wage dispersion, their quantitative importance is yet to be determined. Moreover, shocks to match-specific productivity and/or work effort can be introduced as in Tsuyuhara (2016) and Lamadon (2016). Second, extending the model to study the business cycle seems a natural exercise. When aggregate shocks are present, workers may have further incentives to accumulate assets to smooth consumption. The computational advantage provided by block recursivity of the baseline equilibrium makes it tractable to study business cycles.

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Online Appendix for “Wealth Accumulation, On-the-Job Search and Inequality”

Gaston Chaumont and Shouyong Shi

A Proof of Lemma 1

Maintain assumptions (i) and (ii) in the lemma. We first prove that whenever a worker succeeds in a match, the worker’s wealth at the time of the match can be inferred from the offer wage \hat{w} and the worker’s current job $\eta = (\varepsilon, \omega)$. Then, we prove that after being matched with a worker, a firm can perfectly compute the worker’s future wealth and search targets in the entire duration of the worker’s employment with the firm. This implies that the firm value of a job filled at wage \hat{w} with a worker η is a function of (\hat{w}, η) only. With this result, competitive entry of vacancies into submarkets determines the tightness in each submarket as a function of only (\hat{w}, η) , thus closing the loop of the argument.

Consider a worker with the current job $\eta = (\varepsilon, \omega)$ who chooses to search for wage \hat{w} . For this choice to be the optimal, it must be given by the policy function of optimal search, i.e., $\hat{w} = \tilde{w}_\varepsilon(\omega, \hat{a})$, where \hat{a} is the wealth level at the time of search. Because the policy function $\tilde{w}_\varepsilon(\omega, \hat{a})$ is assumed to be one-to-one for any given ω , its inverse for wealth exists for any η . Write this inverse as $\hat{a} = h(\hat{w}, \eta)$ for some function h .

Now consider a match between a firm offering \hat{w} and a worker from a job $\eta = (\varepsilon, \omega)$. In period τ on the new job, let a_τ be the worker’s wealth at the beginning of the period and w_τ the wage searched by the worker. The firm does not directly observe w_τ , unless the worker succeeds in the new search. The firm does not directly observe a_τ either. However, the firm can infer (a_τ, w_τ) perfectly as follows. Start at $\tau = 1$. The worker’s wealth at the beginning of the period is the same as the wealth level at the time of forming the new match in the previous period. By the calculation above, this is equal to $a_1 = h(\hat{w}, \eta)$. After consumption in period 1 on the new job, the worker’s wealth will be $\hat{a}_1 = \hat{a}_e(\hat{w}, a_1)$. Since this is also the wealth that the worker will bring into the next period if the worker stays at the job, then $a_2 = \hat{a}_e(\hat{w}, a_1)$. In period 1, the worker optimally searches for wage $w_1 = \tilde{w}_e(\hat{w}, a_2)$. Substituting a_1 , we have $a_2 = h_2(\hat{w}, \eta) \equiv \hat{a}_e(\hat{w}, h(\hat{w}, \eta))$ and $w_1 = W_1(\hat{w}, \eta) \equiv \tilde{w}_e(\hat{w}, h_2(\hat{w}, \eta))$. Moving to $\tau = 2$, the worker’s wealth after consumption will be $a_3 = \hat{a}_2 = \hat{a}_e(\hat{w}, a_2)$ and the optimal search target will be $w_2 = \tilde{w}_e(\hat{w}, a_3)$. These can be rewritten as $a_3 = h_3(\hat{w}, \eta) \equiv \hat{a}_e(\hat{w}, h_2(\hat{w}, \eta))$ and $w_2 = W_2(\hat{w}, \eta) \equiv \tilde{w}_e(\hat{w}, h_3(\hat{w}, \eta))$. By induction, in any arbitrary period τ on the job, the worker’s wealth after production and consumption will be $a_{\tau+1} = h_{\tau+1}(\hat{w}, \eta) \equiv \hat{a}_e(\hat{w}, h_\tau(\hat{w}, \eta))$ and the optimal search target will be $w_\tau = W_\tau(\hat{w}, \eta) \equiv \tilde{w}_e(\hat{w}, h_{\tau+1}(\hat{w}, \eta))$. Because the functions $\hat{a}_e(\hat{w}, a)$ and $\tilde{w}_e(\hat{w}, a)$ are assumed to be single-valued, the sequence $\{(a_\tau, w_\tau)\}_{\tau \geq 1}$ is uniquely calculated from the worker’s current wage \hat{w} and the previous job η .

With the above inference on the worker’s future wealth and search targets, the firm can calculate the value of a job at wage \hat{w} filled by an applicant whose current job is $\eta = (\varepsilon, \omega)$. At the beginning of any arbitrary duration τ in the new match, the firm value is

$$J_{alt}(\hat{w}, \eta, \tau) \equiv J(\hat{w}, h_\tau(\hat{w}, \eta)),$$

where the subscript *alt* indicates an alternative expression for J . The dependence of J_{alt} on the worker's previous job η and the duration τ on the current job does not mean that the submarket searched by the firm's employee in period τ is indexed by (η, τ) . Rather, this submarket is indexed by only the offer wage w_τ and the worker's current job $\hat{\eta} \equiv (e, \hat{w})$. Firms recruiting in submarket $(w_\tau, \hat{\eta})$ know that an applicant's wealth is $a_{\tau+1} = h(w_\tau, \hat{\eta})$, where $\hat{\eta} = (e, \hat{w})$. The expression J_{alt} is only for the current employer's internal calculation of the value of a filled job. To compute J_{alt} , note that submarket $(w_\tau, \hat{\eta})$ has tightness $\theta_{alt}(\hat{w}, \eta, \tau) \equiv \theta(w_\tau, a_{\tau+1})$, where $w_\tau = W_\tau(\hat{w}, \eta)$ and $a_{\tau+1} = h_{\tau+1}(\hat{w}, \eta)$. The function J_{alt} obeys the following Bellman equation:

$$\begin{aligned} & J_{alt}(\hat{w}, \eta, \tau) \\ &= y - \hat{w} + (1 - \delta) [1 - \lambda_e p(\theta_{alt}(\hat{w}, \eta, \tau))] \beta J_{alt}(\hat{w}, \eta, \tau + 1). \end{aligned}$$

In particular, the firm value immediately after filling the job is $J_{alt}(\hat{w}, \eta, 1)$, which is a function of only (\hat{w}, η) .

Finally, in submarket (\hat{w}, η) , competitive entry of vacancies implies:

$$q(\theta) J_{alt}(\hat{w}, \eta, 1) \leq k \text{ and } \theta \geq 0,$$

where the two inequalities hold with complementary slackness. This condition determines the tightness θ as a function of only the offer wage \hat{w} and the applicants' current job $\eta = (\varepsilon, \omega)$. **QED**

B Counter-factual: Shutting down on-the-job search

We shut down on-the-job search (OJS) by setting $\lambda_e = 0$ to show that OJS is important for frictional wage dispersion. We recalibrate the model to match the targets in Table 1, provided that they remain valid, except for two parameters. We keep the values of borrowing limit, \underline{a} , and the discount factor, β , as in the baseline calibration. These two parameters are key determinants of the wealth distribution as the distance of the interest rate and discount rate determines incentives to save, and the borrowing limit determines how much credit workers can access. The main change is in the vacancy cost k . The recalibrated value of k and other results of the counter-factual exercise are reported in Table 4.

When on-the-job search (OJS) is shut down, the mean-min wage ratio falls significantly from 1.728 to 1.171. This result shows that most of the frictional wage dispersion in the baseline model comes from on-the-job search. On-the-job search widens wage dispersion mainly by reducing wages that unemployed workers choose to search for. Unemployed workers are willing to lower their search target in the expectation that they can search for higher wages after being employed.

The wealth distribution also changes. Figure 9 shows the density functions of wealth in the baseline model, the model with no OJS and the model with no search. Relative to the baseline, the wealth distribution with no OJS shifts its mean to the right and concentrates around its mean. As job search decisions are less useful as an insurance mechanism, workers need to accumulate on average larger amounts of wealth to insure against unemployment

Table 4. Results of counter-factual exercises

	Baseline	No OJS ($\lambda_e = 0$)	Data
Vacancy cost k	0.32	0.43	—
Unemployment rate	6.50%	6.50%	6.5%
UE transition rate	2.6%	2.6%	2.6%
EE transition rate	2.9%	0%	2.2-3.2%
Elasticity of p_u to θ_u	0.32	0.32	0.27-0.50
Mean-min wage ratio	1.728	1.171	1.7-2.0
Gini: wealth	0.621	0.584	0.80
Gini: earnings	0.05	0.04	0.61
Gini: income	0.05	0.04	0.55
Gini: consumption	0.02	0.02	0.25

risk. In addition, as there is less equilibrium wage dispersion and workers cannot climb the job ladder, there are no workers with strong incentives to accumulate wealth as in the baseline model. This is reflected in a thinner right tale of the wealth distribution.

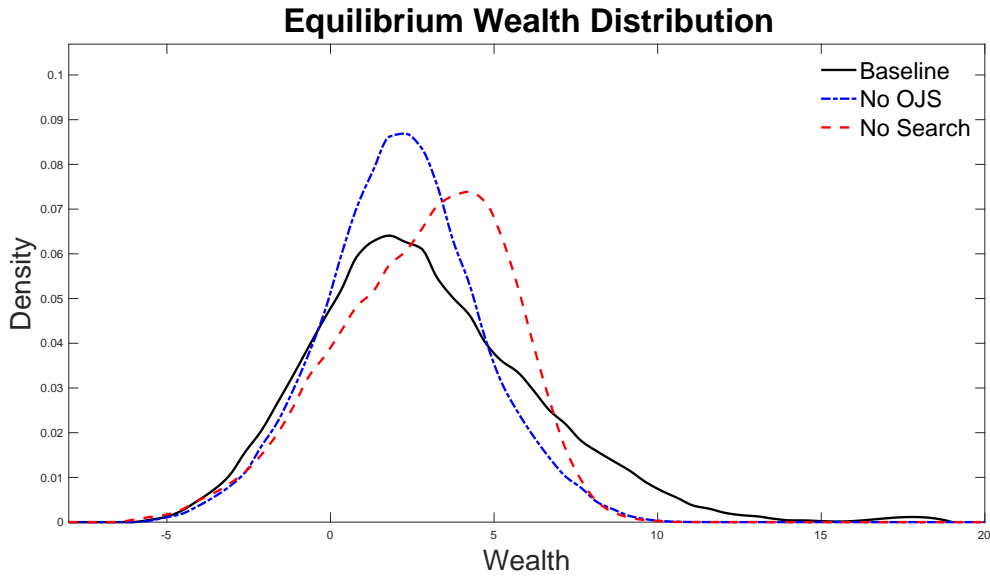


Figure 9. Wealth distribution: baseline vs. two benchmarks.

C Solution Algorithm and Simulation

We solve the model using a nested fixed-point algorithm. First, we define the grids for the spaces of wealth and wages. Wealth is set to be $[\underline{a}, \bar{a}] = [0, 75]$, which is larger than

the equilibrium support of the wealth distribution. Wages are bounded between the home production level b and the total flow of productions per period y . In equilibrium, there will be no wages outside those bounds. The steps of the algorithm are as follows:

- (1) Guess initial value functions of workers and firms.
- (2) Given the value function of firms, solve for the market tightness that is consistent with competitive entry of firms.
- (3) Given the tightness function, solve a worker’s optimization problem and compute optimal savings, consumption, and job search decisions. This step is done by iterating on the worker’s value function until convergence, using the shape-preserving Cubic Hermite interpolation to calculate the policy functions.
- (4) Given workers’ policy functions computed in (3), calculate separation rates of employed workers and update the value function of the firm in each submarket.
- (5) Iterate on the value function of the firm until convergence.

Once the value and policy functions are solved, we simulate the model to obtain the distribution of workers, using $N = 100,000$ workers and $T = 3,100$ time periods (months). We discard the first 2,100 periods to avoid dependence on initial conditions. The average of the last 100 periods of the simulations is used to calculate the stationary distribution.³⁰ To simulate the evolution of workers’ states, we take random draws for separation shocks, the search opportunity (consistent with λ_e) and matching shocks that determine which workers in each submarket are matched. Also, we start the economy by assigning a random state to each of the N workers. Then, we use the equilibrium optimal policy functions and these random shocks to compute the endogenous evolution of the state of each individual.

D Benchmark Models

In this appendix we present two benchmark models to compare with the baseline model in section 2. The first benchmark is a no-search model where search frictions do not exist in the labor market but workers are still exposed to employment uncertainty and a borrowing limit. This model is similar to the model in Aiyagari (1994). The second benchmark is a no-wealth model where workers are hand-to-mouth and search frictions exist in the labor market. This model is the standard model of directed search augmented with on-the-job search and risk aversion.

D.1 No-search model

In this benchmark model, the search stage in each period is replaced by an *iid* employment shock that exogenously determines the employment status of a worker. The shock makes an unemployed worker employed with the probability $p_u \in (0, 1)$ and keeps the worker unemployed with the probability $1 - p_u$. The probability p_u is set to be equal to the average job-finding probability of an unemployed worker in the baseline model. As in the baseline model, a job is hit by an *iid* separation shock with the probability $\delta \in (0, 1)$, in which case the worker becomes unemployed. All employed workers earn the wage \bar{w} that

³⁰The transitions of the wealth distribution might be slow to converge so we need to let the economy run for enough periods to let it converge to the steady state.

sets the profit of a vacancy to zero, and they do not change jobs. There is competitive entry of vacancies, and the vacancy cost is k per period. Individuals face a borrowing limit.

For a worker with the labor market status $\varepsilon \in \{e, u\}$, the optimal decisions on consumption and savings still solve (1) that induces the value function $V_\varepsilon(\omega, a)$. For an unemployed worker with (b, \hat{a}) immediately before the employment shock, the value $R_u(b, \hat{a})$ now is

$$R_u(b, \hat{a}) \equiv p_u V_e(\bar{w}, \hat{a}) + (1 - p_u) [\chi V_u(0, \hat{a}) + (1 - \chi) V_u(b, \hat{a})]$$

Since an employed worker does not search on the job, the worker value after consumption and savings is given as $R_e(\bar{w}, \hat{a})$.

Because employed workers do not search on the job, the probability of a match being destroyed is independent of the employee's wealth. This implies that the firm value of a filled job is independent of the employee's wealth and obeys:

$$J(\bar{w}) = y - \bar{w} + (1 - \delta) \beta J(\bar{w}).$$

Competitive entry of vacancies requires $\beta J(\bar{w}) = k$. Solving $J(\bar{w})$ from the above equation, we can determine the competitive wage by

$$\frac{y - \bar{w}}{1 - (1 - \delta) \beta} = \frac{k}{\beta}.$$

An equilibrium can be defined by adapting the definition in section 3.1. In particular, the market tightness function is irrelevant and the wage rate is given above.

D.2 No-wealth model

In this benchmark, workers cannot accumulate wealth, and so consumption is equal to earnings. As in the baseline model, there are search frictions, workers can search off and on the job, and search is directed. There is competitive entry of vacancies into submarkets. The timing of events in a period is the same as in the baseline model. Since $a = \hat{a} = 0$, we eliminate (a, \hat{a}) from all value functions, policy functions, and the market tightness function. With these changes, the optimal choices of consumption and savings solve (1) and induce the value function $V_\varepsilon(\omega)$. For an unemployed worker, optimal choices of search and participation solve (2) that yields the value $R_u(b)$. For an employed worker, the optimal choice of search solves (3) that yields the value $R_e(w)$. The firm value of a filled job is given by (4) and the tightness of each submarket by (5), with the modified notation. An equilibrium can be defined by adapting the definition in section 3.1.

E Savings Policy Function for All (ω, a)

For the baseline model, we depict the savings policy function for all levels of equilibrium earnings and wealth. In Figure 10, the green dashed line is the savings policy of unemployed workers without unemployment insurance, expressed as $\hat{a}_u(0, a) - a$. The red shaded area is savings policies of unemployed workers receiving unemployment insurance, b , at all

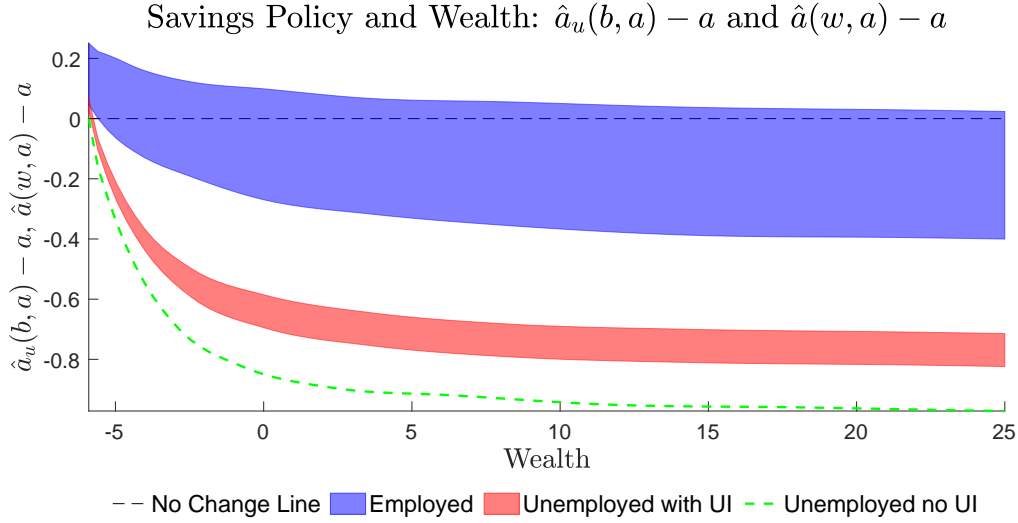


Figure 10. Equilibrium saving policy.

equilibrium values of b , expressed as $\hat{a}_u(b, a) - a$. The blue shaded area is savings policies of employed workers at all equilibrium wages, expressed as $\hat{a}(w, a) - a$.

Unemployed workers decumulate wealth at almost all wealth levels, since $\hat{a}_u(b, a) - a < 0$ for all (b, a) , except for a small fraction of unemployed workers with unemployment insurance that are very close to the borrowing limit. Those few unemployed workers accumulate some wealth in case they lose their unemployment insurance before they find a new job. Most unemployed workers reduce their asset positions to smooth consumption under the expectation of finding a job and getting an increase in income in the future. In contrast, a large part of the blue shaded area in Figure 10 lies above zero, which means that many employed workers accumulate wealth. The savings motive comes from the precaution for exogenous separation into unemployment. In the equilibrium, even the lowest wage of an employed worker is much higher than home production in unemployment. Losing such high earnings represents a large risk which an employed worker wants to insure against by savings. This motive of precautionary savings is particularly strong when an employed worker has low wealth. Even when wealth is high, an employed worker still accumulates wealth if earnings are high. An employed worker decumulates wealth when wealth is high and earnings are low. In this case, the motive of reducing wealth to smooth consumption dominates the motive of precautionary savings. This is because for those workers it is more likely to find a better job in the future than to be separated into unemployment.