Why Don't Elite Colleges Expand Supply?*

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Abstract

While college enrollment has more-than doubled since 1970, elite colleges have barely increased supply, instead reducing admit rates. We show that straightforward reasons cannot explain this behavior. We propose a model where colleges compete on prestige, measured using relative selectivity or relative admit rates. A key comparative static of the model is that higher demand decreases [increases] the admit rate when the weight on prestige is above [below] a critical value, consistent with experience in elite [non-elite] colleges. A calibrated version of the model closely replicates the pattern in the data of declining admit rates at elite colleges while counter-factual simulations without prestige fail. Prestige competition is inefficient. Allowing elite colleges to collude on admissions strategy internalizes the non-pecuniary prestige externality and is Pareto improving.

Keywords: College Education, Supply, Prestige, Admission Rate

JEL Codes: I2, L1, H23

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1 Introduction

In 1979, the incoming class of Yale College freshmen stood at 1,346 students (Waters, 2001). In 2015, the size of the incoming class of Yale College freshmen stood at 1,360 students, an increase of just 14 students. Over the same period, the number of applications to Yale College increased by over 300 percent, from 9,331 students in 1979 to 30,932 in 2015.[1] Across elite colleges, the story is the same—increasing demand for spaces but with only a small increase in supply. In contrast, less elite colleges have largely expanded supply in response to increasing demand.

The large growth in the demand for U.S. college education has been well documented, reflecting growing demand by international students (Bound et al., 2016; Li, 2017) and U.S. women (Goldin, Katz, and Kuziemko, 2006; Malkiel, 2016) as well as increasing returns to education (Katz and Murphy, 1992; Blackburn and Neumark, 1993; Goldin and Katz, 2009; Baum, Ma, and Payea, 2010), including at elite colleges (Hoxby, 1998; Dale and Krueger, 2002, 2011; Long, 2008; MacLeod, Riehl, Saavedra, and Urquiola, 2017).[2]

Consistently, Figure 1 on page 2 shows that *applications* across most colleges have nearly doubled since 2003, the first year in which this data was tracked by the Integrated Postsecondary Education Data System (IPEDS).[3] Notice that applications to highly selective colleges, with selectivity measured by the average SAT score of the students matriculating the college in 2015,[4] grew only slightly faster relative to non-elite colleges. Our model presented later produces this interesting fact, which balances two effects in application strategy: applying to more schools in the face of falling admit rates (Bound et al., 2009) and an increase in the quality threshold for applicants applying to elite colleges in the presence of fixed application costs (Chade et al., 2014).

Supply-side dynamics, the main focus of the current paper, has received less attention than the demand side in the previous literature. The related empirical facts are striking. Figure 2 shows that rising *enrollment is* sharply inversely related to selectivity. For colleges in the Bottom 25% of SAT selectivity, enrollment increased by 61 percent between 1990 and 2015. However, four colleges that regularly rotate as the nation's top ranked college in the widely-cited U.S. News annual survey—Harvard, Princeton, Stanford and Yale

[1] For 2017, Yale conducted a one-time expansion of enrollment by 15%, adding 200 students while still decreasing its admit rate from 6.8% in 2016 to 6.7%. Yale's new enrollment value now puts it at the middle of the Ivy League college enrollments and smaller than most Ivy Plus colleges.

[2] Dale and Krueger (2014) estimate that the returns to attending a selective college is highest for students who are racial minorities.

[3] The IPEDS has been tracking individual college enrollments since 1990 and applicants since 2003.

[4] Using an anchor year prior to 2015 has little impact on the sorting, especially for elite colleges. Since IPEDS tracking of SAT information for non-elite colleges improved over time, 2015 maximizes the number of colleges included in the sorting.

Figure 1: Applicant Ratio (2003 = 1), 2003 - 2015,
Sorted by Average SAT Percentiles

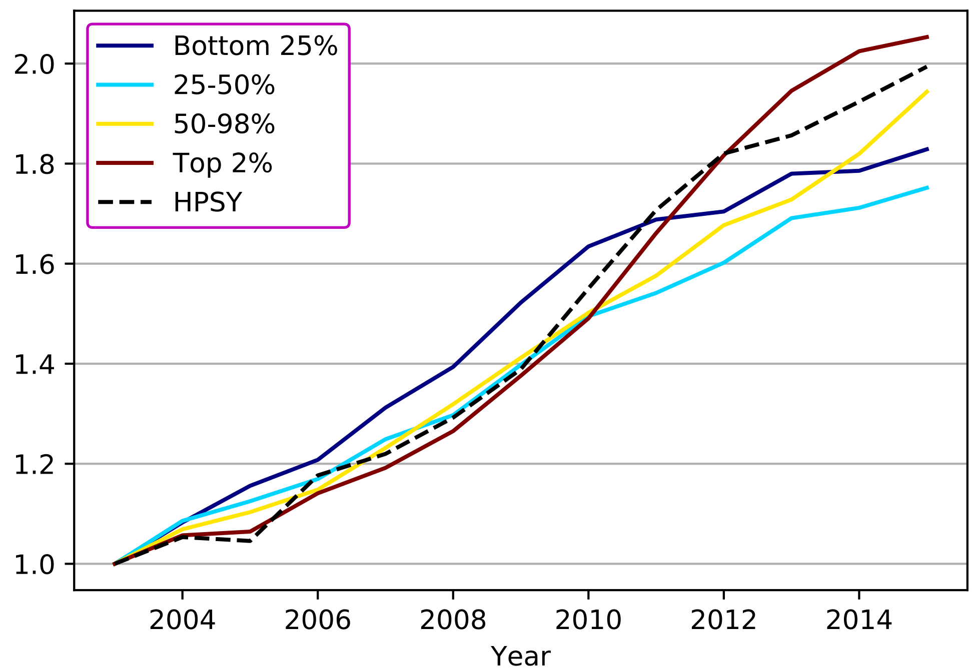
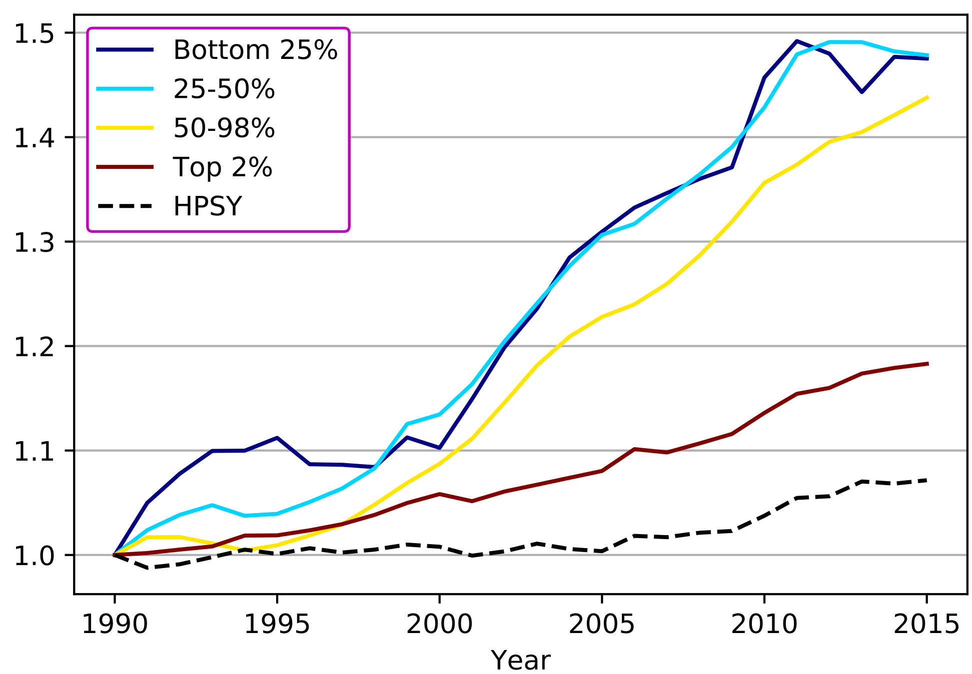


Figure 2: Enrolled Students, Normalized (1990 = 1), 1990 - 2015,
Sorted by Average SAT Percentiles



Source: Authors' calculations based on IPEDS. HPSY colleges are also included in the "Top 2%" group.

(herein "HPSY")—increased their enrollment by only 7 percent. While not tracked by IPEDS, the sorting of enrollment by selectivity appears to extend back to 1970.[5]

Rather than increase supply with demand, elite colleges have chosen to allow their *admit rates* (applications divided by admission offers) to fall. Figures 3 and 4 report plummeting admit rates for Ivy League and "Ivy League Plus" colleges. Figure 5 shows that admit rates, however, have *increased* across non-elite colleges.[6] The enrollment-weighted average admit rate across *all colleges increased* by 36 percent between 2003 and 2015. Falling admit rates, therefore, is mainly an elite college phenomenon.

Consistently, Figure 6 shows that the *share* of aggregate college enrollments captured by elite colleges fell by 40 percent between 1990 and 2015.[7] Elite colleges, therefore, are serving a smaller and smaller share of total college-bound students, becoming more exclusive over time. Elite colleges are now often *distinguished* by their rising SAT scores and falling admit rates. Consider the following headline in the April 8, 2016 edition of *The Harvard Crimson*: "For Fourth Year in a Row, Stanford Beats Harvard's Admissions Rate." Or, as playfully described by *New York Times* columnist, Frank Bruni (2016):

> Cementing its standing as the most selective institution of higher education in the country, Stanford University announced this week that it had once again received a record-setting number of applications and that its acceptance rate—which had dropped to a previously uncharted low of 5 percent last year—plummeted all the way to its inevitable conclusion of 0 percent. ... With no one admitted to the class of 2020, Stanford is assured that no other college can match its desirability in the near future.

At first glance, it might seem that the welfare loss from the rise in exclusivity might be small in the presence of an increasing number of "close substitutes" to elite colleges. The evidence presented in Avery et al. (2012) shows just the opposite: there is a considerable drop-off in applicant utility-based revealed preference by school selectivity. As we show in Appendix A, the loss in consumer (student) surplus from being rejected from a HPSY

[5]Between 1970 and 2015, U.S. Census data indicates that enrollment by full-time undergraduate students increased by more than 200 percent nationally (https://www.census.gov/content/dam/Census/library/visualizations/time-series/demo/fig_6.png). In contrast, during the same 45-year period, enrollment at Yale and Stanford, two of the HSPY colleges for which early historical data is most readily available, increased by 16 percent and 13 percent, respectively (Pierson, 1983; Stanford, 2017).

[6]Calculated on a weighted-average basis. Because of missing enrollment data in earlier years in the IPEDS for some colleges, a decision must be made about how to clean the data. We choose a minimalist procedure. If data were missing for a specific college between years where the data exists for that college, we linearly interpolated to obtained the in-between years data for that college. Otherwise, missing data was dropped for this calculation. In particular, we did not try to fill in missing data for one college based on data from other colleges with similar characteristics, as the random effect is too large.

[7]Recall, by construction, the basket of elite colleges as well as all percentiles are determined by SAT scores in 2015 and, therefore, held fixed going back in time.

Figure 3: Admit Rates in Ivy League, 1987 - 2015

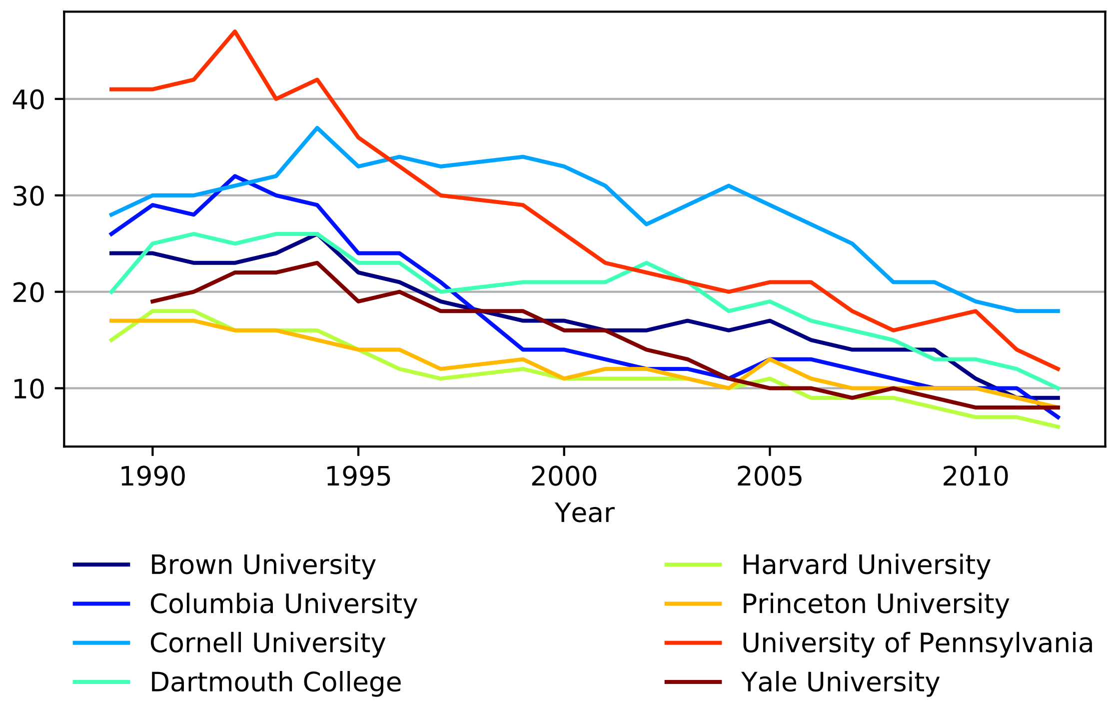
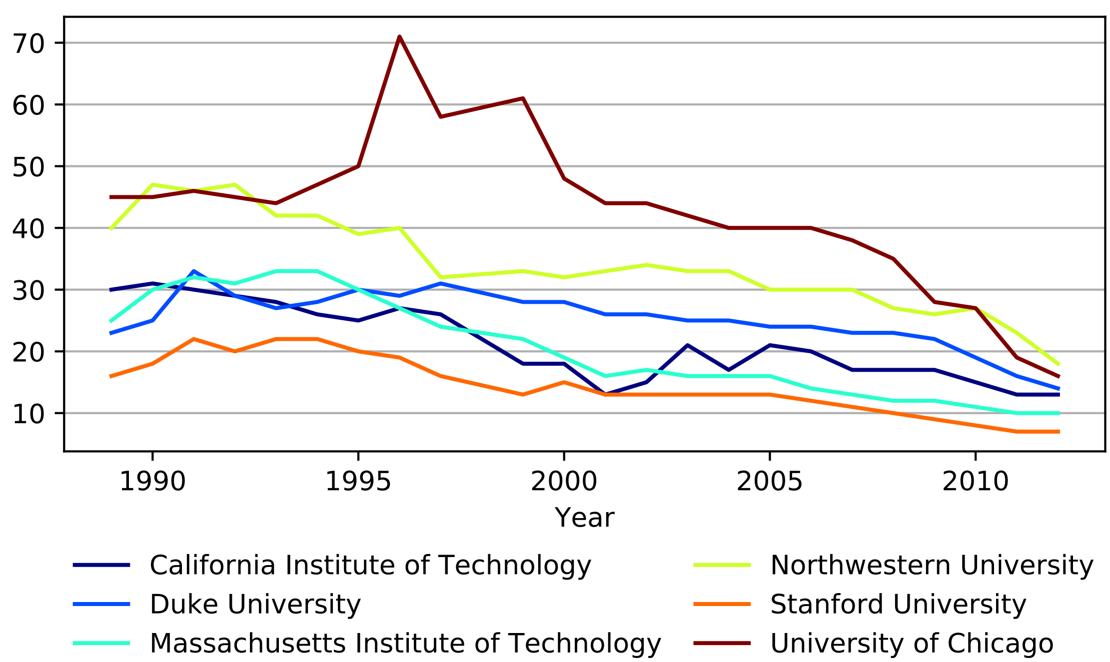


Figure 4: Admit Rates in Selected Ivy Plus, 1987 - 2015



Source: Authors' calculations based on data from IPED and US News and World Report.

Figure 5: Change in Admit Rate,
Weighted within Percentile by its Enrollment, 2003 - 2015

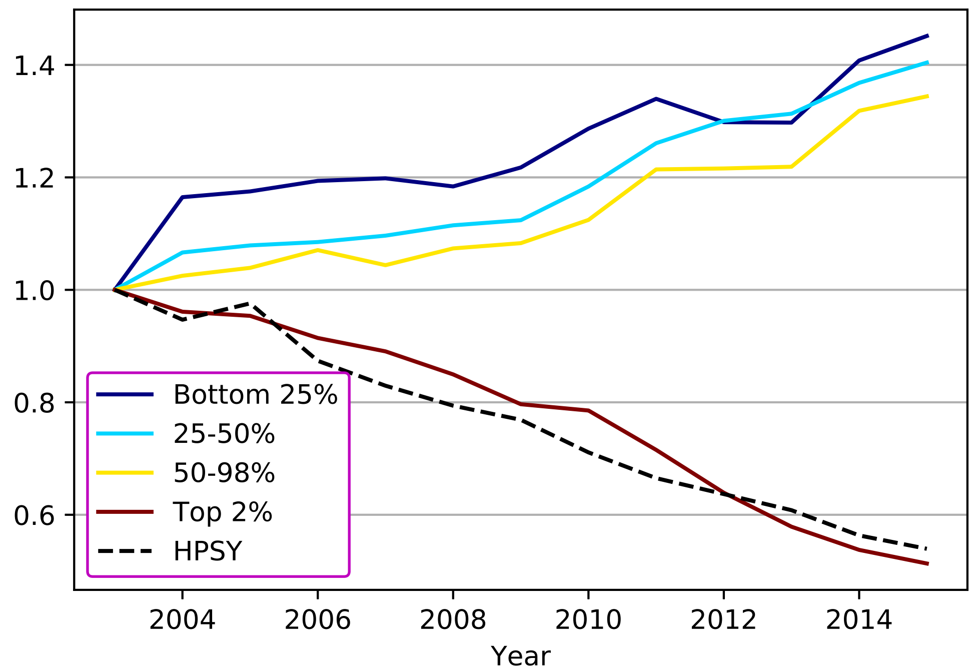
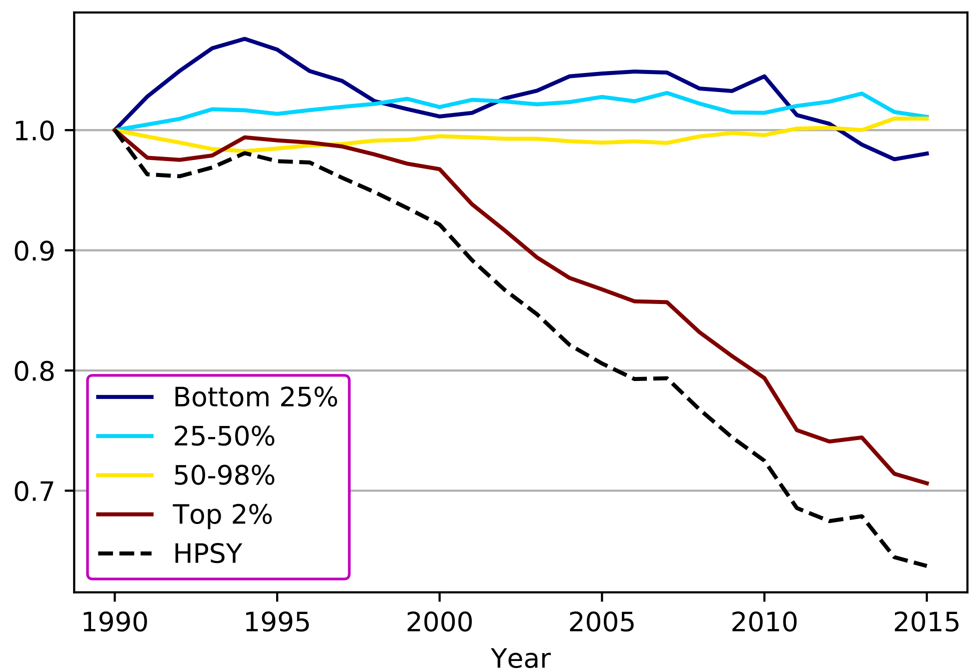


Figure 6: Total Enrollment Shares by Percentile, 1990 - 2015



The shown percentile's share is equal the percentile's total enrollment divided by the total enrollment across all percentiles. Source: Authors' calculations based on data from IPEDS.

college is, *conservatively*, equal to about 140% of the mean total tuition over four years of college in the Avery et al. (2012) sample. This loss is also of the same order of magnitude as recent illegal payments to coaches by parents for the purpose of getting their students into elite universities.

Indeed, if the welfare loss from rising exclusivity were small, falling admit rates would not receive so much attention in the media, including a Washington Post (2016) article that compares modern elite college admissions to the "The Hunger Games." Falling admit rates would also not receive so much attention (and resources) by parents.[8] As Stanford President John Hennessy (2007) wrote in the 2007 Stanford Magazine:

> I have been president for seven years and it is still one of the most difficult parts of the job to explain to parents with gifted children why a son or daughter was denied admission. And at the same time, I must come to terms with the fact that we are denying Stanford the benefit of talent that could contribute to the University and society at large in a significant way.

Before the modern college ratings era begin in earnest in the 1970's, Stanford increased its enrollment—by over 250 percent between 1920 and 1970—similar to many Ivy League colleges at the time (Appendix B). Nonetheless, one could argue that many small college "officials" (or their faculty) settled upon the absolute size of their campus, consistent with some "look and feel" that they want to preserve, at a point in time that coincided with the start of the modern ratings era. Put differently, a falling admit rate is a mere residual of a general rising demand for college. However, Figure 7 revisits the admit rate patterns shown in Figure 2 by restricting the data to "small colleges" with total undergraduate enrollments no larger than Harvard's enrollment in 1990, the largest HPSY college.[9] By construction, colleges in this restricted sample should have a similar "look and feel" of the elite colleges except maybe without the prestige associated with high selectivity. Nonetheless, Figure 7 shows that the same basic pattern previously reported in Figure 2 is preserved, with elite colleges growing substantially slower.[10]

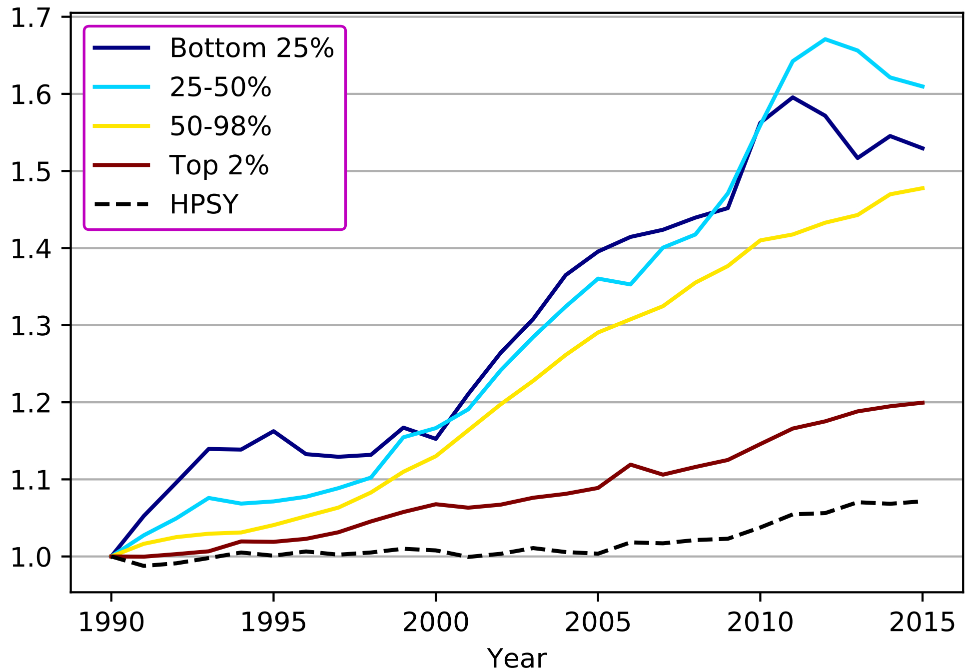
Of course, one might even argue that the taste for status quo "look and feel" is especially pronounced at more *elite* colleges. Of course, the data shows that the appearance of

[8]Parents spent over $1.1 billion in 2019 on tutoring and test preparation for their children, with 25% devoted to college test preparation (IBISWorld, 2020).

[9]The results are robust to capping enrollment at Princeton's value in 1990, the smallest HPSY college. Moreover, while not shown to preserve space, the growth in application ratios for small colleges is very similar to the results shown earlier in Figure 1 for all colleges.

[10]Figure 7 shows that the "Bottom 25%" colleges eventually expanded less than colleges with average SAT scores in the 25-50% range. This crossing, which did not happen in Figure 2 is presumably due to demand-side differences driven by quality differences associated with smaller, less-selective colleges.

Figure 7: Student Ratio (1990 = 1), 1990 - 2015,
Sorted by Average SAT Percentiles,
Smaller Colleges Only



Source: Authors' calculations based on 2015 IPEDS.

Explanation: Data restricted to colleges with total undergraduate enrollment in 1990 no larger than Harvard's 1990 enrollment, the largest of the HPSY colleges. Results are robust to capping enrollment at Princeton's value in 1990, the smallest of the HPSY colleges.

this taste *must* be the case at some definitional (even tautological) level since elite colleges are generally not barred by law from expanding. A more interesting question is, what type of preference is consistent with *falling* admit rates among elite colleges that compete for the best students in the presence of rising demand? A related question is, could such a preference produce social inefficiencies? Our paper address both questions.

We formally prove that if colleges only value the *absolute* skill level of their students, admit rates always *increase* in response to rising demand. Instead, *falling* admit rates require that an elite college also values some *positional* good that compares the skill of its students (or its admit rate) *relative* to peer institutions, which we will refer to as having a concern for "prestige."[11] We also prove that a concern for prestige is socially inefficient.

[11]The exact reason why a college might value prestige is mostly unimportant for our purposes. Like many positional goods, a college might individually rationally value prestige for many reasons that have little overall social benefit: to build its "war chest" (endowment); increase third-party ratings; a compensation "ego" differential for professors, researchers, and even staff. We also do not try to model how some colleges acquired historical "prestige capital" over very long periods of time. While interesting in its own

More specifically, we show that when a college places a weight on prestige above a critical value, an increase in demand *decreases* its admit rate, as observed with *elite* colleges. However, the admit rate *increases* with demand for colleges whose prestige weight falls below this critical value, consistent with the evidence presented above for *non-elite* colleges. For elite colleges, prestige competition generates a non-pecuniary externality that produces a low level of enrollment that makes students and elite colleges worse off.

One sharp policy prediction of our analysis is that allowing elite colleges to coordinate ("collude") their admissions could be Pareto improving. For example, if legalized, the top 200 U.S. colleges could agree to minimum enrollments by college that increase over time, much like maximum carbon caps by country found in the multinational Paris climate accord that decrease over time for some nations. In some countries, (quasi-)government institutions already determine total enrollment at elite colleges that receive public support, internalizing the prestige externality.[12] A similar practice happens with many U.S. public universities at the state level, although state-level action fails to internalize the prestige externality across state boundaries.[13]

This paper is organized as follows. Section 2 provides a literature review. Section 3 presents a simple single-stage theoretical model supporting algebraic solutions that helps build intuition, with proofs provided in Appendix D and various model extensions considered in Appendix C. Section 4 presents an enhanced three-stage simulation model where (Stage 1) each student decides to which colleges to apply in the face of a fixed application cost, (Stage 2) colleges make admissions and financial aid decisions, and (Stage 3) each student picks the actual college to attend among those admitted. Section 5 discusses model calibration. Section 6 brings comparative static predictions of the enhanced model to data. Section 7 concludes. Appendix B considers competing explanations for why elite colleges have expanded so slowly over time: physical capacity constraints; maintaining student or research quality; "knock-on" effects; and, more discussion of "look and feel." We argue that each explanation is inconsistent with the data and often simply imposes an exogenous restriction on outcomes that our model treats as endogenous.

right, that topic is inter-disciplinary in nature (history, geography, and sociology), with likely idiosyncratic factors. Our focus is squarely on the economic effects of this inherited capital when colleges compete on it.

[12] The Australian government, for example, determines enrollment at its elite "Group of Eight" universities. A similar practice exists in Scandinavian countries, including Norway and Sweden (Kirkeboen, Leuven, and Mogstad, 2016; Altmejd, Barrios-Fernández, Drlje, Goodman, Hurwitz, Kovac, Mulhern, Neilson, and Smith, 2021), and in various European and Central America countries (Kirkeboen et al., 2016).

[13] For example, the State of California might wish to protect some of its universities' reputations at the national level.

2 Previous Literature

A large portion of the college markets literature focuses on college demand and matching, with a fixed supply.[14] The demand side of our model builds upon several papers, including: Arcidiacono (2005) and Fu (2014), who estimate structural models of applications, admissions, and college choice; Bound et al. (2009), who shows that applications increase strategically as admits rates fall; and, Chade et al. (2014), who examines student sorting in the presence of fixed application costs and uncertainty in admissions.

Fewer papers allow for elastic supply (enrollment), which is our main focus. A notable exception is Epple et al. (2006) who present an equilibrium model that incorporates student selection, financial aid, enrollment, and educational outcomes. Colleges in their model compete over the *absolute* quality of students, producing a socially *efficient* equilibrium. We build upon their work but allow colleges to compete on a *relative* measure of quality ("prestige"), like a positional good (Charles et al., 2009). The equilibrium in our model is typically socially *inefficient*. On a more technical side, we also derive algebraic results in our base model using a simpler type space.[15] We formally prove equilibrium uniqueness and key comparative statics. We also prove that a *relative* measure is necessary to explain falling admit rates among elite colleges with rising admit rates elsewhere.

Our analysis is also consistent with Avery et al. (2012) who show that most standard college ranking methods, including that used by *US News and World Report*, create strong incentives for colleges to adopt strategic admission policies to improve their rankings. Ranking measures are highly correlated with various potential measures of prestige including selectivity (e.g., average student SAT score) and admit rate.[16] Our analysis demonstrates how this strategic behavior leads to an inefficient low level of supply.We don't seek to directly prove the casual relationship between the modern ratings period and a concern for prestige. Rather, like Avery et al. (2012), we take college preferences, whether for ratings or prestige, as given and demonstrate the competitive outcome.

[14] A partial list includes Arcidiacono (2005); Azevedo and Leshno (2016); Black and Smith (2006); Dillon (2007); Hoxby (2009); Bound et al. (2009); Avery and Levin (2010); Avery et al. (2012); Chade et al. (2014); Fu (2014); Bulman (2015); Hinrichs (2015); Che and Koh (2016); Azevedo and Leshno (2016); Bond et al. (2016), Gordon and Hedlund (2016) and Knight and Schiff (2019). Fu (2014) considers a counter-factual experiment where the government increases the number of spaces at non-elite colleges; but supply is exogenous.

[15] Specifically, Epple et al. (2006) numerically solve a two-dimensional type space where students are differentiated by skill and parental income. Because these two variables are highly correlated (our Section 4), we collapse them into one skill dimension to derive algebraic results.

[16] In contrast, *yield rates* are not a common measure of prestige (or used in rankings) and not nearly as correlated with ratings. Many unranked colleges—for example, BYU, U. of Alaska, U. of Nevada, U. of North Dakota, Yeshiva—have higher yield rates than Ivy League colleges, due to regional and religious preferences. Unlike admit rates, yields are also conditional of being admitted. As a result, elite colleges yield rates are often quite similar to many public schools.

3 Theory

Azevedo and Leshno (2016) shows how stable matching between many students and a discrete number of colleges with fixed supplies can be represented as a standard supply-demand problem. We follow their lead but with additional restrictions that accommodate an endogenous supply with analytical expressions to build intuition. To isolate the role of prestige, we consider monopolist elite colleges facing identical linear demand curves that only compete on prestige. Section 4 provides a richer model that is solved numerically.

3.1 Setting

Colleges and students are arranged spatially by distinct geographic regions.

Regions. There are N geographic regions indexed by $i \in \{1, .., N\}$. Each region has one "elite" college and an arbitrary number of identical non-selective "safety" colleges. Safety colleges produce an elastic supply priced at the constant marginal cost c_s and are not selective. To simplify notation, elite colleges are identical across regions, as are safety colleges.[17] Appendix C generalizes the analytical results presented below to a setting with heterogeneous elite colleges.

Students. Each region contains an identical set of students with abilities uniformly distributed between 0 and 1, $\theta \in \mathcal{U}[0,1]$. The total measure of students in each region is one. Students do not apply to colleges in other regions.[18] A student of type θ gets $V_e(\theta)$ value from an elite college (e.g., present value of increase in future earnings, value of assortative mating, etc.) relative to no college. Similarly, $V_s(\theta)$ is the value of a safety college, where, naturally, $V_s(\theta) < V_e(\theta)$, $\forall\theta$. The willingness-to-pay curve across the student ability type space for the elite college equals $V_e(\theta)$ - $V_s(\theta)$ + P_s, where $P_s = c_s$ is the price of the safety college. Normalize $P_s = c_s = 0$ and set college application costs to zero. All students, therefore, apply to the elite college and at least one safety college in their respective regions. Assume that $V_e(\theta)$ - $V_s(\theta)$ = $b\theta$, for some constant $b > 0$, so that willingness-to-pay is supermodular in ability and college quality, consistent with the standard assumption that student ability and college quality are complements.[19] Then,

[17]We, therefore, only need to track the index i for school Nash *decision rules*, which takes other peer schools $-i$ choices as given. We do not need to track i for preferences or *equilibrium* outcomes.

[18]The regional modeling focuses the model with little impact on the role of prestige. Even in the real world, colleges do not report "takeaway" ratios that count the number of matriculates won by a college for students who also applied to peer colleges. Instead, colleges report metrics such as average SAT scores and admit rates even if drawing from distinct applicant pools.

[19]In particular, the difference $V_e(\theta)$ - $V_s(\theta)$ is increasing in θ, so that higher ability students receive a greater return from an elite college. For example, suppose that the value functions, themselves, increase linearly in θ, $V_e(\theta) = b_e\theta$ and $V_s(\theta) = b_s\theta$, where, naturally, $b_e > b_s$. Then, $b = b_e - b_s$. The more general

$b\theta$ represents the willingness to pay for the elite college by student with ability θ.

Colleges. Each elite college i, therefore, is a monopoly within its region. The value of a student's ability θ is fully observable and the elite college accepts all students with abilities above a cutoff value θ_i, rejecting all other students. Denote $q_i = 1 - \theta_i$ as the number of applicants that are accepted at cutoff θ_i. The student willingness to pay in the *quantity* dimension equals $b\theta = b(1 - q_i) = b - bq_i$, which forms a linear inverse demand curve that we will represent below using more familiar notation of $P_0 - bq_i$, where $P_0 = b$. Elite colleges do not offer financial aid and all admitted students pay the tuition "sticker" price. When drawing from its distinct student pool, each elite college cares about its "prestige," represented by some metric relative to elite colleges in other regions.[20]

3.2 College Admission

Let Q equal the identical number of students (and, hence, applicants) to the elite college located in region i ("college i" for shorthand). Recall, that q_i equals the endogenous number of applicants accepted by elite college i. [21] College i chooses q_i to maximize utility,

$$U_i = \underbrace{(P_0 - bq_i)q_i - cq_i}_{\text{“Profit”}} \quad + \quad \underbrace{r\left[\left(\frac{1 - a_i}{1 - a_{-i}}\right) - 1\right]}_{\text{“Prestige”}}, \tag{1}$$

over its concern for profit and prestige. The first term represents the college's "profit"[22] with term c_i equal to college i's constant marginal cost (also generalized in Section 4).

The second term of equation (1) represent college i's concern for its "prestige." Variable r is an elite college's preference weight on prestige relative to its profit. Variable $a_i \equiv \frac{q_i}{Q}$ is college i's admit rate and $a_{-i} \equiv \frac{\sum_{j \neq i} a_j}{N-1}$ is the average admit rate of its $N - 1$ peers in other regions. Prestige, therefore, is increasing in college i's *rejection* rate, $1 - a_i$, relative to average rejection rate of its peers, $1 - a_{-i}$. The subsequent subtraction of one from this ratio inside $[\bullet]$, while not materially important, conveniently normalizes $[\bullet]$ to zero in *equilibrium* where $a_{-i} = a_i$. While there is considerable evidence that elite colleges care about their admit rates,[23] Section 4 also considers a formulation of prestige based on

condition stated in the text allows for non-linear value functions with equal slopes at θ.

[20] As noted in Section 1, we do not take an exact stand for the basis of prestige. In the current setting, consider "regional pride" that leads to more alumni engagement or faculty pride among peers.

[21] The i index on q_i is needed as choice variable since q_i are identical only in equilibrium.

[22] In practice, this profit could be used to satisfy other mission objectives not directly related to prestige, including various sports, or offsetting public subsidies. For our purposes, we only need another motivation besides prestige.

[23] "One of the ways that colleges are measured is by the number of applicants and their admit rate,

average matriculate test scores relative to those of peer colleges.

We scale the value of r relative to college i's maximum potential social surplus,

$$r = \rho \times SS \equiv \rho \left[\frac{(P_0 - c)^2}{2b} \right], \tag{2}$$

where ρ is the "intrinsic prestige weight" for college i, and SS is, without loss in generality, the maximum social surplus that college could generate, that is, at the competitive solution.[24] Without this normalization, a rising demand for college i (e.g., an increase in $P_{0,i}$) would cause a mechanical decline in the importance of prestige, $r_i[\bullet]$. With this normalization, the intrinsic prestige weight, ρ_i, is scale independent, so that our subsequent comparative statics are driven by prestige effects rather than by mechanical scale effects.

Extending to Dynamics. The values of P_0, b and ρ, of course, are fixed for a single period. Appendix C introduces a two-period inter-temporal model where college i can reduce reduce admissions and profits in order to boost their future intrinsic prestige ρ and increase demand tomorrow. The key results presented below remain unchanged.

3.3 Equilibrium

We start with some definitions:

Definition 1. *a_i^* is the **equilibrium admit rate** that maximizes equation (1), given a_{-i}^*, $\forall i$.*

Definition 2. *$\bar{a} \equiv \frac{P_0 - c}{2bQ}$ is the **no-prestige monopoly admit rate**.*

In particular, it is easy to show that $\bar{a}$ is the admit rate that maximizes each identical elite college's utility (1) with no prestige ($\rho = 0$).

and some colleges do things simply to increase their applicant pool and manipulate those numbers," said Christoph Guttentag, the dean of undergraduate admission at Duke (Pérez-Peña, 2014). For example, Swarthmore eliminated an extra-500 word essay in order to increase applications (Snyder, 2015). According to an investigation by the Wall Street Journal (2019), elite colleges routinely invite high school students to apply that they have no intention of admitting: "Some applicants, in effect, become unknowing pawns." Arcidiacono (2019) presents evidence that "Harvard encourages applications from many students who effectively have no chance of being admitted, and that this is particularly true for African Americans." Moreover, newspaper articles routinely focus on a college's admit rate, including falling rates at elite colleges. Using the New's Bank Access World News database, we searched 460 major U.S. publications for articles since 1978 discussing college admissions versus *yields*. Articles about admit rates outnumbered articles about, for example, yield rates (the fraction of admitted students that matriculated) by a factor of 25 or more, depending on the search term.

[24] The competitive solution sets price equal to marginal cost and ignores both market power and prestige ($\rho = 0$). Other scaling factors could be used with no material change. For example, the social surplus at the monopoly equilibrium without prestige is equal to $\frac{3}{4}SS$, and so the $\frac{3}{4}$ value would get absorbed into ρ. Alternatively, enrollment quantities, rather than surplus, could be used.

Lemma 1. *The admit rate $\overline{a}$ maximizes the sum of college utilities, $\sum_{i=1}^{N} U_i$, in effect, internalizing the prestige externality.*

Theorem 1. *With $0 \leq \rho \leq \frac{1}{\overline{a}}$, there is a unique, feasible and symmetric Nash equilibrium, a^*. Moreover, $a^* < \overline{a}$, and a^* is Pareto inefficient.*

As shown in the proof to Theorem 1, the constraint, $\rho \leq \frac{1}{\overline{a}}$, is required to have a feasible (positive) equilibrium admit rate, $a^* > 0$.[25] A larger rejection rate $(1 - a_i^*)$ by college i generates a non-pecuniary negative externality for college $j \neq i$. There is a symmetric admit rate $\hat{a}$, with $a^* < \hat{a} \leq \overline{a}$, where each college obtains more profit and its *relative* admit rate (prestige) does not change. Hence, each college is better off at $\hat{a}$. But, $\hat{a}$ is not an equilibrium, as each college will deviate downward. Non-admitted students are worse off by revealed preference. The equilibrium is inefficient.

3.4 Traditional Surplus Measures

We drop college index i in this section without risk of ambiguity due to symmetry in equilibrium admit rates. Following conventional terminology, *consumer surplus* is the area between the inverse demand curve and the equilibrium price while *producer surplus* is area between the equilibrium price and the marginal cost curve.

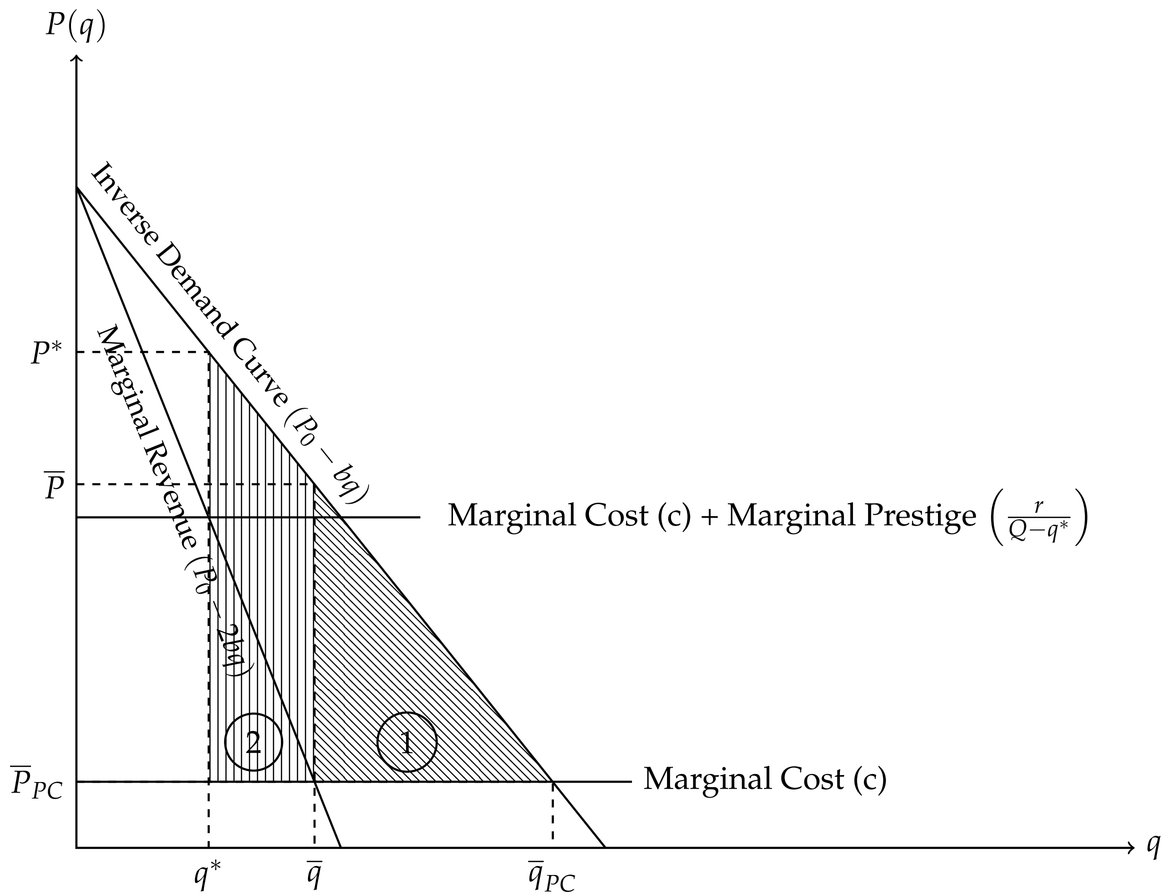
Lemma 2. *The consumer (student) surplus is $CS(a) = \frac{P_0^2 a^2}{2b}$. Similarly, the producer surplus is $PS(a) = \frac{P_0^2}{b}\left(2\overline{a} - a\right) a$, where, recall that $\overline{a}$ is the monopoly admit rate without prestige ($\rho = 0$).*

Also following convention, *social surplus* is the sum of consumer and producer surplus. Define the *traditional* dead-weight loss (DWL) equal to the *competitive social surplus* (i.e., the social surplus calculated at the admit rate where price equals marginal cost) *minus* the *monopoly social surplus without prestige* (i.e., the social surplus calculated at admit rate $\overline{a}$). Hence, DWL corresponds to the usual definition in the industrial organization and public economics literature (Harberger, 1964). Also, define the *prestige* dead-weight loss ($PDWL$) as the monopoly social surplus *minus* the *prestige social surplus* (i.e., the social surplus calculated at a^*).

Figure 8 provides an illustration. As proven in Appendix D, the presence of the prestige externality effectively shifts up the constant marginal cost curve, c, by $\left(\frac{r}{Q-q^*}\right)$, where $q^* = a^* Q$ is the equilibrium number of matriculates. Intuitively, as college i expands its quantity enrollment q_i, college i faces its traditional marginal cost of education delivery

[25]In particular, the marginal revenue term in college's utility (1) is linear whereas the marginal benefit of additional prestige is not. If college i cares too much about prestige then they will not admit a positive level of students.

Figure 8: Social Surplus Loss from Prestige



Explanation: q^* and P^* are the quantity of enrollment and college price, respectively, consistent with the admit rate a^* with prestige ($\rho > 0$). $\overline{q}$ and $\overline{P}$ are the quantity of enrollment and college price, respectively, consistent with the monopoly admit rate $\overline{a}$ without prestige ($\rho = 0$). $\overline{q}_{PC}$ and $\overline{P}_{PC}$ are the quantity of enrollment and college price, respectively, consistent with a competitive admit rate where the produces quantity where marginal costs equals price. The hatched lines in area 1 indicate the traditional dead-weight loss (DWL) associated with monopolistic competition relative to perfect competition (with no prestige effects). The area 2 indicates the prestige dead-weight loss (PDWL) from prestige alone, that is, compared to the monopoly level of production.

plus the marginal cost of reduced prestige in Nash equilibrium where the quantities of the other colleges q_{-i} are taken as given. Since the demand curve is linear, the admit rate of the perfectly competitive college, $2\overline{a}$, is exactly twice as large as that of the monopoly admit rate, $\overline{a}$. The value of DWL is shown in region 1, the shaded area between the monopoly quantity $\overline{q} = \overline{a}Q$ and the perfectly competitive quantity $\overline{q}_{PC} = 2\overline{a}Q$. The value of $PDWL$ is shown in region 2, the area between the equilibrium quantity with prestige $q^* = a^*Q$ and the monopoly quantity without prestige $\overline{q} = \overline{a}Q$.

For elite colleges, the prestige dead-weight loss $PDWL$ can easily be larger than the traditional dead-weight loss DWL.

Theorem 2. *The traditional dead-weight loss is equal to* $DWL \equiv \frac{P_0^2 \bar{a}^2}{2b}$. *The prestige dead-weight loss is equal to* $PDWL \equiv \frac{P_0^2}{2b}(\bar{a} - a^*)(3\bar{a} - a^*)$. *If* $a^* < \bar{a}(2 - \sqrt{2})$, *then* $PDWL > DWL$.

3.5 Comparative Statics

To demonstrate the *change* in admit rate in the presence of rising demand, the following result is useful:

Lemma 3. *The equilibrium admit rate* a^* *is decreasing in the prestige ratio* $\rho > 0$.

We now arrive at our key comparative static:

Theorem 3. *There exists a threshold value of the instrinic prestige ratio,* $\rho^* \in [0, \frac{1}{\bar{a}}]$, *such that an increase in consumer demand* (P_0) *results in a decrease in the admit rate* $\left(i.e., \frac{da^*}{dP_0} < 0\right)$ *if the weight placed on prestige by the group of competing colleges is above this critical value* ($\rho > \rho^*$). *If the weight placed on prestige is below this critical value* ($\rho < \rho^*$), *increases in customer demand result in an increase in the admit rate* $\left(i.e., \frac{da^*}{dP_0} > 0\right)$. *This prediction is broadly consistent with the evidence of falling admit rates at elite colleges and increasing admit rates at non-elite colleges presented in Section 1.*

3.5.1 Remark: A Concern for *Absolute* Metrics

The *relative* nature of prestige is critical for capturing how admit rates fall in applications. Suppose, for example, that college j only cared about the *absolute* value of some metric such as its own admit rate a_j, or, equivalently, rejection rate, $1 - a_j$.

$$U_i = \underbrace{r_i \cdot (1 - a_i)}_{\text{Weight on absolute metric}} + \underbrace{(P_{0,i} - b_i q_i)q_i - c_i q_i}_{\text{“Profit”}}. \tag{3}$$

Since a college's rejection rate is a linear function of the quantity it admits, adding a concern for a college's admit rate simply adjusts the marginal cost function while adding a constant intercept:

$$U_i = \underbrace{r_i}_{\text{Intercept}} + (P_{0,i} - b_i q_i)q_i - \underbrace{\left(c_i + \frac{r_i}{Q}\right)}_{\text{Effective MC}} q_i \tag{4}$$

In fact, this intuition holds for almost any *absolute* measure such as the average "quality"—test scores, GPA, extracurricular skills, diversity, etc.—of students, that can be written as a linear or quadratic function in quantity admitted q. As a result, an increase in demand P_0 fails to lower its admit rate. In sum, a concern by a college over an absolute metric cannot explain a decline in admit rates in the presence of rising demand; the metric must relative to the college's peers.

4 Model

This section presents a richer three-stage model that supports calibration to real-world data under prestige competition along with counter-factual simulations without the presence of prestige (equivalently, prestige is internalized). A student is characterized by skill $\theta \in \Theta$ drawn from the continuous probability distribution $\gamma(\theta)$ over type space Θ. There are N discrete peer colleges indexed by i. In each academic year, students and colleges make decisions across three stages, solved backwards under rational expectations.

Stage 1: Student θ applies to colleges, paying a fixed cost per application.

Stage 2: College i selects students to admit and awards needs-based financial aid.

Stage 3: Each admitted student θ decides which college j to attend.

4.1 Stage 1: Students Choose Where to Apply

A positive application cost $k_i(\theta) > 0$ ensures that all students don't apply to every college.[26] As Chade et al. (2014) notes, in the presence of these costs, if colleges' admit policies were deterministic and common knowledge, students would only apply to colleges to which they would be admitted, trivially producing 100% admit rates. Accordingly, we assume that college i *rejects* student θ with probability $\lambda_i(\theta)$, which is decreasing in θ. Naturally, $\lambda_i(\theta)$ functionals are equilibrium objects.

Let $\eta_i(\theta)$ denote the probability that student θ applies to college i; as shown below, $\eta_i(\theta) \in \{0, 1\}$ (i.e., binary), and so $\eta_i(\theta)$ becomes an indicator function. Student θ picks the college application vector $\vec{\eta}(\theta) = \{\eta_1(\theta), \eta_2(\theta), ..., \eta_N(\theta)\}$ to maximize the expected benefit of her application choices less the application costs:

$$\underbrace{\sum_{i=1}^{N} \Delta_i(\theta)\eta_i(\theta)(1-\lambda_i(\theta))y_i(\theta)\prod_{j=1}^{i-1}\lambda_j(\theta)}_{\text{Expected Benefit of Applications}} - \underbrace{\sum_{i=1}^{N} k_i(\theta)\eta_i(\theta)}_{\text{Cost of Applications}} \tag{5}$$

$\Delta_i(\theta)$ represents type θ's willingness-to-pay for attending college i if accepted. As dis-

[26] Knight and Schiff (2019) provides evidence that reducing application frictions increases the number of college applications.

cussed below, $\Delta_i(\theta)$ is potentially a complicated endogenous operator, but its value can be identified structurally using observed prices. The "yield" $y_i(\theta)$ is the probability that student θ attends college i if admitted, determined in Stage 3, where $\sum_i y_i(\theta) = 1$. The term $\prod_{j=1}^{i-1} \lambda_j(\theta)$ recognizes that the expected value of applying to an *additional* college in the peer group decreases if the student already has a low chance of being rejected to another college in the peer group to which the student has already applied.

It is easy to see that optimal solution for $\eta_i(\theta)$ is a boolean operator given by:

$$\eta_i^*(\theta) = \begin{cases} 1, & \text{if } \Delta_i(\theta)(1-\lambda_i(\theta))y_i(\theta)\prod_{j=1}^{i-1}\lambda_j(\theta) > k_i(\theta) \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

In words, student θ applies to college i if her willingness-to-pay conditional on admission, $\Delta_i(\theta)$, *times* the probability of admission, $1 - \lambda_i(\theta)$, *times* the probability of attending if admitted, $y_i(\theta)$, *times* the marginal value of *this* additional application, $\prod_{j=1}^{i-1} \lambda_i(\theta)$, exceeds the application costs, $k_i(\theta)$. As Chade et al. (2014) notes, while the value of $k_i(\theta)$ might seem small in practice relative to the value of college, the marginal value of an additional application can quickly diminish, which, in fact, occurs in our simulations reported later.

Several technical remarks are in order:

First, there are $N!$ possible college index orderings. With *heterogeneous* colleges, the order that a student applies to colleges, therefore, matters when $k_i > 0$. The optimal application $\vec{\eta}$ vector, therefore, produces the largest value of equation (5) across $N!$ combinations. This technicality, though, turns out to be numerically unimportant below due to the similarity of colleges within a peer group, and so we do not modify our notation presented above to explicitly accommodate the maximum of $N!$ different orderings.

Second, notice that the tuition price does not directly enter equation (6). Following Epple et al. (2006), financial aid enters as first-degree price discrimination where a college extracts consumer surplus. As Epple et al. (2006) argue, this assumption is reasonable for elite colleges with market power who have access to extensive personal finance data provided by its applicants.

Third, as in Chade et al. (2014), we don't take a strong position whether the expected benefit of an additional application corresponds to "expected value" (EV) or "expected utility" (EU). The interpretation of $\Delta_i(\theta)$ determines the technical distinction between these two choices, either as a monetary payoff for attending college i (EV) or in utility units (EU). Unlike many applications where this distinction matters, students in our setting cannot allocate their human capital investments across multiple colleges similar to diversifying financial capital across multiple asset classes. Imposing some curvature over the monetary outcome (to produce EU) would just mainly serve to dampen the number of elite colleges to which a *marginal skill* student applies (their "stretch schools"). But the operator $\prod_{j=1}^{i-1} \lambda_i(\theta)$ also serves this purpose with positive application costs, and so focusing on EV allows us to calibrate $\Delta_i(\theta)$ as the additional net monetary reward that student θ receives from attending college i.

Assumption 1. *Student θ is a price taker in college i's rejection rate, $\lambda_i(\theta)$.*

Assumption 1 says that no single student θ strategically anticipates how his or her *own* application to college i impacts its rejection *rate*. This assumption is standard (e.g., Azevedo and Leshno (2016)) since there are many students relative to colleges.

The total number of applicants to college i equals

$$Q_i = M_i \int_{\Theta} \gamma(\theta)\eta_i(\theta)d\theta \tag{7}$$

where M_i is a scaling parameter that maps the measure-one student type space to empirical values reported below. M_i can be interpreted as the "potential" number of applicants to college i, maybe reflecting student regional, legacy and other idiosyncratic preferences. Unless application costs $k_i(\theta)$ are small, the value of M_i is typically latent and larger than Q_i, and so the value of M_i is inferred as part of model calibration discussed below.

4.2 Stage 2: Colleges Admit Students with Needs-Based Financial Aid

The (expected) number of matriculating students to college i equals.[27]

$$q_i = M_i \int_{\Theta} \gamma(\theta)\eta_i(\theta)(1-\lambda_i(\theta))y_i(\theta)d\theta, \tag{8}$$

College i picks q_i through its rejection function $\lambda_i(\theta)$, a potentially infinite-dimensional equilibrium object. As justified in the calibration section presented below, we consider a rejection function of the form $\lambda_i(\theta) = \lambda_i(\theta; \alpha_i)$, where α_i is a scalar that college i picks to control the shape of λ over the type space, Θ. College i, therefore, chooses α_i to maximize its utility over profits and prestige:[28]

$$\alpha_i^* = \underset{\alpha_i}{\text{argmin}} \; \underbrace{Rev_i - c_i q_i}_{\text{profit}} + \underbrace{R_i\left(\frac{1-a_i}{1-a_{-i}}\right)}_{\text{relative admit rate}} + \underbrace{S_i\left(\frac{\bar{\theta}_i}{\bar{\theta}_{-i}}\right)}_{\text{relative skill}} \tag{9}$$

[27] While q_i is an expected value, it becomes nearly deterministic with enough students. So, we will save on the "expected" terminology throughout.

[28] To enhance concavity and computation speed, the terms $\left(\frac{\bullet}{\bullet}\right)$ in equation (9) can be raised to a power of $\frac{1}{2}$, for example, without much loss in generality.

where

$$Rev_i = M_i \int_\Theta \gamma(\theta)\eta_i(\theta)(1-\lambda_i(\theta))y_i(\theta)\Delta_i(\theta)d\theta,$$

equals total revenue under first-degree price discrimination in a given year. Profit is, therefore, equal to total revenue less total cost, $c_i q_i$, where is c_i the constant average cost.

College i's prestige, however, can depend on its relative admit rate (as before) or its average student ability ("skill") relative to the *average average* skill of its $N-1$ peers. The second term in equation (9) represents college i's prestige from its relative admit rate. As before, R_i is the prestige weight college i puts on its admit rate,

$$a_i = \frac{M_i \int_\Theta \gamma(\theta)\eta_i(\theta)(1-\lambda_i(\theta;\alpha_i))d\theta}{Q_i}, \tag{10}$$

relative to the average admit rate of college i's peers,

$$a_{-i} = \frac{1}{N}\sum_{j\neq 1}^{N} a_j. \tag{11}$$

The third term in equation (9) equals college i's prestige from its average student skill,

$$\overline{\theta}_i = \frac{\int_\Theta \theta\gamma(\theta)\eta_i(\theta)(1-\lambda_i(\theta;\alpha_i))y_i(\theta)d\theta}{\int_\Theta \gamma(\theta)\eta_i(\theta)(1-\lambda_i(\theta;\alpha_i))y_i(\theta)d\theta}, \tag{12}$$

relative to the *average average* student skill of its peers, $\overline{\theta}_{-i}$. Variable S_i is the corresponding prestige weight for college i while $\overline{\theta}_{-i}$ is defined analogously to a_{-i}.

4.3 Stage 3: Students Pick a College

Students now pick a college. If a student is admitted to more than one college, a coin flip as a tie breaker rule is innocuous and standard in game theory if payoffs are nearly homogeneous. The yield of college i for student type θ, conditional on being admitted, is then the inverse of the (expected) number of colleges that admit student θ:

$$y_i^*(\theta) = \frac{1}{1+\sum_{j\neq i}\eta_j(\theta)(1-\lambda_j(\theta;\alpha_j))}. \tag{13}$$

The value of 1 in the denominator reflects the condition that student θ has been admitted to college i. Term $\sum_{j\neq i}[\cdot]$ is the expected number of peer colleges to which student θ applied and is admitted. For heterogeneous colleges, this rule is less innocuous if the values $\Delta_i(\theta)$, $i \in \{1,\ldots,N\}$, differ substantially. Given our choice of peer groups in simulations reported below, the limited heterogeneity is numerically immaterial.

4.4 Equilibrium

Definition 3. *The equilibrium values of $\eta_i^*(\theta)$, α_i^* and $y_i^*(\theta)$ solve equations (6), (9) and (13) for all students $\theta \in [\theta_{min}, \theta_{max}]$ and colleges $i \in \{1, \ldots, N\}$.*

Given Assumption 1, the equilibrium is solved computationally as follows. The student type space is discretized into a fine grid, $\Theta \subset [\theta_{min}, \theta_{max}]$. Then:

- *Step 1*: An initial guess is made for rejection parameters α_i, $i \in \{1, \ldots, N\}$.
- *Step 2*: For each $\theta \in \Theta$ and $i \in \{1, \ldots, N\}$, solve Stages 1 and 3 simultaneously for $\eta_i(\theta)$ and $y_i^*(\theta)$, i.e., taking α_i as given (price taking).
- *Step 3*: Solve Nash game in Stage 2 for the "new value" of α_i, α_i^{new}, $i \in \{1, \ldots, N\}$.
- *Step 4*: Compute the sup norm across the set $\{(\alpha_i - \alpha_i^{new})\}$, $i \in \{1, \ldots, N\}$, and return to Step 2 if not sufficiently small (10^{-12}).

The generality of our model makes it challenging to produce analytical solutions, including proofs of equilibrium existence and uniqueness. Uniqueness is the bigger concern. We (imperfectly) attempt to detect multiple equilibria using globally robust solution methods that can report out multiple stable equilibria. The case study explored in the remainder of this paper focuses on the Harvard-Princeton-Stanford-Yale (HPSY) peer group, where heterogeneity appears to be limited enough to produce a unique solution.

5 Calibration

In the remainder of the paper, we consider a case study of the Harvard-Princeton-Stanford-Yale (HPSY) peer group to examine the role of prestige in limiting admissions between 1990 and 2015. We first calibrate latent model variables in the "baseline" model with prestige competition to match model moments with data. Counter-factual simulations then turn off prestige effects (i.e., $R_i = S_i = 0$) with no changes in the values of other latent parameters. More specifically, we proceed as follows:

Step 1: Functions:	Pick functional forms for skill distribution, willingness-to-pay, college rejection, and costs based on data.
Step 2: Parameters:	Calibrate latent parameters, indexed by college and year, by matching model price cap ("sticker price"), admit rate, and number of matriculates in Nash equilibrium for each year between 1990 and 2015.
Step 3: Model fit:	Report over-identification moments for these same years, including mean skill of matriculates, number of applicants, and mean skill of applicants.
Step 4: Counterfactual:	Turn off just the prestige terms ($R_i = S_i = 0$), and report key model moments related to applications, matriculates, and skills as well as the change in social surplus.

Steps 1 and 2 are reported in this section, followed by a brief discussion that contrasts our "limited-number-but-exact" moment matching exercise against competing calibration methods in an inter-temporal setting. Steps 3 and 4 are addressed Section 6.

5.1 Functional Forms

We first discuss function forms, before discussing calibration of latent parameters:

Student skill distribution, $\gamma(\theta)$. Figure 9 shows that SAT (and ACT converted) scores are normally distributed, which we take as a proxy for a student's skill, θ.[29] SAT scores are integer valued whereas converted ACT scores take non-integer values. More students take the SAT exam, producing spikes at integer values, as shown in Figure 9. The probability distribution $\gamma(\theta)$ for skill θ is, therefore, taken as normal distributed:

$$\gamma(\theta) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(\theta-\mu)^2}{2\sigma^2}\right] \tag{14}$$

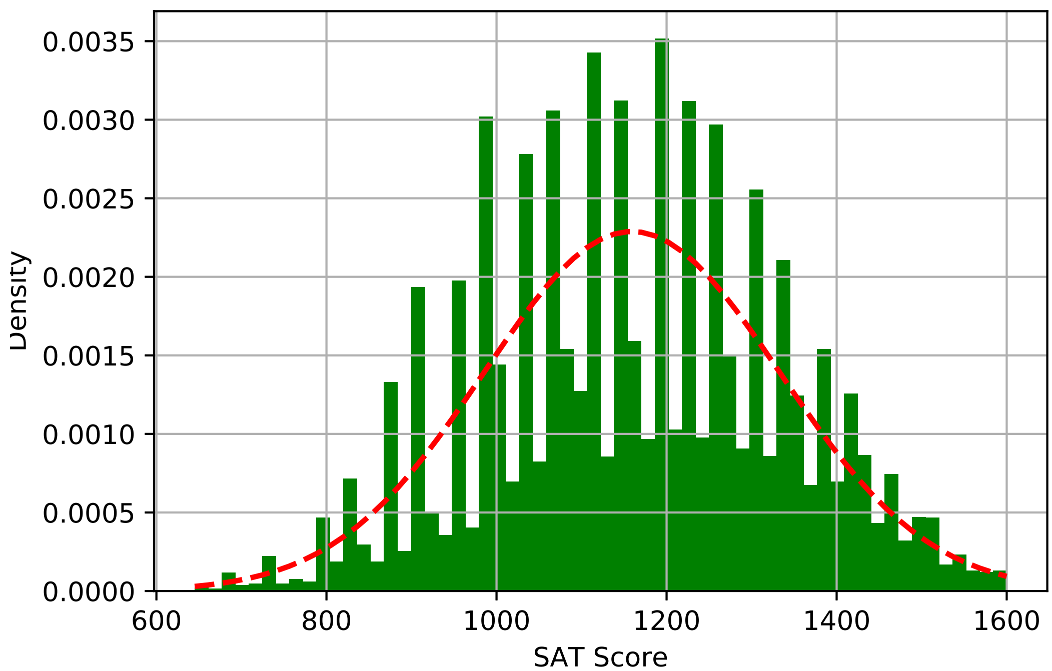
where $\mu = 1160$ and $\sigma = 175$, consistent with Figure 9. The distribution is naturally trimmed at $\theta_{min} = 400$ (minimum SAT score) to $\theta_{max} = 1600$ (maximum SAT score).

Rejection Function, $\lambda(\theta; \alpha_i)$. Rejection follows a power law function decreasing from 1 to λ_{min}, as θ increases from $\theta_{min} = 400$ (minimum SAT) to $\theta_{max} = 1600$ (maximum):

$$\lambda_i(\theta; \alpha_i) = 1 - (1 - \lambda_{min}) \left(\frac{\theta - \theta_{min}}{\theta_{max} - \theta_{min}}\right)^{\alpha_i}. \tag{15}$$

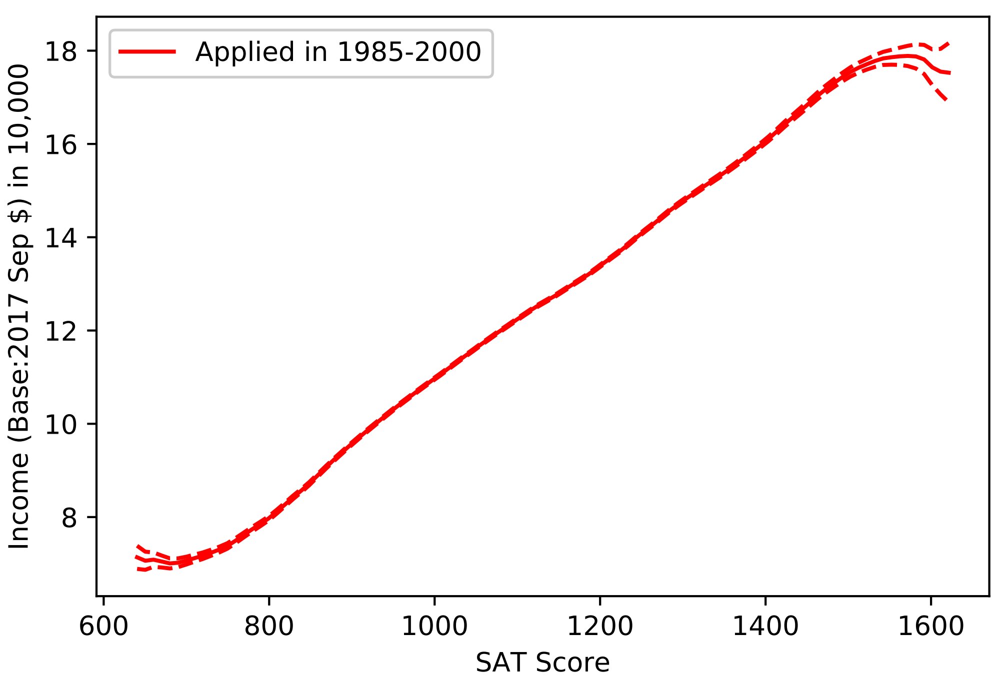
[29] Data comes from the Higher Education Research Institute public-use file of college freshman, 1985 - 2000. ACT scores are converted to their SAT-equivalent values following Chade et al. (2014), with a maximum score of 1600.

Figure 9: SAT Score Density



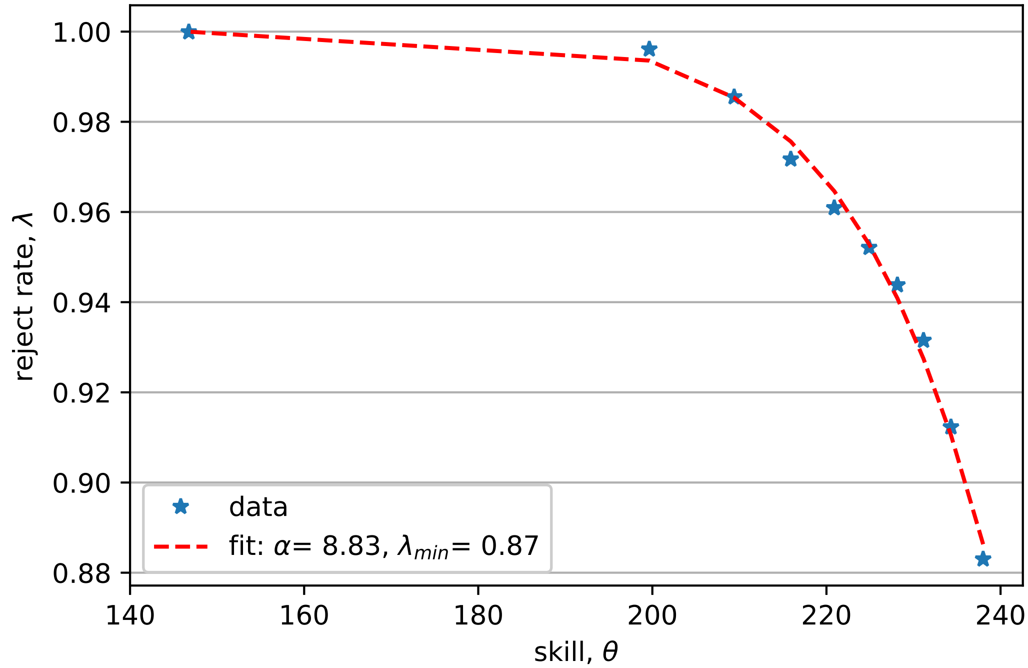
Sources: Higher Education Research Institute: 2001 - 2006.

Figure 10: Household Income vs. (Re-centered) SAT Score



Sources: Higher Education Research Institute: 1985 - 2000.[30]
(Dashed lines indicate 90% confidence intervals.)

Figure 11: Estimates of Harvard Rejection Function, 1996 - 2015



Source: Authors' calculations based on data from Arcidiacono (2019).

λ_{min} is the "minimum" rejection rate for θ_{max} type students, which recognizes that even students with perfect SAT scores are rejected from elite colleges (Pérez-Peña, 2014). The value of α_i is an endogenous operator chosen by college i as its admission policy.[31] The power law specification appears to be a very good approximation for elite college rejection by skill type. Normally, such a function form is hard to validate because colleges do not typically release admit data based on a student skill metric that weights test scores, high school GPA, and other factors. However, as described in more detail in Appendix F, Harvard released such data as part of its recent litigation against Students for Fair Admissions. Harvard gives each applicant an aggregate score between 100 and 240.[32] Naturally, low scoring students are much more likely to be rejected than students with higher scores. However, even a majority of students with perfect scores of 240 will be rejected. Figure 11 indicates a very close fit between a calibrated power-law function and Harvard data.

Costs, c_i and k_i. The marginal cost of college i, c_i, in each year is estimated using the data and the procedure described in Appendix E. The data includes expenditures on

[31]We set θ_{min} at $\mu + 1.0 \cdot \sigma$ since HPSY colleges do not admit students with scores below the national average. Despite the detailed Harvard legal data discussed in text, we typically only observed the admit rate across all applicant skill levels by college, which requires picking a single parameter, namely, the shape value α_i, for calibration. However, the exact choice of θ_{min} does not seem that important in the simulations.

[32]Technically, the score starts as low as 60. However, the released data starts at 100 since 99.99 percent of applicants between 100 - 193.5 where rejected.

instruction and student services for colleges with sufficiently high SAT scores and minimum enrollment. The estimation procedure computes a cost elasticity controlling for fixed effects and potential endogeneity using instrumental variables. The cost of application $k_i(\theta)$ is not type specific and is treated as constant across colleges within a peer group, $k_i(\theta) = k_i = k$.[33]

Willingness to pay, $\boldsymbol{\Delta_i(\theta)}$. The willingness-to-pay by student θ for college i is *proportional* to the returns to college functional form estimated by Dale and Krueger (2011):

$$\Delta_i(\theta) = \overline{\omega_i} e^{\beta_i(\theta-\mu)} \tag{16}$$

where $\overline{\omega_i}$ is the average starting wage of college i. Notice that $d\Delta_i(\theta)/d\theta > 0$, implying that higher skill students get a higher return to college i. Moreover, willingness-to-pay is supermodular in ability and college quality ($\frac{\partial^2 \Delta_i}{\partial\theta\partial\overline{\omega_i}} > 0$). As a result, a higher skill student has a higher willingness-to-pay for an elite college but, of course, by an amount that is less than the total return to the elite college, given potential substitutes. We discuss the corresponding calibration of β_i in the next subsection.

5.2 Moment Targets and Latent Parameters

In the baseline simulation, latent model parameters are calibrated so that the model "exactly" matches three key moments for each college at the Nash equilibrium fixed point in each year: the price cap ("sticker price), admit rate, matriculates.

Beta, β_i**.** Similar to Epple et al. (2006), the deep preference parameter for willingness to pay (here, β_i) is chosen so that $\Delta_i(\theta = 1)$ is consistent with college i's price cap ("tuition"). Unlike Epple et al. (2006), our willingness-to-pay measure only considers student skill (SAT) and not parental income. However, Figure 10 shows that there is strong empirical correlation between parental income and SAT scores; with perfect correlation, β_i is a sufficient statistic across these two dimensions.

Prestige, R_i **and** S_i**.** For any simulation, R_i (prestige weight on relative admit) *or* S_i (prestige weight on relative skill) must be fixed to avoid over-identification.[34] To avoid over-fitting, we simply set $R_i = 0$ and calibrate S_i so that college i's admit rate (through its choice of α_i) in Nash equilibrium equals its empirical value in a given year.[35] This choice is consistent with the focus in Hoxby (2009) on SAT test scores as an indicator of

[33]The value $k = \$90$ by 2015, around the average for HPSY colleges plus score submissions. In practice, some elite colleges offer fee waivers or reductions to lower-income households. These households, however, are also more likely to have lower skill applicants.

[34]Specifically, a loci of points in $[R_i, S_i]$ space can target the same calibrated admit rate.

[35]Simulations exercises indicate that setting $S_i = 0$ and calibrating R_i yield similar results.

college selectivity.

Potential Number of Applicants, M_i. The total number of potential applicants M_i to college i is latent when application costs are positive, $k > 0$. It can be calibrated by matching the model's number of applications Q_i for college i in Nash equilibrium to its observable empirical value, or by matching the model's matriculates q_i against actual data. We target matriculates because it has a longer history in the IPEDS data set, although similar results emerge with targeting applications.

5.3 Discussion: Dynamics

In sum, we calibrate β_i, S_i and M_i in each year so that the model produces an "exact" match for college i's price cap, admit rate, and the number of matriculates in Nash equilibrium equal to their observable empirical values. Other model moments, described in Section 6, are reported as over-identifications relative to their empirical values, where available. This approach allows S_i and willingness-to-pay to grow adaptively over time without imposing an explicit structure on their relationship, thereby cleanly isolating the effects of prestige in our counter-factual simulations.

An alternative approach would model colleges solving an infinite-horizon discounted-utility repeated game. However, a non-arbitrary way is then needed to select a single Nash equilibrium out of an infinite number. Alternatively, we could specify a reduced-form relationship between a "prestige investment" today (e.g., small q_i) and a future payoff (i.e., higher future willingness to pay).[36] GMM could then be used to calibrate latent parameters by matching model moments to available data. However, unlike the other functional forms discussed above, the relationship between a "prestige investment" today and future willingness to pay can't be directly estimated, reducing this appeal.

Moreover, inter-temporal supply-side strategic action implies that HPSY colleges would likely reduce *matriculates* to lower admit rates by an amount that is more than required by increased demand alone. Historically, however, the number of elite college matriculates kept pace with population growth prior to the modern ratings era (Appendix B), leveling off thereafter. HPSY colleges never appreciably reduced matriculates even with the rise of ratings. Instead, prestige appears to evolve adaptively over time, driven by demand. Elite colleges influence the *demand* side by encouraging more applications, including from candidates with little chance of being accepted (see footnote 23).

[36] Recall that we took that approach for the simplified model presented in Section 3 with an extension to dynamics discussed in Appendix C).

6 Simulations

We now report simulation results of how competition over prestige impacted equilibrium enrollments, admit rates, applications, and the level of skill of applicants and matriculates for HSPY colleges between 1990 and 1995. To isolate the effects of prestige competition, we run the simulation model twice. First, we run the model with latent variables calibrated as described in Section 5 with the concomitant results reported in figures below as the "Prestige" model. Second, we run a counter-factual simulation with prestige turned off (i.e., $R_i = S_i = 0$) but all other latent parameters held at their original calibrated values, reported in figures below as the "No prestige" model. This two-step approach ensures that any differences between the simulated and counter-factual values are completely due to the prestige effect.

6.1 Admit Rates and Number of Matriculates

Figures 12 and 13 report the weighted mean admit rates and weighted mean number of new matriculates (freshman) across HPSY colleges for the "Prestige" and "No Prestige" models. These figures also show empirical values ("Data") from the IPEDS.[37] As discussed in Section 5, the *admit rate* and the *number of matriculates* are *targets* of calibration, and so the values from the "Prestige" model match the data in each year by construction. Figure 12 shows that the counter-factual "No Prestige" model produces a much larger admit rate, in fact, up to its maximum limit under first-degree price discrimination.[38] Figure 13 shows that the number of matriculates in the "No prestige" model also increases relative much faster over time, indicating how a concern for prestige suppresses enrollment growth over time.

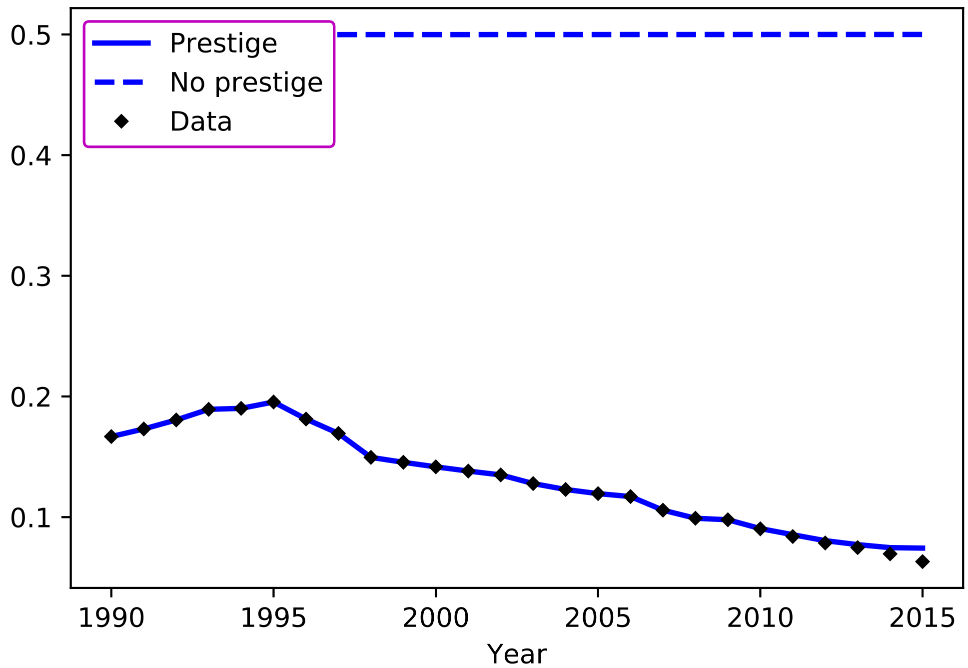
6.2 Matriculate Skills

Unlike the admit rate and the number of matriculates reported above that are calibrated to the data, the mean simulated skill of matriculates of HPSY colleges is an endogenous outcome of the model, shown in Figure 14. With prestige, the simulated skill level does a fairly good job at tracking the increase in selectivity found in the IPEDS data since 1997

[37]The proportion of matriculates from a given HPSY college forms the weight that is used to calculating the shown weighted mean. For the simulations ("Prestige" and "No Prestige"), matriculate count equals the respective simulated value. For "Data," the matriculate count is empirical.

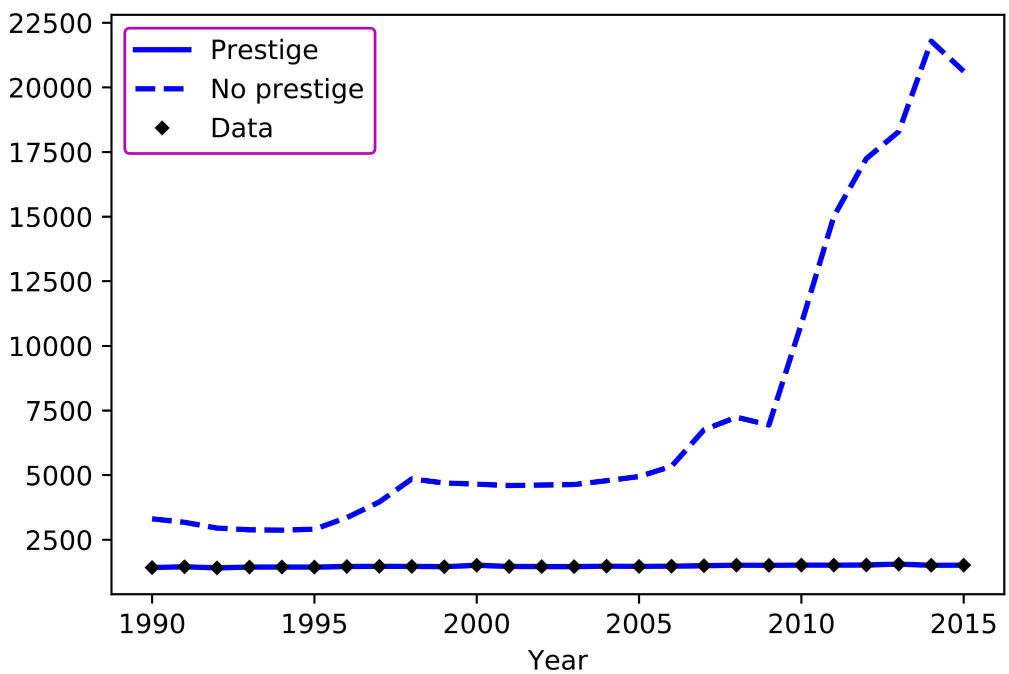
[38]We cap the admit rate at 50 percent to allow for easier visual comparison against the Prestige model, and to reduce computational costs. A cap of 50 percent is more-than adequate for demonstrating the contrast of the counter-factual scenario. In the counter-factual scenario, this cap might bind over time if marginal costs do not increase faster than demand.

Figure 12: Mean Admit Rate



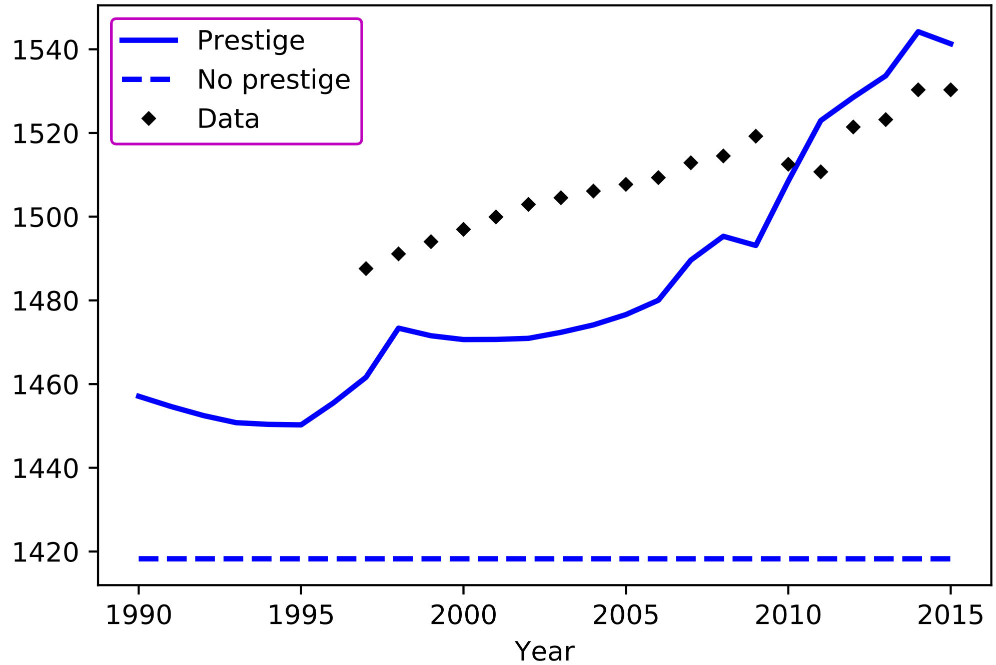
Sources: IPEDS (1990 - 2015) and Model.

Figure 13: Mean Number of Matriculates



Sources: IPEDS (1990 - 2015) and Model.

Figure 14: Mean Skill of Matriculates



Sources: IPEDS (1997 - 2015) and Model.

for HPSY colleges. (IPEDS started collecting SAT data in 1997.) In sharp contrast, the counter-factual simulation without prestige produces no increase in selectivity, consistent with the large and persistent counter-factual admit rate reported in Figure 12.

6.3 Applicants and their Skills

Figure 15 shows (solid line) the simulated weighted mean number of applicants to HPSY colleges, an endogenous outcome of the model. As admits fall, highly-skilled students typically apply to even more HPSY colleges, thereby mechanically lowering admit rates even more, a "knock-on effect" identified by Bound et al. (2009).

Notice that the simulated number of applicants with "Prestige" follows the data fairly closely. In contrast, the simulated number of applicants under the "No Prestige" counter-factual simulation grows substantially faster over time.

As shown earlier in Figure 12, an absence of prestige generates a higher admit rate, and so applying to college is relatively less competitive. At first blush, it, therefore, seems counter-intuitive that the number of applicants would have grown faster over time in Figure 15 in the counter-factual case with no prestige. The explanation is due to strategic application decisions captured by the model. As elite colleges who care about prestige become more selective over time, the average skill of *matriculates* increase. However, so does the average skill of *applicants*, as shown in Figure 16, an effect of application costs previously considered by Chade et al. (2014). In particular, greater selectivity over time shifts rightward the skill distribution of students who are willing to absorb the fix cost k of applying.

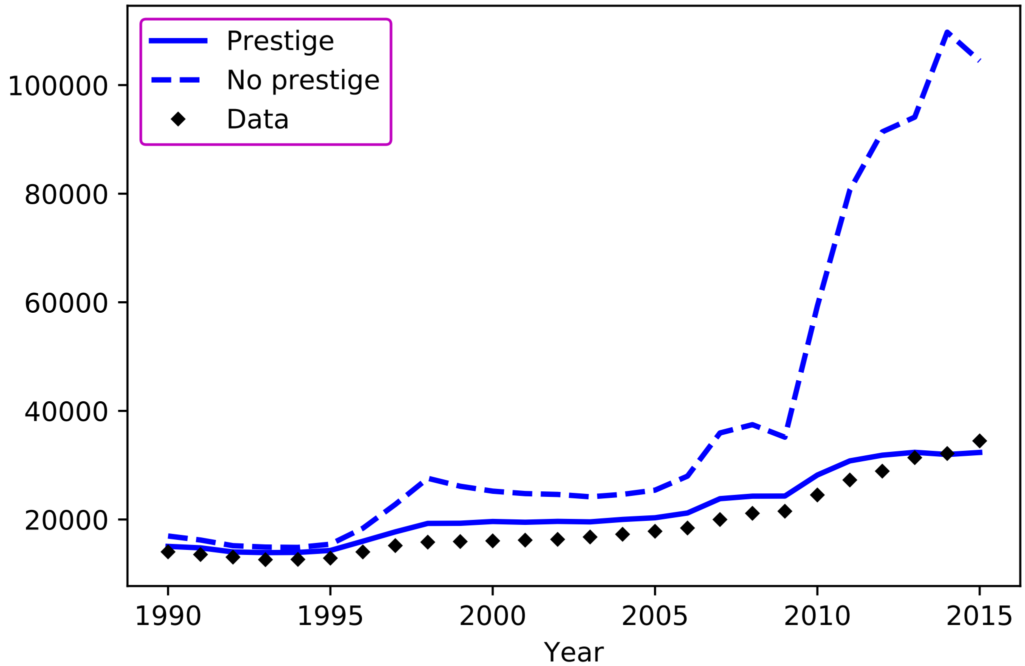
Figure 17 compares the skill of applicants and matriculates under the "Prestige" simulation. Notice that they follow a parallel trend over time. Naturally, simulated applicants have a lower relative mean skill than matriculates. In effect, HPSY colleges represent "stretch colleges" for applicants with "moderately strong" skills who face a lower chance of admission but a chance that is still high enough to absorb the application costs.

Recall that Figure 15 showed that the absolute number of applicants grows slower with "Prestige" than without. However, Figure 18 shows that mean number of applicants *relative to actual positions available* (that is, relative to the number of matriculates) increases substantially over time with "Prestige," consistent with the data for HPSY colleges. In contrast, this ratio remains flat in the "No Prestige" counter-factual simulation. The increasing gap between the "Prestige" and "No Prestige" ratio lines is consistent with the "knock-on" (secondary) effect argued in Bound et al. (2009): A fall in admit rates over time with prestige generates even more applications per position available, which decreases the admit rate even more.

Figure 16 does not report the empirical mean skill of *applicants* to HPSY colleges, since that data is generally unavailable.[39] However, indirect but striking evidence of strategic application decisions can be found using the annual HERI survey of freshman which asks students how their actual college of matriculation ranked within their own personal full choice ranking of colleges to which they previously applied. Figure 19 reports the results for "top private" colleges that, as defined by Chade et al. (2014), had a mean SAT score of 1340 or higher. In 1986, notice that students admitted to their first through fourth top choices had similar SAT scores. However, by the year 2006, SAT scores started to diverge by choice, with students admitted to their highest choice of colleges having relatively *lower* SAT scores. This pattern is consistent with strategic application decisions by students. Students with higher SAT scores are more likely to apply to the very best of the elite colleges. But these colleges also have the lowest admit rates, decreasing even more over time; indeed, as noted earlier, even students with perfect SAT scores are now routinely rejected from top colleges. In contrast, students with relatively weaker SAT scores, while still applying to strong colleges, often skip applying to the very elite colleges, a trend that has increased over time as the very elite colleges have become more competitive.

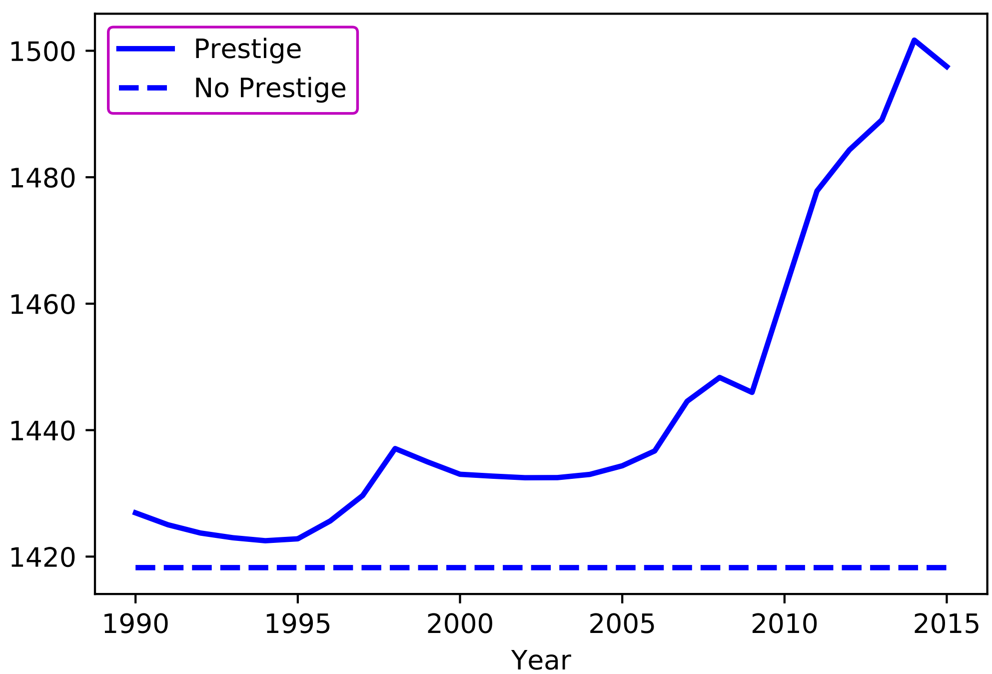
[39]The College Board suspended collecting this data many years ago.

Figure 15: Mean Number of Applicants



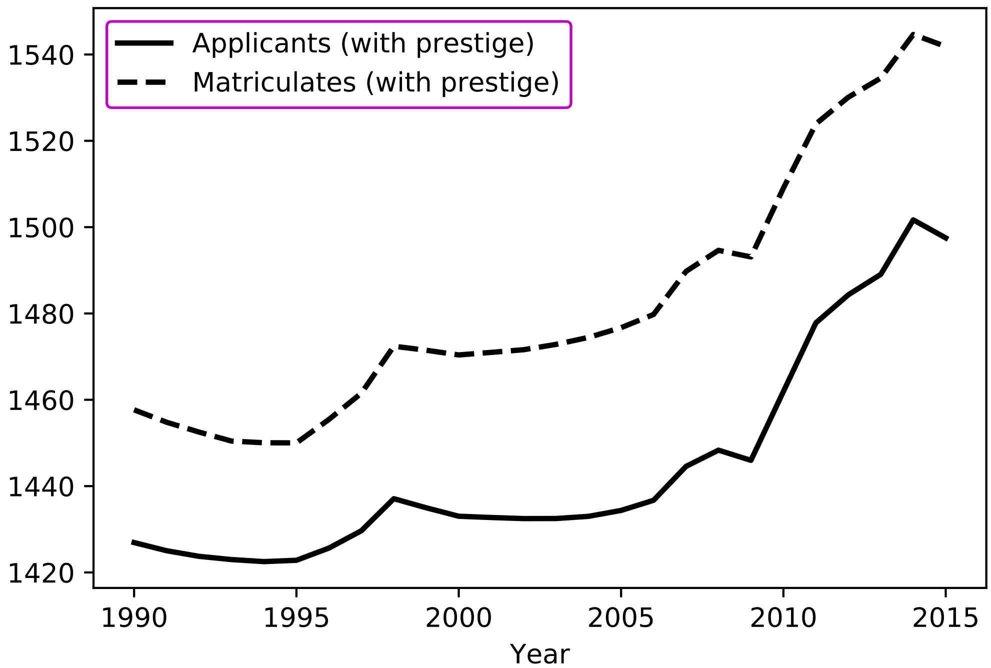
Sources: IPEDS (1990 - 2015) and Model.

Figure 16: Mean Skill of Applicants



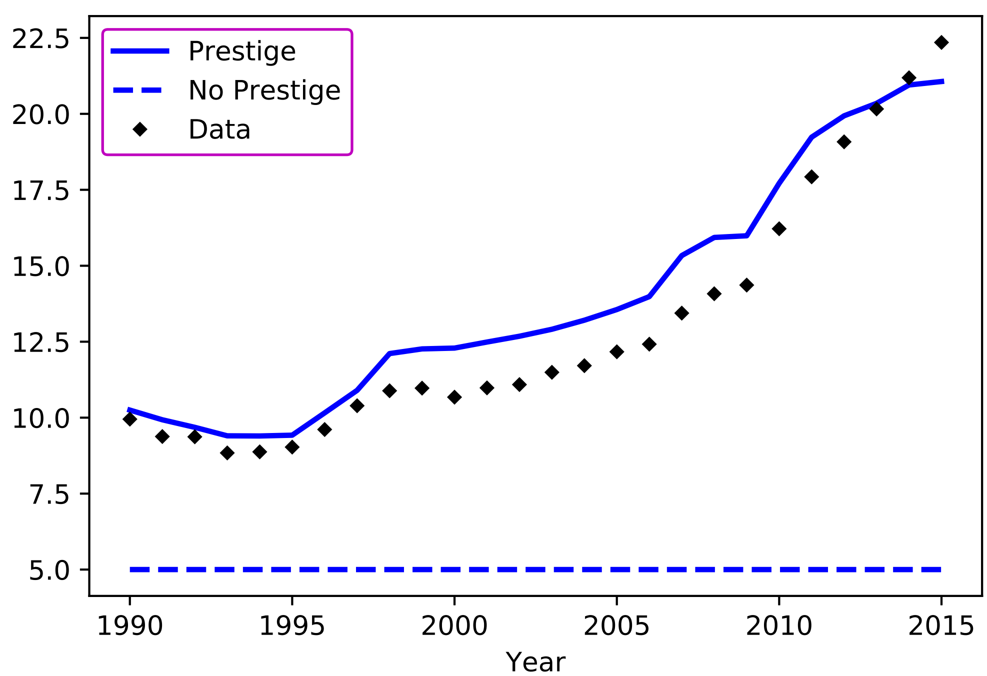
Source: Model.

Figure 17: Mean Skill of Applicants and Matriculates



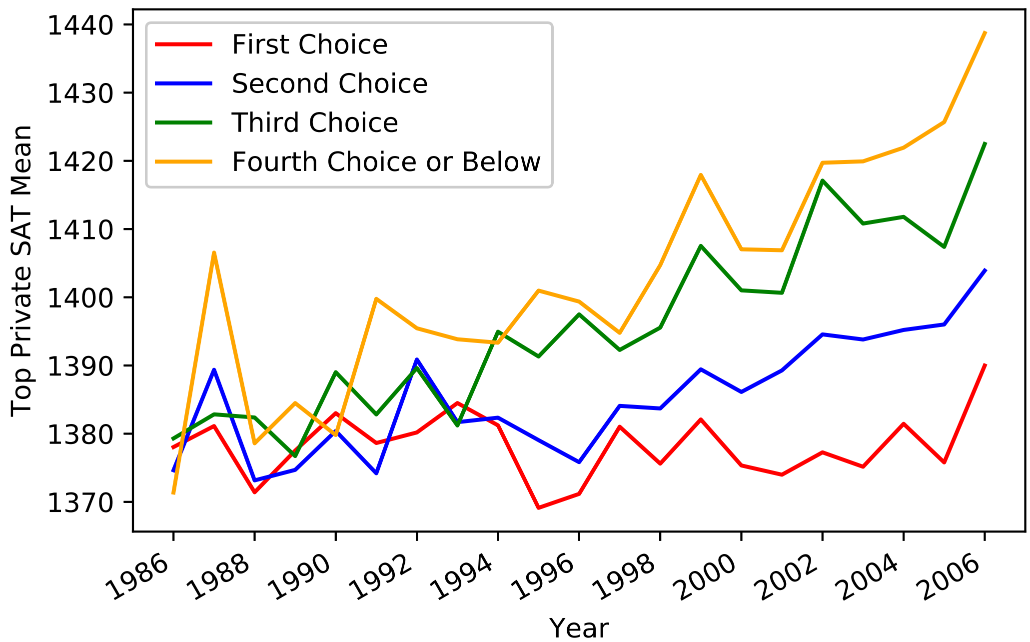
Sources: Model.

Figure 18: Mean Ratio of Applications to Positions



Sources: IPEDS (1990 - 2015) and Model.

Figure 19: Matriculation by Personal Rank of Top Private College (Mean)



Source: HERI, 1986 - 2006.

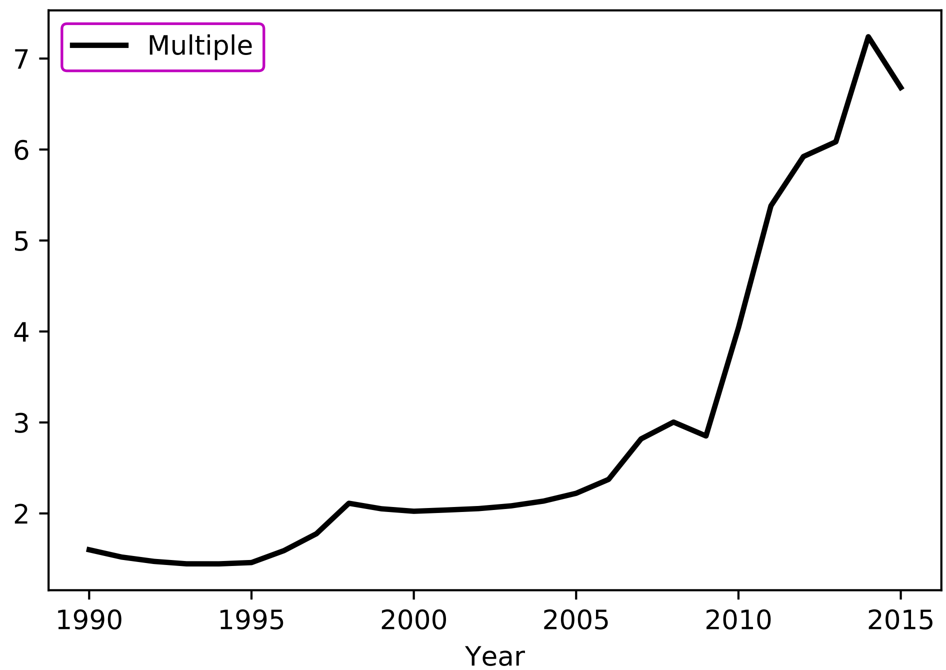
Explanation: Following Chade et al. (2014), "top private" colleges have a mean SAT score of 1340 or higher. For top private college matriculates, the figure shows the mean personal rank of the actual matriculated college of out of all colleges to which they applied. ACT scores were converted to their SAT equivalent values using the method footnoted earlier.

6.4 Loss in Peer-Group Producer Surplus

With first-degree price discrimination, producer surplus is given by the "profits" term in equation (9). Figure 20 shows the weighted mean ratio of producer surplus in the counter-factual simulation without prestige relative to with prestige. Of course, the ratio in 1990 exceeds the value of unity, since the prestige externality is socially inefficient. More importantly, the ratio grew over time, tripling in value by 2015, thereby indicating a sizable increase in the relative loss of social surplus due to the prestige externality. HPSY colleges would be better off if they could legally coordinate their admissions policies, thereby internalizing some of of the prestige externality to *expanding* admissions.[40] To

[40]HPSY colleges might still care about their prestige as a *group* relative to elite but less competitive colleges. However, this concern still leaves considerable room to increase producer surplus from collusion among HPSY colleges since other colleges are not perfect substitutes, as shown in Appendix A.

Figure 20: Producer Surplus Without Prestige Relative to With Prestige



Source: Simulation Model.

Explanation: Figure shows the ratio of producer surplus in the counter-factual simulation without prestige $(R = S = 0)$ relative to producer surplus with prestige, normalized to a value of 1 in 1990.

be sure, the assumption of first-degree price discrimination simplifies many real-world complexities. Appendix A presents an alternative surplus calculation based on Avery et al. (2012) method and data using student preferences. In both cases, the potential loss in surplus from prestige competition is quite large. More generally, a reduction in prestige competition would improve the welfare of both colleges and students, consistent with a Pareto improvement.

7 Conclusion

Total U.S. college enrollment has doubled over the past half century. However, enrollment at elite colleges has barely increased. Even as the quality of students seeking admission has improved markedly over time, elite colleges have instead allowed their admit rates to plummet, often rejecting students that are essentially indistinguishable from admits. Admit rates at HPSY (Harvard, Princeton, Stanford and Yale) colleges—four colleges that regularly rotate the top ranked position in the ubiquitous annual U.S. News college ranking—are today around 5 percent or one-quarter of their 1990 values. Similarly, admit rates at The University of Chicago and Penn, which exceeded 40 percent as late as 1990, stand at less than 10 percent today. In contrast, admit rates among non-elite colleges have increased over time. Falling admit rates is unique to elite colleges.

In Appendix B, we show that seemingly obvious explanations—physical constraints, maintaining student and professor quality, knock-on effects and "look and feel"—don't explain the dramatic trend differences in enrollments between elite and non-elite colleges. Of course, elite colleges value metrics such as their selectivity or admit rate. But, we prove (Section 3) that if colleges only care about the *absolute* value of these metrics, admits rates would have had counter-factually *increased* with demand at elite colleges.

Instead, falling admit rates require that colleges also care about *prestige* which is inherently measured *relative* to peers. Specifically, we show that the decisions of elite colleges is consistent with a model where colleges value prestige, which is measured either as their selectivity (e.g., average SAT score) relative to that of their peer group or by their relative admit rates. When a college's weight on prestige is above a critical value, an increase in demand *decreases* the admit rate, consistent with the experience for *elite* colleges. Conversely, when a college's weight on prestige is below this critical value, an increase in demand *increases* the admit rate, consistent with the evidence observed with *non-elite* colleges. Using HPSY colleges as a case study, a calibrated version of the model closely replicates the empirical evidence (including admit rates, matriculates, skills, and applications), while counter-factual simulations, with prestige turned off, fail.

An important consequence of elite college's preferences over prestige is the under-provision of spaces due to a non-pecuniary prestige externality. As discussed in Section 1, this externality is internalized in several European, Scandinavian and other countries where the government picks the total enrollment for each elite college. In the United States, elite colleges could internalize the externality if they could coordinate their admission decisions, even if just at the *aggregate* level (e.g., the top 200 colleges commit to a minimum admit rate rather than agree on which students to admit). This type of coordination, however, is currently illegal on antitrust grounds under the Sherman Act (Depalma, 1992).

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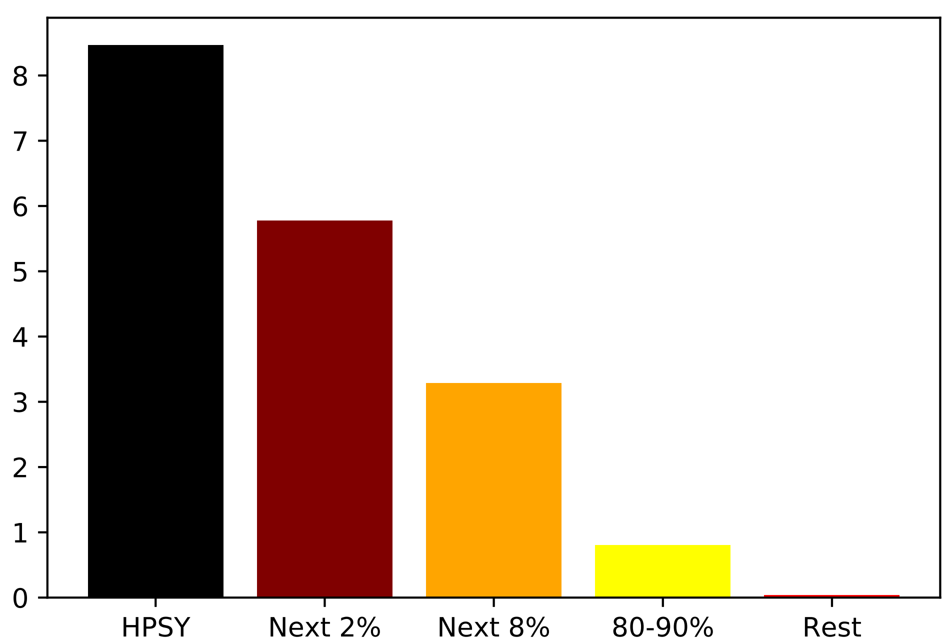
A Illustrative Lower-Bound Consumer Surplus Loss

Avery et al. (2012) use micro-data on the application, admission and matriculation decisions of 3,240 randomly selected high achieving students from schools with a history of placing multiple students in selective colleges. In their empirical model, the indirect utility u_{ij} of student i choosing college j is a function of the subjective mean quality of college (θ_j) as well as ten other factors ($x_{i,j}$), including sticker tuition price, features of the financial aid offer (cash grants offered, loans offered, amount of work study offered), familial connections to the university (whether father, mother or sibling is an alum of the college), and location (whether the college is in the home state or home region and the distance of the college from the student's home):

$$u_{ij} = \theta_j + \sum_{k=1}^{10} x_{ijk}\delta_{ik} + \epsilon_{ij} \tag{17}$$

Figure 21 reports the enrollment-weighted value of θ_j estimated by Avery et al. (2012) for HPSY colleges as well as the "Next 2%" of colleges with the highest θ. For completeness, Figure 21 also reports the θ values for other colleges as well, including the next 8%, the next 10% and, finally, the average remainder of other colleges.

Figure 21: Avery et al. (2012) θ,
Weighted mean within percentile by enrollment



Source: Avery 2013

For the purposes of our back-of-the envelope calculation, we consider the average utility drop off from HPSY to the "Next 2%" top colleges, which is about 2.5 utility (θ) points. This estimate conservatively assumes that a student who is rejected from a HPSY college is accepted with certainty at a "Next 2%" top college.

The Avery et al. (2012) analysis allows us to "dollarize" this drop in θ by asking: how much reduction in financial aid would an average student rejected from a HPSY college be willing to forfeit (or, more generally, pay out-of-pocket if greater than total possible financial aid) to obtain the same utility as attending a HPSY college? Totally differentiate equation (17) and set to zero to get

$$\begin{aligned} du_{ij} = 0 &= \frac{\partial u_{ij}}{\partial \theta_j} d\theta_j + \frac{\partial u_{ij}}{\partial \tilde{x}_{ij}} dx_{ij2} \\ &= d\theta_j + \delta_i dx_{ij2}, \end{aligned}$$

where x_{ij2} is the value of cash grants offered to student i by college j. Hence:

$$dx_{ij2} = -\frac{1}{\delta_{i2}} d\theta_j \tag{18}$$

Table 4 of Avery et al. (2012) provides a point estimate of $\delta_2 = 0.087$, averaged across students, with a standard error of $= 0.007$, per $\$1,000$ of grants per year. Combining with $d\theta_j = -2.5$, the dollar-equivalent loss in consumer (student) surplus of being rejected from a HPSY college is

$$dx_{ij2} = \frac{1}{0.087} \times 2.5 \times 4\$1,000 = \$114,942 \tag{19}$$

with a standard error of \$9,248.[41] The 95% confidence interval is $[\$96,816, \$133,068]$. Larger losses would emerge if we also included that a student rejected from a HPSY college is not accepted to a "Next 2%" college with certainty. Even so, this calculated loss in consumer surplus is comparable to 140% of the mean total tuition over four years of college in the Avery et al. (2012) sample. It is also of the same order of magnitude as illegal payments to coaches by parents for the purpose of getting their students into elite universities.[42]

[41]Given $\Delta U(\delta_2) = \frac{A}{\delta_2}$, where δ_2 is the conversion factor and A is the constant of proportionality. The standard error on $se(\Delta U(\delta_2)) = \sqrt{var(\delta_2)[\Delta U'(\delta_2)]^2} = \sigma_{\delta_2} \times \frac{\Delta U(\delta_2)}{\delta_2} = 0.007 \times \frac{114{,}942}{0.087} = 9,248$.

[42]For example, former Stanford sailing coach, John Vandemoer, was paid \$270,000 to classify two students as prospective sailors in order to boost their admissions chances. Both were denied admission. An unnamed family paid Rick Singer, the mastermind behind the college admissions fraud, \$1.2M to have their daughter classified as a women's soccer recruit at Yale. She was admitted.

B Potential Alternative Explanations

Before presenting our prestige-based model, we consider potential competing explanations of supply constraints at elite colleges that are not based on prestige.

B.1 Costs

Fixed physical constraints are an unlikely explanation for why elite colleges have differentially failed to expand. Colleges like Stanford and Duke have plenty of land and have added significant building capacity over the past decade. Even colleges with less land within their traditional boundaries have added substantial physical capacity, what The New York Times (Martin, 2012) referred to as a "decade-long spending binge." Brown, Princeton, Penn, and MIT have added numerous new buildings within their existing footprints. Harvard, Yale, Columbia, and Cornell added new campuses.

As we show later (Section 4), marginal costs, however, *have* risen over time. Still, the demand for admission has increased by substantially more. If there were no concern for prestige, we show that admit rates at even elite research-based colleges would be much higher today even with rising marginal costs over time (Section 4).

In fact, our estimates of rising marginal costs embellish their true costs in a counterfactual where prestige were not important. Elite research colleges could rely on additional, inexpensive adjunct faculty for their teaching needs while still remaining competitive with less elite colleges. Nationwide, less than 40 percent of students are now taught by tenured or tenure-track professors, down from over 75 percent several decades ago (Carey, 2012). In fact, Figlio et al. (2015) find that students, especially average and less-qualified ones, learn relatively *more* from *non-tenure track* professors in their introductory courses at Northwestern. Our own experience is that undergraduates rarely understand the difference between adjunct and tenure-track faculty; they also do not seem to care upon learning the distinction. Additional students could, therefore, be easily priced in excess of marginal costs for universities who really cared about research instead of prestige. However, such a shift in teaching responsibilities would lead to a penalty in rankings.[43]

Hurricane Katrina provided a natural experiment in 2005. In response to displaced students from universities like Tulane, Ivy Plus colleges accommodated several hundred

[43] In the ubiquitous *US News and World Report* college rating, for example, two factors—faculty salaries and the proportion of faculty who have obtained the highest degree in their field—compose 10 percent of the total score received by a college. Since top rankings are often separated by small differences, this weighting creates substantial pressure to have research faculty teach core courses, despite evidence that non-tenure track teachers often make better teachers (Figlio et al., 2015). In contrast, the student-faculty ratio, a metric that tends to favor using more adjuncts, counts only 1 percent.

additional students (Associated Press, 2015). Students could stay until graduation.

B.2 Maintaining Student Quality

Are elite colleges slow to expand because they are simply "holding the line" in student quality? Hoxby (2009), however, shows that re-centered SAT and ACT scores actually significantly increased at elite colleges over time, which are now admitting students close to the maximum scores. From 1970 - 2000, the number of applicants to Stanford with Math scores exceeding 700 increased by a factor of 2.3 (Hennessy, 2007). By 2016, just 12, 8 and 13 percent of students with perfect scores on their SAT reading, math and writing exams, respectively, where admitted to Stanford, as were only 6 percent with 4.0+ high school GPA's.[44] At Yale College, the inter-quartile range of the SAT Math of admitted students increased from 620-730 (1975) to 710-790 (2014) and likewise for SAT Verbal from 670-780 (1975) to 710-800 (2014).[45] If elite colleges were simply maintain student quality, their total enrollments would have doubled or tripled since just 1990.

B.3 "Knock-On" Effects

Many elite colleges, with the notable exception of University of Chicago, started accepting the Common Application in the 1980's, thereby reducing the effective cost of applying to elite colleges.[46] As Bound et al. (2009) nicely shows, this primary increase in applications has a secondary effect—a "knock-on effect"— of increasing applications per student even more due to the falling admit rates from the primary effect. Our extended model, presented later, captures the knock-on effect. As we show, the knock-on effect would not have existed if elite colleges simply increased available slots in proportion to the increase in the number of high-quality *students* (as opposed to applications). Instead, as discussed above, the average student quality increased substantially at elite colleges.

[44] http://admission.stanford.edu/apply/selection/profile16.html

[45] These statistics account for the re-centering of the SAT in 1995.

[46] Most of the colleges shown in Section 1 started accepting the Common Application a decade or more before the data period shown. One exception is The University of Chicago, which, in the middle 1990's, also deviated from the decreasing admit trend, largely because it resisted driving up its application count (Hoover, 2010). As its admission dean from 1989 to 2009, Theodore A. O'Neill, said: "It is important to signal something true and meaningful about yourself. The more signals, the more honest you're being, and doing that does limit the applications." Later leadership, however, emphasized increasing applications, including accepting the Common Application.

Figure 22: Change in number of Verbal and Math SAT scores in 700 800 Range, 1996 - 2015

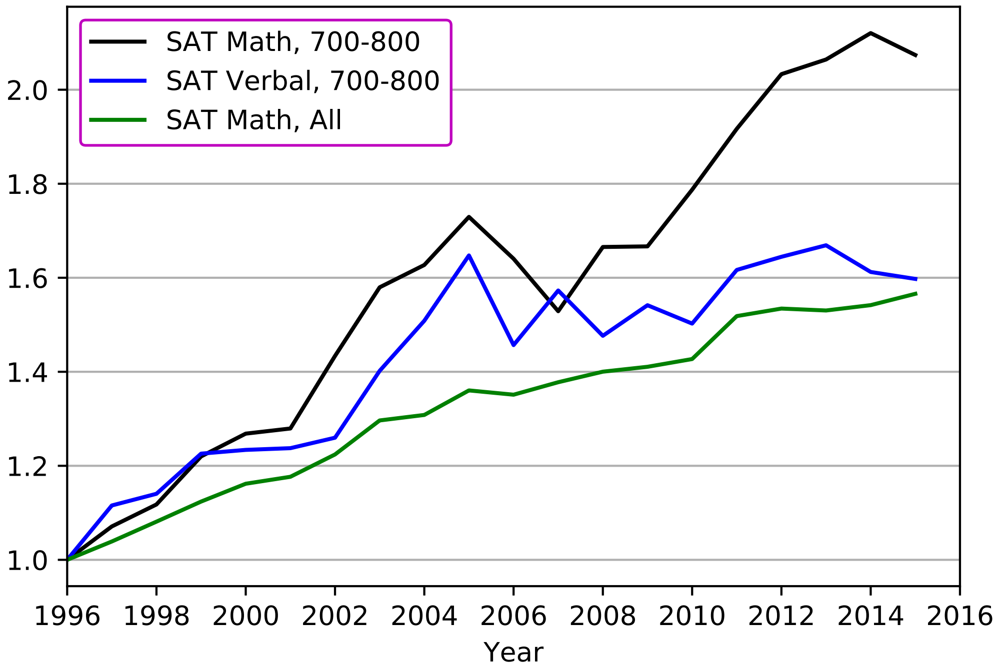
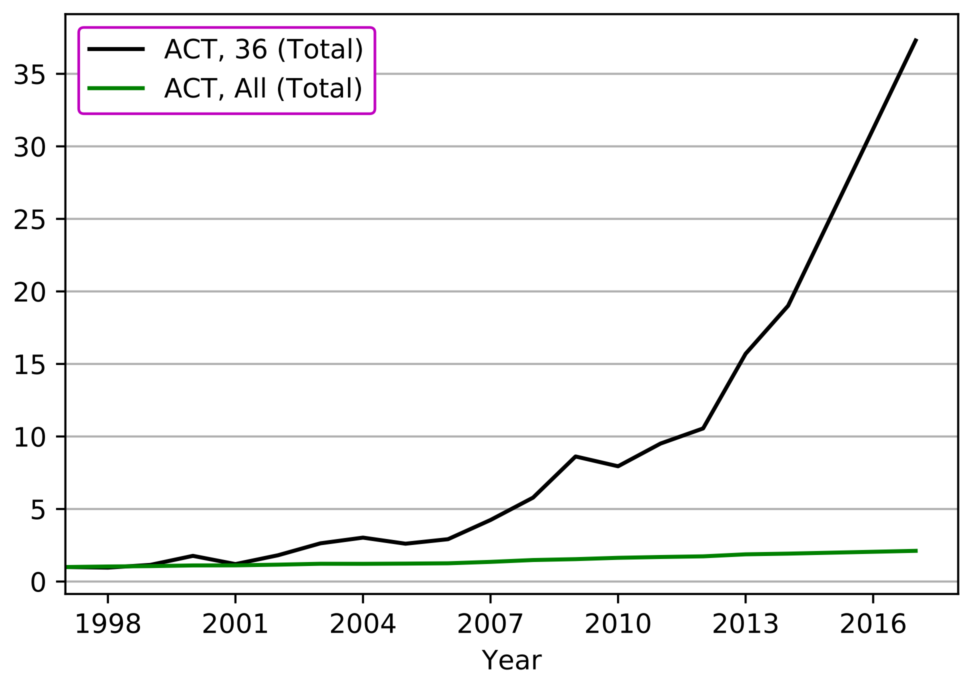
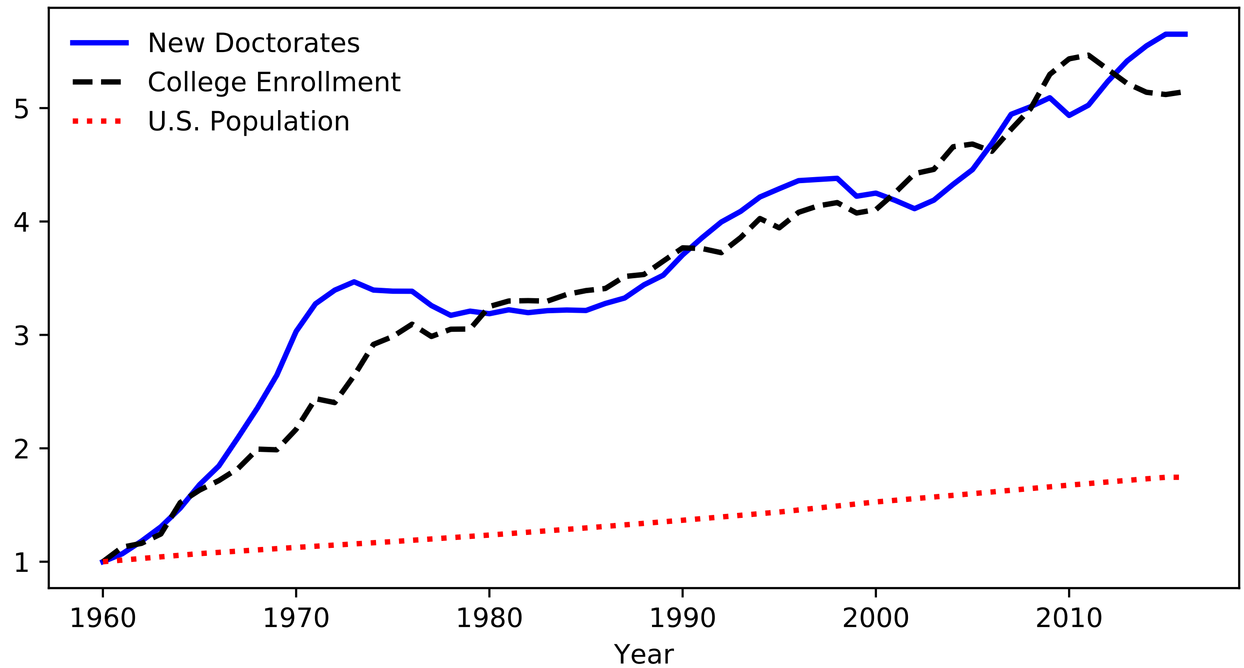


Figure 23: Change in number of ACT scores equal to 36, 1997 - 2017



Source: Authors' calculations based on data from IPED.

Figure 24: Doctorates, College Enrollment, Population Ratios (1960 = 1), 1960 - 2015



Sources: National Science Foundation; U.S. Census.

B.4 Maintaining Research Quality

Maybe elite institutions are instead "holding the line" on professor quality, including research potential. There are several problems, however, with this theory as well.

First, as Figure 24 shows, the quantity of doctoral degrees awarded have largely kept up with college enrollment, both of which have risen much faster than general population growth. Moreover, quality of doctoral students has also increased over time. Using data collected from U.S. News World Report for various graduate programs over the past two decades, we find that average test (GMAT, GRE and MCAT) scores of matriculates generally increased within the top 10 programs ranked in 1994, the first year in which U.S. News World Report started tracking this data for select fields.

Second, enrollments at elite but less-research intensive liberal arts colleges increased by only 14.7% between 1990 - 2015—in fact, slightly *less* than the "Top 2%" colleges shown earlier in Figure 2—thereby producing falling acceptance rates.[47] If research were the binding constraint rather than prestige, we would expect these enrollments to have increased with the national average.

[47]For this calculation, we selected the 10 highest ranked liberal arts colleges in the 2018 Best National Liberal Arts Colleges, U.S. News and World Report: Williams, Amherst, Swarthmore, Wellesley, Middlebury, Pomona, Carleton, Claremont McKenna, Davidson, and Washington and Lee.

Third, empirically, it appears that "size matters" empirically for maximizing research, as the largest universities produce the most research. Only two Ivy League universities rank within the top 10 universities by total spending on research and development. Large, public universities comprise half of the top 10 as well as two-thirds of the top 50 (National Science Foundation, 2018). To be sure, *research per faculty* member might be higher at more elite colleges. But, not only is this relationship obviously endogenous (the best colleges attract the best researchers), it is unclear why any school would want to maximize its reported research per faculty member, unless it were for prestige, potentially including the ability to influence grant-writing organizations in the presence of imperfect information. Maximizing research per faculty member is almost certainly globally (socially) inefficient. It might even be inefficient locally (for the competing peer college group) as well.

Indeed, faculty members at elite research colleges routinely argue with college administrators for more faculty research slots. Resistance usually comes from college administrators who cite limited teaching needs based on student body size.

B.5 "Look and Feel"

Another possible explanation is that colleges have settled in on the "look and feel" of their current campuses. Section 1 already discusses this issue in some detail. This subsection provides some additional details.

Interestingly, before the modern college ratings era begin in earnest in the 1970's, elite colleges readily expanded their enrollment. Between 1920 and 1970, both Yale and Stanford increased enrollment by over 250 percent. Indeed, since its start in 1701 and 1970, Yale increased its enrollment by about two percent per year on average, slightly faster than the general population growth rate (Pierson, 1983). Before the Civil War, Harvard and Yale actually competed to be the largest colleges in the nation (*ibid*). In other words, the slowdown in enrollments at elite colleges is a fairly modern phenomenon.

Moreover, even since just 1990, the "look and feel" of campuses have changed dramatically, including with the rise in apartment-like dormitories, luxury recreational and dining facilities (e.g., Duke recently spent $90 million updating its West Union dining hall), and, more generally, a growing appearance of shifting from traditional *loco parentis* toward "students as customers" (The New York Times Editors, 2010). Moreover, MBA programs at elite colleges, including Harvard and Wharton, have also doubled in size since 1990, even as the size of their undergraduate programs have flat-lined.[48]

[48]The primary measure of desirability to MBA students and used in many MBA ratings—the average starting wage upon graduation—appears to have been largely unaffected by growth in MBA enrollment at top programs. In contrast, average starting wage is not a common metric used to measure undergraduate

C Extensions to Base Model Presented in Section 3

Adding Dynamics

We now extend the base model to accommodate dynamics. According to the Higher Education Research Institute, the top reason high school seniors pick a college is due to its academic reputation HERI (2013). Reputation is especially important for elite colleges. To capture this feature of student preferences, we now allow a student's willingness-to-pay for a college to be a function of its prestige. We also allow for a college's prestige weight to increase in its demand, reflected in the growth of its applications.

Consider a two-period model in which a college chooses the number of students to admit in both periods, q_t and q_{t+1}, to maximize its two period utility:

$$\begin{aligned} U = &\underbrace{r_{i,t}\left(\frac{1-a_{i,t}}{1-a_{-i,t}}\right) + (P_{0,i,t} - b_{i,t}q_{i,t})q_{i,t} - c_{i,t}q_{i,t}}_{\text{Period } t \text{ Utility}} \\ &+ \underbrace{\beta\left[r_{i,t+1}\left(\frac{1-a_{i,t+1}}{1-a_{-i,t+1}}\right) + (P_{0,i,t+1} - b_{i,t+1}q_{i,t+1})q_{i,t+1} - c_{i,t+1}q_{i,t+1}\right]}_{\text{Period } t+1 \text{ Utility}} \end{aligned} \tag{20}$$

where the prestige scaling parameter takes the familiar form for both periods:

$$r_{i,t} = \rho_{i,t}\left[\frac{(P_{0,i,t} - c_{i,t})^2}{2b_{i,t}}\right]. \tag{21}$$

The maximum willingness to pay in period $t+1$, $P_{0,i,t+1}$, is given by:

$$P_{0,i,t+1} = \underbrace{(1+g)P_{0,i,t}}_{\text{Secular Growth}} + \underbrace{\epsilon P_{0,i,t}\left(\frac{1-a_{i,t}}{1-a_{-i,t}} - 1\right)}_{\text{Prestige}}, \tag{22}$$

where g is the secular growth rate (potentially zero) of demand common to all colleges in the peer group, arising, for example, from population growth or an increasing skill premium. The second term in equation (22) shows that willingness to pay is also increasing in a college's selectivity relative to its peers. For consistency, we measure selectivity using the college's relative acceptance rate, but a college's selectivity could just as easily be

institutions since starting salaries differ substantially with the choice of major. For example, colleges *outside* of the Ivy League, typically with strong engineering and nursing programs, dominate the top-25 ranking of highest starting salaries (PayScale, 2017).

measured by the average skill of its students (Section 4). A one percentage point increase in relative selectivity shifts demand out by $100 \times \epsilon$ percent.

The prestige weight ρ_{t+1} of college i changes over time:

$$\rho_{t+1} = \rho_t + \nu(Q_{i,t+1} - Q_{i,t}), \tag{23}$$

where $(Q_{i,t+1} - Q_{i,t})$ is the difference in the number of applicants between period t and period $t+1$, which grows by the gradient ν. Intuitively, prestige increases in the change in demand. Since colleges observe applications before making admissions decisions, ρ_{t+1} is a function of contemporaneous demand, without loss of generality.

For a given prestige weight ρ_{t+1}, the comparative statics are qualitatively identical to the static model discussed above:

Definition 4. *Let* $\rho^d_{i,t} \equiv \rho_i \left(1 + \frac{\epsilon \beta P_{0,i,t} q_{i,t+1}}{r_{i,t}}\right)$ *be the* dynamic prestige weight.

Theorem 4. *For N identical colleges with prestige* $\rho_{t+1} > 0$ *and* $\rho^d_t > 0$*, there is a unique feasible Nash equilibrium of the two-period dynamic game* (a_t, a_{t+1}) *that is also Pareto inefficient (as in Theorem 1). Moreover,* $\exists\, \rho^{d*}_t \in [0, \frac{1}{\bar{a}_t}]$ *such that* $\forall\, \rho^d_t \in [0, \rho^{d*}_t)$ $\frac{da_t}{dP_0} > 0$*, and* $\forall\, \rho^d_t \in (\rho^{d*}_t, \frac{1}{\bar{a}_t}]$ $\frac{da_t}{dP_0} < 0$.

However, the presence of dynamics allows for the prestige weight to change over time, which reduces the admit rate by even more than predicted by the previous static model where students were not willing to pay more for a more prestigious college ($\epsilon = 0$):

Theorem 5. *The equilibrium admit rate* a_t *is decreasing in* $\epsilon > 0$*, i.e.* $\frac{da_t}{d\epsilon} < 0$*. In words, if the willingness to pay by students is increasing in prestige, colleges admit fewer students in equilibrium.*

Heterogeneous Colleges

The base model presented in Section 3 also assumed that schools were homogeneous ($\rho_i = \rho$, $P_{0,i} = P_0$, and $c_i = c$). For succinctness, we present results for the simple case of N=2 schools. The utility function for college 1 is given by:

$$U_1 = \underbrace{r_1 \left[\left(\frac{1-a_1}{1-a_2}\right) - 1\right]}_{\text{Prestige}} + \underbrace{(P_{0,1} - b_1 q_1) q_1 - c_1 q_1}_{\text{"Profit"}}. \tag{24}$$

and the utility function for college 2 is given by:

$$U_2 = \underbrace{r_2 \left[\left(\frac{1-a_2}{1-a_1} \right) - 1 \right]}_{\text{Prestige}} + \underbrace{(P_{0,2} - b_2 q_2) q_2 - c_2 q_2}_{\text{“Profit”}}. \tag{25}$$

The following two theorems are similar to those provided earlier for homogeneous schools:

Theorem 6. *For $\rho_1 > 0$, and $\rho_2 > 0$, there exists a unique feasible Nash equilibrium (a_1^*, a_2^*). Moreover $a_i^* < \overline{a}_i$ for $i = 1, 2$, i.e., the admit rates with prestige are smaller than their corresponding values without prestige.*

Theorem 7. *An increase in $P_{0,i}$* ***decreases*** *the equilibrium admit rate $a_{i,L}$, i.e. $\frac{da_i}{dP_{0,i}} < 0$, if and only if:*

$$\rho_i > \frac{1 - a_{-i}^*}{2\overline{a}_i}. \tag{26}$$

The following two theorems are more unique to the case with heterogeneous colleges, in essence showing the “cross-partial” derivatives associated with competition.

Theorem 8. *For $\rho_i > 0$ and $\rho_{-i} > 0$, $\frac{da_i}{d\rho_i} < 0$ and $\frac{da_i}{d\rho_{-i}} < 0$.*

Theorem 9. *Increases in demand for competitor school is proportional to increases in own demand:*

$$\frac{da_i}{dP_{0,-i}} = -(\overline{a}_i - a_i) \frac{da_i}{dP_{0,i}}. \tag{27}$$

D Proofs

Lemma 1

Consider the admit rate that is chosen when each school fully internalizes the effect of its supply decision on the supply decision of its competitors:

$$U^{FB} \equiv \sum_{i=1}^{N} \left[r_i \left(\frac{1-a_i}{1-a_{-i}} \right) + (P_{0,i} - b_i q_i) q_i - c_i q_i \right]. \tag{28}$$

College j chooses an amount q_j to maximize its utility. This generates the following FOC:

$$\overbrace{\underbrace{\frac{-r_j}{Q_j(1-a_{-j})}}_{\text{Direct Effect}} + \underbrace{\sum_{i \neq j} \left[r_i \frac{1-a_i}{(1-a_{-i})^2} \frac{1}{(N-1)Q_j} \right]}_{\text{Indirect Effect}}}^{\text{Marginal Prestige}} + \overbrace{(P_{0,j} - 2b_j q_j) - c_j}^{\text{Marginal Profit}} = 0. \tag{29}$$

Increasing the number of students q_j has a direct effect on relative prestige – it lowers relative prestige by lowering college j's rejection rate. The indirect effect of increasing the quantity supplied by college j is that it results in other institutions being relatively more selective than college j, which increases the utility for each of j's peers. When colleges are identical, in equilibrium $a_i = a_{-i} = a_j \ \forall i$ and the direct and indirect effect of increasing quantity exactly cancel. The resulting FOC is the standard profit condition of marginal revenue equals marginal cost, which produces the monopoly quantity for colleges and hence the monopoly admit rate, $\bar{a} = \frac{P_0 - c}{2bQ}$

Theorem 1

Solving for Nash Equilibria

The first order condition for school i is:

$$-r_i \left(\frac{1}{(1 - a_{-i})Q} \right) + P_{0,i} - c_i - 2b_i q_i = 0. \tag{30}$$

$$\implies \underbrace{P_{0,i} - 2b_i q_i}_{\text{Marginal Revenue}} = \underbrace{c_i + r_i \left(\frac{1}{(1 - a_{-i})Q} \right)}_{\text{Marginal Cost + Marginal Prestige}} \tag{31}$$

From this first order condition we generate the following best response function:

$$a_i(a_{-i}) = \bar{a}_i - \frac{\rho_i \bar{a}_i^2}{1 - a_{-i}} \tag{32}$$

where:

$$\bar{a}_i \equiv \frac{P_{0,i} - c_i}{2b_i Q} \tag{33}$$

and ρ_i is the prestige weight defined in equation (2). Since the N peer colleges are identical, we consider the case of a symmetric Nash equilibrium in which $a_i = a_{-i} \equiv a$. Any such Nash equilibria must satisfy the following quadratic equation:

$$a^2 - a(1 + \bar{a}) - \bar{a}(\rho \bar{a} - 1) = 0. \tag{34}$$

The two solutions that solve the Nash condition are:

$$a_L = \frac{(1 + \bar{a}) - \sqrt{(1 + \bar{a})^2 + 4\bar{a}(\rho \bar{a} - 1)}}{2} \tag{35}$$

and:

$$a_H = \frac{(1+\overline{a}) + \sqrt{(1+\overline{a})^2 + 4\overline{a}(\rho\overline{a}-1)}}{2} \tag{36}$$

We prove that $0 < a_L^* < \overline{a}$ is the unique feasible equilibrium using a proof by contradiction. First we show that $a_H < 1$ implies that $\rho < 0$, which is also a contradiction; hence a_H is *not* feasible. Using equation (36), the condition $a_H < 1$ requires:

$$(1+\overline{a}) + \sqrt{(1+\overline{a})^2 + 4\overline{a}(\rho\overline{a}-1)} < 2. \tag{37}$$

Rearranging this expression we obtain:

$$\sqrt{(1+\overline{a})^2 + 4\overline{a}(\rho\overline{a}-1)} < 1 - \overline{a} \tag{38}$$

$$(1+\overline{a})^2 + 4\overline{a}(\rho\overline{a}-1) < (1-\overline{a})^2 \tag{39}$$

$$4\rho\overline{a}^2 < 0 \tag{40}$$

$$\implies \rho < 0. \tag{41}$$

Second, we show that a_L is feasible by showing that it is less than the monopoly admit rate $\overline{a}$, which is by construction feasible. To prove this part of the proposition, we show that $a_L > \overline{a}$ implies that $\rho < 0$, which is a contradiction; hence $a_L < \overline{a}$. From equation (35), the condition of $a_L > \overline{a}$ requires:

$$(1+\overline{a}) - \sqrt{(1+\overline{a})^2 + 4\overline{a}(\rho\overline{a}-1)} > 2\overline{a}. \tag{42}$$

Rearranging this expression:

$$(1-\overline{a}) > \sqrt{(1+\overline{a})^2 + 4\overline{a}(\rho\overline{a}-1)} \tag{43}$$

$$(1-\overline{a})^2 > (1+\overline{a})^2 + 4\overline{a}(\rho\overline{a}-1) \tag{44}$$

$$0 > 4\rho\overline{a}^2 \tag{45}$$

$$\implies \rho < 0. \tag{46}$$

Finally, we show that $a_L > 0$ is consistent with $\rho > 0$. From equation (35), the condition $a_L > 0$ requires:

$$(1+\bar{a}) > \sqrt{(1+\bar{a})^2 + 4\bar{a}(\rho\bar{a}-1)} \tag{47}$$
$$0 > 4\bar{a}(\rho\bar{a}-1) \tag{48}$$
$$0 > \rho\bar{a}-1 \tag{49}$$
$$\implies \rho < \frac{1}{\bar{a}}. \tag{50}$$

Since $\bar{a} > 0$, the condition $\rho < \frac{1}{\bar{a}}$ is consistent with $\rho > 0$.

Lemma 2

For an equilibrium quantity q, or equivalently an admit rate $a = \frac{q}{Q}$, the consumer surplus is the area between the inverse demand curve and the equilibrium price:

$$CS(a) \equiv \frac{1}{2}[P_0 - P(q)]q \tag{51}$$
$$= \frac{1}{2}[P_0 - P(Qa)]Qa \tag{52}$$
$$= \frac{1}{2}(P_0 - P_0 + bQa)Qa. \tag{53}$$
$$= \frac{P_0^2 a^2}{2b}, \tag{54}$$

where $Q = \frac{P_0}{b}$ because of zero application cost. The producer surplus at this quantity the area between the equilibrium price and the marginal cost curve:

$$PS(a) = [P(q) - c]q \tag{55}$$
$$= [P(Qa) - c](Qa) \tag{56}$$
$$= [P_0 - c - bq]Qa \tag{57}$$
$$= \left[\frac{P_0 - c}{bQ} - a\right] bQ^2 a \tag{58}$$
$$= (2\bar{a} - a)\frac{P_0^2}{b}a \tag{59}$$
$$= \frac{P_0^2}{b}(2\bar{a} - a)\,a \tag{60}$$

where, recall, $\bar{a}$ is the no-prestige ($\rho = 0$) admit rate.

Theorem 2

The total social surplus at the equilibrium admit rate a is the sum of the consumer surplus of equation (54) and the producer surplus of equation (60):

$$SS(a) = \frac{P_0^2}{2b}(4\bar{a} - a)a. \tag{61}$$

The standard dead weight loss (DWL) is the difference between the social surplus at the $\rho = 0$ competitive admit rate $2\bar{a}$ and the $\rho = 0$ monopoly admit rate $\bar{a}$:

$$DWL = SS(2\bar{a}) - SS(\bar{a}) = \frac{P_0^2}{2b}. \tag{62}$$

The prestige dead weight loss (PDWL) is the difference between the social surplus generated from the admit rate $\bar{a}$ with $\rho = 0$ and the $\rho > 0$ equilibrium admit rate $a^* < \bar{a}$:

$$PDWL = SS(\bar{a}) - SS(a) = \frac{P_0^2}{2b}(\bar{a} - a^*)(3\bar{a} - a^*). \tag{63}$$

The condition $PDWL > DWL \implies PDWL - DWL > 0$:

$$\underbrace{\frac{P_0^2}{2b}(\bar{a} - a^*)(3\bar{a} - a^*)}_{\text{PDWL}} - \underbrace{\frac{P_0^2\bar{a}^2}{2b}}_{\text{DWL}} > 0. \tag{64}$$

This inequality simplifies to:

$$a^{*2} - 4a^*\bar{a} + 2\bar{a}^2 > 0 \implies (2\bar{a} - a^*)^2 > 2\bar{a}^2. \tag{65}$$

Hence $a^* < \bar{a}(2 - \sqrt{2})$ or $a^* < \bar{a}(2 + \sqrt{2})$. Because $a^* < \bar{a}$ holds, then $a^* < \bar{a}(2 - \sqrt{2})$ will certainly bind for all $a^* \in [0, \bar{a}]$.

Lemma 3

The low admission equilibrium in equation (35) is parametrized by the prestige ratio ρ. Taking the derivative of a_L with respect to ρ we find:

$$\frac{da_L}{d\rho} = -\frac{2\bar{a}^2}{\sqrt{(1 + \bar{a})^2 + 4\bar{a}(\rho\bar{a} - 1)}} < 0. \tag{66}$$

Theorem 3

The derivative of the equilibrium admit rate a_L with respect to P_0 is given by:

$$\frac{da}{dP_0} = \frac{1}{2}\frac{d\overline{a}}{dP_0}g(\rho), \tag{67}$$

where:

$$g(\rho) \equiv \left(1 - \frac{2(1+\overline{a}) + 8\rho\overline{a} - 4}{\sqrt{(1+\overline{a})^2 + 4\overline{a}(\rho\overline{a} - 1)}}\right) \tag{68}$$

Since $\frac{d\bar{a}}{dP_0} > 0$, to show $\exists\ \rho^* \in [0, \frac{1}{\bar{a}}]$ such that $\forall\ \rho \in [0, \rho^*)$ $\frac{da}{dP_0} > 0$, and $\forall\ \rho \in (\rho^*, \frac{1}{\bar{a}}]$ $\frac{da}{dP_0} < 0$, it suffices to show:

1. g(ρ) is continuous on interval $\rho \in [0, \frac{1}{\bar{a}}]$,
2. $lim_{\rho\to 0}$ g(ρ) > 0,
3. $lim_{\rho\to\frac{1}{\bar{a}}}$ g(ρ) < 0, and
4. g(ρ) is a monotonic decreasing function on interval $\rho \in [0, \frac{1}{\bar{a}}]$, i.e. g$'(\rho) < 0$.

The first three conditions, by the Intermediate Value Theorem, guarantee the existence of at least one value of ρ^* such that $g(\rho^*) = 0$. The fourth property of monontonicity guarantees uniqueness of ρ^* and gives us that $\frac{da}{dP_0} > 0$ for $\rho \in [0, \rho^*)$ and $\frac{da}{dP_0} < 0$ for $\rho \in (\rho^*, \frac{1}{\bar{a}}]$.

The first condition, continuity of g(ρ) follows trivially. The limiting behavior of g(ρ) at the end of the interval follows from the following calculations:

$$lim_{\rho\to 0}\left(1 - \frac{2(1+\overline{a}) + 8\rho\overline{a} - 4}{\sqrt{(1+\overline{a})^2 + 4\overline{a}(\rho\overline{a} - 1)}}\right) = \left(1 - \frac{2(a-1)}{\sqrt{(1-\bar{a})^2}}\right) = \left(1 + \frac{2(1-\bar{a})}{(1-\bar{a})}\right) > 0 \tag{69}$$

$$lim_{\rho\to\frac{1}{\bar{a}}}\left(1 - \frac{2(1+\overline{a}) + 8\rho\overline{a} - 4}{\sqrt{(1+\overline{a})^2 + 4\overline{a}(\rho\overline{a} - 1)}}\right) = \left(1 - \frac{2(1+\bar{a}) + 4}{(1+\bar{a})}\right) = \left(-1 - \frac{4}{(1+\bar{a})}\right) < 0 \tag{70}$$

To check that g(ρ) in monotone decreasing we show g$'(\rho) < 0$:

$$\frac{dg}{d\rho} = \frac{d}{d\rho}\left(1 - \frac{2(1+\bar{a}) + 8\rho\bar{a} - 4}{\sqrt{(1+\bar{a})^2 + 4\bar{a}(\rho\bar{a}-1)}}\right) \tag{71}$$

$$= \frac{(-8\bar{a})\sqrt{(1+\bar{a})^2 + 4\bar{a}(\rho\bar{a}-1)} + (2(1+\bar{a}) + 8\rho\bar{a} - 4)\frac{2\bar{a}^2}{\sqrt{(1+\bar{a})^2+4\bar{a}(\rho\bar{a}-1)}}}{(1+\bar{a})^2 + 4\bar{a}(\rho\bar{a}-1)} \tag{72}$$

$$= \frac{\left[4\left[(1+\bar{a})^2 + 4\bar{a}(\rho\bar{a}-1)\right] - \bar{a}(2(1+\bar{a}) + 8\rho\bar{a} - 4)\right]\frac{-2\bar{a}}{\sqrt{(1+\bar{a})^2+4\bar{a}(\rho\bar{a}-1)}}}{(1+\bar{a})^2 + 4\bar{a}(\rho\bar{a}-1)} \tag{73}$$

$$= \frac{\left[4\left[(1-\bar{a})^2 + 4\bar{a}^2\rho\right] - \bar{a}(2(\bar{a}-1) + 8\rho\bar{a})\right]\frac{-2\bar{a}}{\sqrt{(1-\bar{a})^2+4\bar{a}^2\rho}}}{(1-\bar{a})^2 + 4\bar{a}^2\rho} \tag{74}$$

$$= \frac{\left[4(1-\bar{a})^2 + 16\bar{a}^2\rho + 2\bar{a}(1-\bar{a}) - 8\bar{a}^2\rho\right]\frac{-2\bar{a}}{\sqrt{(1-\bar{a})^2+4\bar{a}^2\rho}}}{(1-\bar{a})^2 + 4\bar{a}^2\rho} \tag{75}$$

$$= \frac{\left[4(1-\bar{a})^2 + 8\bar{a}^2\rho + 2\bar{a}(1-\bar{a})\right]\frac{-2\bar{a}}{\sqrt{(1-\bar{a})^2+4\bar{a}^2\rho}}}{(1-\bar{a})^2 + 4\bar{a}^2\rho} < 0 \tag{76}$$

The last inequality to sign $g'(\rho)$ follows from feasiblity constraint on the monopoly admissions rate $0 < \bar{a} < 1$ and the conditions on prestige weight for a unique Nash equilibrium in Theorem 1, i.e. $\rho \in [0, \frac{1}{\bar{a}}]$.

Theorem 4

We solve this problem using backwards induction. Starting in the last period, $t+1$, the first order condition of college i is given by:

$$\frac{dU}{dq_{t+1}} = -\left(\frac{r_{i,t+1}}{Q_{t+1}(1-a_{-i,t+1})}\right) + P_{0,i,t+1} - c_{i,t+1} - 2b_{i,t+1}q_{i,t+1} = 0. \tag{77}$$

This results in a best response function:

$$a_{i,t+1} = \frac{P_{0,i,t+1} - c_{i,t+1}}{2b_{i,t+1}Q_{i,t+1}} - \left(\frac{r_{i,t+1}}{2b_{i,t+1}Q^2_{i,t+1}(1-a_{-i,t+1})}\right) \tag{78}$$

After simplifying, the best response function becomes:

$$a_{i,t+1} = \bar{a}_{i,t+1} - \frac{\rho_{i,t+1}\bar{a}^2_{i,t+1}}{1-a_{i,t+1}}, \tag{79}$$

where:
$$\bar{a}_{i,t+1} = \frac{P_{0,i,t+1} - c_{i,t+1}}{2b_{i,t+1}Q_{t+1}} \tag{80}$$

For the case of identical colleges, $a_{i,t+1} = a_{t+1}$ and $\rho_{i,t+1} = \rho_{t+1}$ $\forall i$; hence the equilibrium admit rate in $t+1$ is given by:

$$a_{t+1} = \frac{(1+\bar{a}_{t+1}) \pm \sqrt{(1+\bar{a}_{t+1})^2 + 4\bar{a}_{t+1}(\rho_{t+1}\bar{a}_{t+1} - 1)}}{2}. \tag{81}$$

This is identical to the result for the static case (equations (35) and (36), with the replacement of $\bar{a}_{i,t+1}$ for $\bar{a}$. From Theorem 1, we know that:

$$a_{t+1} = \frac{(1+\bar{a}_{t+1}) - \sqrt{(1+\bar{a}_{t+1})^2 + 4\bar{a}_{t+1}(\rho_{t+1}\bar{a}_{t+1} - 1)}}{2}, \tag{82}$$

is the unique feasible Nash equilibrium for $\rho_{t+1} > 0$. Moreover, we also know from Theorem 1 that $a_{t+1} < \bar{a}_{t+1}$. Further it follows from Theorem (3) that $\exists\, \rho^* \in [0, \frac{1}{\bar{a}_{t+1}}]$ such that $\forall\, \rho \in [0, \rho^*)$ $\frac{da_{t+1}}{dP_0} > 0$, and $\forall\, \rho \in (\rho^*, \frac{1}{\bar{a}_{t+1}}]$ $\frac{da_{t+1}}{dP_0} < 0$.

Continuing with the first period t, the first order condition of college i is given by:

$$\frac{dU}{dq_{i,t}} = -\left(\frac{r_{i,t}}{Q_{i,t}(1-a_{-i,t})}\right) + P_{0,i,t} - c_{i,t} - 2b_{i,t}q_{i,t} - \beta\left[\left(\frac{\epsilon P_{0,i,t}q_{i,t+1}}{Q_{i,t}(1-a_{-i,t})}\right)\right] = 0. \tag{83}$$

This results in the following period t best response function:

$$a_t = \frac{P_{0,i,t} - c_{i,t}}{2b_{i,t}Q_{i,t}} - \left(\frac{r_{i,t} + \epsilon\beta P_{0,i,t}q_{i,t+1}}{2b_{i,t}Q_{i,t}^2(1-a_{-i,t})}\right) \tag{84}$$

$$a_t = \bar{a}_{i,t} - \frac{\rho_i\bar{a}_{i,t}^2}{1-a_{-i,t}}\left(1 + \frac{\epsilon\beta P_{0,i,t}q_{i,t+1}}{r_{i,t}}\right) \tag{85}$$

where:
$$\bar{a}_{i,t} = \frac{P_{0,i,t} - c_{i,t}}{2b_{i,t}Q_{i,t}}. \tag{86}$$

To express the period t best response function of equation (85) in the standard form, we define a dynamic prestige weight:

$$\rho_{i,t}^d \equiv \rho_i\left(1 + \frac{\epsilon\beta P_{0,i,t}q_{i,t+1}}{r_{i,t}}\right), \tag{87}$$

where we use a_{t+1} from equation (90) to define $q_{i,t+1} = a_{t+1}Q_{t+1} = a_{t+1}\left(\frac{P_0(1+g)}{b_{t+1}}\right)$. With this simplification we obtain the following period 1 best response function:

$$a_{i,t} = \bar{a}_{i,t} - \frac{\rho^d_{i,t}\bar{a}^2_{i,t}}{1 - a_{-i,t}}. \tag{88}$$

For the case of identical colleges, $a_{i,t} = a_t$ and $\rho^d_{i,t} = \rho^d_t$ $\forall i$; hence the equilibrium admit rate in t is given by:

$$a_{i,t} = \frac{(1+\bar{a}_{i,t}) \pm \sqrt{(1+\bar{a}_{i,t})^2 + 4\bar{a}_t(\rho^d_t\bar{a}_t - 1)}}{2}. \tag{89}$$

This is identical to the result for the static case (equations (35) and (36), with the replacement of $\bar{a}_t$ for $\bar{a}$. From Theorem 1, we know that:

$$a_t = \frac{(1+\bar{a}_t) - \sqrt{(1+\bar{a}_t)^2 + 4\bar{a}_t(\rho^d_t\bar{a}_t - 1)}}{2}, \tag{90}$$

is the unique feasible Nash equilibrium for $\rho^d_t > 0$. Moreover, it follows from Theorem 1 that $a_t < \bar{a}_t$. Further it follows from Theorem (3) that $\exists\, \rho^* \in [0, \frac{1}{\bar{a}_t}]$ such that $\forall\, \rho \in [0, \rho^*)$ $\frac{da_t}{dP_0} > 0$, and $\forall\, \rho \in (\rho^*, \frac{1}{\bar{a}_t}]$ $\frac{da_t}{dP_0} < 0$.

Theorem 5

Equation (87) guarantees $\epsilon > 0 \implies \rho^d > \rho$.[49] According to Lemma 3 the admit rate is decreasing in the prestige weight, hence $\rho^d > \rho \implies$ the admit rate is lower in the dynamic model than in the static model.

Theorem 6

The best response for college 1 is given by:

$$a_1(a_2) = \bar{a}_1 - \frac{\rho_1\bar{a}^2_1}{1 - a_2} \tag{91}$$

The best response for college 2 is given by:

$$a_2(a_1) = \bar{a}_2 - \frac{\rho_2\bar{a}^2_2}{1 - a_1} \tag{92}$$

[49] This assumes that $\beta, P_0, r, q_{t+1} > 0$, which are necessarily true given that the discount factor, the maximum WTP, the prestige scaling factor, and the expected future number of admits are positive.

To solve for a_1 as a function of the parameters, we insert the best response function for college 2 into the best response function for college 1:

$$a_1 = \bar{a}_1 - \frac{\rho_1 \bar{a}_1^2}{1 - \left(\bar{a}_2 - \frac{\rho_2 \bar{a}_2^2}{1-a_1}\right)} \tag{93}$$

The Nash equilibria of this game must satisfy the following quadratic equation:

$$a_1^2 - a_1 \left[1 + \bar{a}_1 + \frac{\rho_2 \bar{a}_2^2 - \rho_1 \bar{a}_1^2}{1 - \bar{a}_2}\right] + \frac{\rho_2 \bar{a}_2^2 \bar{a}_1 - \rho_1 \bar{a}_1^2}{1 - \bar{a}_2} + \bar{a}_1 = 0. \tag{94}$$

The two values of a_1 that satisfy the Nash equilibrium condition are:

$$a_{1,L} = \frac{\left[1 + \bar{a}_1 + \frac{\rho_2 \bar{a}_2^2 - \rho_1 \bar{a}_1^2}{1-\bar{a}_2}\right] - \sqrt{\left[1 + \bar{a}_1 + \frac{\rho_2 \bar{a}_2^2 - \rho_1 \bar{a}_1^2}{1-\bar{a}_2}\right]^2 - 4\left(\frac{\rho_2 \bar{a}_2^2 \bar{a}_1 - \rho_1 \bar{a}_1^2}{1-\bar{a}_2} + \bar{a}_1\right)}}{2} \tag{95}$$

$$a_{1,H} = \frac{\left[1 + \bar{a}_1 + \frac{\rho_2 \bar{a}_2^2 - \rho_1 \bar{a}_1^2}{1-\bar{a}_2}\right] + \sqrt{\left[1 + \bar{a}_1 + \frac{\rho_2 \bar{a}_2^2 - \rho_1 \bar{a}_1^2}{1-\bar{a}_2}\right]^2 - 4\left(\frac{\rho_2 \bar{a}_2^2 \bar{a}_1 - \rho_1 \bar{a}_1^2}{1-\bar{a}_2} + \bar{a}_1\right)}}{2} \tag{96}$$

By symmetry, the solution for college 2 is identical to the solution for college 1, with the interchange of the index 1 for 2:

$$a_{2,L} = \frac{\left[1 + \bar{a}_2 + \frac{\rho_1 \bar{a}_1^2 - \rho_2 \bar{a}_2^2}{1-\bar{a}_1}\right] - \sqrt{\left[1 + \bar{a}_2 + \frac{\rho_1 \bar{a}_1^2 - \rho_2 \bar{a}_2^2}{1-\bar{a}_1}\right]^2 - 4\left(\frac{\rho_1 \bar{a}_1^2 \bar{a}_2 - \rho_2 \bar{a}_2^2}{1-\bar{a}_1} + \bar{a}_2\right)}}{2} \tag{97}$$

$$a_{2,H} = \frac{\left[1 + \bar{a}_2 + \frac{\rho_1 \bar{a}_1^2 - \rho_2 \bar{a}_2^2}{1-\bar{a}_1}\right] + \sqrt{\left[1 + \bar{a}_2 + \frac{\rho_1 \bar{a}_1^2 - \rho_2 \bar{a}_2^2}{1-\bar{a}_1}\right]^2 - 4\left(\frac{\rho_1 \bar{a}_1^2 \bar{a}_2 - \rho_2 \bar{a}_2^2}{1-\bar{a}_1} + \bar{a}_2\right)}}{2} \tag{98}$$

We prove that $0 < a_{i,L}^* < \bar{a}_i$ is the unique feasible equilibrium using a proof by contradiction. First we show that $a_{i,H} < 1$ implies that $\rho_{-i} < 0$, which is also a contradiction; hence $a_{i,H}$ is *not* feasible. With out loss of generality we prove this for $i = 1$ and $-i = 2$ and by symmetry it must hold for $i = 2$ and $-i = 1$. Using equation: (98), the condition $a_{1,H} < 1$

requires:

$$(1+\bar{a}_1+\psi_{1,2})+\sqrt{(1+\bar{a}_1+\psi_{1,2})^2-4(v_1+\bar{a}_1)}<2. \tag{99}$$

where we have made the following definitions:

$$\psi_{1,2}\equiv\frac{\rho_2\bar{a}_2^2-\rho_1\bar{a}_1^2}{1-\bar{a}_2}, \tag{100}$$

and

$$v_{1,2}\equiv\frac{\rho_2\bar{a}_2^2\bar{a}_1-\rho_1\bar{a}_1^2}{1-\bar{a}_2} \tag{101}$$

Rearranging the expression in equation (99) we obtain:

$$\sqrt{(1+\bar{a}_1+\psi_{1,2})^2-4(v_{1,2}+\bar{a}_1)}<1-\bar{a}-\psi_{1,2} \tag{102}$$

$$(1+\bar{a}_1+\psi_{1,2})^2-4(v_{1,2}+\bar{a}_1)<(1-\bar{a}_1-\psi_{1,2})^2 \tag{103}$$

$$4(\bar{a}_1+\psi_{1,2})<4(v_{1,2}+\bar{a}_1) \tag{104}$$

$$\implies \psi_{1,2}\leq v_{1,2}. \tag{105}$$

$$\implies \frac{\rho_2\bar{a}_2^2-\rho_1\bar{a}_1^2}{1-\bar{a}_2}<\frac{\rho_2\bar{a}_2^2\bar{a}_1-\rho_1\bar{a}_1^2}{1-\bar{a}_2} \tag{106}$$

$$\implies \rho_2\bar{a}_2(1-\bar{a}_1)<0 \tag{107}$$

$$\implies \rho_2<0. \tag{108}$$

Second, we show that $a_{i,L}$ is feasible by showing that it is less than the monopoly admit rate $\bar{a}_i$, which is by construction feasible. To prove this part of the proposition, we show that $a_{i,L}>\bar{a}_i$ implies that $\rho_i<0$, which is a contradiction; hence $a_{i,L}<\bar{a}_i$. Without loss of generality, we show this for $i=1$ and$-i=2$. From equation (95), the condition of $a_{1,L}>\bar{a}_1$ requires:

$$(1+\bar{a}_1+\psi_{1,2})-\sqrt{(1+\bar{a}_1+\psi_{1,2})^2-4(v_{1,2}+\bar{a}_1)}>2\bar{a}_1. \tag{109}$$

Rearranging this expression in equation (109):

$$(1-\bar{a}_1+\psi_{1,2})^2>(1+\bar{a}_1+\psi_{1,2})^2-4(v_{1,2}+\bar{a}_1) \tag{110}$$

$$\implies v_{1,2}>\bar{a}_1\psi_{1,2} \tag{111}$$

$$\implies \rho_1\bar{a}_1^2(\bar{a}_1-1)>0 \implies \rho_1<0. \tag{112}$$

Finally, we show that $a_{1,L} > 0$ requires an upper bound on $\rho_1 < \frac{1-\bar{a}_2+\rho_2\bar{a}_2^2}{\bar{a}_1}$. From equation (95), the condition $a_{1,L} > 0$ requires:

$$
\begin{aligned}
(1+\bar{a}+\psi_{1,2})^2 &> (1+\bar{a}+\psi_{1,2})^2 - 4(v_{1,2}+\bar{a}_1) && (113)\\
\implies \bar{a}_1 &> -v_{1,2} && (114)\\
\implies \bar{a}_1 &> \frac{\rho_1\bar{a}_1^2 - \rho_2\bar{a}_2^2\bar{a}_1}{1-\bar{a}_2} && (115)\\
\implies \rho_1 &< \frac{1-\bar{a}_2+\rho_2\bar{a}_2^2}{\bar{a}_1}. && (116)
\end{aligned}
$$

This upper bound is also greater than 0 because $\bar{a}_2 < 1$ and $\rho_2 > 0 \implies 1-\bar{a}_2+\rho_2\bar{a}_2^2 > 0$. Therefore $\rho_1 > 0$ is consistent with $a_{1,L}$ as the feasible unique equilibrium.

Theorem 7

We show the result for college 1. By symmetry the result for college 2 is the same as that for college 1 with the interchange of the indices 1 to 2. The admissions rate for college 1 is given by:

$$a_1 = \frac{(1+\bar{a}_1+\psi_{1,2}) - \sqrt{(1+\bar{a}_1+\psi_{1,2})^2 - 4(v_1+\bar{a}_1)}}{2}. \quad (117)$$

where we have made the following definitions:

$$\psi_{1,2} \equiv \frac{\rho_2\bar{a}_2^2 - \rho_1\bar{a}_1^2}{1-\bar{a}_2}, \quad (118)$$

and

$$v_{1,2} \equiv \frac{\rho_2\bar{a}_2^2\bar{a}_1 - \rho_1\bar{a}_1^2}{1-\bar{a}_2} \quad (119)$$

The derivative of the equilibrium admit rate a_1 with respect to $P_{0,1}$ is given by:

$$\frac{da_1}{dP_{0,1}} = \frac{1}{2}\left(\frac{d\bar{a}_1}{dP_{0,1}}\right) g(\vec{\rho},\vec{a}), \quad (120)$$

where:

$$g(\vec{\rho},\vec{a}) \equiv \left((1+\phi_{1,2}) - \frac{2(1+\phi_{1,2})(1+\bar{a}_1+\psi_{1,2}) - 4(1+\eta_{1,2})}{\sqrt{(1+\bar{a}_1+\psi_{1,2})^2 - 4(v_{1,2}+\bar{a}_1)}}\right) \quad (121)$$

Since $\frac{d\bar{a}_1}{dP_{0,1}} > 0$, to show $\exists\ \rho_1^* \in \left[0, \frac{1-\bar{a}_2+\rho_2\bar{a}_2^2}{\bar{a}_1}\right]$ such that $\forall\ \rho_1 \in [0,\rho_1^*)$ $\frac{da}{dP_0} > 0$, and $\forall$ $\rho_1 \in \left(\rho_1^*, \frac{1-\bar{a}_2+\rho_2\bar{a}_2^2}{\bar{a}_1}\right]$ $\frac{da_1}{dP_{0,1}:} < 0$, it suffices to show:

1. $g(\vec{\rho},\vec{a})$ is continuous on interval $\rho_1 \in \left[0, \frac{1-\bar{a}_2+\rho_2\bar{a}_2^2}{\bar{a}_1}\right]$

2. $lim_{\rho_1 \to 0}\, g(\vec{\rho},\vec{a}) > 0$,

3. $lim_{\rho_1 \to \frac{1-\bar{a}_2+\rho_2\bar{a}_2^2}{\bar{a}_1}}\, g(\vec{\rho},\vec{a}) < 0$, and

4. $g(\vec{\rho},\vec{a})$ is a monotonic decreasing function of ρ_1 on interval $\rho_1 \in \left[0, \frac{1-\bar{a}_2+\rho_2\bar{a}_2^2}{\bar{a}_1}\right]$

   $\rho_1 \in [0, \frac{1}{\bar{a}}]$, i.e. $\frac{\partial g}{\partial \rho_1}(\vec{\rho},\vec{a}) < 0$.

The first three conditions, by the Intermediate Value Theorem, guarantee the existence of at least one value of ρ_1^* such that $g(\rho_1 = \rho_1^*) = 0$. The fourth property of monotonicity guarantees uniqueness of $\rho_!^*$ and gives us that $\frac{da_1}{dP_{0,1}} > 0$ for $\rho_1 \in [0, \rho_1^*)$ and $\frac{da_1}{dP_{0,1}} < 0$ for $\rho_1 \in \left(\rho_1^*, \frac{1-\bar{a}_2+\rho_2\bar{a}_2^2}{\bar{a}_1}\right]$. The first condition, continuity of $g(\cdot)$ follows trivially. The limiting behavior of $g(\cdot)$ at the ends of the interval follows from the following calculations:

$$lim_{\rho \to 0}\left(1 - \frac{2(1+\bar{a}) + 8\rho\bar{a} - 4}{\sqrt{(1+\bar{a})^2 + 4\bar{a}(\rho\bar{a}-1)}}\right) = \left(1 - \frac{2(a-1)}{\sqrt{(1-\bar{a})^2}}\right) = \left(1 + \frac{2(1-\bar{a})}{(1-\bar{a})}\right) > 0 \tag{122}$$

$$lim_{\rho \to \frac{1}{\bar{a}}}\left(1 - \frac{2(1+\bar{a}) + 8\rho\bar{a} - 4}{\sqrt{(1+\bar{a})^2 + 4\bar{a}(\rho\bar{a}-1)}}\right) = \left(1 - \frac{2(1+\bar{a})+4}{(1+\bar{a})}\right) = \left(-1 - \frac{4}{(1+\bar{a})}\right) < 0 \tag{123}$$

To check that $g(\rho)$ in monotone decreasing we show $g'(\rho) < 0$:

$$
\begin{aligned}
\frac{dg}{d\rho} &= \frac{d}{d\rho}\left(1 - \frac{2(1+\bar{a}) + 8\rho\bar{a} - 4}{\sqrt{(1+\bar{a})^2 + 4\bar{a}(\rho\bar{a}-1)}}\right) && (124)\\
&= \frac{(-8\bar{a})\sqrt{(1+\bar{a})^2 + 4\bar{a}(\rho\bar{a}-1)} + (2(1+\bar{a}) + 8\rho\bar{a} - 4)\frac{2\bar{a}^2}{\sqrt{(1+\bar{a})^2+4\bar{a}(\rho\bar{a}-1)}}}{(1+\bar{a})^2 + 4\bar{a}(\rho\bar{a}-1)} && (125)\\
&= \frac{\left[4\left[(1+\bar{a})^2 + 4\bar{a}(\rho\bar{a}-1)\right] - \bar{a}(2(1+\bar{a}) + 8\rho\bar{a} - 4)\right]\frac{-2\bar{a}}{\sqrt{(1+\bar{a})^2+4\bar{a}(\rho\bar{a}-1)}}}{(1+\bar{a})^2 + 4\bar{a}(\rho\bar{a}-1)} && (126)\\
&= \frac{\left[4\left[(1-\bar{a})^2 + 4\bar{a}^2\rho\right] - \bar{a}(2(\bar{a}-1) + 8\rho\bar{a})\right]\frac{-2\bar{a}}{\sqrt{(1-\bar{a})^2+4\bar{a}^2\rho}}}{(1-\bar{a})^2 + 4\bar{a}^2\rho} && (127)\\
&= \frac{\left[4(1-\bar{a})^2 + 16\bar{a}^2\rho + 2\bar{a}(1-\bar{a}) - 8\bar{a}^2\rho\right]\frac{-2\bar{a}}{\sqrt{(1-\bar{a})^2+4\bar{a}^2\rho}}}{(1-\bar{a})^2 + 4\bar{a}^2\rho} && (128)\\
&= \frac{\left[4(1-\bar{a})^2 + 8\bar{a}^2\rho + 2\bar{a}(1-\bar{a})\right]\frac{-2\bar{a}}{\sqrt{(1-\bar{a})^2+4\bar{a}^2\rho}}}{(1-\bar{a})^2 + 4\bar{a}^2\rho} < 0 && (129)
\end{aligned}
$$

The last inequality to sign $g'(\rho)$ follows from feasiblity constraint on the monopoly admissions rate $0 < \bar{a} < 1$ and the conditions on prestige weight for a unique Nash equilibrium in Theorem 1, i.e. $\rho \in [0, \frac{1}{\bar{a}}]$.

Theorem 8

With out loss of generality we consider the comparative static for $i = 1$ since the case for $i = 2$ is identical under the exchange of indices. From equation (95), we compute the following derivative:

$$
\frac{da_1}{d\rho_1} = -\frac{\left(\frac{\bar{a}_1}{1-a_2}\right)}{\left[1 + \frac{(\bar{a}_1-a_1)(\bar{a}_2-a_2)}{(1-a_1)(1-a_2)}\right]}. \tag{130}
$$

Since $a_i \leq \bar{a}_i \leq 0.5$, it follows that $\frac{da_1}{d\rho_1} < 0$. Moreover, by symmetry of interchanging $i = 1$ and $i = 2$, we have $\frac{da_2}{d\rho_2} < 0$.

We compute $\frac{da_2}{d\rho_1}$ from equation (92):

$$
\frac{da_2}{d\rho_1} = \frac{\rho_2\bar{a}_2^2}{(1-a_2)^2}\frac{da_1}{d\rho_1}. \tag{131}
$$

Since $\frac{da_1}{d\rho_1} < 0$ from equation (130), it follows that $\frac{da_2}{d\rho_1} < 0$. Moreover, by symmetry $\frac{da_1}{d\rho_2} < 0$.

Theorem 9

The best response function of college i is given by (see equation (91) & (92)):

$$a_i = \bar{a}_i - \frac{\rho_i \bar{a}_i^2}{(1 - a_{-i})} \tag{132}$$

Taking the derivative of equation (132) with respect to competitors demand:

$$\frac{da_i}{dP_{0,i}} = \frac{\rho_i \bar{a}_i^2}{(1 - a_i)^2} \frac{da_i}{dP_{0,-i}} \tag{133}$$

We then use the best-response function in equation (132) to simplify the expression in equation (134) $\frac{\rho_i \bar{a}_i^2}{(1-a_i)^2}$ with $\frac{\bar{a}_i - a_i}{(1-a_i)}$:

$$\frac{da_i}{dP_{0,i}} = \frac{\bar{a}_i - a_i}{(1 - a_i)} \frac{da_i}{dP_{0,-i}}. \tag{134}$$

E Estimating Marginal Cost

We use the variation in total costs in the IPEDS data across elite colleges and across time to estimate a cost function for this group that depends on the number of enrolled undergraduate students and other observable attributes. We then infer the marginal cost of each school in the sample by evaluating the derivative of the cost function at the number of enrolled undergraduate students.

E.1 Constructing the Data

Because elite colleges might have a distinct cost structure relative to non-elite colleges, we restrict our sample to colleges with: (i) more than 400 full-time undergraduate students (i.e., an average entering freshmen class of 100 students or more); (ii) an SAT Math 25^{th} percentile score that exceeds 650 during at least one year during our time frame and (iii) an average SAT Math 25^{th} percentile score that is in excess of 600 over the time frame with test data (2002 - 2015).[50,51] Table 1 reports the colleges in our sample and their maximum SAT Math 25^{th} percentile score during the sample period 1987 - 2015.

The two important variable costs in our data are expenditures on instruction and expenditures on student services. To obtain total variable costs, we take the sum of expen-

[50] The time frame in which with test data is available is shorter than the time frame with cost data used in the estimation. But we use the shorter time frame with test data to help select the colleges included in the cost analysis using the longer time panel.

[51] Recall that for the data presented in Section 4, we followed Chade et al. (2014) and defined a "top private" school has having a mean SAT score of 1340 or higher, using the HERI data set. However, the IPEDS data set does not track average or combined SAT scores. Instead, IPEDS tracks scores at the 25^{th} and 75^{th} percentiles, at the subject level. The restrictions noted in the text appear to produce a similar quality of colleges, although IPEDS contains more colleges in total in its data set.

ditures on student services and a weighted measure of expenditure on instruction. We weight the expenditure on instruction to account for the fact that instructional expenditure in the IPEDS data captures faculty salaries paid for both teaching and research. Since undergraduate instruction is primarily a teaching endeavor, we use the ratio of net tuition revenue[52] divided by the sum of total revenue from net tuition plus revenue from research, as our *teaching weight*. By this measure, colleges which have no research revenue have a teaching weight of one, whereas colleges with research revenue and no revenue from net tuition have a teaching weight of zero. Colleges with a mix of both research and tuition revenue have a teaching weight that is between zero and one. Finally, we adjust this total variable cost by the fraction of a college's (or university's) full-time undergraduate population, computed by dividing the number of full-time undergraduates by the total number of full-time students plus one half times the number of part-time students.

Table 1: Colleges in the Sample

College	Max SAT Math (25%)
California Institute of Technology	780
Massachusetts Institute of Technology	750
Rice University	750
Harvey Mudd College	740
Cornell University	720
University of Chicago	720
Vanderbilt University	720
Washington University in St Louis	720
Harvard University	710
Princeton University	710
Yale University	710
Carnegie Mellon University	700
Columbia University in the City of New York	700
Duke University	700
Northwestern University	700
Pomona College	700
Stanford University	700
Claremont McKenna College	690
Dartmouth College	690
Johns Hopkins University	690
Tufts University	690
University of Pennsylvania	690
Amherst College	680
Bowdoin College	680
Brown University	680

[52] According to the IPEDS variable glossary: "net tuition revenue is the amount of money the institution takes in from students after institutional grant aid is provided." This number is not the same as the net tuition number available in IPEDS that is net of all discounts and allowances applied to tuition and fees.

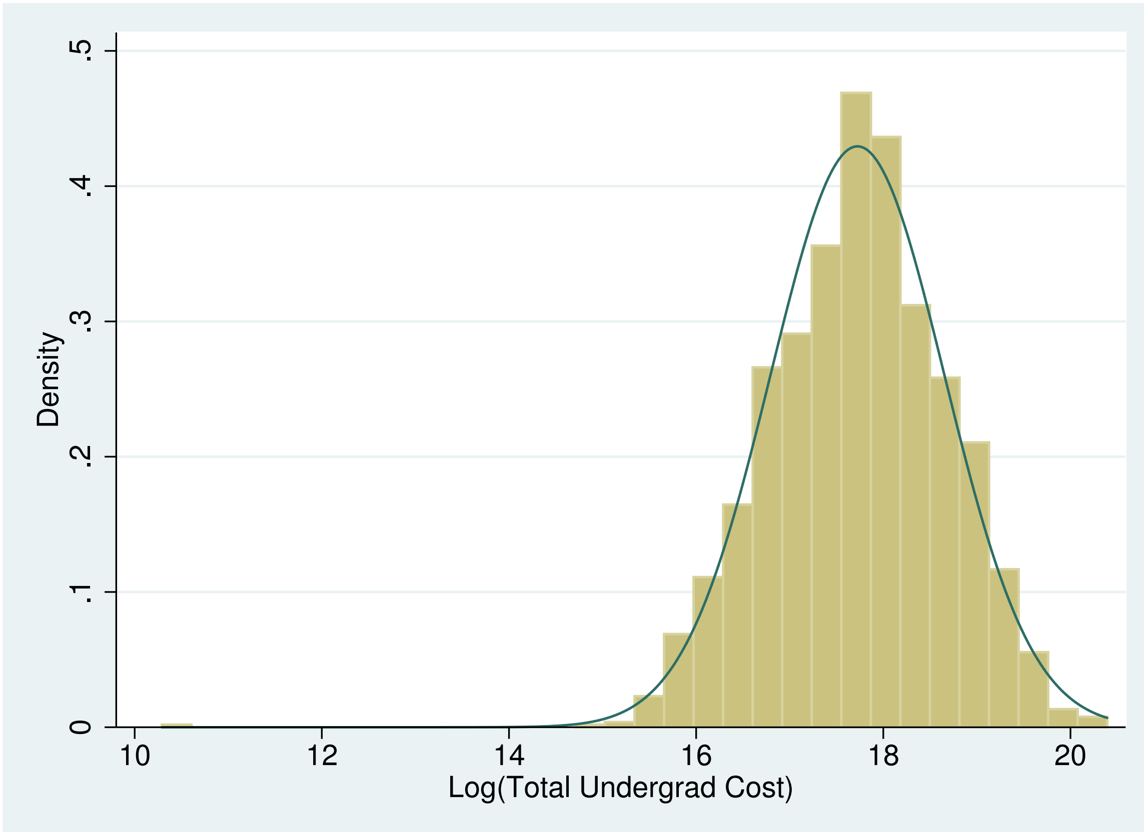
Colleges in the Sample, Continued	
College	Max SAT Math (25%)
Carleton College	680
Georgia Institute of Technology-Main Campus	680
Lehigh University	680
Middlebury College	680
Northeastern University	680
Rensselaer Polytechnic Institute	680
Swarthmore College	680
University of Notre Dame	680
Case Western Reserve University	670
Emory University	670
University of Southern California	670
Williams College	670
Oberlin College	664
Brandeis University	660
Georgetown University	660
Grinnell College	660
Hamilton College	660
Haverford College	660
Stevens Institute of Technology	660
University of Michigan-Ann Arbor	660
Washington and Lee University	660
Wesleyan University	660
Bates College	650
Boston College	650
Colgate University	650
Colorado School of Mines	650
Cooper Union for the Advancement of Science and Art	650
Polytechnic Institute of New York University	650
Tulane University of Louisiana	650
University of California-Berkeley	650
University of Rochester	650
Vassar College	650
Wellesley College	650

E.2 Estimating the Cost Function

Figure 25 plots the histogram of the log of total undergraduate educational costs for the schools in our restricted sample over-laid with a normal pdf.[53] Since total costs appear to

[53]The corresponding IPEDS variables are shown in italics: (1) total full-time undergraduate students: *total_full_time_undergraduates*; (2) total part-time students: *total_part_time*; (3) net tuition: *nettuition01*; (4) expenditure on instruction: *instruction01*; (5) expenditure on student services: *studserv01*; (6) revenue from

Figure 25: Log of Education Costs



Source: IPEDS, 1987 - 2015

follow a log-normal distribution, we estimate our total cost function in logs:

$$\log(c_{i,t}) = \alpha_0 + \alpha_1 \log(Q^{ugrad}_{i,t}) + \alpha_2 \log(Q^{grad}_{i,t}) + \theta Z_i + \rho t + \epsilon_{i,t}, \tag{135}$$

where: $c_{i,t}$ is total cost of college i at time t, as constructed in Section E.1; $Q^{ugrad}_{i,t}$ is the quantity of full-time undergraduate students enrolled in college i at time t; $Q^{grad}_{i,t}$ is the quantity of full-time graduate students; Z_i is a time-invariant vector of college i attributes; t is a time trend variable; and, $\epsilon_{i,t}$ captures idiosyncratic college cost variation. The attributes in vector Z_i are: (i) a dummy variable that equals 1 for flagship state colleges; (ii) a dummy variable that equals 1 for private colleges; and, (iii) the maximum SAT Math 25^{th} percentile score of the university between 2002 -2015.[54] The corresponding marginal

research: *research01*; (7) total full-time graduate students: *total_full_time-total_postbacc*

[54]As noted in Section E.1, IPEDS does not track SAT scores prior to 2002. Including annual SAT math scores in our actual cost estimation, therefore, would decrease our sample size by half. Including the maximum 25^{th} percentile math SAT score, computed over the time period 2002 - 2015, as a time-invariant control variable, therefore, preserves the sample size.

cost is then given by

$$MC_{i,t} = \left[\frac{\partial c_{i,t}}{\partial Q^{ugrad}}\right] = \alpha_1 \frac{c_{i,t}}{Q^{ugrad}_{i,t}}, \tag{136}$$

where α_1 is the elasticity of cost with respect to the number of undergraduate students.

Table 2 reports the estimates of α_1 from three different regression specifications. Column 1 reports the results from the simple ordinary-least squares (OLS) regression shown in equation (135). For robustness, the fixed effects (FE) model of Column 2 then adds U.S. state dummy variables to control for a college's state location, as there is heterogeneity in the cost of living across states. In additional to state-level dummy variables, Column 3 then uses the GMM procedure to estimate an instrumental variables (IV) model, where the number of undergraduate students in the current period is instrumented by the number of undergraduate students from the prior period; similarly, for the number of graduate students. This IV procedure controls for the potential for simultaneity bias arising from the fact that both the total cost and number of students are determined in the same year. The identifying assumption is that the previous year's number of students is predetermined and exogenous to the current period's number of students.

Table 2: Estimation of Undergraduate Cost Function

	(OLS)	(FE)	(IV+FE)
Log Full-time Undergraduates	0.897	0.936	0.948
	(0.023)**	(0.021)**	(0.032)**
Log Full-time Graduate Students	0.032	0.015	0.003
	(0.021)	(0.018)	(0.032)
Flagship University	0.684	1.158	1.173
	(0.062)**	(0.090)**	(0.060)**
Private Non-Profit Institution	1.147	1.683	1.694
	(0.043)**	(0.067)**	(0.041)**
Max SAT Math 25(%)	0.001	0.001	0.001
	(0.000)**	(0.000)*	(0.000)
Time Trend	0.050	0.050	0.049
	(0.001)**	(0.001)**	(0.001)**
Constant	7.462	7.070	7.237
	(0.236)**	(0.274)**	(0.346)**
R^2	0.88	0.92	0.92
N	1,260	1,260	1,204

* $p < 0.05$; ** $p < 0.01$

Table 2 indicates that the value of the cost elasticity, α_1, ranges from 0.90 to 0.95. The signs on all of the control variables are as expected: Flagship public colleges and private colleges have a higher total cost than public non-flagship schools; schools with higher quality, as measured by SAT Math scores, spend more educating their students; and, the cost of education is increasing over time (by an average of 5% per year). We also find a small positive but insignificant relationship between the number of graduate students and the total cost of educating undergraduate students. Given the expression for marginal cost in equation (136), our regression results suggest that the marginal cost for the colleges in our sample is between 90% to 95% of each college's average variable cost. Our preferred estimate is $\alpha_1 = 0.949$ from column (3), because it controls for state fixed-effects and addresses the potential for simultaneity.

F Harvard Admissions Data

On November 17, 2014 Students for Fair Admissions sued Harvard on the grounds Harvard discriminates against Asian-American students in its admissions process. During the discovery phase of this lawsuit, Harvard released individual application and admissions data for students who applied for admissions in 2010-2015 as well as aggregate admissions data covering students admitted in 1996-2015. This data included the application files of 150,701 students and details on Harvard's internal scoring of the applications along several dimensions that are relevant to admissions.

"Exhibit X" from the Plaintiff's brief reports the admissions rate for applicants to Harvard College by "academic index" (AI) decile. The AI is constructed by Harvard as part of its admissions process and is a weighted combination of the students highest SAT I/ACT score, highest 2 SAT II scores and the class rank/GPA. Table 3 report the upper limit and lower limit of the academic index by decile.[55] We define skill θ by decile as the average of the lower and upper limit of the decile's shown AI score. We choose λ_{min} and α to fit the rejection function (15) to the rejection rates shown in Table 3, assuming differences between predicted and data are i.i.d. noise (Figure 11).[56]

[55] Technically, the AI score starts as low as 60. However, the released data starts at 100 since 99.99 percent of applicants between 100 - 193.5 where rejected.

[56] Both OLS estimation and MLE produce similar results. The show fit uses OLS. The shown data for all races.

Table 3: Data from "Exhibit X" in Arcidiacono (2019)

Academic Index (AI) Decile	AI Lower Limit	AI Upper Limit	Reject Rate
1	100.0	193.5	99.99
2	193.8	205.5	99.61
3	205.8	213.0	98.55
4	213.3	218.5	97.17
5	218.8	223	96.09
6	223.3	226.5	95.21
7	226.8	229.5	94.38
8	229.8	232.5	93.15
9	232.8	235.8	91.23
10	236.0	240.0	88.30