Culture, Institutions & the *Long Divergence*[*]

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Abstract

During the medieval and early modern periods the Middle East lost its economic advantage relative to the West. Recent explanations of this historical phenomenon—called the *Long Divergence*—focus on Middle Eastern (over-)reliance on religious legitimacy and political centralization. We study these features in a political economy model of the interactions between rulers, secular and clerical elites, and civil society. The model induces a joint evolution of culture and political institutions (delegation of power from rulers to elites) converging to one of two distinct stationary states: a *religious* and a *secular* regime. We then map qualitatively parameters and initial conditions characterizing the West and the Middle East (separation between state and religion, initial political power of clerical elites and predominance of religious values in the population) into the implied model dynamics to show that they are consistent with the *Long Divergence* as well as with several key stylized political and economic facts highlighted in the historical narrative. Most notably, this mapping suggests non-monotonic political economy strategies, in terms of legitimacy and political decentralization, in both regions which indeed characterize their history.

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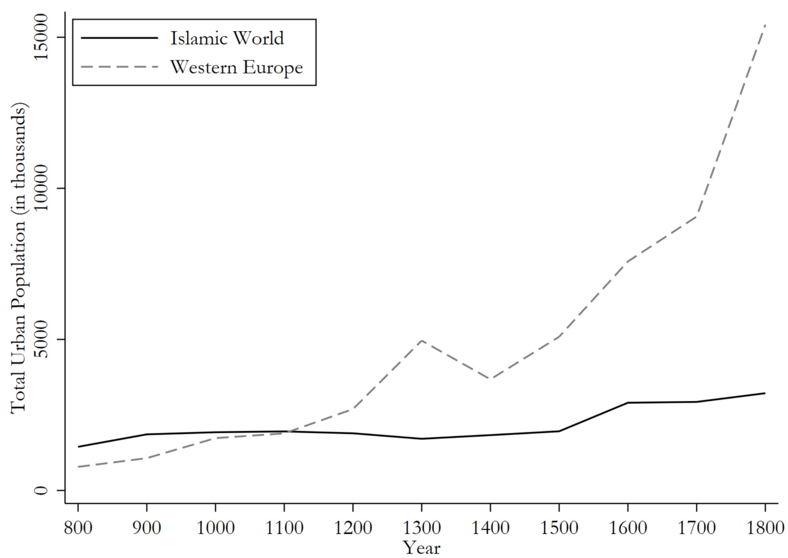
1 Introduction

Around the year 1000 C.E., the Muslim Middle East was far ahead of Christian Western Europe in terms of socio-economic development. By the dawn of the industrial period (circa 1750), however, the Middle East severely lagged behind along several dimensions, including technology, innovation, literacy, wages, and financial development (Bosker, Buringh and Van Zanden 2013, Kuran 2011, Mokyr 1990, Özmucur and Pamuk 2002, Rubin 2017). In the course of the medieval and early modern periods, economic institutions in the Middle East failed to keep pace with those of the West. This is what Timur Kuran (2011) calls the *Long Divergence*. The different urbanization trends, as proxies for socio-economic development, present a clear illustration of the divergence; see Figure 1.

Figure 1: Urban Population in the Islamic World and Western Europe, 800–1800



Data source: Bosker, Buringh and Van Zanden (2013).

Several important factors have been shown to contribute the Long Divergence. Kuran (2011) identifies the root cause of Middle East stagnation in the religious legal system (Islamic law or Sharia) in governing most economic activities. Certain aspects of Islamic law, such as its inheritance system and partnership law, placed impediments that were difficult for economic actors to overcome, especially as the world changed and opportunities for long-distance exchange flourished. Well into the 19th century, the Ottoman "institutional complex" discouraged demand for institutional, legal, and economic change, even as outside economic conditions changed dramatically. Rubin (2017) argues that the persistence of Islamic law is at least partly a consequence of the role of the political power ceded to Muslim religious authorities due to their ability to provide legitimacy to the ruler; that is, to support a belief system whereby citizens have a moral obligation to obey the ruler. This power was used to block important socio-economic advancements, a leading example being the printing press, and more generally to limit the political role of elites advocating for laws and policies more favorable to economic development. In Europe, on the other hand, where the Catholic Church had a much weaker legitimating role, novel ideas and reforms spread more quickly (thanks also to the printing press), economic elites in parliaments developed laws and policies that favored economic development, and long-run economic growth resulted. Blaydes and Chaney (2013) concentrate on the different constraints faced by rulers in the Muslim world and Western Europe. In particular, they argue that the relative weakness of Western European rulers, who had to rely on feudal institutions for tax collection and military recruitment, led to a balance of power more favorable to local (feudal) elites, which promoted economic growth in the long run. Muslim sultans, on the other hand, relied more heavily on centralized political power, derived in large part due to their access to slave soldiers, to satisfy both fiscal and military needs. This limited the political power of economic elites and instead furthered the socio-economic power of religious elites.

All the common and interdependent themes underlying these narratives fundamentally (and rather consistently) interpret economic growth in Western Europe and the Middle East as the outcome of the development of institutional and technological progress brought about or hindered by different strategies rulers adopted to sustain their political support and to enlarge fiscal capacity. Motivated by these historical narratives, in this paper we propose a political economy model of the interactions between rulers, secular and clerical elites, and civil society. This model provides a unitary account of multiple stylized facts associated with the Long Divergence between Western Europe and the Middle East in the period from approximately 1000–1800 C.E. In doing so, the model elucidates the historical mechanisms which might have contributed to the divergent growth paths of Western Europe and the Middle East since the late medieval period. Most importantly, the model provides novel insights into these mechanisms which can be qualitatively mapped into relevant historical facts and narratives on the subject.

Our political economy model focuses on the distribution of power between rulers, religious elites (who provide legitimacy to rulers in return for religious infrastructures),[1] and secular elites (who provide tax revenue as a commitment mechanism which constrains the ruler from extracting resources ex post).[2] It also identifies a fundamental role of religious identity as a somewhat missing component in the historical narratives of the Long Divergence. Indeed, the profile of religious values in the population is bound to crucially affect the choices of rulers, e.g., to seek religious legitimacy from the clerics or else to seek political decentralization to secular elites.

More specifically, our model captures three fundamental elements of the socio-economic environment under study. The first is a complementarity between religious legitimacy and the profile of religious values in the population. Religious elites provide services to the religious component of civil society, which shape civil society's moral beliefs that support a moral obligation to obey the ruler, in turn lowering the subjective tax rate for the religious. Moreover, institutional changes delegating power to clerics reinforce the incentives of religious individuals to transmit their values across generations,[3] increasing their relative share in the population. A higher fraction of religious individuals in the population in turn augments the political incentives for the ruler to delegate power to clerics to increase legitimacy. The socio-economic dynamics of society is fundamentally shaped by this complementarity between the dynamics of religious values (culture) and institutions.

The second element is a trade-off between religious legitimacy and religious proscriptions with respect to the size of the taxable surplus. On the one hand, religious legitimacy increases the size of the taxable surplus by incentivizing the religious component of civil society to exert more effort (or expend less effort in evading taxation). On the other hand, religious proscriptions (e.g., usury laws) dampen economic activity; in the spirit of Kuran (2011), we envision the dominance of Islamic law in business affairs as a ubiquitous pro-

[1] The study of political legitimacy has a long history in the social sciences. Perhaps most famously, Weber (1947) defined political legitimacy as either charismatic, traditional, or legal-rational. Our definition follows more closely in the footsteps of the definition of political legitimacy employed by Lipset (1959, p. 86): "the capacity of a political system to engender and maintain the belief that existing political institutions are the most appropriate or proper ones for the society." For similar definitions of political legitimacy, see Hurd (1999), Tyler (2006), Gilley (2006), Levi, Sacks and Tyler (2009), Greif and Tadelis (2010), Rubin (2017), and Greif and Rubin (2021). More specifically, in our context, see also Auriol and Platteau (2017), Coşgel, Miceli and Rubin (2012), Coşgel and Miceli (2009), Lewis (1974, 2002), Platteau (2017), Rubin (2011), and Kuru (2019).

[2] See, for example, Acemoglu, Johnson and Robinson (2005*a*) and Bisin and Verdier (2017) for a general class of models where institutional change is driven in large part from the lack of commitment of political authorities.

[3] See Bisin and Verdier (2001, 2009, 2011) for economic models of cultural transmission.

scription with far-reaching, path dependent economic consequences. There is therefore a trade-off between religious legitimacy and religious proscriptions which is shaped by their relative importance.

The third element is a trade-off between religious legitimacy and political decentralization with respect to the state's fiscal capacity. Decentralization is akin to delegation of political power from rulers to secular elites, who enforce tax compliance and share the proceeds of fiscal collection with the ruler. Political decentralization increases tax revenues by placing constraints on the ruler's ability to extract resources in societies where the ruler lacks commitment.[4] However, this may come at the cost of undermining the efficacy of religious legitimacy, as in this case the incentives to transmit religious values in the population tend to decline, limiting the clerical elites' legitimating capacity.

In this socio-economic environment, our model provides numerous insights into the dynamic relations between rulers and (religious and secular) elites. First, the complementarity between the cultural and institutional dynamics is such that the incentives of rulers to delegate power and acquire religious legitimacy may become reinforced over time, giving rise to a *lock-in* effect in spite of religious proscriptions damaging economic activity, as religious cultural values reinforce the political power of religious elites, and vice versa. This induces joint dynamics of religious beliefs and institutions which display two types of stationary states: i) a *religious regime* where clerics have substantial political power, they legitimate the ruler, and religious cultural values are predominant in the population; and ii) a *secular regime* where clerics have little political power and secular beliefs are predominant. Second, allowing for political decentralization induces a further characterization of the *secular regime* which tends to display decentralization of political power in favor of the secular elites. Third, the dynamics converging to both the religious and the secular stationary states are not necessarily monotonic. In a subset of the basin of attraction of the religious state, for instance, and specifically when religious values are not predominant initially, rulers will not search for legitimacy from religious authorities for some time, to change strategy only after religious values are spread enough in the population. Conversely, non-monotonic dynamics in which rulers delegate power to clerical elites for some time before turning, e.g., to decentralize power to secular elites, occur in the basin of attraction of the secular stationary state when religious values are initially predominant. In both cases,

[4]For more on the connection between fiscal capacity and executive constraint, see Acemoglu and Robinson (2012, 2019), Acemoglu, Johnson and Robinson (2005*b*), Besley and Persson (2009, 2010), Bisin and Verdier (2017), Dincecco (2009), Johnson and Koyama (2017), Mann (1986), North and Weingast (1989), Tilly (1990).

the dynamics are characterized and determined by a "horse race" between cultural and institutional change.

Several stylized facts highlighted in the historical literature with regards to the socio-economic environment of Western Europe and the Middle East in the early Middle Ages allow for a qualitative mapping of structural parameters and initial conditions into the basins of attraction of the different dynamics identified by the model. First of all, after the fall of the Roman Empire, Christianity was relatively weak at legitimating rule (Feldman 1997, Rubin 2011, 2017). The opposite was true for Islam in the Middle East. This was due specifically to the contexts in which these religions were born. Christianity was born in the Roman Empire, its followers being a persecuted minority, and hence was in no position to legitimate the emperor. Early Christian doctrine is clearly reflective of the low legitimating capacity of Christianity (Feldman 1997, Rubin 2011). Meanwhile, Islam formed conterminously with an expanding empire, and numerous important Islamic dictates specify the righteousness of following leaders who act in accordance with Islam (Hallaq 2005, Rubin 2011, 2017).

Secondly, with respect to initial religious cultural values, Christianity was widespread in the former Roman lands (i.e., religious cultural beliefs were widespread), while this was not the case for Islam in the Middle East. Islamic political power spread rapidly—spanning the Iberian Peninsula to South Asia within a century of Muhammad—but the population living under Islamic regimes were largely non-Muslim for the first few Islamic centuries (Bessard 2020, Saleh 2018). This tension between the structural ability of religious elites to provide legitimacy and the initial fraction of the population with religious beliefs—for both the Middle East and the West— suggests a mapping into the non-monotonic convergence dynamics the model allows for: the incentives to seek religious legitimacy were initially high in the Christian West, to be then overtaken because of the limited legitimating ability of Christianity; while the opposite was the case in the Islamic Middle East.

While the map we obtain between parameters and initial conditions into different dynamics for the West and the Middle East is purely qualitative, we show that it is indeed consistent with several fundamental aspects of the historical narrative. First of all, the different stationary states attracting the Middle East and the West are a representation of the *Long Divergence*. The Middle East, in a religious stationary state, is expected to be less economically vibrant in the long-run, due to the effects of religious proscriptions on socio-economic activity. Such proscriptions may be narrowly targeted, as in the case of usury laws or printing restrictions (Coşgel, Miceli and Rubin 2012, Rubin 2011), or they can affect entire sectors of the economy, as was the case when most commercial transactions were subject to religious law (Kuran 2005, 2011).[5] Furthermore, the main mechanisms driving the convergence to the distinct stationary states are i) the persistent use of religious legitimacy in the Middle East but not in Western Europe (Kuru 2019, Platteau 2017, Rubin 2011, 2017); and ii) the lack of political decentralization of the Middle East relative to the West (Blaydes and Chaney 2013).[6]

Finally, the non-monotonicity of the dynamic paths implied by our mapping is consistent with the historical political economy patterns in the two regions. While we will discuss this in much more detail in Section 4.3, we note here that in Western Europe, following the fall of the Roman Empire, in the Germanic "follower kingdoms," rulers either converted to Christianity or promoted it, as e.g., was the case of Frankish king Clovis (r. 481–509) (Tierney 1970, Rubin 2017, pp. 62–63). These strategies characterized Western Europe until the 11th century, when the re-birth of commerce gave rise to independent cities and increased tensions between the religious and secular elite (Angelucci, Meraglia and Voigtländer 2020, Rubin 2011). In the Middle East, early Islamic rulers did not delegate power to the clerics at the outset of the empire—and indeed they claimed to have religious authority vested in themselves (Crone and Hinds 1986). Only after the religious establishment consolidated in the ninth century (Coşgel, Miceli and Ahmed 2009), and especially after the rise of the madrasa system in the 11th century (Kuru 2019), religious authorities were the primary agents capable of determining whether rulers acted in accordance with Islam.

The paper proceeds as follows. In Section 2 we lay out the basic socio-economic environment we study in the paper, in terms of preferences and technologies of the ruler, the clerics, and the civil society; as well as the space of policy interventions available in society. In Section 3 we study the societal equilibrium for each generation t (Section 3.1) and the processes of institutional and cultural change across generations (Sections 3.2 and 3.3, respectively). In Section 4 we map the model into historical facts and narratives.

[5]Importantly, as Kuran (2011) points out, such proscriptions can also be *path-dependent*, preventing future advancements from taking place and reinforcing institutional lock-in. See also Berman (2000), Carvalho (2013), Seror (2018). Kuran (2011) argues that religious proscriptions were self-reinforcing because they stifled demand for institutional and economic change, which meant that little change was supplied. This argument complements ours, as we focus on the supply-side of institutional change. It also should be noted that religious proscriptions can have welfare-enhancing features, especially in the case of religious minorities; see Iannaccone (1992).

[6]Relatedly (and complementarily), Platteau (2017) and Auriol and Platteau (2017) argue that reforms portending long-run economic growth in the West might have been easier to pursue than in the Middle East thanks to the centralized nature of Christianity. A decentralized body of clerics, as in the case of Islam, makes autocratic regimes more unstable with respect to economic reforms.

In Section 5 we extend the model to study equilibrium and dynamics when we allow for political decentralization to secular elites. In Section 6 we conclude.[7]

2 Ruler, Clerics, and Civil Society

We consider a political economy model of the distribution of power between three types of agents in a society, a ruler, religious clerics, and the civil society.[8] Religious legitimacy, in the model, is an equilibrium phenomenon. It results from the ruler's optimal delegation of power and it depends on the profile of religious values of the civil society in the population, on the efficiency of the clerics' "legitimating" technology, and on the restrictiveness of religious proscriptions imposed by clerics.

Let $t = 0, 1, \ldots$ index generations. All agents only live for one generation. As a consequence, the game played between the ruler, clerics, and civil society is a series of one-shot games in which behavior is not forward looking with respect to institutional or cultural evolution.[9]

Civil Society. Each generation consists of a continuum $[0, 1]$ of citizens. Civil society is composed of two types i of citizens: religious individuals ($i = Re$) in proportion q_t in generation t, and secular individuals ($i = S$) in proportion $1 - q_t$. Citizens employ effort in production activities. Total production is $q_t e_{Re,t} + (1 - q_t) e_{S,t}$, where $e_{i,t},\ i = Re, S$ is the per-capita work effort employed by an individual of type i in generation t.

Ruler and Clerics. The ruler lives off taxing civil society at a tax rate τ_t. The tax base which the ruler has access to is the total production of citizens: $E_t = q_t e_{Re,t} + (1 - q_t) e_{S,t}$.

[7] In the Appendix, we further extend the model to consider the role of religion and religious legitimacy in inhibiting innovation and technological change (Bénabou, Ticchi and Vindigni 2015, 2020, Coşgel, Miceli and Rubin 2012, Davids 2013, Mokyr 1990, 2010, 2016, Squicciarini 2020, White 1972, 1978). More generally, it is certainly not the case that religion as a whole has always a negative impact on economic development; see Barro and McCleary (2003) and McCleary and Barro (2019) for an overview of the literature and a theory of the positive associations between religion and economic development.

[8] In Section 5 we shall extend the model to study the relationship, at equilibrium, between religious legitimacy and political decentralization.

[9] This is in line with the conceptualization of institutional change proposed in Greif and Laitin (2004) and Greif (2006), in which institutions are exogenous to the players at any given point in time but evolve over time in response to the actions taken by the players at that time in response to institutional and cultural incentives. A fully forward looking model of institutional change is analytically intractable when joined with cultural dynamics; see Bisin and Verdier (2017) for a discussion and Acemoglu, Egorov and Sonin (2015), Lagunoff (2009) for forward looking institutional change. Some historical motivation for myopic institutional change in the study of the emergence of democracy is in Treisman (2020).

The ruler also contributes to building and maintaining religious infrastructures, $m_t \geq 0$, for the clerics to provide religious services. The total religious services provided for the society are $m_t \cdot \alpha_{c,t}$, where $\alpha_{c,t} \geq 0$ is the effort of the (representative) cleric at t. The building of religious infrastructures has cost $C(m_t)$ that the ruler pays for. Meanwhile, clerics pay for the daily maintenance costs $F(m_t)$ of these infrastructures.[10]

Legitimacy. Clerics can provide the ruler with *legitimacy* through religious services which facilitate governance and obedience for religious individuals.[11] Citizens are more likely to defer to tax authorities when governance is viewed as legitimate, and they likewise may feel better about paying taxes to a divinely sanctioned political authority. We capture this by assuming that religious individuals, when taxed by the ruler, subjectively perceive a tax rate $\tau^e_{Re,t}$ smaller than the actual τ chosen by the ruler and decreasing in the religious effort of the clerics, $\alpha_{c,t}$:

$$\tau^e_{Re,t} = \tau_t(1 - \theta\alpha_{c,t}). \tag{1}$$

The parameter $\theta \in [0, 1]$ represents the efficiency of the "legitimating" technology of the clerics. Likewise, θ can be interpreted as the efficiency of religious legitimacy in encouraging compliance with authority (or, similarly, discouraging tax evasion) (Coşgel and Miceli 2009, Greif and Rubin 2021). For secular individuals, $\tau^e_{S,t} = \tau$.

Proscriptions. Religious services have an indirect cost, by imposing *proscriptions* (i.e., regulations and constraints) on individual behavior for both religious and secular individuals. We capture this effect by assuming that the cost of individual production effort is

$$c(\alpha_{c,t})\Phi(e_{i,t}), \text{ with } \Phi(e_{i,t}) = \frac{e^2_{i,t}}{2} \text{ and } c(\alpha_{c,t}) = 1 + \phi\alpha_{c,t},\ i = Re, S. \tag{2}$$

The parameter $\phi > 0$ represents the degree of restrictiveness of religious proscriptions on economic activities.

[10] These costs are assumed to be increasing in m_t and sufficiently convex to satisfy a regularity condition, needed to ensure that when religious clerics have a high political weight λ_t in the institutional structure, the policy problem associated to institutional design is well behaved, and provides a finite equilibrium provision of m.

[11] There are many ways to capture this relationship in a model. Legitimacy affects compliance with all sorts of policies, and it lowers the likelihood of revolt (Bentzen and Gokmen 2020, Cantoni, Dittmar and Yuchtman 2018, Chaney 2013, Gill 1998, Gilley 2008, Greif and Rubin 2021, Guo 2003, Hechter 2009, Hurd 1999, Johnson and Koyama 2019, Platteau 2017, Rubin 2017, Tyler 2006). Following many studies on legitimacy in the political science and economics literatures (e.g., Coşgel and Miceli 2009, Levi and Sacks 2009, Wintrobe 1998), we focus on the role legitimacy plays in *tax collection*.

We assume the cost functions $C(.)$, $F(.)$ and $\Psi(.)$ are increasing and convex in their argument.[12] We denote $\bar{\tau}$ the maximum feasible tax rate which the ruler can impose.

Preferences. Preferences of the agents in this society at any generation t are as follows. The ruler has utility

$$U_r(m_t) = \tau_t E_t - C\left(m_t\right). \tag{3}$$

Clerics derive utility $m_t \cdot \alpha_{c,t}$ from religious services, at effort cost $\Psi(\alpha_{c,t})$.[13] The utility of the clerics therefore is

$$U_c(m_t, \alpha_{c,t}) = m_t \cdot \alpha_{c,t} - \Psi(\alpha_{c,t}) - F(m_t). \tag{4}$$

Finally, the utility of agents of type $i = Re, S$ in civil society is

$$U_i(e_{i,t}) = e_{i,t}(1 - \tau^e_{i,t}) - c(\alpha_{c,t})\Phi(e_{i,t}), \quad i = Re, S. \tag{5}$$

This setup establishes—somewhat starkly—one of the model's fundamental building blocks: the trade-off between religious legitimacy and religious proscriptions with respect to the size of the taxable surplus. Legitimacy increases the incentive to provide effort for the religious (or alternatively, lowers their incentive to evade taxation), but comes at the cost of lowered productivity due to proscriptions.

Policy. Choosing the tax rate τ_t is the sole responsibility of the ruler. But we assume that this is not the case for the choice of religious infrastructures m_t, which are rather the outcome of a collective choice problem in any given generation t, reflecting the political power and preferences of the three groups. The relative political power of the groups is captured by their respective weight in the social welfare function W_t which is the objective of policy choices. In other words, the social welfare function W_t can be thought as the objective of a "fictitious policy-maker" and policies are the outcome of a "bargain" implicit in the institutional structure of society.

[12] We also assume that $F'(m) < C'(m)$ for all $m > 0$; i.e., that the marginal cost of infrastructure maintenance is smaller than the marginal cost of building infrastructures.

[13] In various times and places, such as "Golden Age" Islam or medieval Europe, religious authorities were also directly involved in economic activities. Although we do not explicitly model this possibility here, it follows from our setup that religious authorities can benefit from a greater economic surplus since it provides more revenue for expenditure on religious services.

Specifically, the social welfare function W_t to be maximized by the policy choice m_t is:

$$W_t = \frac{1}{2}U_r(m_t) + \frac{\lambda_t}{2}U_c(m_t, \alpha_{c,t}) + \frac{1-\lambda_t}{2}\left[q_t U_{Re,t}(e_{Re,t}) + (1-q_t)U_S\left(e_{S,t}\right)\right]. \qquad (6)$$

Fixing the relative power of the ruler (to $\frac{1}{2}$),[14] the power of the clerics and of civil society is, respectively, $\frac{\lambda_t}{2}$ and $\frac{1-\lambda_t}{2}$ with $\lambda_t \in [0,1]$.

Each generation's *societal equilibrium* will obtain as the ruler, the clerics, and agents in civil society choose, respectively, τ_t, $\alpha_{c,t}$, and $e_{i,t}$ (for $i = Re, S$,) to maximize their utility given by (3), (4), and (5), respectively. The policy choice m_t is chosen to maximize (6). At a *societal equilibrium* in each generation t, the ruler, the policy-maker, the clerics, and civil society, take as given i) the distribution of power between the groups in society, λ_t; as well as ii) the distribution of religious and secular types in civil society, q_t. But both the distribution of power and the distribution of types in civil society are endogenously determined in society. In the next section, we study first the *societal equilibrium* for any t and then the dynamics of λ_t and q_t in the model.

3 Societal Equilibrium and Dynamics

At any time t, for a given institutional power structure of the different groups and a given population profile of religious and secular individuals, the *societal equilibrium* is a Nash equilibrium of the simultaneous game between the ruler, the policy-maker, the clerics, and civil society. The non-cooperative nature of choices captures the idea of a public choice environment plagued by externalities and lack of commitment, whereby policy-makers and agents do not internalize the full impact of their behavior on society.

On the other hand, institutional change arises as a mechanism to internalize the externalities associated with the political process, given the changing cultural composition of society (Acemoglu and Robinson 2019, Bisin and Verdier 2017, Iyigun, Rubin and Seror 2021). Cultural dynamics derive from purposeful inter-generational transmission, emanating from parental socialization and imitation of society at large (Bisin and Verdier 2001, 2017).

[14] This is just for simplicity and concreteness: all that is needed is that the ruler has large enough power with respect to the other members of society.

3.1 Societal Equilibrium

At a *societal equilibrium* for generation t, the choices of τ_t, $\alpha_{c,t}$, and $e_{i,t}$ $(i = Re, S)$ and m_t constitute a Nash equilibrium, denoted by $\{\tau_t(\lambda_t), m_t(\lambda_t), \alpha_{c,t}(\lambda_t), e_{S,t}(\lambda_t), e_{Re,t}(\lambda_t)\}$.[15]

At equilibrium, the optimal tax rate $\tau_t(\lambda_t)$ is equal to its maximum possible value $\bar{\tau}$. As the ruler has a higher political weight that the citizens, taxation is fully extractive. In order to simplify notation, we write τ instead of $\bar{\tau} = \tau_t(\lambda_t)$ in the remainder of the paper. The comparative statics at equilibrium in any period t are summarized in the following Lemma. For notational convenience, we suppress the time subscript t in the rest of this section.

Lemma 1 ***Religious infrastructures:*** *The equilibrium investment in religious infrastructures, $m(\lambda)$, and the equilibrium effort of the clerics, $\alpha_c(\lambda)$, are increasing in λ and independent from θ and ϕ.*

When the weight of the clerics in social choice increases, so does the marginal benefits of provisioning the religious infrastructure m. In turn, the clerics increase their own effort in provisioning religious services $\alpha_c(\lambda)$. Since the weight of the clerics in social choice is $\frac{\lambda}{2}$, both $\alpha_c(\lambda)$ and $m(\lambda)$ increase with λ.

In the model, clerics do not derive utility from imposing proscriptions on economic activity nor from legitimating the ruler. Hence, the investment in religious infrastructure $m(\lambda)$ and the provision of the religious services $\alpha_c(\lambda)$ are independent from θ and ϕ.

Lemma 2 ***Labor effort:*** *The equilibrium effort of secular individuals $e_S(\lambda)$ is decreasing in λ and ϕ and is independent from θ. On the other hand, as long as $\theta \geq \frac{\phi(1-\tau)}{\tau}$, the equilibrium effort of religious individuals $e_{Re}(\lambda)$ is increasing in λ and θ, and is decreasing in ϕ.*

When the efficiency of the clerics to legitimate the ruler θ increases, so does the effort of religious individuals who subjectively perceive a lower tax rate. By contrast, the efficiency of the legitimizing technology has no effect on the effort of secular individuals. An increase in the degree of restrictiveness of religious proscriptions, ϕ, leads to lower efforts from both

[15] The equilibrium is fully characterized in the Appendix. Since there is a complementarity between the provision of the religious good m_t and the investments of the clerics in religious infrastructures $\alpha_{c,t}$, the uniqueness of the equilibrium is not guaranteed. Under mild conditions, however, the equilibrium is uniquely determined.

the religious and secular individuals, as harsher proscriptions decrease individuals' labor productivity.

The political weight of the clerics affects the labor efforts through $\alpha_c(\lambda)$, the equilibrium effort of the clerics. While more effort from the clerics $\alpha_c(\lambda)$ makes secular individuals reduce their own labor effort—through costly regulations and prohibitions ϕ—when $\theta \geq \frac{\phi(1-\tau)}{\tau}$, clerics have the opposite effect on the labor effort of religious individuals e_{Re}. This is because when clerics provide more effort, the religious individuals perceive a lower tax rate. Despite costly religious regulations, they increase their effort due to higher investments in religious infrastructures. In order to make this key difference between secular and religious individuals stark, we make the following Assumption:

Assumption 1 $\theta \geq \frac{\phi(1-\tau)}{\tau}$.

We denote the tax base as $E(\lambda) = qe_{Re}(\lambda) + (1-q)e_S(\lambda)$. From the two previous Lemmas, we deduce the following result:

Lemma 3 ***Tax base:*** *Under Assumption 1, the tax base is increasing in q and θ, and it is decreasing in ϕ. It increases with λ as long as $q \geq \frac{\phi(1-\tau)}{\tau\theta}$.*

While religious infrastructures increase the scope of religious proscriptions, they also positively affect the effort of the religious individuals under Assumption 1. Hence, when religious individuals are sufficiently numerous, the latter effect dominates, and the tax base $E(\lambda)$ increases with the effort of the clerics $\alpha_c(\lambda)$, so it increases with λ. Similarly, since θ positively affects the labor effort of religious individuals, it also positively affects the tax base. Religious proscriptions ϕ negatively affect the tax base, as they decrease labor efforts. The tax base increases with the fraction of religious q, who provide greater effort than their secular counterparts.

3.2 Institutional Dynamics

Each generation brings about institutional change in the relative power to be delegated to clerics and civil society in the future; that is, at the end of any generation t, λ_{t+1} is chosen from the point of view of the social welfare function with weight λ_t.[16] In other words,

[16] We assume that institutional design is myopic, anticipating only socio-economic outcomes one generation ahead. This implies that the institutional structure does not internalize institutional "slippery slopes," whereby moving to a different structure of decision rights may in turn trigger subsequent institutional changes leading to undesirable outcomes from the point of view of the initial structure. See Bisin and Verdier (2017) for a discussion of how this issue can be accounted in the model.

institutions are exogenous from the perspective of all players at any point in time but change over time to reduce externalities associated with the decisions made by policymakers.[17] More formally, at any time t, given institutions λ_t, future institutions λ_{t+1} are designed as the solution to:

$$\max_{\lambda_{t+1}} \frac{1}{2} U_r(m_t(\lambda_{t+1})) + \frac{\lambda_t}{2} U_c(m_t(\lambda_{t+1}), \alpha_{c,t}(\lambda_{t+1})) + \\ \frac{1-\lambda_t}{2} \left[q_t U_{Re}(e_{Re,t}(\lambda_{t+1})) + (1-q_t) U_S \left(e_{S,t}(\lambda_{t+1}) \right) \right]. \quad (7)$$

Institutional change between periods t and $t+1$ therefore internalizes two externalities that are not taken into account by the optimal decisions characterizing the Nash equilibrium of period t. The first one relates to the fact that the provision of religious infrastructures m grants *legitimacy* to the ruler, reducing the subjectively perceived tax rate for religious individuals. The second is the fact that it also has a depressing effect on labor productivity via proscriptions. Hence, increased provision of the religious good m not only affects the utility of the clerics, but also feeds back into the utility of both the ruler and the citizens. Solving the optimization problem (7), we obtain the following result:

Proposition 1 *The optimization problem (7) admits a unique solution $\lambda_{t+1} \in [0, 1]$. The solution is characterized by a threshold $\overline{q}(\lambda_t) \in [0, 1]$ such that,*

$$\lambda_{t+1} > \lambda_t \text{ (resp. } \leq \text{), if } q_t > \overline{q}(\lambda_t) \text{(resp. } \leq \text{).}$$

Furthermore, the threshold $\overline{q}(\lambda_t)$ is decreasing in θ and increasing in ϕ.

The uniqueness result follows from the convexity of the optimization problem. Whether more power is delegated to the clerics over time depends on the fraction of religious individuals q_t. If the religious are sufficiently numerous, then a larger weight to the clerics $\lambda_{t+1} > \lambda_t$ increases their effort $\alpha_c(\lambda_{t+1})$. This in turn increases both the utility of the ruler U_r—who benefits from a larger tax base (Lemma 3)—and the total welfare of the citizens $q_t U_{Re} + (1-q_t) U_S$. Civil society can also benefit from higher effort from the clerics—despite

[17] In this sense, our conception of institutional change follows in the spirit of Greif and Laitin (2004), Greif (2006), and Bisin and Verdier (2017) in that institutions change over time in response to the actions taken by the relevant players at a point in time given the incentives they face at that time. As in our conception of λ_t, such "quasi-parameters" (to use the term coined in Greif and Laitin (2004)) are exogenous to all players in period t but change over time in response to their actions.

religious proscriptions—as religious individuals are better off when they perceive a lower tax rate.

Relative to the comparative static results, when the strength of religious proscriptions ϕ increases, so does the cost for the ruler of using religious legitimacy as a means of extracting resources from the population. The parameter space over which λ_t increases shrinks as $\overline{q}$ increases. On the other hand, when clerics are efficient at legitimating the ruler, i.e. when θ increases, then delegating power to the clerics is more beneficial and $\overline{q}$ decreases.

3.3 Cultural Dynamics

Cultural dynamics are modeled as purposeful inter-generational transmission (Bisin and Verdier 2001, 2017) through parental socialization and imitation of society at large. *Direct vertical* socialization to the parent's trait $i \in \{Re, S\}$ occurs with probability d_i. If a child from a family with trait i is not directly socialized, which occurs with probability $1-d_i$, he/she is *horizontally/obliquely* socialized by picking the trait of a role model chosen randomly in the population.[18] The probability P_{ij} that a child in group i is socialized to trait j writes as:

$$\begin{aligned} P_{ii} &= d_i + (1-d_i)q_i \\ P_{ij} &= (1-d_i)q_j; \end{aligned} \tag{8}$$

with $q_{Re} = q$ and $q_S = 1-q$. We assume that the probability of direct socialization d_i is the solution of a parental socialization problem[19] in which: a) parents are paternalistic (i.e., imperfectly altuistic) and have a bias for children sharing their own cultural trait; b) such paternalistic bias writes as $\Delta V_i(\lambda_t) = V_{ii}(\lambda_t) - V_{ij}(\lambda_t)$, where $V_{ij}(\lambda_t) = U_i(e_j(\lambda_t))$ is the utility perceived by a type i parent of having a type j child, for $i, j \in \{Re, S\}$ and $j \neq i$; c) parents of type $i \in \{Re, S\}$ have socialization costs that are increasing and convex in d_i; d) religious infrastructures m_t may act as complementary inputs to the transmission effort d_{Re} of religious families in the socialization of children to the religious trait.

More specifically, denote $h_{Re}(d_{Re}, m_t)$ the socialization cost of religious families and $h_S(d_S)$ the socialization cost of secular families. Then religious parents solve the following

[18] Vertical, horizontal, and oblique transmission are the core mechanisms in the dual-inheritance theory of cultural evolution. For more, see Cavalli-Sforza and Feldman (1981) and Boyd and Richerson (1985).

[19] See Bisin and Verdier (1998, 2000, 2001) for a similar approach in different contexts and Bisin and Verdier (2011) for a survey of the economic literature on cultural transmission.

socialization problem:

$$\max_{d_{Re}} -h_{Re}(d_{Re}, m_t) + P_{ReRe} \cdot V_{ReRe}(\lambda_t) + P_{ReS} \cdot V_{ReS}(\lambda_t), \tag{9}$$

while secular parents solve the following socialization problem:

$$\max_{d_S} -h_S(d_S) + P_{SS} \cdot V_{SS}(\lambda_t) + P_{SRe} \cdot V_{SRe}(\lambda_t). \tag{10}$$

As shown in the appendix, the solution of (9) provides the equilibrium socialization effort of religious families $d^*_{Re,t} = D_{Re}\left[(1-q_t)\Delta V_{Re}(\lambda_t), m(\lambda_t)\right]$, which is an increasing function of both $(1-q_t)\Delta V_{Re}(\lambda_t)$ and $m(\lambda_t)$. Similarly, the solution of (10) defines the equilibrium socialization effort of secular families $d^*_{S,t} = D_S\left[q_t \Delta V_S(\lambda_t)\right]$, which is an increasing function of $q_t \Delta V_S(\lambda_t)$. In addition, the dynamics of the proportion of the population with the religious trait is characterized by the following "cultural replicator" dynamics:

$$q_{t+1} - q_t = q_t(1-q_t)\{d^*_{Re,t} - d^*_{S,t}\}. \tag{11}$$

In equation (11), the term

$$D(q_t, \lambda_t) = d^*_{Re,t} - d^*_{S,t} = D_{Re}\left[(1-q_t)\Delta V_{Re}(\lambda_t), m(\lambda_t)\right] - D_S\left[q_t \Delta V_S(\lambda_t)\right],$$

can be interpreted as the relative "cultural fitness" of the religious trait in the population. This term is frequency dependent (i.e., it depends on the state of the population q_t). It is also affected by the institutional environment λ_t, as this variable interacts with the process of parental cultural transmission both through paternalistic motivations $\Delta V_i(\lambda_t)$, and through the provision of religious infrastructures $m_t = m(\lambda_t)$ as a complementary input to religious family socialization.

In other words, there is a complementarity between religious legitimacy and the profile of religious values in the population. We deduce the following result:

Proposition 2 *There exists a threshold $q^*(\lambda_t)$ such that*

$$q_{t+1} < q_t \text{ (resp. } \geq \text{) if } q_t > q^*(\lambda_t) \text{(resp. } \leq \text{).}$$

Furthermore, the threshold $q^*(\lambda_t)$ is increasing in θ and λ_t and decreasing in ϕ.

Because the process of cultural transmission (8) is characterized by cultural substitution between vertical and oblique transmission, the relative "cultural fitness" of the religious trait $D(q_t, \lambda_t)$ is decreasing in the frequency q_t of religious individuals in the population (Bisin and Verdier 2001). Consequently, the proportion $q^*(\lambda_t)$ such that $D(q^*(\lambda_t), \lambda_t) = 0$ is the unique attractor of the cultural dynamics in (11). When the fraction of religious individuals q_t is above (resp. below) $q^*(\lambda_t)$, then it decreases (resp. increases) in order to converge in the direction of $q^*(\lambda_t)$.

The dependence of the threshold $q^*(\lambda_t)$ on the institutional environment λ_t and comparative statics on the parameters θ and ϕ depends on how the relative "cultural fitness" $D(q_t, \lambda_t)$ of the religious trait is affected by changes in such features.

An increase in the political weight of the clerics λ_t affects cultural transmission in two ways, through its effect on socialization incentives $\Delta V_{Re}(\lambda_t)$ and $\Delta V_S(\lambda_t)$ and through its effect on religious infrastructures, $m = m(\lambda_t)$. On the one hand, an increase in λ_t promotes the clerics' effort $\alpha_c(\lambda_t)$ and consequently leads to a lower perceived tax rate τ^e_{Re} by religious individuals. The labor effort choice of religious and secular individuals is therefore further apart and, consequently, the incentives of parents to socialize their children to their own cultural trait, $\Delta V_{Re}(\lambda_t)$ and $\Delta V_S(\lambda_t)$, are larger in both groups.[20] However when the socialization effort of religious parents is more sensitive to these incentives than the effort of secular parents, the religious trait is relatively more successfully transmitted than the secular trait, and $D(q_t, \lambda_t)$ is shifted up with an increase in λ_t. On the other hand, an increase in λ_t also increases the amount of religious infrastructures $m = m(\lambda_t)$. When such infrastructures enter as complementary inputs in the socialization process of the religious trait, then again religious parents tend to socialize more intensively than secular ones when m increases. The religious trait has consequently higher cultural fitness than the secular trait and again $D(q_t, \lambda_t)$ is shifted up with λ_t. In either situation, the diffusion of the religious trait is favored by an increase in λ_t, and $q^*(\lambda_t)$ becomes larger.

A change in the other parameters θ and ϕ affects the relative cultural fitness of the religious trait only through their induced changes on the paternalistic motives $\Delta V_{Re}(\lambda_t)$ and $\Delta V_S(\lambda_t)$. For instance, a higher efficiency of the clerics θ tends to widen the gap between the optimal work effort of a religious individual compared to that of a secular individual. As a consequence, an increase in θ shifts up both $\Delta V_{Re}(\lambda_t)$ and $\Delta V_S(\lambda_t)$. As mentioned above, when religious parents are more sensitive to paternalistic motives

[20] Given the quadratic specification of the utility function $U_i(e_i)$, and substituting the optimal labor efforts in the utility of the citizens, one finds that $\Delta V_{Re}(\lambda_t) = \Delta V_S(\lambda_t) = \frac{(\tau\theta\alpha_c(\lambda_t))^2}{2(1+\phi\alpha_c(\lambda_t))}$, which is increasing in λ_t.

than secular parents, these shifts lead religious parents to socialize more intensively than secular parents, and religious values are passed from generation to generation with a higher intensity. This results in a higher value of $q^*(\lambda_t)$. Conversely, a higher value of religious proscriptions ϕ dampens the impact of work effort on economic outcomes. Consequently, behavioral differences induced by cultural traits are less relevant from a utility point of view. This in turn reduces the paternalistic motives $\Delta V_{Re}(\lambda_t)$ and $\Delta V_S(\lambda_t)$ of religious and secular parents. The effect of a change in proscriptions ϕ on cultural evolution is then qualitatively the opposite of that of a change in θ.

4 Model Dynamics and Historical Narrative

In this section we draw the implications of the model with regards to the joint dynamics of culture and institutions and match them with various salient elements of the historical narrative regarding Middle Eastern and Western political economy during the medieval and early modern periods. To this end we proceed as follows.

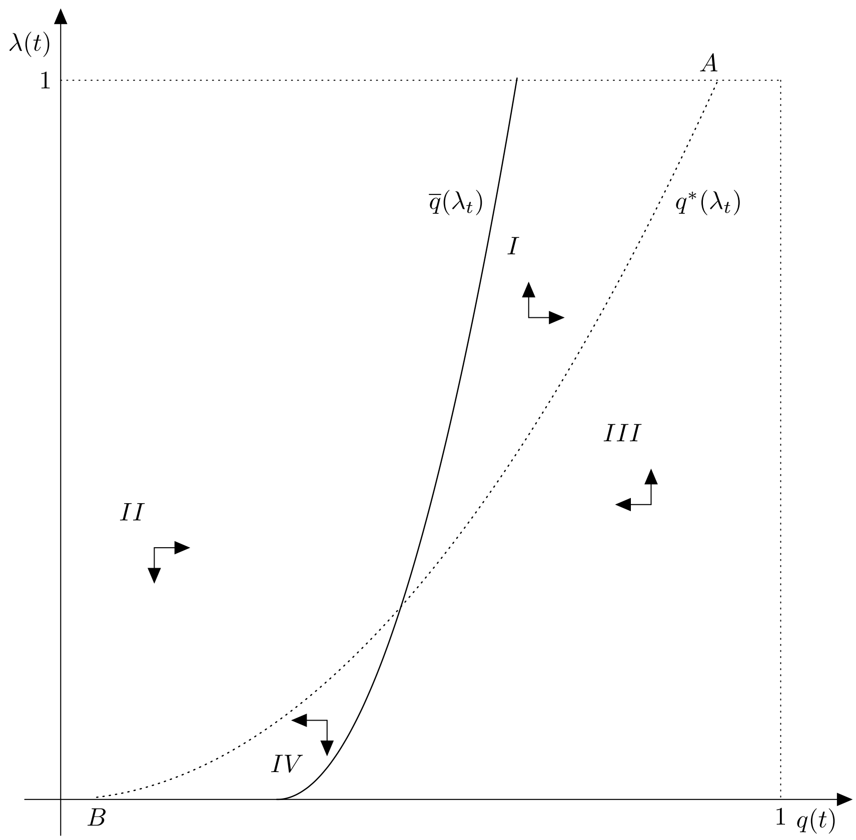
In Section 4.1 we represent the dynamics of the model by a phase diagram. To this end we exploit the characterization we obtained in the previous section of the dynamics' stationary states, their stability properties, and their basins of attraction, as a function of structural parameters and initial conditions. In Section 4.2 we exploit relevant historical information to draw a qualitative mapping of structural parameters and initial conditions for the Middle East and the West into the basins of attraction of the different dynamics identified by the model. Finally, in Section 4.3 we match the model's implied dynamics for these two regions to the historical narrative regarding the Long Divergence as well as other characteristics of the political economy patterns of the history of these regions.

4.1 The Joint Dynamics of Culture and Institutions

Under the conditions of Propositions 1 and 2, we can represent the joint cultural and institutional dynamics in the phase diagram of Figure 2. The solid black line represents the threshold of the institutional dynamics $\overline{q}(\lambda_t)$. The dotted line represents the threshold $q^*(\lambda)$ associated with the cultural dynamics.[21] The arrows in Figure 2 depict the joint dynamics of culture and institutions, given our results in Propositions 1 and 2.

[21] It can be shown that $q^*(0) = 0$, and that $\overline{q}(0) > 0$ with $\overline{q}'(0) > 0$. Under parametric conditions ensuring that $\overline{q}(1) < q^*(1)$, continuity of $\overline{q}(\lambda)$ and $q^*(\lambda)$ implies that $\overline{q}(\lambda)$ necessarily cuts from below $q^*(\lambda)$ characterizing an interior steady state point (q^*, λ^*) as shown in Figure 2. Such a point can be shown to

Figure 2: Joint Dynamics of Culture and Institutions



Stationary states. As described in the figure, the joint dynamics of culture and institutions in this society display two steady states.[22] The first one could be characterized as a *religious regime* represented by point A in Figure 2, where the ruler is legitimated by religion, clerics have significant political power (λ_t is high), taxation is high (the tax rate τ is maximal and the tax base E is high), and the share of religious individuals in civil society is high (q is high). The second steady state, point B in Figure 2, could be characterized as a *secular regime* where the ruler is not legitimated by religion, clerics have little political power (λ_t is zero), taxation is limited (the tax rate τ is maximal but the tax

be a saddle point steady state of the joint dynamics of culture and institutions, leading formally to the possibility of institutional divergence away from (q^*, λ^*). See Appendix A.6 for details.

[22] $\overline{q}(\lambda)$ and $q^*(\lambda)$ may intersect more than once at some interior point. This would provide other steady states whose dynamic stability will alternate between saddle points and stable points. The qualitative discussion of our analysis about institutional and cultural divergence between secular and a religious steady states are not affected by these possibilities.

base E is small), and civil society is secular (q is small). Two mechanisms characterize the dynamics.

Monotonic convergence paths. In regions I and IV of Figure 2, the ruler's option to rely on religious legitimacy to increase tax capacity induces a fundamental *complementarity* between religious legitimacy and the profile of religious values in the population. On the one hand, religious elites provide services to the religious component of civil society, which shape civil society's moral beliefs that support a moral obligation to obey the ruler, which in turn lowers the subjective tax rate for the religious. Institutions delegating power to clerics (i.e., high λ_t) therefore reinforce the incentives of religious individuals to transmit their values. This in turn increases the relative share of the religious in the population. In addition, a higher fraction of religious individuals in the population augments the political incentives for the ruler to delegate power to clerics to increase legitimacy. This complementarity then operates to produce dynamics converging to the *religious regime*, as represented by point A in Figure 2 or to the *secular regime*, as represented by point B. In these regions, the complementarity between culture and institutions *locks-in* society to one of the two stable equilibria.

Non-monotonic convergence paths. In regions II and III of Figure 2, the dynamics are not characterized by complementarity and hence by monotonicity. In these regions of the phase diagram, a "horse race" arises between cultural and institutional change. The "winner" of the horse race determines which stable equilibrium—religious or secular—emerges in the long run. In region II, for example, religious individuals are insufficiently numerous and λ_t decreases over time. At the same time, religious values grow: as the religious trait is not widespread, religious individuals invest more in direct socialization (Bisin and Verdier 2001). Depending on the speed of institutional change relative to cultural change, the joint dynamics can either reach region I or region IV. Region II may give rise to a transitory path to the religious equilibrium when the religious population grows fast despite the political weight of the clerics decreasing over time. In this case, religious individuals become sufficiently numerous at some point that the course of institutional change is reversed, and the political power of religious clerics starts to grow after a transitory period. In region III, religious individuals are sufficiently numerous for the political power of the religious clerics to increase over time. But the religious population is too large, so secular individuals invest more in direct socialization. Again, depending on the speed of institutional change relative to cultural change, either region I or region IV could be reached by the joint dynamics.

If the religious population decreases faster than religious institutions grow, we can expect the joint dynamics to reach region IV. In this case, the religious population becomes so low after a transitory period that the political weight of the clerics decreases over time and equilibrium B is reached in the long-run.

Comparative dynamics. The basin of attraction of each stationary state—the subset of initial conditions from which the dynamical system converges to this state in the phase diagram in Figure 2—depends on the parameters of the society. Since the size of each basin of attraction can be interpreted as a likelihood of reaching that stationary state, it is important for our analysis to characterize their dependence from the efficiency of the "legitimating" technology of the clerics, θ, and the restrictiveness of the religious proscriptions imposed by the clerics, ϕ. The following Proposition combines the results established in our analysis of the institutional and cultural dynamics in Propositions 1 and 2.

Proposition 3 ***Joint dynamics of culture and institutions:*** *The size of the basin of attraction of the religious (resp. secular) stationary state is increasing (resp. decreasing) in religious legitimacy θ and decreasing (resp. increasing) in the restrictiveness of religious proscriptions ϕ.*

As an illustration, consider the basin of the religious state (the one of the secular is the complement). A higher efficiency of the clerics θ—by definition—decreases the subjectively perceived tax rate of the religious. As a consequence, religious parents have a higher willingness to transmit their cultural values inter-generationally. At the same time, clerics become more important in the institutional apparatus, as they increase social welfare by (i) lowering the perceived cost of effort and (ii) increasing the rents extracted by the ruler. Therefore, the complementarity between the spread of religious values and institutional changes delegating power to the clerics is reinforced when θ is higher. On the other hand, when religious proscriptions ϕ increase, the cost for the ruler from using religious legitimacy as a means of extraction increases. The threshold $\overline{q}(\lambda_t)$ also increases. Similarly, greater religious proscriptions makes the religious trait less resilient, as the threshold $q^*(\lambda_t)$ associated with the cultural dynamics decreases. This explains why the complementarity between the spread of religious values and institutional changes delegating power to the clerics is weakened.

4.2 Historical Parameters and Initial Conditions

In the historical context we study—Western Europe and the Middle East over the period 1000–1800—the historical literature has identified several key differences between the regions.

Parameters θ and ϕ. Muslim religious authorities had greater capacity to legitimate than their Christian counterparts. This is due to the environment in which the religions were born. Christianity was born in the Roman Empire and was in no position to legitimate the emperor. Early Christian doctrine is reflective of the low legitimating capacity of Christianity (Feldman 1997, Rubin 2011). For instance, Jesus famously said "Render unto Caesar the things which are Caesar's, and unto God the things that are God's" (Matthew 22:21). Meanwhile, Islam formed conterminously with expanding empire, and there are numerous important Islamic dictates specifying the righteousness of following leaders who act in accordance with Islam (Hallaq 2005, Rubin 2011, 2017). Although early Islamic rulers claimed to have religious authority vested in themselves (Crone and Hinds 1986), after the religious establishment consolidated in the ninth century (Coşgel, Miceli and Ahmed 2009), and certainly after the rise of the madrasa system in the 11th century (Kuru 2019), religious authorities were the primary agents capable of determining whether rulers acted in accordance with Islam. In the context of our model, we map these historical differences into a higher θ for the Islamic Middle East.

Secondly, economically-inhibitive religious proscriptions existed in both Christianity and Islam, but were more pervasive in the latter. For instance, Kuran (2005, 2011) cites how Islamic law regarding partnerships and inheritance combined to discourage long-lived or large business ventures. Partnerships would be split among numerous heirs upon the death of any partner, any of whom could dissolve the enterprise. Another well-known set of proscriptions are those related to usury, which existed in both Islam and Christianity, but were interpreted much more restrictively in the former (Noonan 1957, Rubin 2011, 2017). More generally, Islamic law, as formulated in the first few centuries of Islam, covers numerous aspects of commercial life. We map these differences into a higher ϕ in the Islamic Middle East.[23]

[23] While Islamic law did change over time to address economic exigencies, this change was slower than changes that occurred to secular law in Europe (Berman 1983, Hallaq 1984, 2005). We do not model institutional dynamics as changes in the restrictiveness of religious proscriptions. Adding this dimension to our analysis would not change its main results. Indeed, according to Kuran (2005, 2011), these proscriptions persisted *because they worked*: with few people violating the proscriptions, there was little demand to remove them,

Initial conditions q and λ. The "Islamic world" was not thoroughly Muslim for at least a few centuries after the initial spread of Islam and it first spread along trade routes before spreading into other Muslim-controlled territory (Michalopoulos, Naghavi and Prarolo 2016, 2018). Though Islam spread quickly, reaching Spain in the west and the Indian subcontinent in the east within its first century under the Umayyad Caliphate (661–750), "Muslims still formed a small part of the populace... [Umayyad] authorities, who realized that this would deprive them of much-needed tax revenue, did not encourage conversion" (Bessard 2020, p. 18).[24] The mapping into a high initial λ in the Middle East is instead a direct consequence of the environment in which Islam was born; that is, as we already noted, conterminously with the empire, to the point that the political and religious authority was concentrated on the ruler. After the first Caliphate (632–661), whose rulers were companions of the Prophet Muhammad, the Sunni successor empires (the Umayyad Empire and the Abbasid empire (750–1258)) employed Islamic religious authorities to legitimate rule, provide jurisprudence, and administer imperial rule. These historical characteristics of the Islamic Middle East in the early Middle Ages can be mapped, in the in the context of our model, into "low q, high λ" initial conditions.

The historical characteristics of Western Europe, following the fall of the Roman Empire, were somewhat orthogonal to those we identified for the Middle East. First of all, the Roman population had largely become Christianized in the fourth and fifth centuries, so that Christianity was vastly predominant in the Germanic "follower kingdoms." On the other hand, again as a consequence of the the environment in which Christianity was born, the political power of the church was relatively small, to the point that the Germanic "follower kingdoms" were not initially ruled by Christians. We map therefore these historical characteristics of Western Europe into "high q, low λ" initial conditions in the model.

Qualitatively, the structure of the parameters and the initial conditions we have identified from the historical narratives suggest a mapping into region II of Figure 2 for the Islamic Middle East and into region III of Figure 2 for the Christian West.

4.3 Matching Model Dynamics and Historical Trajectories

We consider the two regions in turn.

since e.g., business remained small and there was thus little demand for the types of laws amenable to bigger business.

[24]For more on the role that tax revenue, particularly the *jizya* tax on non-Muslim subjects, played in conversion goals, see Saleh and Tirole (2021).

Christian West. Our mapping of the Christian West into region III of Figure 2 implies that the West could have converged to either the *secular* or the *religious* stationary state in the long-run. The implied dynamics from this region are sensitive to slight variations in their initial conditions and they depend on the relative speed of cultural and institutional change. Most importantly, the paths to convergence are not monotonic: they allow for historical trajectories characterized by early institutional changes whereby rulers delegated power to religious clerics to gain religious legitimacy in the face of a largely religious civil society, before turning back to secular institutional structures.

These transitory, non-monotonic dynamics of institutions indeed characterized Western Europe until the 11th century (although not in Northern Europe, which was Christianized between the 8th and 12th centuries). Following the fall of the Roman Empire, the vast predominance of a Christian civil society provided a strong incentive for Germanic rulers to either convert to Christianity or promote Christianity. For instance, the Frankish king Clovis (r. 481–509) converted and employed Christianity to legitimate his Frankish expansion into new territory (Tierney 1970, Rubin 2017, pp. 62–63). Around 1000 CE, the re-birth of commerce gave rise to independent cities and increased tensions between the religious and secular elite (Angelucci, Meraglia and Voigtländer 2020, Rubin 2011). Although we do not model the re-emergence of trade endogenously, it had clear implications for the institutional and cultural dynamics at the heart of the model. The rebirth of commerce entailed that religious proscriptions (ϕ in our model), such as the ban on usury, were more economically harmful. In the absence of widespread trade prior to the Commercial Revolution, such proscriptions had little dampening effect on the economy. Yet, they became increasingly harmful as trade flourished (Rubin 2011). The increase in ϕ combined with the relatively low θ of Christian religious authorities encouraged rulers to break with the Church as a primary means of legitimation. The most important event in this break was the Investiture Controversy (1075–1122), a conflict between various secular rulers and the papacy over the role of the former in religious affairs. The Investiture Controversy culminated in European rulers seeking alternative justifications for their rule (i.e., lowering λ) (Tierney 1988, pp. 33–95). They found these alternative justifications in the universities, where leading scholars provided justification for secular rule based on Aristotelian thought, while others helped codify various branches of secular law such as merchant law, feudal law, and manorial law (Berman 1983, Cantoni and Yuchtman 2014, Hollenbach and Pierskalla 2020). Indeed, Blaydes, Grimmer and McQueen (2018) find that it was precisely in this period that European political advice texts began to de-emphasize religious appeals.

As a whole, these events helped place much of Western Europe on a path towards the more "secular" equilibrium described in our model. Institutional change in the direction of more political power to the Church did not arise fast enough, especially after the Investiture Controversy gave local rulers greater suzerainty over their lands. In the context of the model, Western Europe thus ultimately ended up in region IV of Figure 2. In this region, the declining political power of religious clerics reinforced cultural changes that placed less emphasis on religious values. These reinforcing mechanisms resulted in lock-in, whereby there was little role for religious authorities in legitimating political rule, and more political power rested in civil society. The Reformation, in particular, played a key role in further secularizing civil society. In the context of the model, such secularization is necessary for a society to reach region IV of Figure 2. In England, Greif and Rubin (2021) argue that following the Reformation, the political power of religious authorities dropped significantly and the law (as formed in Parliament) became a key source of royal legitimacy. In Germany, Cantoni, Dittmar and Yuchtman (2018) find that, following the Reformation, there was a massive reallocation of resources and education from religious to secular purposes. In other words, where the Reformation undermined the political power of the Church (i.e., lowered λ), less cultural capital was invested in religious pursuits. This is precisely the type of lock-in the model predicts will arise in a society in region IV.

Islamic Middle East. Our qualitatively mapping of the Middle East into region II of Figure 2 suggests historical trajectories somewhat specular with respect to those of the West: convergence to the *religious* stationary state in the long-run but through historical trajectories characterized by early institutional changes whereby rulers limited the power of religious clerics early on, before turning back to a strategy of delegation in exchange for legitimacy which led society to a religious stationary state.

Institutional change away from a strong clerical class was certainly, though slowly, transpiring in the Islamic Middle East at the outset of its growth. The merchant class saw a rise in its economic and political power in the first few centuries of Islam (Bessard 2020, ch. 9). A common currency and political institutions facilitated a massive expansion of trade. The Umayyad and Abbasid states sponsored markets and provided privileges for leading merchants, directly involving themselves in urban retailing to "establish their power and legitimacy from the first decades of the eighth century" (Bessard 2020, p. 5).

This was not just a period of economic growth; it was also the "Golden Age" of rationalist Islamic thought. Islamic science, technology, mathematics, architecture, and medicine were the envy of Western Eurasia. Hence, there were indeed forces pushing against the political power of religious elites (i.e., lower λ, as is predicted in region II).

Yet, these forces did not result in a long-run "secular" equilibrium. Throughout the Middle East and North Africa, religious elites retained their key role in various ruling coalitions. The first four caliphs as well as Umayyad and Abbasid rulers claimed religious powers and linked themselves to the Prophet Mohammad, constructing theocratic arguments to legitimate their rule (Bessard 2020, Crone and Hinds 1986). Our model sheds light on this outcome. Despite the initially low q in the Middle East, religious authorities provided stability and essential services. Hence, the religious establishment maintained some degree of political power despite inroads made by other groups. After the religious establishment consolidated in the eighth and ninth centuries, they were able to provide legitimacy by providing judgments and new interpretations of law that supported the state (Hallaq 2005, Kuru 2019, Rubin 2017). Over time, the Muslim population grew (i.e., q increased), and the religious elite were also able to provide legitimacy by associating the ruler's name with piety.

Importantly, as posited in our model, the Middle East became Islamicized *prior to* an unraveling of political power for religious clerics. In the context of Figure 2, this placed much of the Muslim Middle East on a path towards region I. In this region, religious culture reinforces clerical political power, and a religious stationary state becomes locked-in in the long run. This equilibrium was characterized by a massive expansion in madrasas (Chaney 2016, Kuru 2019), less frequent "rationalist" interpretation of Islam in favor of traditionalist interpretation (i.e., the "closing of the gate of *ijtihād*" (Coulson 1969, Hallaq 1984, 2001, Schacht 1964, Weiss 1978)), and almost zero political bargaining power for the economic elite (Pamuk 2004*a*,*b*).

Two examples from two different periods and regions highlight the reinforcement of Muslim institutions and culture in a "high q, high λ" world. First, Chaney (2013) finds that medieval Egyptian religious authorities were more secure in their rule (e.g., higher λ) when the Nile flooded or there was a drought. This is precisely when a ruler would most need religious legitimacy, both because the tax base would be lower and because there was a greater threat of revolt. Moreover, this was a period of increasing Islamization of the Egyptian population (i.e., q was increasing). Saleh (2018) finds evidence of massive conversions of lower socio-economic status Copts into Islam: by 1200, Muslims were 80% of the Egyptian population, and by 1500 they were over 90% of the population.[25] Conversion elsewhere happened more rapidly. By the late ninth century, around 80% of the populations of Iraq, Iran, and Syria had converted (Bessard 2020, p. 19). Combined, these two studies reveal a "high q, high λ" equilibrium, with cultural and institutional forces reinforcing each other.

A second example comes from the Arab provinces of the Ottoman Empire, where the population had largely converted to Islam centuries prior to Ottoman expansion (i.e., q was high). In the late 15th century, the Ottomans brought the religious establishment into the state, establishing the office of the Grand Mufti (chief religious jurist). This gave the Ottomans significant power to formulate controversial decisions in a manner consistent with Islam (Imber 1997). Meanwhile, the reinforcement of institutions and culture strengthened after the Ottomans conquered the Egyptian Mamluk Empire (in 1517) and took control over Mecca and Medina, the two holy cities of Islam. This further enhanced the capacity of clerics to confer legitimacy by associating the ruler with Islamic piety (e.g., mentioning the name of the legitimate ruler in each Friday sermon or supporting obedience to the ruler in judicial rulings) (Hallaq 2005, ch. 8). Thus, the high level of religious legitimacy (θ) provided by Muslim clerics resulted in a "high q, high λ" equilibrium for much of Ottoman history.

4.4 The Long Divergence through Lens of the Model

Our model squares two of the leading theories of the "Long Divergence," and in doing so directly addresses one stylized fact highlighted in the literature: the persistence of religious legitimacy in the Middle East and the secularization of politics in Western Europe. The model suggests that the diverging long-run paths of the economies of these two regions—"high q, high λ" in the Middle East and "low q, low λ" in Western Europe—were in part a result of the relatively high efficacy of religious legitimacy (θ) in the Islamic world. This meant that the two regions had different responses to religious proscriptions (ϕ). In Western Europe, once commerce revived in the 11th and 12th centuries, religious proscriptions were sufficiently economically damaging to push society on the path that ultimately resulted in a low q, low λ equilibrium. On the other hand, in the Islamic world such religious proscriptions may have been even more economically damaging at the time, given that

[25] Saleh (2018) argues that negative selection among Copts was due to the poll tax that non-Muslims had to pay; those that could not afford it simply converted to Islam.

the Islamic world was ahead of Europe. However, the relatively high θ in Middle Eastern societies helps account for the presence of strict religious proscriptions in a "high q, high λ" equilibrium. Although proscriptions diminish the attractiveness of religious legitimacy to rulers and of passing down religious traits to one's child, proscriptions are mitigated for the ruler if religious legitimacy is effective enough (i.e., θ is high) and enough of the population is religious (i.e., q is high). Hence, supporting economically-inhibitive religious doctrine is more than worth it for a ruler in a high-q society when θ is also large.

These insights therefore unify Kuran's theory emphasizing religious proscriptions with theories emphasizing religious legitimacy (Kuru 2019, Platteau 2017, Rubin 2017). Kuran's theory centers not just on the fact that religious proscriptions existed in Islamic law, but that they *persisted* for so long after they were useful. Our theory sheds light on the how religious culture reinforced clerical political power, and vice versa, which resulted in the persistence of religious proscriptions. Meanwhile, an emphasis on religious proscriptions reveals why legitimating arrangements changed over time in Europe.

More importantly, these insights shed light on a second stylized fact central to the literature: the long-run economic vibrancy of Western Europe relative to the Middle East. Even though there are welfare-enhancing properties of religious legitimacy (as highlighted in the model), these welfare gains can be overwhelmed by religious proscriptions. As Kuran (2011) points out, such proscriptions can have unforeseeable, path dependent consequences for economic growth. For instance, Islamic partnership law and inheritance law jointly discouraged larger enterprises, which ultimately stifled the creation of anything remotely resembling the corporate form (Kuran 2005, 2011). Meanwhile, the persistent dominance of Islamic law over commercial transactions entailed the slow (or non-) adoption of new organizational forms and financial instruments from abroad, which itself had numerous unforeseeable economic consequences (Kuran and Rubin 2018, Rubin 2010, 2017).

So far, our model does not account for the third major theory of the long divergence: Middle Eastern rulers had more unconstrained power relative to other elites. As such, it cannot account for an important stylized fact mentioned in the introduction: the political decentralization of Western Europe but not the Middle East. Blaydes and Chaney (2013) ascribe the relatively greater power of Middle Eastern rulers to their access to slave soldiers, which gave rulers access to coercive power without ceding political power. Meanwhile, weaker European rulers had greater incentive to negotiate with their economic (i.e., feudal) elites for revenue and military power, since they had little capacity to rule otherwise (Duby 1982). Throughout Europe, rulers also ceded power to urban burghers, who had relative freedom from imperial rule (Angelucci, Meraglia and Voigtländer 2020, Mann 1986, Putnam, Leonardi and Nanetti 1994, Schulz 2020). More generally, this meant that Muslim rulers had less constraint on their power, which a large literature suggests is harmful for economic growth (Acemoglu and Robinson 2012, Acemoglu, Johnson and Robinson 2005*b*, North and Weingast 1989, North, Wallis and Weingast 2009, van Zanden, Buringh and Bosker 2012). Our model currently does not permit the ruler to share power with other (secular) elites that may constrain her, so it cannot speak to the conditions under which this occurs. In the next section, we extend the model to consider how political decentralization interacts with the various parameters of importance in our model (namely, θ and ϕ).

5 Religious Legitimacy and Political Decentralization

In this section we extend and enrich the model introduced in Section 2 by considering the emergence of political decentralization. Pre-modern states tended to have little fiscal capacity or capacity to provide law and order to regions far away from the capital. Administrative capacity tended to be quite weak in most parts of the world, meaning that rulers could not easily implement their desired policies (Besley and Persson 2014, González de Lara, Greif and Jha 2008, Greif 2008, Ma and Rubin 2019). As such, there was a limit to the potential tax revenue available to rulers that was well below the optima on a Laffer curve (Besley and Persson 2009, 2010, Dincecco 2009, Johnson and Koyama 2017). This issue is (implicitly) central to the framework proposed by Blaydes and Chaney (2013). Without the capacity to collect revenue on their own, pre-modern rulers had to delegate tax collection to powerful people. Such powerful people could deter tax evasion via force and more easily assess taxable surpluses.

The degree to which rulers had to delegate tax collection (and, more generally, the administrative functions of the state) depended on their own power vis-à-vis other elites. According to Blaydes and Chaney (2013), Muslim rulers had to delegate less because they had access to slave soldiers. This meant they did not need local elites for military service or, oftentimes, tax collection. Meanwhile, the feudal arrangement in medieval Europe was such that local taxes were collected by powerful local elites and in return rulers received military service and, occasionally, tax revenue.

We study the interactions between rulers and local elites in a political economy model where political power is divided between three groups: the ruler, religious clerics, and a secular elite (e.g., feudal lords, parliament, or the military). This allows us to incorporate into the model a fundamental elements of the socio-economic environment under study, as discussed in the Introduction: a tradeoff between religious legitimacy and political decentralization with respect to the state's fiscal capacity. This, in turn, allows us to study the conditions under which the ruler decentralizes political institutions by sharing political power with the secular elite, who has the capacity to collect taxes.

We treat secular elites as representatives of the citizenry. In terms of the distribution of power between groups, we assign the "ruling coalition" the combined weight of the ruler and the secular elites, $\frac{1}{2} + \frac{1-\lambda}{2} = 1 - \frac{\lambda}{2}$, in social welfare. This is similar to the baseline model, with the citizenry being replaced by the secular elites. In other words, if the ruler and the secular elites are the "ruling coalition" (as in North, Wallis and Weingast 2009), then $1 - \frac{\lambda}{2}$ is the total weight of the coalition. Clerics have weight $\frac{\lambda}{2}$ and citizens have no political power (i.e., zero weight).[26]

The secular elite enforces tax compliance and it shares with the ruler the proceeds of tax collection. The share of the tax revenues accruing to the ruler vis-a-vis the secular elites is $\beta \in [0, 1]$. As a simple illustration, a regime where $\lambda = 1$ can be interpreted as a theocracy, while $\lambda = 0$ is a dictatorship when $\beta = 1$, and a republic when $\beta = 0$, as the ruler does not benefit from tax revenue in the latter case. It is therefore the tradeoff between β and λ that determines the state's fiscal capacity.

We denote $\alpha_l \in [0, \overline{\alpha}_l]$ the enforcement effort of the secular elites, with $\overline{\alpha}_l > 0$. Let $\mu\frac{\alpha_l^2}{2}$, $\mu > 0$, be a quadratic cost associated with this effort. The utility of the secular elites can be expressed as:

$$U_l(m, \alpha_l) = (1 - \beta)[\tau E - C(m)] - \mu\frac{\alpha_l^2}{2}. \tag{12}$$

Consider now the utility of the ruler. We assume the ruler faces a cost $\rho\alpha_l$ when letting the secular elite enforce tax compliance α_l. For instance, medieval European rulers provided feudal lords with lands to administer. Tax enforcement was accompanied with the hiring and building of a force capable of violence by these lords. These elements suggest that the more the ruler cedes to lords the power of tax enforcement, the larger is the military power of the lords, which may eventually be turned against the ruler herself. The cost $\rho\alpha_l$ is a simple way to capture such threats. We maintain the assumption that the maintenance

[26] This is a simplification to reduce the dimensionality of the dynamics of institutions while expanding the qualitative features of the narrative of the interactions between ruler, clerics, and citizens we analyzed in Section 3.

cost of religious infrastructures paid by the clerics is $F(m)$. The utility of the ruler is then

$$U_r(m) = \beta(\tau E - C(m)) - \rho\alpha_l,$$

and the utility of the clerics is:

$$U_c(m, \alpha_c) = m\alpha_c - \Psi(\alpha_c) - F(m).$$

In order to focus on the institutional implications of endogenous tax enforcement, we also simplify the production structure of the economy. More precisely, we assume that all citizens are now endowed with one unit of resource out of which they produce $\frac{1}{1+\phi\alpha_{c,t}}$ of the consumption good. They then face the dichotomous choice of complying or not with tax collection. When an individual of type $i \in \{Re, S\}$ complies with taxation, he pays the effective tax rate τ on his output, while enjoying from a welfare point of view, a "perceived" tax rate $\tau_{i,t}$, with as before $\tau_{Re,t} = \tau(1 - \theta\alpha_{c,t})$ and $\tau_{S,t} = \tau_t$. When the individual decides to evade tax collection, he faces an expected consumption penalty which depends on two factors: i) the capacity of tax enforcement on the part of the elites, and ii) the capacity of that individual to escape taxation. More precisely, denote by $\epsilon(\alpha_{l,t})$ a measure of the capacity of tax enforcement by the elites, increasing in the elite's tax collection effort $\alpha_{l,t}$.[27] Assume as well that each individual has an idiosyncratic (inverse) capacity to evade taxes c drawn from a uniform distribution on a segment $[0, \overline{c}]$, with $\overline{c} > 0$. An individual with characteristic c who does not comply with tax collection incurs an expected consumption penalty $c\epsilon(\alpha_{l,t})$.[28] In this modified version of the model, the expected utility of an individual belonging to type $i \in \{Re, S\}$ with an (inverse) evasion capacity c is then:[29]

$$U_i = \begin{cases} \frac{1-\tau_{i,t}}{1+\phi\alpha_{c,t}} & \text{if the individual complies} \\ \frac{1-c\epsilon(\alpha_{l,t})}{1+\phi\alpha_{c,t}} & \text{otherwise} \end{cases} . \tag{13}$$

[27]For analytical convenience, we assume $\epsilon(\alpha_{l,t}) = \frac{\epsilon_0}{1-\alpha_{l,t}}$, so that $\epsilon_0 \in (0, 1)$ is the enforcement level when the secular elites are not providing an effort ($\alpha_{l,t} = 0$). For simplicity, we also assume that the maximum enforcement level that the secular elite can undertake $\overline{\alpha}_{l,t}$ is less than $1 - \epsilon_0$, so that $\epsilon(\alpha_{l,t})$ always lies in the interval $[\epsilon_0, 1]$.

[28]This consumption penalty is "burned out" and not recovered by tax collectors.

[29]With this specification of production, we highlight the distortions associated with the extensive margin of taxation, rather than the intensive margins of labor effort as in the base model. Introducing the intensive margin of production effort does not change the qualitative conclusions of this section, at the cost of increased analytical complexity.

5.1 Societal Equilibrium and Dynamics

The *societal equilibrium* in generation t is a Nash equilibrium of the game between the ruler, clerics, secular elite, and civil society. In this equilibrium, the religious infrastructures m are chosen to maximize social welfare,

$$\left(1-\frac{\lambda_t}{2}\right)[U_r(m_t)+U_l(m_t,\alpha_{l,t})]+\frac{\lambda_t}{2}U_c(m_t,\alpha_{c,t})). \tag{14}$$

The clerics and the secular elite choose, respectively, $\alpha_{c,t}$ and $\alpha_{l,t}$. We denote $\{m_t(\lambda_t),$ $\alpha_{c,t}(\lambda_t),\ \alpha_{l,t}(\lambda_t,\beta_t)\}$ the equilibrium. In the rest of this section, we omit the time indices when not necessary. Solving the equilibrium in any period t, we obtain the following results:

Lemma 4 ***Religious infrastructures:*** *The equilibrium investments in religious infrastructures $m(\lambda)$ and the optimal effort of the clerics $\alpha_c(\lambda)$ are increasing in λ, and independent from β, θ, and ϕ.*

Lemma 5 ***Tax enforcement:*** *The equilibrium enforcement effort of the secular elite $\alpha_l(\lambda,\beta)$ is decreasing in β, λ, q, θ, and ϕ.*

Lemma 4 is similar to Lemma 1 in the previous model and has the same intuition. Lemma 5 highlights several results. First, when the ruler receives a larger share of the tax revenues β, the secular elite invests less in enforcing tax collection. Second, since individuals subjectively perceive a lower tax rate when clerics provide more effort, they also comply more with taxation, reducing the need for the secular elite to supply their own enforcement effort. Additionally, more effort from the clerics implies more religious proscriptions, which depress citizens' labor productivity, and decreases the proceeds of the tax collection. This also decreases the effort provided by the secular elite in enforcing the tax collection. Hence for both reasons, the clerics' legitimizing effort α_c, and the secular elite tax enforcement effort α_l are strategic substitutes with respect to building up the tax base of society. Consequently, given that clerics provide more effort when they are more powerful (i.e. when λ is higher), the secular elite is conversely less willing to enforce the tax collection in such a case: (i.e., $\alpha_l(\lambda,\beta)$ decreases with λ).

The same intuition explains both the effect of a higher frequency q of religious individuals and of more efficient clerics θ on the effort of the secular elite α_l. Finally, when religious proscriptions ϕ get stronger, the proceeds of the tax collection are reduced, so the secular elite also provides less tax enforcement effort.

We now turn to the analysis of institutional change, i.e., the change in the structure of political weights across society. The ruler can delegate power to the clerics λ and also constrain herself to share more revenues with the secular elites by decreasing her own fraction β of fiscal revenues.

Institutional change internalizes two types of externalities that are not taken into account by equilibrium individual decisions. First, as in the previous model, the religious provision m grants *legitimacy* to the ruler, reducing the subjectively perceived tax rate of religious individuals while at the same time depressing labor productivity because of religious proscriptions. Second, institutions now also respond to the externality implied by the enforcement effort α_l of the secular elite on the fiscal revenue received by the ruler. By committing to share the proceeds of tax collection, the ruler can indirectly induce greater fiscal capacity for her own benefit. This is the trade-off at the heart of this extension of the model.

Hence, given the current institutional structure (λ_t, β_t), future institutions $(\lambda_{t+1}, \beta_{t+1})$ are designed as the solution to:

$$\max_{\lambda_{t+1},\beta_{t+1}} \left(1 - \frac{\lambda_t}{2}\right) [U_r(m_t(\lambda_{t+1}), \alpha_{l,t}(\lambda_{t+1}, \beta_{t+1})) + U_l(m_t(\lambda_{t+1}), \alpha_l(\lambda_{t+1}, \beta_{t+1}))] + \frac{\lambda_t}{2} U_c(m_t(\lambda_{t+1}), \alpha_{c,t}(\lambda_{t+1})), \tag{15}$$

with $\{m_t(\lambda_{t+1}), \alpha_{c,t}(\lambda_{t+1}), \alpha_{l,t}(\lambda_{t+1}, \beta_{t+1})\}$ denoting the Nash equilibrium of period t, as evaluated under an institutional set-up $(\lambda_{t+1}, \beta_{t+1})$. Solving this optimization problem, we deduce the following results which characterize the institutional dynamics:

Proposition 4 *When $C(m)$ and $F(m)$ are sufficiently convex, the optimization problem (15) admits a unique solution $(\lambda_{t+1}, \beta_{t+1}) \in [0, 1]^2$ and:*

there exists a threshold $\overline{q}_d(\lambda_t) \in [0, 1]$ such that if $q_t > \overline{q}_d(\lambda_t)$, then $\lambda_{t+1} > \lambda_t$. Otherwise, $\lambda_{t+1} \leq \lambda_t$. Moreover $\overline{q}_d(\lambda_t)$ is decreasing in λ_t;

there exists a threshold $\tilde{q}_d(\lambda_t, \beta_t) \in [0, 1]$ with $\tilde{q}_d(\lambda_t, 1) = 1$ such that if $q_t > \tilde{q}_d(\lambda_t, \beta_t)$, then $\beta_{t+1} > \beta_t$. Otherwise, $\beta_{t+1} \leq \beta_t$. Moreover, the threshold $\tilde{q}_d(\lambda_t, \beta_t)$ is decreasing in λ_t and increasing in β_t.

The uniqueness result follows from the convexity and the separability of the two dimensions of the optimization problem (15). This result highlights the trade-off between religious legitimacy and political decentralization with respect to the state's fiscal capacity, and the role that the cultural profile (q_t) plays in tipping the balance of this trade-off. As before, whether the ruler delegates more power to clerics over time depends on the fraction of religious individuals q_t. If the religious are sufficiently numerous, then more weight to the clerics $\lambda_{t+1} > \lambda_t$ increases their effort $\alpha_{c,t}(\lambda_{t+1})$. This will increase the utility of the ruler, who benefits from a larger tax base (Lemma 4). Second, when the religious are sufficiently numerous, the political weight of the secular elite relative to the ruler tends to decrease, $\beta_{t+1} > \beta_t$. As the ruler becomes more reliant on religious legitimacy to raise revenues, he also faces weaker incentives to delegate power to the secular elite and to build fiscal capacity.

Cultural evolution of the religious and secular traits is driven by some process of intergenerational transmission emanating from paternalistic parents and oblique social role models. The formal features of the cultural dynamics need, however, to be amended to the new specification of production and taxation as outlined above.[30] Again one may compute the paternalistic motives ΔV_{Re} and ΔV_S to transmit the religious and the secular trait in this context. As shown in the appendix, due to the quadratic specification of the expected payoff functions, these paternalistic motives simply write as functions of the state variables λ_t, β_t, and q_t such that $\Delta V_S = \Delta V_{Re} = \Delta V(\lambda_t, \beta_t, q_t)$.[31] The dynamics of the frequency of the religious trait is again characterized by the following "cultural replicator" dynamics:

$$q_{t+1} - q_t = q_t(1 - q_t)D(q_t, \lambda_t, \beta_t). \tag{16}$$

where again

$$\begin{aligned} D(\lambda_t, \beta_t, q_t) &= d^*_{Re,t} - d^*_{S,t} \\ &= D_{Re}\left[(1 - q_t)\Delta V(\lambda_t, \beta_t, q_t), m_t(\lambda_t)\right] - D_S\left[q_t \Delta V(\lambda_t, \beta_t, q_t)\right] \end{aligned}$$

is the relative "cultural fitness" of the religious trait in the population, and in general depends on the three state variables λ_t, β_t, and q_t. When the cultural substitutability between vertical and oblique transmission is strong enough, the relative "cultural fitness"

[30] When deciding on their optimal socialization effort, parents take into account that their children will draw in their adult life an idiosyncratic evasion capacity c, which matters for their decision to comply or not with taxation.

[31] Because the equilibrium tax collection effort $\alpha_l(\lambda, \beta, q)$ of the secular elite enters into the paternalistic motives, we may note that $\Delta V(\lambda_t, \beta_t, q_t)$ now also depends on q_t and is actually an increasing function of q_t (see the appendix).

of the religious trait $D(\lambda_t, \beta_t, q_t)$ is decreasing in the frequency q_t of religious individuals in the population and we deduce the following result:

Proposition 5 *With strong enough cultural substitution between vertical and horizontal cultural transmission, there exists a unique threshold $q_d^*(\lambda_t, \beta_t)$ such that*

$$q_{t+1} < q_t \text{ (resp. } \geq \text{) if } q_t > q_d^*(\lambda_t, \beta_t) \text{ (resp. } \leq \text{).}$$

As before, the threshold $q_d^*(\lambda_t, \beta_t)$ is the unique attractor of the cultural dynamics (16). Hence, when the fraction of religious individuals q_t is above (resp. below) the threshold $q_d^*(\lambda_t, \beta_t)$, it tends to decrease (resp. increase).

5.2 Model Dynamics and Historical Trajectories

The joint dynamics of culture and institutions in this society are now three dimensional: the two institutional parameters, λ_t and β_t, and the cultural component q_t evolve jointly, as characterized in Propositions 4 and 5. A full characterization of this dynamic system is difficult. Still one can derive some insights on the forces behind the joint dynamics by investigating how the thresholds $\overline{q}_d(\lambda_t)$, $\tilde{q}_d(\lambda_t, \beta_t)$, and $q_d^*(\lambda_t, \beta_t)$, which characterize respectively the dynamics of λ_t, β_t, and q_t, are themselves affected by these state variables.

As in the benchmark model, there is a fundamental complementarity between the dynamics of culture and institutions. To see that, note first that because $\overline{q}_d(\lambda_t)$ is decreasing in λ_t, from Proposition 4, the political weight of the religious clerics λ_t keeps increasing (resp. decreasing) over time as soon as it is above (resp. below) a threshold $\overline{\lambda}(q_t)$ defined by $\overline{q}_d(\lambda) = q_t$. A strong (resp. weak) clerics' institutional representation is reinforced (resp. weakened) over time. This feature creates a force towards an institutional steady state characterized as a *religious institutional regime* with $\lambda = 1$, or on the contrary a *secular institutional regime* with $\lambda = 0$. Also, given that the threshold $\overline{\lambda}(q_t)$ is decreasing in q_t, the reinforcing dynamics for the religious institutional regime are facilitated (resp. weakened) when the religious (resp. secular) trait is already well disseminated in society.

Conversely, from Proposition 5, $q_d^*(\lambda_t, \beta_t)$ is increasing in the institutional weight λ_t of the clerics. As before, a religious institutional regime with a high value of λ_t stimulates more religious infrastructures and reinforces the incentive of religious individuals to pass their values inter-generationally. Religious values are more widely diffused within a religious institutional regime, while secular values widely prevail under an secular institutional regime.

With respect to the dynamics of political centralization β_t, Proposition 4 reveals that β_t is more likely to increase as q_t and λ_t become larger. Indeed, as the threshold $\tilde{q}_d(\lambda_t, \beta_t)$ is decreasing in λ_t and increasing in β_t, the condition for $\beta_{t+1} - \beta_t \gtrless 0$ rewrites as $\beta_t \lesseqgtr \widetilde{\beta}_d(\lambda_t, q_t)$ with $\widetilde{\beta}_d(\lambda_t, q_t)$ increasing both in λ_t and q_t. This feature underlines a force for the system to move in the direction of a steady state level of political centralization $\widetilde{\beta}_d^*$ that is increasing both in the level of institutional power λ of the clerics, and the extent q of religious values prevailing in the society. The more religious the state and the more diffused the religious values in the population, the larger the religious legitimacy enjoyed by the ruler, and the lower the need to empower the secular elite for fiscal consolidation.

Qualitatively, the previous discussion indicates that the joint dynamics of culture and institutions entails the possibility of two steady states. The first is a *religious regime with political centralization*, where the ruler has a strong say on fiscal revenues (β is high) and is legitimated by religion, while the clerics have significant political power ($\lambda = 1$). Fiscal capacity is low, as the secular elite have minimal incentives to enforce tax collection. The share of religious individuals in civil society is high (q is high). The second steady state is a *secular regime with political decentralization.* The ruler is fiscally weak while the secular elite is strong (β is low). The clerics have little political power ($\lambda = 0$), while fiscal capacity is high given that secular elites have strong incentive to enforce tax collection. At the same time, the share of religious individuals is low (q is low).

In the appendix, we show that the previous discussion can be made analytically more precise in the case where the threshold of the cultural dynamics $q_d^*(\lambda_t, \beta_t)$ does not depend on β_t. The dynamics of λ_t and q_t are then decoupled from the dynamics of β_t and follow the same pattern as in the benchmark model. Depending on the initial conditions (λ_0, q_0), (λ_t, q_t) converge towards *a religious regime* $(1, q_d^*(1))$ or a *secular regime* $(0, q_d^*(0))$. Associated with these dynamics, political centralization then converges towards strong state centralization with $\beta_1^* = \widetilde{\beta}_d(1, q_d^*(1))$, or weak state centralization $\beta_0^* = \widetilde{\beta}_d(0, q_d^*(0)) < \beta_1^*$.

As in the benchmark model, a ruler's option to rely on religious legitimacy induces a fundamental complementarity between the dynamics of culture and institutions. When a ruler relies more on religious legitimacy to raise revenues, she also faces increasingly lower incentives to delegate power to the secular elite and to consolidate fiscal capacity. As she becomes fiscally stronger relative to the secular elite, she also commits to an institutional set-up delegating more power to the clerics, leading to increased diffusion of religious values in the society. In turn, the predominance of religious individuals augments the political incentives to bias the institutional structure towards both the clerics and the ruler. This dynamic complementarity between institutions and culture then operates to produce a process converging towards a *religious regime with political centralization.*

Alternatively, when a ruler relies less on religious legitimacy to raise revenues, she also faces stronger incentives to delegate power to the secular elite, who consequently consolidate fiscal capacity. As the ruler becomes more reliant on her secular elite to collect taxes, she accordingly faces lower incentives to commit to an institutional set-up where religious clerics are powerful. Both the political weight of the clerics and the value of passing religious values inter-generationally decrease. A lower predominance of religious individuals in society and a lower legitimacy to directly raise taxes further augments the political incentives to consolidate fiscal capacity by empowering the secular elite. Eventually, the joint dynamics of culture and institutions converge towards a *secular regime with political decentralization.*

5.3 Matching Model Dynamics and Historical Trajectories

This extension allows us to unify the three main theories of the "Long Divergence." It takes seriously the idea that rulers can be constrained by other powerful elites in society and searches for the conditions under which this is likely to happen. Importantly, it does so in the context of the previously-established framework in which religious legitimacy and religious proscriptions play a role in determining the joint evolution of institutions and culture. Indeed the dynamics implied by this extension are in accord with the historical record and shed light on some further stylized facts of the Long Divergence.

We first consider the relationship between constraint on executive power and fiscal capacity. This relationship is central to the extension proposed in Section 5. Recall that Western Europe decentralized politically (via parliaments and other organizations that constrained executive power) in the medieval and early modern periods but the Middle East did not.

There is a large literature claiming that states in which fiscal capacity and the "power of the purse" are held by groups outside of the central executive are able to collect more taxes due to greater constraints on executive power (Besley and Persson 2009, Dincecco 2009, Karaman and Pamuk 2013, Ma and Rubin 2019, North and Weingast 1989, Stasavage 2011, 2020). Our model adds additional insight to this literature by shedding light on the process through which political decentralization, as we define it, engenders cultural change (i.e., secularization) that reinforces the state's fiscal capacity. One of our primary insights is that rulers will only decentralize political authority when the returns from religious legitimacy are sufficiently low. This in turn triggers cultural change to a more secular society. On the contrary, when society is religious, the returns from religious legitimacy may be high even when religious proscriptions impinge on productive effort. In this case, culture and institutions evolve in tandem and society becomes more religious over time.

Section 5 highlights multiple reasons why European political institutions became decentralized in the medieval period. First, following the fall of the Western Roman Empire, European rulers had relatively little fiscal power relative to other elites. In the terms of our model, their initial level of β was low. This also follows from the framework of Blaydes and Chaney (2013), who argue that European rulers were weak relative to other elites because they lacked access to independent sources of military power, unlike Muslim rulers who could employ slave soldiers.

However, an explanation relying solely on executive constraint leaves a major question unanswered. If Muslim rulers were so strong relative to other elites, why should they have feared decentralizing some of their power to those "secular" elites, which could have yielded more tax revenue? Even as late as the early modern period, Ottoman tax collection was notoriously low (Karaman and Pamuk 2013). Why did the Ottomans not give more power to local notables, who would have almost certainly had more capacity to collect taxes? These elites should not have been a threat to Muslim rulers. After all, rulers had slave soldiers and local elites did not.

Our model provides insights which help solve this puzzle. It suggests that the ruler's fiscal power relative to other elites (β) *interacted* with the greater legitimating capacity of Muslim religious authorities. Muslim rulers failed to decentralize political power not because they feared that other elites would become too strong. They did so because political decentralization would have resulted in a weakening of the efficacy of religious legitimacy. Granting more power to secular authorities would have encouraged a cultural shift to a more secular state, yielding religious legitimacy less effective. Given the relative efficacy of religious legitimacy, this would not have been an optimal strategy for a Muslim ruler. This was exacerbated by access to slave soldiers, which gave rulers more power vis-à-vis other elites. However, as the model indicates, this relative power (β) changes endogenously over time. Just because Muslim rulers had an initial advantage vis-à-vis other elites does not explain why it persisted.

The opposite was true in medieval Western Europe. The relatively weak initial power of rulers combined with the relatively weak legitimating capacity of the Church incentivized rulers to decentralize political power. This ultimately yielded a secular equilibrium in which religious proscriptions barely impinged on economic development.

These insights accord well with the historical record. Medieval European economic and political institutions were highly decentralized. Feudal institutions gave lords—secular lords as well as powerful bishops—great power over their local domains, and in return the lords provided military service and tax revenue to their sovereign (Duby 1982). Over the course of the late medieval and early modern periods, parliaments became the primary institution which bargained with European rulers (Angelucci, Meraglia and Voigtländer 2020, van Zanden, Buringh and Bosker 2012). Parliaments allowed the economic elite to gain representation at the political bargaining table, and they generally included three classes: the landed nobility, powerful churchmen, and commercial/urban elite. As warfare became more expensive, European rulers ceded more to these elites, who could provide them with revenue (Gennaioli and Voth 2015, North and Weingast 1989, Stasavage 2011, Tilly 1990). Ultimately, parliaments became the main tool for constraining rulers, which resulted in a massive increase in fiscal capacity (Dincecco 2009, Johnson and Koyama 2017, North and Weingast 1989, Tilly 1990, van Zanden, Buringh and Bosker 2012).[32]

On the other hand, in the Middle East economic power was decentralized but *political power* remained centralized (Coşgel and Miceli 2005, Karaman and Pamuk 2013, Karaman 2009). In early Islam, under the Umayyad Caliphate, regional governors subject to imperial control administered and collected taxes. This differed both from feudal European as well as the pre-existing Byzantine systems in that these governors were *not* locally dominant aristocratic families subject to little discretion from the center. They were not as powerful and had relatively little fiscal independence from the center (Bessard 2020, p. 37–38). Centuries later this was still the case. At the height of Ottoman power in the fifteenth and early sixteenth centuries, the sultan derived two-thirds to three-quarters of his revenue through the *tımar* system, a military lease contract whereby the provincial cavalry collected agricultural taxes directly from the peasantry as remuneration for their military services to the state (Coşgel and Miceli 2005). The *tımar* system was similar to the tax collection system of feudal Europe, where local feudal lords controlled revenues in return for military service. However, a key difference between the two is that European feudal lords also had

[32] For theoretical treatments of the rise of state capacity and its affect on economic development, see Acemoglu (2005) and Besley and Persson (2009, 2010, 2014).

political power: their families ruled over their domains for generations, providing local law and order, collecting taxes, and representing them in parliament. On the other hand, *tımar* holders were rotated every few years *precisely* so that they would not acquire local political power. All political power remained vested in the sultan and key religious authorities, not *tımar* holders. Unlike European elites, who were ultimately able to constrain their rulers and receive concessions in return for revenue, *tımar* holders never organized collectively in any manner close to resembling a parliament, and Ottoman rulers remained relatively unconstrained (Balla and Johnson 2009). As a result, the economic elite rarely had any real political power in the Ottoman Empire (Pamuk 2004*a*,*b*). Meanwhile, religious legitimacy remained important (as discussed in Section 4.3), and as a result sultans ceded purview over commercial law to religious authorities, and the associated proscriptions dampening economic activity lasted for centuries (Kuran 2011).

These insights also help account for another stylized fact of the Long Divergence: Middle Eastern fiscal capacity was much greater than in Western Europe in the centuries following the spread of Islam, but there was ultimately a reversal of fortunes, with Western European fiscal capacity well-outpacing that of the Ottoman Empire in the early modern period. According to Stasavage (2020, p. 12), the Abbasid Empire was able to extract around 7% of GDP in tax revenues in 850 CE, whereas centuries later England and France were only about to extract about 1% of GDP (in 1300). However, by 1700, the leading economies of Western Europe (England, the Dutch Republic, and France) were able to extract many times more of per capita GDP than the Ottoman Empire (Karaman and Pamuk 2013). Our model highlights one reason for this reversal of fortunes. In the early medieval period, prior to the rise of European parliaments and the reduction in sovereign political power that came with it, European states received little revenue from decentralized tax collection, much of which remained in the pockets of local feudal lords. Meanwhile, centralized Middle Eastern states benefited from religious legitimacy, which increased the tax base and thus the revenues taken in by the central state. Indeed, religious authorities and institutions were employed to facilitate tax collection in many cities, including Basra, in the Umayyad period, with mosques playing a central role (although this role ultimately came under the purview of military and economic elites) (Bessard 2020, pp. 205–06, 256). After the rise of European parliaments and the reduction of clerical influence in politics, economic elites gained significant political power (i.e., λ and β were low). In this setting, there was much incentive for the elite (i.e., parliaments) to raise taxes because those taxes were spent on their policy preferences. In other words, the benefits of decentralization outweighed the benefits of religious legitimation with respect to tax revenue collected by the state.

6 Conclusion

In this paper we provide an explanation for an important historical phenomenon: the *Long Divergence* between Middle Eastern and Western European economies during the medieval and early modern periods. We provide an explanation in terms of a model of institutional and cultural change. In doing so, we unify prevailing theories based on religious legitimacy, religious proscriptions, and decentralization of political power. In the process, our model resolves many puzzles left unaddressed in the literature.

In the context of the Long Divergence, the model centers on the power dynamics of rulers, clerics, and secular elites in framing institutions in a religious environment. It highlights three central historical features of these power dynamics: rulers derive *legitimacy* from the religious elites, religious authorities impose *proscriptions* that impinge on economic development, and constraints on executive power and the decentralization of political power have a fundamental role in inducing economic growth. Most importantly, the model highlights how the institutions resulting from the power dynamics of rulers, clerics, and secular elites interact with the spread of culture (religious beliefs) in civil society. Political centralization interacts with religious legitimacy and religious proscriptions to determine its long-run economic and political paths. Citizens remain religious or not in the face of religious proscriptions, depending on the feedback between religious institutions and cultural evolution. The religious legitimacy of the political system depends crucially on the prominence of religious values in society.

Our analysis concentrates on the role of religious proscriptions, legitimacy, and political centralization as the main components of the Long Divergence between the Middle East and the West. In the appendix, we show how our framework can also accommodate the role of innovation and technological change as another key driver, interacting with religion and religious legitimacy in the process of institutional and cultural divergence. In particular, we discuss how our model is consistent with recent theories which argue that culture (Davids 2013, Mokyr 1990, 2010, 2016, White 1972, 1978), and religious proscriptions in particular (Bénabou, Ticchi and Vindigni 2015, 2020, Coşgel, Miceli and Rubin 2012, Squicciarini 2020) can inhibit technological change.

More generally, our approach can be seen as an illustration of the explanatory power of a class of models centered on some simple, general, and yet minimal components: i) institutions as reflective of the relative political power of different groups in society to affect policy decisions, ii) institutional change as a mechanism to internalize externalities and other distortions characterizing the equilibrium, iii) the cultural profile of values and preferences in society as evolving according to various socioeconomic incentives.[33] In this type of set-up, the interdependence between institutions and culture is a fundamental factor, along with technology, driving socio-economic change and long term institutional paths. We hope that this methodology is a stepping stone for further theoretical and empirical analyses in economic history, projecting along those lines significant historical processes of the evolution of power and social structures across groups and individuals.

[33] See Acemoglu, Johnson and Robinson (2005*a*), Acemoglu, Egorov and Sonin (2021), Bisin and Verdier (2021), and Persson and Tabellini (2021) for surveys of this class of models.

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Appendices

A.1 Extension: Religious Legitimacy and Technological Progress

Although not highlighted in the central theories of the Long Divergence, nearly every theory of Britain's (and eventually Europe's) industrialization asks why Britain eventually became technologically advanced beginning of the 18th century (Allen 2009, Mokyr 1990, 2010, 2016). While not all the advancements of the Industrial Revolution were science based—especially inventions in textile production—many were, including the quintessential invention of the period, the steam engine. That Europe pulled ahead in science and technology is puzzling: for centuries after the spread of Islam, the Middle East had a massive technological and scientific lead on Western Europe (Chaney 2016). What happened? Why was there a reversal of scientific and technological fortunes between the two regions?

In this appendix, we consider an extended version of our framework to sketch and discuss another potentially important driver of the Long Divergence between the Middle East and the West, namely technological and scientific progress. As in the main text, we first sketch the formal model and then discuss the historical stylized patterns. Proofs of the propositions and mathematical derivations are provided afterwards.

A.1.1 A model of institutional and cultural divergence with technological progress

Again, we consider an extended version of the model where political power is divided between religious clerics and the ruler. But now we study the conditions under which the ruler allows an endogenous technological choice or adoption of a scientific innovation, which is a source of productivity gains although it sometimes erodes religious beliefs.

More specifically, let the ruler and the clerics have political weights $1-\lambda$ and λ respectively. Let also the parameter $\alpha_I \in [0, \alpha_{\max}]$ denote a variable characterizing the technology level of the society. We assume that the level of technology is a policy instrument bounded by the knowledge frontier $\alpha_{\max}$.

Given that our primary interest is to study the joint evolution of culture, institutions, and technology, we consider again a reduced form model where the political power of the citizens is set to zero. The ruler now has utility

$$U_r = \tau E - C(m);$$

and religious clerics have utility

$$U_c(m, \alpha_c) = m\alpha_c - \Psi(\alpha_c) - F(m).$$

We now consider religious legitimacy as a function of technology. Specifically, the religious legitimacy of the ruler, $\theta(\alpha_I) = \theta_0 - k\alpha_I$, is a decreasing function of the level of technology α_I.[34] In other words, adoption of innovative and sophisticated technologies erodes traditional religious beliefs where the ruler is seen as legitimate. This can be inherent to the process of innovative or scientific discoveries, which question the relationship between people and the natural world (Bénabou, Ticchi and Vindigni 2020, Mokyr 1990, Squicciarini 2020).[35] Finally, we assume that labor productivity is proportional to the technology level: $a = \alpha_I$.

As in Section 5, citizens do not necessarily comply with tax collection and differ in their (inverse) evasion capacity c. We fix now the taxation enforcement measure to $\epsilon_0 < 1$.

Equilibrium: At any time t, society reaches an equilibrium of the game between the ruler, the clerics, and civil society. Following the same line of reasoning as in Section 3 in the main text, the tax base of the economy is:

$$E = E(\alpha_I, \alpha_c, q_t) = \frac{\alpha_I}{1 + \phi\alpha_c}\{1 - \frac{\tau(1 - q_t\theta(\alpha_I) \cdot \alpha_c)}{\epsilon_0\bar{c}}\}$$

The policy choices, that is the religious infrastructure m and the technology level α_I are collectively chosen so as to maximize social welfare:

$$W = (1 - \lambda_t)U_r(m, \alpha_I, \alpha_c, q_t) + \lambda_t U_c(m, \alpha_c); \tag{A.1}$$

[34] To avoid some cumbersome taxonomy, we assume that $k\alpha_{\max} < \theta_0 < 2k\alpha_{\max}$. The first inequality ensures that religious legitimacy can always be produced at any potential technological level. The second inequality ensures that maximum knowledge $\alpha_{\max}$ is sufficiently large not to always constrain the equilibrium technology choice by society.

[35] Religious precepts are not always antithetical to scientific advancement. Indeed, White (1972, 1978) and Davids (2013) argue that certain medieval European technologies were complementary to the Church's interest. For the sake of this extension, we focus on technologies that are antithetical to the interests of religious authorities. Mokyr (1990) argues that this more often than not the case with new and disruptive technologies.

while the clerics choose α_c. Solving the equilibrium:

$$\alpha_c = m, \; - C'(m) + \lambda \alpha_c = 0, \tag{A.2}$$

$$\alpha_I\left(\alpha_c, q_t\right) = \min\left[\frac{\epsilon_0 \overline{c} - \tau(1 - q_t \theta_0 \alpha_c)}{2\tau q_t k \alpha_c}, \alpha_{\max}\right]. \tag{A.3}$$

The equilibrium choice of technology reflects the trade-off with respect to the tax base of an increase in labor productivity and the erosion of religious legitimacy provided by the clerics. It can also be seen that the optimal level of technology $\alpha_I\left(\alpha_c, q_t\right)$ is decreasing in q_t and in α_c. When the religious are more numerous and/or clerics undertake higher religious efforts, the ruler is more reliant on religious legitimacy to raise revenues. Consequently, he is also more reluctant to adopt innovative activities that may erode such legitimacy.

The solution to (A.2) and (A.3) provides the equilibrium values $m\left(\lambda_t\right)$, such that $C'(m) = \lambda_t m$, $\alpha_c\left(\lambda_t\right) = m\left(\lambda_t\right)$, and $\alpha_I\left(\lambda_t, q_t\right) = \alpha_I\left(m\left(\lambda\right), q_t\right)$.

Institutional Dynamics. We allow the ruler to delegate power to the clerics λ. Institutional change again internalizes the externality that is not taken into account by individual decisions in equilibrium. As in the benchmark model, the provision of religious infrastructures m grants legitimacy to the ruler, reducing the subjectively perceived tax rate of religious individuals, while at the same time depressing labor productivity because of increased religious proscriptions. As will be clear below, this interacts with the choice of the optimal technology level adopted by society.

More specifically, given institutions λ_t, future institutions λ_{t+1} are designed as the solution to:

$$\max_{\lambda_{t+1}}(1 - \lambda_t)\left[U_r(m(\lambda_{t+1}), \alpha_I(\lambda_{t+1}), \alpha_c\left(\lambda_{t+1}\right), q_t)\right] + \lambda_t U_c(m\left(\lambda_{t+1}\right), \alpha_c(\lambda_{t+1})), \tag{A.4}$$

with $\{m(\lambda_{t+1}), \alpha_c(\lambda_{t+1}), \alpha_I(\lambda_{t+1})\}$ the equilibrium of period $t+1$, as evaluated under the institutional set-up λ_t. Solving this optimization problem, we deduce that:

Proposition 6 *The optimization problem (A.4) admits a unique solution* $(\lambda_{t+1}) \in [0, 1]$. *Furthermore, there exists a threshold* $\overline{q}_I(\lambda_t)$ *such that*

$$\lambda_{t+1} > \lambda_t \text{ (resp. } \leq \text{) if } q_t > \overline{q}_I(\lambda_t) \text{ (resp. } \leq \text{).}$$

The uniqueness result follows from the convexity of the optimization problem (A.4). Whether the ruler delegates more power to clerics over time depends again on the fraction of religious individuals q_t. If the religious are sufficiently numerous, then religious legitimacy matters relatively more than technology for the ruler's tax base. Consequently, more weight to the clerics $\lambda_{t+1} > \lambda_t$ is provided, as this increases their effort $\alpha_c(\lambda_{t+1})$. The ruler consequently benefits from a larger tax base.

Cultural Dynamics. As in the main text, cultural dynamics are driven by inter-generational transmission decisions from the citizens, and we have the following result:

Proposition 7 *There exists a unique threshold $q_I^*(\lambda_t)$ such that*

$$q_{t+1} < q_t \text{ (resp. } \geq \text{) if } q_t > q_I^*(\lambda_t) \text{ (resp. } \leq \text{).}$$

Furthermore, the threshold $q_I^*(\lambda_t)$ is increasing in λ_t.

The cultural dynamics are still as in (11) and the threshold value $q_I^*(\lambda_t)$ is their unique attractor. Hence, when the fraction of religious individuals q_t is above (resp. below) $q_I^*(\lambda_t)$, it tends to decrease (resp. increase).

Joint Dynamics. There are two steady states. In the *religious regime* equilibrium, the ruler is legitimated by religion. The clerics have significant power (λ is high) and religious beliefs are widespread (q is high). For both reasons, the technology level implemented in society is low, as this threatens the religious legitimacy generated in this religious state. Because, innovation adoption and scientific activity is limited, labor productivity is low, as are fiscal revenues despite extractive taxation. The second steady state is a *secular innovative regime* where a high level of technology close to the knowledge frontier is adopted. Clerics are weak, given that innovations limit their capacity to legitimate the ruler (λ is zero) and the share of religious individuals is low (q is low). Fiscal revenues can be substantial, given that a process of scientific innovation leads to an overall increase in labor productivity.

Complementarity. Again, a ruler's option to rely on religious legitimacy induces a fundamental complementarity of the dynamics of culture and institutions. Along the path towards a religious steady state, the ruler relies more on religious legitimacy to raise revenues. She also faces increasingly lower incentives to adopt efficient innovations that erode her legitimacy. The ruler then commits to an institutional set-up delegating an increasingly large share of power to the clerics, reinforcing the incentive of religious individuals to pass their values inter-generationally. In turn, this further decreases the incentive of the ruler to adopt innovative technologies. Labor productivity stays low, given that technology is limited. Finally, taxes are increasingly more extractive given that the population becomes more religious but labor productivity remains low.

On the other hand, as a ruler relies less on religious legitimacy to raise revenues, she also faces stronger incentives to adopt innovations that increase labor productivity and consequently the fiscal base. As the ruler becomes more reliant on innovative activities to raise revenues, her religious legitimacy erodes, so she faces less incentive to commit to an institutional set-up where the religious clerics are powerful. Both the political weight of the clerics and the value of passing religious values inter-generationally decrease. A lower predominance of religious individuals further augments the political incentives to commit and change the institutional set-up so as to adopt more efficient technologies, leading to a substantial increase over time in labor productivity and fiscal revenues. Eventually, the joint dynamics of culture and institutions converge to a *secular regime* where the implemented technology is not constrained by political forces, but only by the existing knowledge frontier.

A.1.2 The Historical Stylized Pattern

One of the great mysteries of the Long Divergence is the reversal of fortunes between Middle Eastern and Western European science and technology. Data presented in Chaney (2016) reveal that not only were scientific topics among the most ubiquitous in the corpus of Islamic writings up through the 11th century, but up to that point the Islamic world well out-paced Europe in scientific output. At some point in the 11th and 12th centuries, however, a reversal of fortunes occurred. Islamic scientific production began to wane around the 12th century. This was not simply a matter of the Islamic world falling behind relative to Europe; it fell behind in *absolute* terms relative to what had once been. At the same time, scientific works became much more prevalent in Western Europe. By the end of the medieval period, Western Europe had a technological and scientific lead, and this would only grow in subsequent centuries. Can this reversal of fortunes be explained by our model?

Our model, along with the history overviewed in Section 4, suggests that the reversal of technological and scientific fortunes was a consequences of a changing equilibrium in which Muslim religious authorities became increasingly important for legitimating the state while European rulers sought alternative forms of legitimacy. In the Middle East, the 11th century saw the rise of the madrasa system (Chaney 2016, Kuru 2019). This institutionalized the political role that had increasingly been played by religious authorities since their consolidation under the Abbasids in the 9th and 10th centuries (Coşgel, Miceli and Ahmed 2009, Rubin 2017). In this equilibrium, as we describe in Section A.1, religion played an important role in legitimating rule (λ was large), society was largely religious (q was large), and science and technology were impeded. As in Bénabou, Ticchi and Vindigni (2020), technological stagnation mutually benefited religious authorities and the state: the former lost power when alternative means of discovering truths or interpreting the world were present, and the latter was harmed when one of its key sources of legitimacy was undermined.

In the context of Middle Eastern history, this logic sheds light on both why madrasas were allowed to thrive in spite of their negative effects on scientific production and why rulers throughout the Muslim world banned one of the most important technologies of the late medieval period: the printing press. Coşgel, Miceli and Rubin (2012) argues that the Ottomans banned the press for over 240 years after first hearing of it precisely because it threatened the religious establishment. By the 15th century, religious authorities across the Islamic world (not just in the Ottoman Empire) had set up high barriers to entry. The largest of these barriers was the years of training required to know various religious texts and interpretations of those texts. These barriers raised the status of the religious elite, further entrenching the "high-λ, high-q" equilibrium. The printing press threatened to undermine these barriers and the equilibrium they helped uphold. Had printing become widespread, a much larger share of the population would have had access to the great religious and non-religious texts of the Islamic world (and beyond). This would have undermined one of the very features that gave Muslim religious authorities the power to legitimate in the first place. Hence, as our model predicts, heavy restrictions were placed on this vastly important technology.

Muslim religious authorities had good reason to fear the spread of printing. They only needed to look to Europe, where the press helped facilitate one of the great movements against Church power in the history of Christianity: the Protestant Reformation (Boerner, Rubin and Severgnini 2021, Dittmar and Seabold 2020, Rubin 2014). Unlike Ottoman religious authorities, the Church was not able to stop the spread of the printing press. The reason why this was the case follows from the logic of the model. As noted in Section 4.3, the Church had already lost much of its legitimating power in Europe prior to the spread of printing. Alternative sources of legitimacy had emerged in the form of universities (which provided a theoretical justification for monarchical rule) and parliaments (which brought together elites who could legitimate rule in return for a seat at the political bargaining table). By 1200 or so, religious authorities had lost their monopoly over the printed word as well; book demand and supply was increasingly found in university towns and urban centers (Buringh and Van Zanden 2009). As a result, there was little the Church could have done to stop the spread of printing had it wanted to. By the mid-15th century, Europe was in a "low-λ, low-q" equilibrium. Our model suggests that this should also entail few restrictions on technology—at least those technologies that damage the capacity of religious authorities to legitimate. The history of printing suggests that this was the case.

The Christian world was hardly uniform in the degree to which religious legitimacy was part of the broader political equilibrium. This was especially true after the Reformation, which fundamentally undermined the role of religious authorities in the ruling coalition (Rubin 2017). This had consequences for the spread of science and technology. Bénabou, Ticchi and Vindigni (2020) summarize many of the scientific and technological advances blocked or suppressed by the Church, including the works of Galileo, the Copernican Revolution, Newtonism, the Scientific Revolution, and technical education in schools. These restrictions were much more widely applied in Catholic areas than Protestant ones. According to Mokyr (2016), it was the "culture of growth" supported by the Republic of Letters that permitted the spread of the new, rational thinking of those like Bacon and Newton. While the Republic of Letters was a pan-European phenomenon, there was little resistance in the leading Protestant lands (England and the Dutch Republic). Meanwhile, even after the first wave of industrialization, the Church attempted to limit secular education and curriculum in schools (Squicciarini 2020).

In short, this extension helps explain both the technological and scientific reversal of fortunes between Western Europe and the Middle East as well as the the divergence within Europe. In Protestant Europe, new inventions and scientific ideas were allowed to spread relatively unimpeded. This is what the model predicts would be the case in a "low-λ, low-q" equilibrium. The equilibrium in Catholic Europe was one of higher λ and q, and as a result some (though certainly not all) scientific and technological advances were suppressed. In the "high-λ, high-q" equilibrium that pervaded most of the medieval and early modern Middle East (at least, after the 11th century), scientific and technological advancements were even more restricted. Our model explains these outcomes not solely as reflecting the desires of religious authorities, but also their place in their society's broader political-economy and cultural equilibria.

A.1.3 Proofs of Extension A.1

- **Proof of Proposition 6**

We consider that the policymaker chooses the amount of religious infrastructures m, and level of technology $\alpha_I \in [0, \alpha_{\max}]$ to maximize

$$W(m, \alpha_I, \alpha_c, \lambda, q) = (1-\lambda)\left[U_r(m, \alpha_I, q)\right] + \lambda U_c(m, \alpha_c)$$

while the cleric maximizes $U_c(m, \alpha_c)$ with respect to α_c with

$$\begin{aligned} U_r(m, \alpha_I, q) &= \tau E(\alpha_I, \alpha_c, q) - C(m) \\ U_c(\alpha_c, m) &= m\alpha_c - \frac{\alpha_c^2}{2} - C(m) \end{aligned}$$

(we assume for convenience that the cost of the religious infrastructures $C(m)$ is paid as a lump-sum cost by all segments of society) with

$$E(\alpha_I, \alpha_c, q) = \frac{\alpha_I}{1+\phi\alpha_c}\{1 - \frac{\tau(1 - q_t\theta\alpha_c)}{\epsilon_0 \bar{c}}\}$$

where religious legitimacy is decreasing in the innovation effort: $\theta = \theta(\alpha_I) = \theta_0 - k\alpha_I$. We assume $k\alpha_{\max} < \theta_0 < 2k\alpha_{\max}$ Given the institutional framework λ, one immediately gets

$$\alpha_c = m, \ -C'(m) + \lambda\alpha_c = 0$$

and α_I determined by the FOC:

$$\alpha_I(\alpha_c, q) = \min\left[\frac{1 - \frac{\tau(1 - q\theta_0\alpha_c)}{\epsilon_0\bar{c}}}{\frac{2\tau qk\alpha_c}{\epsilon_0\bar{c}}}, \alpha_{\max}\right]$$

This gives the equilibrium values $m(\lambda)$, such that $C'(m) = \lambda m$ and $\alpha_c(\lambda) = \alpha_c(\lambda) = m(\lambda)$ and $\alpha_I(\lambda, q) = \alpha_I(m(\lambda), q)$. (We assume that $C'(0) = 0$ and $C''(m) > 1$ to ensure the existence of a unique equilibrium for all $\lambda \in [0, 1]$. This provides also $\alpha_c(\lambda) = m(\lambda)$, As in the related proofs of Propositions 1 and 4, we first demonstrate that the optimization problem (A.4) admits a unique solution $\lambda_{t+1} \in [0,1]$:

$$\max_{\lambda_{t+1}}(1-\lambda_t)\left[U_r(m(\lambda_{t+1}),\alpha_I(\lambda_{t+1}),q_t)\right]+\lambda_t U_c(m\left(\lambda_{t+1}\right),\alpha_c(\lambda_{t+1}))) \tag{A.5}$$

In order to solve this maximization problem, we solve the following related optimization problem:

$$\max_{m,\alpha_I} \widetilde{W}\left(m,\alpha_I,\lambda_t,q_t\right)=(1-\lambda_t)\left[U_r(m,\alpha_I,q_t)\right]+\lambda_t U_c(m), \tag{A.6}$$

where the solution, denoted $(\tilde{m}(\lambda_t,q_t),\widetilde{\alpha}_I(\lambda_t,q_t))$ maximizes the social welfare when the externalities are internalized, so given that $U_c(m)=U_c(m,\alpha_c(m))=\frac{1}{2}m^2-C(m)$, as $\alpha_c(m)=m$. $U_r(m,\alpha_I,q_t)=\tau E(m,\alpha_I,q_t)-C(m)$, with

$$E(m,\alpha_I,q_t)=\frac{\alpha_I}{1+\phi m}\{1-\frac{\tau(1-q_t\left[\theta_0-k\alpha_I\right]m)}{\epsilon_0\overline{c}}\}. \tag{A.7}$$

We also assume that in the previous optimization problem, the choices of both the religious provision m and of the effort of the innovators α_I are made by a ruler who has a policy commitment capacity, internalizing the externalities associated with the policy choice problem described in the main text. We find that $(\tilde{m}(\lambda_t,q_t),\widetilde{\alpha}_I(\lambda_t,q_t))$ solves the following equations:

$$\begin{cases}\frac{\partial\widetilde{W}}{\partial m}=\lambda_t m-C'(m)+(1-\lambda_t)\frac{\alpha_I}{1+\phi m}\left[\frac{-\phi}{1+\phi m}\left[1-\frac{\tau(1-q[[\theta_0-k\alpha_I]]m)}{\epsilon_0\overline{c}}\right]+\frac{\tau q[\theta_0-k\alpha_I]}{\epsilon_0\overline{c}}\right]=0,\\ \frac{\partial\widetilde{W}}{\partial\alpha_I}=\frac{(1-\lambda_t)}{1+\phi m}\{\alpha_I\left[1-\frac{\tau(1-q_t\theta m)}{\epsilon_0\overline{c}}\right]-\frac{k\alpha_I\tau q_t m}{\epsilon_0\overline{c}}\}=0.\end{cases} \tag{A.8}$$

From the second FOC equation we again get the optimal level of technology:

$$\alpha_I\left(m,q_t\right)=\min\left[\frac{1-\frac{\tau(1-q_t\theta_0 m)}{\epsilon_0\overline{c}}}{\frac{2\tau q_t km}{\epsilon_0\overline{c}}},\alpha_{\max}\right]$$

which rewrites as

$$\begin{aligned}\alpha_I\left(m,q_t\right)&=\frac{\frac{\epsilon_0\overline{c}}{\tau}-1}{2kq_t m}+\frac{\theta_0}{2k}=\alpha_I^{op}\left(m,q_t\right)\quad\text{when}\quad\frac{A}{q_t}\leq m\\&=\alpha_{\max}\quad\text{when}\quad\frac{A}{q_t}\geq m\end{aligned}$$

with

$$A=\frac{\frac{\epsilon_0\overline{c}}{\tau}-1}{2k\alpha_{\max}-\theta_0}>0$$

Note that $\alpha_I(m,q)$ is decreasing in q_t and m. Now the characterization of $\tilde{m}(\lambda_t,q_t)$ is obtained from

$$\Theta(m)=\frac{\partial\widetilde{W}}{\partial m}\left(m,\alpha_I(m,q_t),\lambda_t,q_t\right)\leq 0 \quad\text{and}\quad m\geq 0$$

When $C(m)$ is sufficiently convex, $\Theta(m)$ is decreasing in m. Moreover given that

$$\Theta(0)=(1-\lambda_t)\alpha_{\max}\left[-\phi\left[1-\frac{\tau}{\epsilon_0\overline{c}}\right]+\frac{\tau q_t\left[\theta_0-k\alpha_{\max}\right]}{\epsilon_0\overline{c}}\right]$$

we have $\Theta(0)>0$ when

$$q_t>\overline{q}=\frac{\phi}{\left[\theta_0-k\alpha_{\max}\right]}\left[\frac{\epsilon_0\overline{c}}{\tau}-1\right]$$

Thus $\tilde{m}(\lambda_t,q_t)=0$ for $q_t\leq\overline{q}$ and $\tilde{m}(\lambda_t,q_t)>0$ for $q_t>\overline{q}$. Substitution provides $\widetilde{\alpha}_I(\lambda_t,q_t)=\alpha_I\left(\tilde{m}(\lambda_t,q_t),q_t\right)$.

Moreover as

$$\begin{aligned}\frac{\partial^2\widetilde{W}}{\partial m\partial q} &= (1-\lambda_t)\frac{\alpha_I}{1+\phi m}\left[\frac{-\phi}{1+\phi m}\left[\frac{\tau\left[\left[\theta_0-k\alpha_I\right]\right]m}{\epsilon_0\overline{c}}\right]+\frac{\tau\left[\theta_0-k\alpha_I\right]}{\epsilon_0\overline{c}}\right]\\ &= (1-\lambda_t)\frac{\alpha_I}{\left[1+\phi m\right]^2}\frac{\tau\left[\theta_0-k\alpha_I\right]}{\epsilon_0\overline{c}}>0\end{aligned}$$

Then $\tilde{m}(\lambda_t,q_t)$ is increasing in q_t. As well $\tilde{m}(\lambda_t,q_t)\geq m(\lambda_t)$ if and only if

$$\frac{-\phi}{1+\phi m(\lambda_t)}\left[1-\frac{\tau(1-q_t[[\theta_0-k\alpha_I(m(\lambda_t),q_t)]]m)}{\epsilon_0\overline{c}}\right] + \frac{\tau q_t[\theta_0-k\alpha_I(m(\lambda_t),q_t)]}{\epsilon_0\overline{c}}\geq 0$$

or

$$\phi\left[\frac{\epsilon_0\overline{c}}{\tau}-1\right]\leq q_t\left[\theta_0-k\alpha_I\left(m\left(\lambda_t\right),q_t\right)\right] \tag{A.9}$$

$q_t\left[\theta_0-k\alpha_I\left(m\left(\lambda_t\right),q_t\right)\right]$ is an increasing function of q_t and decreasing function of λ_t. Condition (A.9) can be rewritten as a threshold condition $q_t\geq\overline{q}_I(\lambda_t)$ for $\overline{q}_I(\lambda_t)\in(0,1]$ with $\overline{q}_I(\lambda_t)$ is a decreasing function of λ_t.

Summarizing we get $\tilde{m}(\lambda_t,q_t)\geq m(\lambda_t)$ if and only if $q_t\geq\overline{q}_I(\lambda_t)$ for $\overline{q}_I(\lambda_t)\in(0,1]$.

Since $(\tilde{m}(\lambda_t, q_t), \tilde{\alpha_I}(\lambda_t, q_t))$ maximizes the social welfare when the externalities are internalized, λ_{t+1} solves the optimization problem (A.4) when:

$$\begin{cases} \tilde{m}(\lambda_t, q_t) = m(\lambda_{t+1}), \text{ and} \\ \tilde{\alpha}_I(\lambda_t, q_t) = \alpha_I\left(m(\lambda_{t+1}), q_t\right) \end{cases} \tag{A.10}$$

Given the first equality, it is immediate to see that the second equality is automatically satisfied from the definition of $\alpha_I\left(m, q_t\right)$. Given this the institutional dynamics of λ_t is uniquely determined. Observe as well that $\tilde{m}(\lambda_t, q_t) \geq m\left(\lambda_t\right)$ if and only if $q_t \geq \overline{q}_I(\lambda_t)$. This can be rewritten as $m(\lambda_{t+1}) \geq m\left(\lambda_t\right)$ if and only if $q_t \geq \overline{q}_I(\lambda_t)$. Given the fact that $m\left(\lambda\right)$ is increasing in λ, we deduce the following result:

$$\lambda_{t+1} \geq \lambda_t \text{ if and only if } q_t \geq \overline{q}_I(\lambda_t)$$

This concludes the proof of Proposition 6.

- **Proof of Proposition 7**

The paternalistic motives have to be amended to take into account the fact that productivity is optimally determined by the endogenous choice of technology: More precisely we have:

$$\begin{cases} V_{ReRe}(\lambda, q) = \frac{(1-\tau_{Re})\alpha_I(\lambda,q)}{1+\phi\alpha_c(\lambda)} \int_{\tau_{Re}/\epsilon_0}^{\overline{c}} \frac{dc}{\overline{c}} + \int_0^{\tau_{Re}/\epsilon_0} \frac{(1-c\epsilon_0)}{1+\phi\alpha_c(\lambda)} \frac{dc}{\overline{c}} \\ V_{Re\ S}(\lambda, q) = \frac{(1-\tau_{Re})\alpha_I(\lambda,q)}{(1+\phi\alpha_c(\lambda))} \int_{\tau/\epsilon_0}^{\overline{c}} \frac{dc}{\overline{c}} + \int_0^{\tau/\epsilon_0} \frac{(1-c\epsilon_0)}{1+\phi\alpha_c(\lambda)} \frac{dc}{\overline{c}}, \end{cases} \tag{A.11}$$

Hence,

$$\Delta V_{Re}(\lambda, \beta, q) = \frac{(\tau\theta\alpha_c(\lambda))^2 \alpha_I\left(\lambda, q\right)}{2\overline{c}\epsilon_0(1+\phi\alpha_c(\lambda))}. \tag{A.12}$$

Similarly, we find that

$$\Delta V_S(\lambda, \beta, q) = \Delta V_{Re}(\lambda, \beta, q) = \Delta V(\lambda, \beta, q) = \frac{(\tau\theta\alpha_c(\lambda))^2 \alpha_I\left(\lambda, q\right)}{2\overline{c}\epsilon_0(1+\phi\alpha_c(\lambda))}. \tag{A.13}$$

Again the result that $\Delta V_s(\lambda, \beta, q) = \Delta V_{re}(\lambda, \beta, q)$ follows from the quadratic specification of the expected payoff functions. Note as well that because $\alpha_I\left(\lambda, q\right))$ depends on q (ie. is a decreasing function in q), $\Delta V(\lambda, \beta, q)$ also depends on q and is decreasing function of q

Now, the cultural dynamics write as

$$q_{t+1} - q_t = q_t(1 - q_t)D(\lambda_t, q_t). \tag{A.14}$$

with

$$D(\lambda_t, q_t) = d^*_{Re} - d^*_S = D_{Re}\left[(1 - q_t)\Delta V(\lambda_t, q_t), m(\lambda_t)\right] - D_S\left[q_t \Delta V(\lambda_t, q_t)\right]$$

can be interpreted as the relative "cultural fitness" of the religious trait in the population. Again simple inspection shows

$$D(\lambda_t, 0) = D_{Re}\left[\Delta V(\lambda_t, 0), m(\lambda_t)\right] > 0$$

and

$$D(\lambda_t, 1) = -D_S\left[\Delta V(\lambda_t, 1)\right] < 0$$

From this it follows that there exists a threshold $q^*_I(\lambda_t) \in (0, 1)$ such that

$$D(\lambda_t, q^*_I(\lambda_t)) = 0 \tag{A.15}$$

Compared to the benchmark model, $D(\lambda_t, q_t)$ may not be always decreasing function in q_t, as $\Delta V(\lambda_t, q_t)$ is decreasing in q_t and the uniqueness of the threshold $q^*_d(\lambda_t)$ is not necessarily ensured. When however $q\Delta V(\lambda, q)$ is increasing function of q,[36] simple inspection shows that $D(\lambda_t, q_t)$ is a decreasing function of q_t and that $q_{t+1} < q_t$ if and only if $q_t > q^*_I(\lambda_t, \beta_t)$, as stated in proposition 7. **QED.**

[36]This is ensured when $1 > \frac{\tau^2}{\bar{c}\epsilon_0} \max\left(\frac{\theta}{\phi}, 1\right)$

A.2 Proofs of Lemmas 1, 2 and 3

In order to prove the three Lemmas of the main text, we solve the equilibrium, where the ruler chooses the amount of religious infrastructures m so as to maximize the social welfare W, with $e_{Re}(\lambda)$

$$W = \frac{1}{2}U_R(m) + \frac{\lambda}{2}U_c(m, \alpha_c) + \frac{1-\lambda}{2}\left[qU_{Re}(e_{Re}) + (1-q)U_S\left(e_S\right)\right]. \tag{A.16}$$

The clerics and the individuals choose, respectively, α_c and $e_i,\ i = Re, S$ to maximize their utility. The equilibrium is denoted $\{\tau(\lambda), m(\lambda), \alpha_c(\lambda), e_S(\lambda), e_{Re}(\lambda)\}$. Since $\lambda \leq 1$, $\tau(\lambda)$ is equal to $\overline{\tau} \equiv \tau$ and the remaining first-order conditions are:

$$\begin{cases} -C'(m) - \lambda F'(m) + \lambda \cdot \alpha_c = 0 \\ m - \Psi'(\alpha_c) = 0 \\ (1-\tau_{Re}) - (1+\phi\alpha_c)e_{Re} = 0 \\ (1-\tau) - (1+\phi\alpha_c)e_S = 0, \end{cases} \tag{A.17}$$

or after substitution:

$$\begin{cases} C'(m) + \lambda F'(m) = \lambda\alpha_c \\ \Psi'(\alpha_c) = m \\ e_{Re} = \frac{1-\tau+\tau\theta\alpha_c}{1+\phi\alpha_c} \\ e_S = \frac{1-\tau}{1+\phi\alpha_c} \end{cases} \tag{A.18}$$

Assuming that the marginal cost functions $C'(.)$, $F'(.)$ and $\Psi'(.)$ are increasing convex functions (ie. $C'''(.) \geq 0, F'''(.) \geq 0$ and $\Psi'''(.) \geq 0$) with at least one of these cost derivatives strictly convex), and the limit condition $\lim_{x\to\infty} F''(x) > 1$, and $F''(0)\Psi''(0) < 1$, then the first two equations of (A.18) simply characterize a unique equilibrium couple $m(\lambda) > 0$ and $\alpha_c(\lambda) > 0$ when $\frac{C''(0)\Psi''(0)}{1-F''(0)\Psi''(0)} < \lambda$, while $m(\lambda) = \alpha_c(\lambda) = 0$ for $\lambda \leq \frac{C''(0)\Psi''(0)}{1-F''(0)\Psi''(0)}$.

Lemma 1: Differentiating the previous first-order conditions, it is easy to note that the optimal provision of religious infrastructure $m(\lambda) > 0$ and the effort of the clerics $\alpha_c(\lambda) > 0$ are both increasing in λ and independent from θ and ϕ. This concludes the proof of Lemma 1.

Lemma 2: The equilibrium production efforts are obtained as

$$\begin{cases} e_{Re}(\lambda) = \frac{1-\tau+\tau\theta\alpha_c(\lambda)}{1+\phi\alpha_c(\lambda)} \\ e_S(\lambda) = \frac{1-\tau}{1+\phi\alpha_c(\lambda)} \end{cases} \tag{A.19}$$

The equilibrium secular effort $e_S^*(\lambda)$ is decreasing in clerics activities α_c^* and thus, it is decreasing in λ. It is independent from ϕ and θ

Additionally, from the equation above, $e_{Re}(\lambda)$ increases with θ and decreases with ϕ. The effect of $\alpha_c(\lambda)$ on $e_{Re}(\beta, \lambda)$ is ambiguous. By deriving $e_{Re}(\lambda)$ with respect to α_c, we find that when $\theta > \frac{1-\tau}{\tau}\phi$, then $e_{Re}(\lambda)$ increases with $\alpha_c(\lambda)$, in which case $e_{Re}(\lambda)$ increases with λ. This concludes the proof of Lemma 2.

Lemma 3: The equilibrium tax base of the ruler writes as

$$E(\lambda) = q \cdot e_{Re}(\lambda) + (1-q) \cdot e_s(\lambda), \tag{A.20}$$

so

$$E(\lambda) = \frac{1-\tau+\tau\theta q \cdot \alpha_c(\lambda)}{1+\phi\alpha_c(\lambda)}. \tag{A.21}$$

By deriving the previous expression with respect to $\alpha_c(\lambda)$, we find that the tax base is increasing in the clerics' effort if and only if $q \geq \frac{1-\tau}{\tau\theta}\phi$. Hence, when the previous condition is satisfied, $E(\lambda)$ is increasing in λ. Finally, from (A.21), $E(\lambda)$ is increasing in q and θ, and decreasing in ϕ. This concludes the proof of Lemma 3.

A.3 Proof of Proposition 1

- First, we demonstrate that the optimization problem (7) rewritten below admits a unique solution $\lambda_{t+1} \in [0, 1]$:

$$\max_{\lambda_{t+1}} \frac{1}{2}U_r(m(\lambda_{t+1})) + \frac{\lambda_t}{2}U_c(m(\lambda_{t+1}), \alpha_c(\lambda_{t+1})) + \frac{1-\lambda_t}{2}\left[q_t U_{re}(e_{Re}(\lambda_{t+1})) + (1-q_t)U_s\left(e_s(\lambda_{t+1})\right)\right]. \tag{A.22}$$

In order to solve this maximization problem, we consider the following related optimization problem:

$$\max_{m} W(m, q_t) = \frac{1}{2}\{U_r(m) + \lambda_t \widetilde{U}_c(m) + (1-\lambda_t)\left[q_t \widetilde{U}_{re}(m) + (1-q_t)\widetilde{U}_s(m)\right]\}, \quad \text{(A.23)}$$

with

$$\begin{cases} \widetilde{\alpha}_c(m) = \psi'^{-1}(m) \\ E(m) = \frac{1-\tau+\tau\theta q\widetilde{\alpha}_c(m)}{1+\phi\widetilde{\alpha}_c(m)} \\ U_r(m) = \tau E(m) - C(m) \\ \widetilde{U}_c(m) = \widetilde{\alpha}_c(m)m - \psi(\widetilde{\alpha}_c(m)) - F(m) \\ \widetilde{U}_{Re}(m) = \frac{[1-\tau+\tau\theta\widetilde{\alpha}_c(m)]^2}{2(1+\phi\widetilde{\alpha}_c(m))} \\ \widetilde{U}_S(m) = \frac{(1-\tau)^2}{2(1+\phi\widetilde{\alpha}_c(m))}. \end{cases} \quad \text{(A.24)}$$

In the optimization problem (A.23), the choice of the religious infrastructure m is made by a ruler able to commit to the provision of m, and therefore internalizing the two externalities detailed in the main text. We find that:

$$2\frac{\partial W}{\partial m} = \lambda_t\left[\widetilde{\alpha}_c(m) - F'(m)\right] - C'(m) + \tau E'(m) + (1-\lambda_t)[q_t \widetilde{U}'_{Re}(m) + (1-q_t)\widetilde{U}'_S(m)]. \quad \text{(A.25)}$$

When $C(.)$ and $F(.)$ are sufficiently convex, the function W is concave in m, and the previous optimization admits a unique solution $\tilde{m}(\lambda_t, q_t) \geq 0$.

Note that $\alpha_c(\lambda) = \widetilde{\alpha}_c(m(\lambda))$, $U_i(e_i(\lambda)) = \widetilde{U}_i(m(\lambda))$ for $i = \{Re, S\}$, and $U_c(m(\lambda), \alpha_c(\lambda)) = \widetilde{U}_c(m(\lambda))$. Given that $\tilde{m}(\lambda_t, q_t)$ maximizes the social welfare when the externalities are internalized, the solution λ_{t+1} of the optimization problem (7), should be such as to induce an equilibrium choice $m(\lambda_{t+1})$ as close to $\tilde{m}(\lambda_t, q_t)$ as possible:

$$\lambda_{t+1} = \begin{cases} \lambda \text{ s.t } m(\lambda) = \tilde{m}(\lambda_t, q_t) & \text{if } \tilde{m}(\lambda_t, q_t) \in (m(0), m(1)) \\ 1 & \text{if } \tilde{m}(\lambda_t, q_t) > m(1) \\ 0 & \text{if } \tilde{m}(\lambda_t, q_t) < m(0). \end{cases} \quad \text{(A.26)}$$

When the clerics have power λ_{t+1} given by (A.26), institutions are designed for $t+1$ so as to induce a choice $m(\lambda_{t+1})$ in that period that maximizes the social welfare of period t. Given that $m(\lambda)$ is increasing in λ,this solution λ_{t+1} of problem (7) is unique and the institutional dynamics are well defined.

Note that $\frac{\partial \tilde{m}}{\partial q} = -\frac{\partial^2 W}{\partial m \partial q} / \frac{\partial^2 W}{\partial m^2}$ and has the sign of $\frac{\partial^2 W}{\partial m \partial q}$ (as W is concave in m). As

$$2\frac{\partial^2 W}{\partial m \partial q} = (1-\lambda_t)[\widetilde{U}'_{Re}(m) - \widetilde{U}'_S(m)]$$

and

$$\widetilde{U}_{Re}(m) - \widetilde{U}_S(m) = \frac{\tau\theta\widetilde{\alpha}_c(m)\left[2(1-\tau) + \tau\theta\widetilde{\alpha}_c(m)\right]}{2(1+\phi\widetilde{\alpha}_c(m))}$$

is an increasing of $\widetilde{\alpha}_c(m)$ and therefore an increasing function of m. It follows that $\widetilde{U}'_{Re}(m) - \widetilde{U}'_S(m) > 0$ and $\frac{\partial^2 W}{\partial m \partial q} > 0$, from which we conclude that $\tilde{m}(\lambda_t, q_t)$ is increasing in q_t.

- In the second step of the proof, we demonstrate that there exists a threshold $\overline{q}(\lambda_t)$ such that if $q_t > \overline{q}$, then $\lambda_{t+1} > \lambda_t$. Otherwise, $\lambda_{t+1} \leq \lambda_t$.

In order to demonstrate this claim, we first show the following intermediary result:

Lemma 6 *$\lambda_{t+1} > \lambda_t$ if and only if $\tilde{m}(\lambda_t, q_t) > m(\lambda_t)$.*

Proof: Indeed, $\tilde{m}(\lambda_t, q_t) > m(\lambda_t)$ means that if the ruler had the capacity to commit, in period t, to provide religious infrastructures m, then he would chooses a level $\tilde{m}(\lambda_t, q_t)$ strictly above what he actually provides in equilibrium. Since $m(.)$ is an increasing function (Lemma 1), we deduce that λ_{t+1} is such that $\lambda_{t+1} > \lambda_t$.[37] **QED.**

Lemma 7 *$\tilde{m}(\lambda_t, q_t) > m(\lambda_t)$ if and only if $q_t > \overline{q}(\lambda_t)$, with:*

$$\overline{q}\left(\lambda_t\right) = \frac{1}{\tau\theta}\frac{\phi\left(1-\tau\right)\left[\tau + \left(1-\lambda_t\right)\frac{1-\tau}{2}\right]}{\tau + \left(1-\lambda_t\right)\left[1-\tau+\tau\theta\alpha_c^*(\lambda_t)\left(1+\frac{\phi}{2}\alpha_c^*(\lambda_t)\right)\right]} \tag{A.27}$$

Proof: From the proof of Lemma 1 above, the first-order condition associated with the determination of $m(\lambda)$ is:

$$\lambda_t\left[\widetilde{\alpha}_c(m) - F'(m)\right] - C'(m) = 0, \tag{A.28}$$

given that $\widetilde{\alpha}_c(m) = \psi'^{-1}(m)$.

The first order condition for the determination of $\tilde{m}(\lambda_t, q_t)$ writes as $\frac{dW}{dm} = 0$, with

$$\frac{dW}{dm} = \frac{1}{2}\left[\lambda_t\left[\widetilde{\alpha}_c(m) - F'(m)\right] - C'(m) + \tau E'(m) + (1-\lambda_t)[q_t\widetilde{U}'_{Re}(m) + (1-q_t)\widetilde{U}'_S(m)]\right]. \tag{A.29}$$

[37] When an interior solution exists, λ_{t+1} solves $\tilde{m}(\lambda_t) = m(\lambda_{t+1})$. Hence, if $\tilde{m}(\lambda_t) > m(\lambda_t)$ then $\lambda_{t+1} > \lambda_t$.

Consider the expression

$$H(m) = \tau \cdot E'(m) + (1-\lambda)[q_t \widetilde{U}'_{Re}(m) + (1-q_t)\widetilde{U}'_S(m)]\}.$$

Given the two FOCs above, we deduce that $\tilde{m}(\lambda_t, q_t) > m(\lambda_t)$ if and only if $H(m(\lambda_t)) > 0$. We show that condition $H(m(\lambda_t)) > 0$ is equivalent to a condition over the possible values of q.

$$\begin{aligned} E'(m) &= q_t \cdot \frac{de_{Re}}{dm} + (1-q_t) \cdot \frac{de_S}{dm} && \text{(A.30)} \\ &= \frac{q_t\tau\theta - (1-\tau)\phi}{[1+\phi\widetilde{\alpha}_c(m)]^2} \frac{d\widetilde{\alpha}_c(m))}{dm}, && \text{(A.31)} \end{aligned}$$

$$\begin{aligned} U'_{Re}(m) &= e_{Re}(m)\left[\theta\tau - \phi \cdot \frac{e_{Re}(m)}{2}\right]\frac{d\widetilde{\alpha}_c(m)}{dm} && \text{(A.32)} \\ &= \frac{1-\tau+\tau\theta\widetilde{\alpha}_c(m)}{1+\phi\widetilde{\alpha}_c(m)}\left[\theta\tau - \phi \cdot \frac{1}{2}\frac{1-\tau+\tau\theta\widetilde{\alpha}_c(m)}{1+\phi\widetilde{\alpha}_c(m)}\right]\frac{d\widetilde{\alpha}_c(m)}{dm} && \text{(A.33)} \end{aligned}$$

and

$$\begin{aligned} U'_s(m) &= -\frac{\phi\,(e_S(m))^2}{2}\frac{d\widetilde{\alpha}_c(m)}{dm} && \text{(A.34)} \\ &= -\phi \cdot \frac{1}{2}\left[\frac{1-\tau}{1+\phi\widetilde{\alpha}_c(m)}\right]^2 \cdot \frac{d\widetilde{\alpha}_c(m)}{dm} && \text{(A.35)} \end{aligned}$$

Thus,

$$2\frac{dW}{dm} = \lambda\widetilde{\alpha}_c(m) - C'(m) - \lambda F'(m) + H(m),$$

with

$$[1+\phi\widetilde{\alpha}_c(m)]^2 \frac{H(m)}{\frac{d\widetilde{\alpha}_c(m)}{dm}} = \tau \cdot (q_t\tau\theta - (1-\tau)\phi) + (1-\lambda)G(m)$$

and

$$\begin{aligned}
G(m) &= q_t(1-\tau+\tau\theta\widetilde{\alpha}_c(m))\left[\theta\tau\left(1+\phi\widetilde{\alpha}_c(m)\right)-\frac{\phi}{2}(1-\tau+\tau\theta\widetilde{\alpha}_c(m))\right] \\
&\quad -(1-q_t)\frac{\phi}{2}\left[1-\tau\right]^2 \\
&= q_t\tau\theta\left[(1-\tau)+\tau\theta\widetilde{\alpha}_c(m)\left(1+\frac{\phi}{2}\widetilde{\alpha}_c(m)\right)\right]-\frac{\phi}{2}\left[1-\tau\right]^2
\end{aligned}$$

Then the condition $H(m(\lambda_t)) > 0$ writes as

$$\tau\cdot(q_t\tau\theta-(1-\tau)\phi)+(1-\lambda)\left[\begin{array}{c} q_t\tau\theta\left[(1-\tau)+\tau\theta\widetilde{\alpha}_c(m)\left(1+\frac{\phi}{2}\widetilde{\alpha}_c(m)\right)\right] \\ -\frac{\phi}{2}\left[1-\tau\right]^2 \end{array}\right]\geq 0$$

or using $\alpha_c(\lambda) = \widetilde{\alpha}_c(m(\lambda)) = \Psi'^{-1}(m(\lambda))$ and rearranging terms $H(m(\lambda_t)) > 0$ if and only if $q_t > \overline{q}(\lambda_t)$ with

$$\overline{q}\left(\lambda\right)=\frac{1}{\tau\theta}\frac{\phi\left(1-\tau\right)\left[\tau+(1-\lambda)\frac{1-\tau}{2}\right]}{\tau+(1-\lambda)\left[1-\tau+\tau\theta\alpha_c(\lambda)\left(1+\frac{\phi}{2}\alpha_c(\lambda)\right)\right]}, \tag{A.36}$$

and $\alpha_c(\lambda)$ is an increasing function of λ. We conclude that $\tilde{m}(\lambda_t, q_t) > m(\lambda_t)$ if and only if $q_t > \overline{q}(\lambda_t)$.**QED.**

Combining the results established in Lemmas 6 and 7, it follows that $\lambda_{t+1} > \lambda_t$ if and only if $q > \overline{q}(\lambda_t)$.

Finally, from (A.36), we deduce that $\overline{q}(\lambda_t)$ is decreasing in θ and ϕ. This concludes the proof of the first point of Proposition 1.

A.4 Proof of Proposition 2

As mentioned in the main text, cultural dynamics are modeled as purposeful inter-generational transmission (Bisin and Verdier (2001), Bisin and Verdier (2017)), through parental socialization and imitation of society at large. *Direct vertical* socialization to the parent's trait $i \in \{Re, S\}$ occurs with probability d_i. If a child from a family with trait i is not directly socialized, which occurs with probability $1 - d_i$, he/she is *horizontally/obliquely* socialized by picking the trait of a role model chosen randomly in the population. The probability

P_{ij} that a child in group i is socialized to trait j writes as:

$$\begin{cases} P_{ii} = d_i + (1 - d_i)q_i \\ P_{ij} = (1 - d_i)q_j \end{cases} \tag{A.37}$$

with $q_{Re} = q_t$ and $q_S = 1 - q_t$. Let $V_{ij}(\lambda_t) = U_i(e_j(\lambda_t))$ denote the utility to a cultural trait i parent of a type j child, with $i, j \in \{Re, S\}$. We denote the paternalistic bias of a parent of type i as $\Delta V_i(\lambda_t) = V_{ii}(\lambda_t) - V_{ij}(\lambda_t)$, for $j \neq i$. The socialization cost $h_{Re}(d_{Re}, m)$ of a parent of type Re (respectively S) is assumed to be a smooth function with $\frac{\partial h_{Re}(d_{Re},m)}{\partial d_{Re}} \geq 0$; $\frac{\partial^2 h_{Re}(d_{Re},m)}{\partial d_{Re}^2} > 0$ (ie. $h_{Re}(d_{Re}, m)$ is increasing convex in d_{Re}) and the Inada conditions $h_{Re}(0, m) = \frac{\partial h_{Re}(0,m)}{\partial d_{Re}} = 0$, $\lim_{d\to 1} h_{Re}(d, m) = \lim_{d\to 1} \frac{\partial h_{Re}(d,m)}{\partial d_{Re}} = +\infty$ Similarly, the socialization cost $h_S(d_S)$ of a parent of type S satifies $h'_S(d_S) \geq 0$; $h''_S(d_S) > 0$ (ie. $h_S(d_S)$ is increasing convex in d_S (ie.), and $h_S(0) = h'_S(0) = 0$, $\lim_{d\to 1} h_S(d) = \lim_{d\to 1} h'_S(d) = +\infty$.

Furthermore, to reflect the fact religious infrastructures may enter as a complementary input to parental effort for transmission of the religious trait, we assume that $\frac{\partial h_{Re}(d_{Re},m)}{\partial m} \leq 0$ and $\frac{\partial^2 h_{Re}(d_{Re},m)}{\partial d_{Re}\partial m} \leq 0$, (ie. m affects negatively the cost and the marginal cost of socialization of religious parents). Following Bisin and Verdier (2001), direct socialization d^*_{Re} of religious parents is the solution to the following socialization problem:

$$\max_{d_{Re}} -h_{Re}(d_{Re}, m_t) + P_{ReRe} \cdot V_{ReRe}(\lambda_t) + P_{ReS} \cdot V_{Re\ S}(\lambda_t), \tag{A.38}$$

while direct socialization d^*_S of secular parents is the solution to the following socialization problem:

$$\max_{d_S} -h_S(d_S) + P_{SS} \cdot V_{SS}(\lambda_t) + P_{SRe} \cdot V_{SRe}(\lambda_t), \tag{A.39}$$

The FOCs of the previous programs determine the optimal socialization efforts as:

$$\frac{\partial h_{Re}(d^*_{Re}, m_t)}{\partial d_{Re}} = (1 - q_t)\Delta V_{Re}(\lambda_t) \text{ and } h'_S(d^*_S) = q_t \Delta V_s(\lambda_t)$$

which can be rewritten as $d^*_{Re}(q_t, \lambda_t) = D_{Re}((1 - q_t)\Delta V_{Re}(\lambda_t), m(\lambda_t))$ and $d^*_S(q_t, \lambda_t) = D_S(q_t \Delta V_S(\lambda_t))$.

Note that by the Inada conditions on $h_{Re}(\cdot,\cdot)$, $d_{Re}^{*} \in [0,1]$, and $D_{Re}(0,m) = 0$. As well $D_{Re}(\cdot,\cdot)$ is an increasing function of both arguments $(1-q_t)\Delta V_{Re}(\lambda_t)$ and m, as we have:

$$\frac{\partial d_{Re}^{*}}{\partial(1-q_t)\Delta V_{Re}(\lambda_t))} = \frac{1}{\frac{\partial^2 h_{Re}}{\partial d_{Re}^2}} > 0 \text{ and } \frac{\partial d_{Re}^{*}}{\partial m_t} = -\frac{\frac{\partial^2 h_{Re}}{\partial d_{Re}\partial m}}{\frac{\partial^2 h_{Re}}{\partial d_{Re}^2}} > 0$$

Similarly the Inada conditions on $h_S(\cdot)$ ensure that $d_S^{*} \in [0,1]$, $D_S(0) = 0$. As well $d_S^{*} = D_S(q_t \Delta V_S(\lambda_t))$ is an increasing function of $q_t \Delta V_S(\lambda_t)$ as

$$\frac{\partial d_S^{*}}{\partial(q_t \Delta V_S(\lambda_t))} = \frac{1}{h_S''} > 0$$

Using the Law of Large Numbers, one easily obtains the intergenerational evolution of the frequency of the religious trait q_t in the population as

$$q_{t+1} = q_t \cdot P_{ReRe} + (1-q_t) \cdot P_{SRe}$$

or after substitution of (A.37) and the values of d_{Re}^{*} and d_S^{*},

$$q_{t+1} - q_t = q_t(1-q_t)\{d_{Re}^{*}(q_t,\lambda_t) - d_S^{*}(q_t,\lambda_t)\}. \tag{A.40}$$

As mentioned in the main text, in equation (A.40), the term

$$\begin{aligned} D(q_t,\lambda_t) &= d_{Re}^{*}(q_t,\lambda_t) - d_S^{*}(q_t,\lambda_t) \\ &= D_{Re}\left[(1-q_t)\Delta V_{Re}(\lambda_t), m(\lambda_t)\right] - D_S\left[q_t \Delta V_S(\lambda_t)\right] \end{aligned}$$

can be interpreted as the relative "cultural fitness" of the religious trait in the population. This term is frequency dependent (ie. depends on the state of the population q_t). Moreover simple inspection shows that $D(q_t,\lambda_t)$ is a decreasing function of q_t, with $D(0,\lambda_t) = D_{Re}\left[\Delta V_{Re}(\lambda_t), m(\lambda_t)\right] > 0$ and $D(1,\lambda_t) = -D_S\left[\Delta V_S(\lambda_t)\right] < 0$. From this it follows that there exists a unique threshold $q^*(\lambda_t) \in (0,1)$ such that

$$D(q^*(\lambda_t),\lambda_t) = 0 \tag{A.41}$$

Inspection of equation (A.40) and the fact that $D(q_t,\lambda_t)$ is a decreasing function of q_t provides immediately that $q_{t+1} < q_t$ if and only if $q_t > q^*(\lambda_t)$, proving therefore proposition in the main text. **QED.**

A.5 Comparative statics on the cultural threshold $q^*(\lambda_t)$

The relative "cultural fitness" of the religious trait $D(q_t, \lambda_t)$ is affected by the institutional environment λ_t, as this variable interacts with the process of parental cultural transmission both through paternalistic motivations $\Delta V_i(\lambda_t)$, and through the provision of religious infrastructures $m_t = m(\lambda_t)$ as a complementary input to religious family socialization. Therefore the dependence of the threshold $q^*(\lambda_t)$ on the institutional environment λ_t and the comparative statics on the parameters θ and ϕ depends on how the relative "cultural fitness" $D(q_t, \lambda_t)$ of the religious trait is affected by changes in such features.

It is first useful to note that with the quadratic specification for the utility functions $U_i(.)$ of workers, the paternalistic motives .$\Delta V_{Re}(\lambda_t)$ and $\Delta V_S(\lambda_t)$ are equal and take a simple form. Indeed we have:

$$\begin{cases} V_{ReRe}(\lambda) = \frac{(1-\tau_{Re})^2}{2(1+\phi\alpha_c(\lambda))} \\ V_{Re\ S}(\lambda) = (1-\tau_{Re})\frac{1-\tau}{1+\phi\alpha_c(\lambda)} - \frac{1}{2}(1+\phi\alpha_c(\lambda))\frac{(1-\tau)^2}{(1+\phi\alpha_c(\lambda))^2}. \end{cases} \tag{A.42}$$

Hence,

$$\Delta V_{Re}(\lambda) = V_{ReRe}(\lambda) - V_{Re\ S}(\lambda) = \frac{(\tau\theta\alpha_c(\lambda))^2}{2(1+\phi\alpha_c(\lambda))}. \tag{A.43}$$

Similarly, we find that

$$\begin{cases} V_{SS}(\lambda) = \frac{(1-\tau)^2}{2(1+\phi\alpha_c(\lambda))} \\ V_{SRe}(\lambda) = (1-\tau)\frac{1-\tau_{Re}}{1+\phi\alpha_c(\lambda)} - \frac{1}{2}(1+\phi\alpha_c(\lambda))\frac{(1-\tau_{Re})^2}{(1+\phi\alpha_c(\lambda))^2}. \end{cases}$$

and

$$\Delta V_S(\lambda) = V_{SS}(\lambda) - V_{SRe}(\lambda) = \frac{(\tau\theta\alpha_c(\lambda))^2}{2(1+\phi\alpha_c(\lambda))} \tag{A.44}$$

Thus posing $\Delta V(\lambda) = \frac{(\tau\theta\alpha_c(\lambda))^2}{2(1+\phi\alpha_c(\lambda))}$, we get $\Delta V_{Re}(\lambda) = \Delta V_S(\lambda) = \Delta V(\lambda)$ and the relative "cultural fitness" of the religious trait $D(q_t, \lambda_t)$ rewrites as:

$$D(q_t, \lambda_t) = D_{Re}\left[(1-q_t)\Delta V(\lambda_t), m(\lambda_t)\right] - D_S\left[q_t \Delta V(\lambda_t)\right]$$

Now, considering the functions $D_{Re}(x, y)$ and $D_S(z)$ that respectively characterize the optimal socialization behavior of religious parents as

$$d^*_{Re}(q_t, \lambda_t) = D_{Re}\left[(1-q_t)\Delta V(\lambda_t), m(\lambda_t)\right], \text{and } \ d^*_S(q_t, \lambda_t) = D_S(q_t \Delta V_S(\lambda_t))$$

define the sensitivity of parents' socialization to paternalistic motives by the following elasticities:

$$\epsilon_{Re}(q,\lambda) = \frac{\partial D_{Re}(x,y)}{\partial x} \cdot \frac{x}{D_{Re}} \text{ and } \epsilon_S(q,\lambda) = \left.\frac{\partial D_S}{\partial z} \cdot \frac{z}{D_{Re}}\right|$$

evaluated respectively at $x = (1-q)\Delta V(\lambda)$ and $y = m(\lambda)$, and $z = q\Delta V_S(\lambda)$.

Differentiation of (A.41) then provides with $d^*(\lambda_t) = d^*_{Re}(q^*(\lambda_t),\lambda_t) = d^*_S(q^*(\lambda_t),\lambda_t)$

$$q^{*\prime}(\lambda_t) = \frac{[\epsilon_{Re}(q^*,\lambda_t) - \epsilon_S(q^*,\lambda_t)]\, d^*(\lambda_t) \cdot \frac{\Delta V'(\lambda_t)}{\Delta V(\lambda_t)} + \frac{\partial D_{Re}}{\partial m} \cdot m'(\lambda_t)}{-\frac{\partial D}{\partial q}(q^*(\lambda_t),\lambda_t)} \tag{A.45}$$

Given that $\frac{\partial D}{\partial q}(q^*(\lambda_t),\lambda_t) < 0$, $\frac{\partial q^*}{\partial \lambda_t}$ has the sign of the numerator. This numerator is composed of two terms reflecting the two channels through which the institutional environment λ_t affects cultural transmission. The first term $K(\lambda_t) = [\epsilon_{Re}(q^*,\lambda_t) - \epsilon_S(q^*,\lambda_t)]\, d^*(\lambda_t) \cdot \frac{\Delta V'(\lambda_t)}{\Delta V(\lambda_t)}$ is the paternalistic motive channel. As $\Delta V'(\lambda_t) > 0$, both types of parents increase the intensity of socialization to their own traits. The sign of $K(\lambda_t)$ depends on the relative sensitivity of parents' socialization to paternalistic motives. It is positive when $\epsilon_{Re}(q^*,\lambda_t) > \epsilon_S(q^*,\lambda_t)$, namely when the socialization rate of religious parents d^*_{Re} is more sensitive to paternalistic motives than the one of secular parents d^*_S.

The second term $\frac{\partial D_{Re}}{\partial m} \cdot m'(\lambda_t)$ is positive. It reflects the fact that by promoting religious infrastructures that enter as complementary inputs in the socialization process of the religious trait, an increase in the clerics weight λ_t makes the religious trait to be relatively more successfully transmitted than the secular trait.

From this discussion it follows that when religious parents' socialization efforts are more sensitive to paternalistic motives than secular parents (ie. $\epsilon_{Re}(q,\lambda_t) > \epsilon_S(q,\lambda_t)$) , and (or) when religious infrastructures are strong enough complementary inputs to socialization to the religious trait, then the numerator of (A.87) is positive and $q^*(\lambda_t)$ is increasing in λ_t.

As can be seen from (A.43) and (A.44), a change in the other parameters θ (the efficiency of the clerics) and ϕ (the restrictiveness of religious proscriptions) affects the relative cultural fitness of the religious trait only through their induced changes on the paternalistic motive $\Delta V(\lambda_t)$, with $\Delta V(\lambda)$ increasing in θ,and decreasing ϕ. It follows that

$$\frac{\partial q^*(\lambda_t)}{\partial \theta} = \frac{K(\lambda_t) \cdot \frac{\partial \Delta V(\lambda_t)}{\partial \theta}}{-\frac{\partial D}{\partial q}(q^*(\lambda_t),\lambda_t)} \text{ and } \frac{\partial q^*(\lambda_t)}{\partial \theta} = \frac{K(\lambda_t) \cdot \frac{\partial \Delta V(\lambda_t)}{\partial \phi}}{-\frac{\partial D}{\partial q}(q^*(\lambda_t),\lambda_t)}$$

When religious parents are more sensitive to paternalistic motives than secular parents, one has $K(\lambda_t) > 0$ and a positive shift in θ (negative shift of ϕ) leads to a higher value of $q^*(\lambda_t)$. This provides the comparative statics discussion on $q^*(\lambda_t)$ in the main text. **QED**.

- **Example with constant elasticity socialization cost functions:**

Consider the following socialization cost functions:

$$\begin{cases} h_{Re}(d) = \frac{d^{1+\eta_{re}}}{1+\eta_{re}} \cdot \frac{1}{m^{\gamma}} \text{ and} \\ h_s(d) = \frac{d^{1+\eta_s}}{1+\eta_s}, \end{cases} \tag{A.46}$$

with $\eta_s \geq \eta_{re} > 0$ and $\gamma > 0$. The optimal socialization efforts are such that:

$$\begin{cases} d^*_{Re}(q_t, \lambda_t) = ((1-q_t)\Delta V(\lambda_t))^{\frac{1}{\eta_{re}}} \cdot m(\lambda_t)^{\frac{\gamma}{\eta_{re}}} \\ d^*_S(q_t, \lambda_t) = (q_t \Delta V(\lambda_t))^{\frac{1}{\eta_s}}. \end{cases} \tag{A.47}$$

and in this constant elasticity specification $\epsilon_{Re}(q, \lambda) - \epsilon_S(q, \lambda) = \frac{1}{\eta_{re}} - \frac{1}{\eta_s} \geq 0$. Rewriting the cultural dynamics equation (11), we deduce that:

$$q_{t+1} - q_t = q_t(1-q_t)\{((1-q_t)\Delta V(\lambda_t))^{\frac{1}{\eta_{re}}} \cdot m(\lambda_t)^{\frac{\gamma}{\eta_{re}}} - (q_t \Delta V(\lambda_t))^{\frac{1}{\eta_s}}\}, \tag{A.48}$$

which admits two unstable steady states $q = 0$ and $q = 1$, and a unique interior attractor, which we denote $q^*(\lambda_t)$ such that:

$$\frac{q^*(\lambda_t)^{\frac{1}{\eta_s}}}{(1-q^*(\lambda_t))^{\frac{1}{\eta_{re}}}} = \Delta V(\lambda_t)^{\frac{\eta_S - \eta_{re}}{\eta_S \eta_{re}}} \cdot m(\lambda_t)^{\frac{\gamma}{\eta_{re}}} \tag{A.49}$$

given that $\eta_S \geq \eta_{re}$, we deduce that $q^*(\lambda_t)$ is increasing in θ, λ_t, and decreasing in ϕ.

A.6 Existence and Stability Analysis of interior steady states

Let Γ, The set of interior steady state of the joint dynamics of culture and institutions:

$$\Gamma = \left\{(\lambda, q) \in (0,1)^2 \mid q = \overline{q}(\lambda) \text{ and } q = q^*(\lambda)\right\}$$

namely the set of interior intersection points of the institutional and cultural manifolds $q = \overline{q}(\lambda)$ and $q = q^*(\lambda)$.

- *When $\overline{q}(1) < q^*(1)$, the set Γ is not empty.*

Proof: First note that $q^*(0) = 0$. Indeed the thresholds $q^*(\lambda)$ is the solution of (A.48):

$$D(q,\lambda) = d^*_{Re}(q,\lambda) - d^*_S(q,\lambda) = 0$$

Now given that religious infrastructures m are an essential input in the socialization of religious individuals and that $m(0) = 0$,

$$d^*_{Re}(q,0) = D_{Re}\left[(1-q)\Delta V_{Re}(0), m(0)\right] = D_{Re}\left[(1-q)\Delta V_{Re}(0), 0\right] = 0$$

while $d^*_S(0,\lambda) = D_S[0] = 0$, Thus $D(0,0) = 0$ and therefore $q^*(0) = 0$.

The thresholds $\overline{q}(\lambda)$ is characterized by (A.36):

$$\overline{q}(\lambda) = \frac{\phi(1-\tau)}{\tau\theta} \frac{\tau + (1-\lambda)\frac{1-\tau}{2}}{\tau + (1-\lambda)\left[1 - \tau + \tau\theta\alpha_c(\lambda)\left(1 + \frac{\phi}{2}\alpha_c(\lambda)\right)\right]}$$

Hence $\overline{q}(0) = \frac{\phi(1-\tau^2)}{2\tau\theta} \in (0,1)$ under assumption 1. As well differentiation of $\overline{q}(\lambda)$ provides

$$\begin{aligned}
\overline{q}'(0) &= \frac{\phi(1-\tau)}{\tau\theta}\left\{-\frac{1-\tau}{2} - \left(\frac{1+\tau}{2}\right)\cdot\left(-[1-\tau] + \tau\theta\cdot\alpha'_c(0)\right)\right\} \\
&= \frac{\phi(1-\tau)\tau}{2\tau\theta}\left\{(1-\tau) - (1+\tau)\theta\cdot\alpha'_c(0)\right\}
\end{aligned}$$

the function $\Lambda(\lambda) = q^*(\lambda) - \overline{q}(\lambda)$ is continuous and such that $\Lambda(0) = -\overline{q}(0) < 0$, and $\Lambda(1) = q^*(1) - \overline{q}(1) > 0$. Thus there is a $\lambda^* \in (0,1)$ such that $\Lambda(\lambda^*) = 0$ and given that $q^* = q^*(\lambda^*) < 1$, the point $(\lambda^*, q^*) \in \Gamma$ and the set Γ is non empty.**QED**

- *Condition $\overline{q}(1) < q^*(1)$*

Note that $\overline{q}(1) = \frac{\phi(1-\tau)}{\tau\theta} > \frac{\phi(1-\tau^2)}{2\tau\theta} = \overline{q}(0)$. Moreover the condition $\overline{q}(1) < q^*(1)$ is equivalent to $D(\overline{q}(1), 1) > 0$, or

$$D_{Re}\left[(1-\overline{q}(1))\Delta V(1), m(1)\right] > D_S\left[\overline{q}(1)\Delta V(1)\right]$$

with $\Delta V(1) = \frac{(\tau\theta\alpha_c(1))^2}{2(1+\phi\alpha_c(1))}$. We know that when religious parents are more sensitive to paternalistic motives than secular parents, $q^*(\lambda_t)$ is increasing in θ and decreasing in ϕ. This hold in particular for $q^*(1)$. Given that $\overline{q}\,(1)$ is a decreasing function of θ and an increasing function of ϕ, it follows that the condition $\overline{q}(1) < q^*(1)$ is more likely to be satisfied when θ is large enough and ϕ small enough. In the parametrization with constant elasticity socialization cost functions, the condition for $\overline{q}(1) < q^*(1)$ writes as:

$$\frac{\overline{q}(1)^{\frac{1}{\eta_s}}}{(1-\overline{q}(1))^{\frac{1}{\eta_{re}}}} < \left[\frac{(\tau\theta\alpha_c(1))^2}{2(1+\phi\alpha_c(1))}\right]^{\frac{\eta_S-\eta_{re}}{\eta_S\eta_{re}}} \cdot m\,(1)^{\frac{\gamma}{\eta_{re}}}$$

which will hold when $m\,(1)$ is large enough.

- *Saddle node steady state in the joint dynamics of culture and institutions:*

- Let denote the interior steady state $(\lambda_E^*, q_E^*) \in \Gamma$ such that $\lambda_E^* = \min\left\{\lambda \in (0,1) \mid \overline{q}\,(\lambda) = q^*(\lambda)\right\}$ and $q_E^* = \overline{q}\,(\lambda_E) = q^*(\lambda_E)$. (λ_E^*, q_E^*) is the "lowest" interior steady state of the system. It is clear that because of the smoothness of the function $\Lambda(\lambda) = q^*(\lambda) - \overline{q}\,(\lambda)$, one should have $\Lambda'(\lambda_E^*) > 0$ or $q^{*\prime}(\lambda_E^*) > \overline{q}'\,(\lambda_E^*)$.

Consider now the local dynamics around the interior steady state (λ_E^*, q_E^*). Inside the interior of $[0,1]^2$, the joint dynamics of institutions and culture write as

$$\begin{aligned} \lambda_{t+1} - \lambda_t &= m^{-1}\left[\tilde{m}(\lambda_t, q_t)\right] - \lambda_t \\ q_{t+1} - q_t &= q_t(1-q_t)D(q_t, \lambda_t)\}. \end{aligned}$$

At the continous time limit, this provides the differential equations system :

$$\begin{aligned} \dot{\lambda} &= m^{-1}\left[\tilde{m}(\lambda, q)\right] - \lambda \\ \dot{q} &= q(1-q)D(q, \lambda)\}. \end{aligned} \tag{A.50}$$

Note first that the threshold $q = \overline{q}\,(\lambda)$ is obtained from the relationship $\tilde{m}(\lambda, q) = m(\lambda)$, while the threshold $q^*(\lambda)$ is obtained from the relationship $D(q,\lambda) = 0$. From this we obtain that the slopes of the manifolds:

$$\overline{q}'\,(\lambda_E^*) = \frac{m'(\lambda_E^*) - \tilde{m}'_\lambda(\lambda_E^*, q_E^*)}{\tilde{m}'_q(\lambda_E^*, q_E^*)} \text{ and } q^{*\prime}(\lambda_E) = \frac{D_\lambda(q_E^*, \lambda_E^*)}{-D_q(q_E^*, \lambda_E^*)}$$

Now the jacobian matrix of the system (A.50) at the steady state (λ_E^*, q_E^*) is given by

$$J = \begin{pmatrix} \frac{\tilde{m}_\lambda(\lambda_E^*, q_E^*)}{m'(\lambda_E^*)} - 1 & \frac{\tilde{m}_q(\lambda_E^*, q_E^*)}{m'(\lambda_E^*)} \\ q_E^*(1-q_E^*)D_\lambda(q_E^*, \lambda_E^*) & q_E^*(1-q_E^*)D_q(q_E^*, \lambda_E^*) \end{pmatrix}$$

Given that $m'(\lambda_E^*) > 0$, the sign of the determinant Δ of that jacobian is the same as the sign of

$$\left[\tilde{m}_\lambda(\lambda_E^*, q_E^*) - m'(\lambda_E^*)\right] D_q(q_E^*, \lambda_E^*) - \tilde{m}_q(\lambda_E^*, q_E^*) D_\lambda(q_E^*, \lambda_E^*)$$

Now the condition $q^{*\prime}(\lambda_E^*) > \overline{q}'\,(\lambda_E^*)$ rewrites as

$$\frac{D_\lambda(q_E^*, \lambda_E^*)}{-D_q(q_E^*, \lambda_E^*)} > \frac{m'(\lambda_E^*) - \tilde{m}'_\lambda(\lambda_E^*, q_E^*)}{\tilde{m}'_q(\lambda_E^*, q_E^*)}$$

or (given that $D_q(q_E^*, \lambda_E^*) < 0$ and $\tilde{m}'_q(\lambda_E^*, q_E^*) > 0$, as $\tilde{m}(\lambda, q)$ is increasing in q)

$$\tilde{m}'_q(\lambda_E^*, q_E^*) D_\lambda(q_E^*, \lambda_E^*) > \left[\tilde{m}_\lambda(\lambda_E^*, q_E^*) - m'(\lambda_E^*)\right] D_q(q_E^*, \lambda_E^*)$$

which means that the sign of the determinant Δ of the jacobian at the steady state (λ_E^*, q_E^*) is negative and consequently (λ_E^*, q_E^*) is a saddle node of the joint dynamics of culture and institutions.

- *Many steady states and stability*

Assume that $\overline{q}(1) < q^*(1)$ and therefore $\Lambda(1) > 0$. When the set Γ includes more than one point (say N), one may order the various steady states by increasing order of their institutional values λ_i^* for $i \in [1, N]$. Moreover λ_i^* for $i \in [1, N]$ are the zeros of the smooth function $\Lambda(\lambda)$ in $[0, 1]$ with $\Lambda(0) < 0 < \Lambda(1)$. Therefore N is necessarily odd and $N = 2K + 1$. Recall that in such a case the steady state associated to $\lambda_1^* = \lambda_E^*$ is a saddle and $\Lambda'(\lambda_{2k+1}^*) > 0$ for $k \in [0, K]$ odd and $\Lambda'(\lambda_{2k}^*) < 0$ for $k \in [1, K]$. Then we have:

- *For $k \in [1, K]$, the steady states $\left(\lambda_{2k+1}^*, q_{2k+1}^*\right)$ are saddle nodes, and the steady state $(\lambda_{2k}^*, q_{2k}^*)$ are locally stable.*

Proof: The jacobian matrix at a steady state λ_i^*, q_i^* $i \in [1, N]$ is

$$J_i = \begin{pmatrix} \frac{\tilde{m}_\lambda(\lambda_i^*, q_i^*)}{m'(\lambda_i^*)} - 1 & \frac{\tilde{m}_q(\lambda_i^*, q_i^*)}{m'(\lambda_i^*)} \\ q_i^*(1-q_i^*)D_\lambda(q_i^*, \lambda_i^*) & q_i^*(1-q_i^*)D_q(q_i^*, \lambda_i^*) \end{pmatrix}$$

Given that $m'(\lambda_i^*) > 0$, the sign of the determinant Δ_i of that jacobian is the same as the sign of

$$\overline{\Delta}_i = [\tilde{m}_\lambda(\lambda_i^*, q_i^*) - m'(\lambda_i^*)]\, D_q(q_i^*, \lambda_i^*) - \tilde{m}_q(\lambda_i^*, q_i^*) D_\lambda(q_i^*, \lambda_i^*)$$

Recalling the fact that at an interior steady state λ_i^* :

$$\Lambda'(\lambda_i^*) = q^{*\prime}(\lambda_i^*) - \overline{q}'\,(\lambda_i^*) = \frac{D_\lambda(q_i^*, \lambda_i^*)}{-D_q(q_i^*, \lambda_i^*)} - \frac{m'(\lambda_i^*) - \tilde{m}'_\lambda(\lambda_i^*, q_i^*)}{\tilde{m}'_q(\lambda_i^*, q_i^*)} = \frac{-\overline{\Delta}_i}{-D_q(q_i^*, \lambda_i^*)\tilde{m}'_q(\lambda_i^*, q_i^*)}$$

and because $\tilde{m}'_q > 0$, and $D_q < 0$, it follows that the sign of $\overline{\Delta}$ (and therefore Δ)at a steady state λ_i^* is the opposite to the sign of $\Lambda'(\lambda_i^*)$.

From this we conclude that:

i) for $k \in [1, K]$ as $\Lambda'(\lambda_{2k+1}^*) = q^{*\prime}(\lambda_{2k+1}^*) - \overline{q}'\left(\lambda_{2k+1}^*\right) > 0$, the sign of Δ_{2k+1} is negative and the steady state $\left(\lambda_{2k+1}^*, q_{2k+1}^*\right)$ is a saddle node.

ii) For $k \in [1, K]$ as $\Lambda'(\lambda_{2k}^*) = q^{*\prime}(\lambda_{2k}^*) - \overline{q}'\,(\lambda_{2k}^*) < 0$, the sign of Δ_{2k} is positive. Moreover given that

$$\overline{q}'\,(\lambda_{2k}^*) = \frac{m'(\lambda_{2k}^*) - \tilde{m}'_\lambda(\lambda_{2k}^*, q_{2k}^*)}{\tilde{m}'_q(\lambda_{2k}^*, q_{2k}^*)} > q^{*\prime}(\lambda_{2k}^*) > 0$$

and thus $m'(\lambda_{2k}^*) - \tilde{m}'_\lambda(\lambda_{2k}^*, q_{2k}^*) > 0$. Morever $D_q(q_{2k}^*, \lambda_{2k}^*) < 0$ and $m'(\lambda_{2k}^*) > 0$. It follows that the trace $Tr\,(J_{2k})$ of the jacobian J_{2k} is negative as

$$Tr\,(J_{2k}) = \underbrace{\frac{\tilde{m}'_\lambda(\lambda_{2k}^*, q_{2k}^*)}{m'(\lambda_{2k}^*)} - 1}_{-} + \underbrace{q_{2k}^*(1 - q_{2k}^*)D_q(q_{2k}^*, \lambda_{2k}^*)}_{-} \quad < 0$$

From this it follows that the steady state $(\lambda_{2k}^*, q_{2k}^*)$ is locally stable for $k \in [1, K]$. **QED.**

A.7 Proof of Proposition 3

The likelihood of reaching the religious equilibrium is increasing in θ: From Proposition 1, $\overline{q}(.)$ is decreasing in θ. From Proposition 2, $q^*(.)$ is increasing in θ. Hence, the measure of parameters for which there is a complementarity between the spread of religious values and an increase in the political weight of the clerics is larger. This explains why the likelihood of reaching the religious equilibrium increases.

The likelihood of reaching the religious equilibrium is decreasing in ϕ: From Proposition 1, $\overline{q}(.)$ is increasing in ϕ. From Proposition 2, $q^*(.)$ is decreasing in ϕ. Hence, the measure of parameters for which there is a complementarity between the spread of religious values and an increase in the political weight of the clerics is lower.

A.8 Proof of Lemmas 4 and 5:

In order to prove the two Lemmas, we first derive the tax base E. Since an individual of type $i \in \{re, s\}$ complies only when

$$\frac{1-\tau_i}{1+\phi\alpha_c} > \frac{1-c\epsilon}{1+\phi\alpha_c}, \tag{A.51}$$

with $\epsilon = \frac{\epsilon_0}{1-\alpha_l}$, the fraction of individuals of type i that comply is:

$$\int_{\tau/\epsilon}^{\overline{c}} \frac{dc}{\overline{c}} = 1 - \frac{\tau_i(1-\alpha_l)}{\epsilon_0\overline{c}}. \tag{A.52}$$

Summing the taxes that are collected in the two cultural groups, we find that the tax base is:

$$E = \frac{1}{1+\phi\alpha_c}\{1 - \frac{\tau(1-q\theta\alpha_c)(1-\alpha_l)}{\epsilon_0\overline{c}}\}. \tag{A.53}$$

We are now able to solve the equilibrium. As a matter of simplification, we assume throughout the extension that $\psi(\alpha_c)$ is quadratic with $\psi(\alpha_c) = \alpha_c^2/2$.

The first-order conditions associated with the determination of $m(\lambda)$, $\alpha_l(\lambda, \beta, q)$, and $\alpha_c(\lambda)$ are respectively:

$$\begin{cases} -(1-\frac{\lambda}{2})C'(m) + \frac{\lambda}{2}\left(\alpha_c - F'(m)\right) = 0, \\ -\alpha_l + (1-\beta)\tau\frac{\partial E}{\partial \alpha_l} \leq 0, \text{ and} \\ m - \alpha_c = 0. \end{cases} \tag{A.54}$$

The equilibrium is unique, when the marginal cost functions $F'(.)$ and $C'(.)$ are strictly increasing convex functions and $\lim_{m\to\infty} F''(m) > 1 > F''(0) + C''(0)$. Typically $m(\lambda) = \alpha_c(\lambda) = 0$ when $\lambda \leq 2\frac{C''(0)}{C''(0)+1-F''(0)}$, and $m(\lambda) = \alpha_c(\lambda) > 0$ is the positive solution of

$$(1-\frac{\lambda}{2})C'(m) + \frac{\lambda}{2}F'(m) = \frac{\lambda}{2}m, \tag{A.55}$$

when $\lambda > 2\frac{C''(0)}{C''(0)+1-F''(0)}$. From this, we deduce that $m(\lambda)$ and $\alpha_c(\lambda)$ are increasing in λ, when $F'(m) < C'(m)$ and is independent from β, θ, and ϕ. This concludes the proof of Lemma 4.

Substituting (A.53) in the the second FOC above, we find

$$\alpha_l(\lambda,\beta,q) = \begin{cases} (1-\beta)\frac{\tau^2(1-q\theta\alpha_c(\lambda))}{(1+\phi\alpha_c(\lambda))\overline{c}\epsilon_0} \text{ if } (1-\beta)\frac{\tau^2(1-q\theta\alpha_c(\lambda))}{(1+\phi\alpha_c(\lambda))\overline{c}\epsilon_0} < \overline{\alpha}_l \text{ and} \\ \overline{\alpha}_l \text{ otherwise.} \end{cases} \tag{A.56}$$

We deduce that $\alpha_l(\lambda,\beta,q)$ is decreasing in β, λ q, θ and ϕ. This concludes the proof of Lemma 5.

A.9 Proof of Proposition 4

As in the related proof of Proposition 1, we first demonstrate that the optimization, problem (15) – rewritten below – admits a unique solution $(\lambda_{t+1},\beta_{t+1}) \in [0,1]^2$:

$$\begin{aligned}\max_{(\lambda_{t+1},\beta_{t+1})}(1-\frac{\lambda_t}{2})\{U_r(m(\lambda_{t+1}),\alpha_l(\lambda_{t+1},\beta_{t+1},q_t)) + U_l(m(\lambda_{t+1}),\alpha_l(\lambda_{t+1},\beta_{t+1},q_t))\} \\ + \frac{\lambda_t}{2}U_c(m(\lambda_{t+1}),\alpha_c(\lambda_{t+1})),\end{aligned}$$

In order to solve this maximization problem, we solve the following related optimization problem:

$$\max_{m,\alpha_l} W(m,\alpha_l,\lambda_t) = (1-\frac{\lambda_t}{2})\{U_r(m,\alpha_l) + U_l(m,\alpha_l)\} + \frac{\lambda_t}{2}U_c(m). \tag{A.57}$$

The solution, denoted $(\tilde{m}(\lambda_t,q_t),\tilde{\alpha}_l(\lambda_t,q_t))$,[38] maximizes the social welfare when the externalities are internalized, with

$$\begin{aligned} U_c(m) &= m\alpha_c(m) - \psi(\alpha_c(m)) - F(m) = \frac{1}{2}m^2 - F(m) \\ U_r(m,\alpha_l) &= \beta_t(\tau E(m,\alpha_l,q_t) - C(m)) - \rho\alpha_l \\ U_l(m,\alpha_l) &= (1-\beta_t)(\tau E(m,\alpha_l,q_t) - C(m)) - \frac{\alpha_l^2}{2} \end{aligned}$$

[38] making now explicit the dependence on the state variables (λ_t,q_t).

and

$$E(m, \alpha_l, q_t) = \frac{1}{1+\phi m}\{1 - \frac{\tau(1-q_t\theta m)(1-\alpha_l)}{\epsilon_0 \bar{c}}\}. \quad \text{(A.58)}$$

The previous optimization problem can be rewritten:

$$\max_{m,\alpha_l} W(m, \alpha_l, \lambda_t) = (1-\frac{\lambda_t}{2})\{\tau E(m, \alpha_l, q_t) - C(m) - \rho\alpha_l - \frac{\alpha_l^2}{2}\} + \frac{\lambda_t}{2}\{\frac{1}{2}m^2 - F(m)\}, \quad \text{(A.59)}$$

In this optimization problem, the choices of both the religious provision m and of the effort of the secular elite α_l are made by a ruler who can commit, and hence that internalizes the externalities detailed in the main text. We find that the solution $(\tilde{m}(\lambda_t, q_t), \tilde{\alpha}_l(\lambda_t, q_t))$ of (A.57) solves the following equations:

$$\begin{cases} \frac{\partial W}{\partial m} = \frac{\lambda_t}{2}(m - F'(m)) - (1-\frac{\lambda_t}{2})C'(m) + (1-\frac{\lambda_t}{2})\{\frac{-\phi}{(1+m\phi)^2}[1 - \frac{\tau(1-q\theta m)(1-\alpha_l)}{\epsilon_0\bar{c}}] \\ \qquad + \frac{1}{1+m\phi}\frac{\tau q\theta(1-\alpha_l)}{\epsilon_0\bar{c}}\} = 0, \\ \text{and} \\ \frac{\partial W}{\partial \alpha_l} = -\alpha_l - \rho + \frac{\tau^2(1-q_t\theta m)}{\bar{c}\epsilon_0(1+\phi m)} = 0. \end{cases} \quad \text{(A.60)}$$

We deduce the following lemma which characterizes the solution $(\tilde{m}(\lambda_t, q_t), \tilde{\alpha}_l(\lambda_t, q_t))$ of (A.57)

Lemma 8 *the solution $(\tilde{\alpha}_l(\lambda_t, q_t), \tilde{m}(\lambda_t, q_t))$ is uniquely determined when $C(.)$, and $F(m)$ are sufficiently convex (ie $W(m, \alpha_l, \lambda_t)$ is concave in m, α_l).*

Proof: Specifically, it is a simple matter to see that

$$\begin{aligned} \frac{\partial^2 W}{\partial m^2} &= \frac{\lambda_t}{2}(1 - F''(m)) - (1-\frac{\lambda_t}{2})C''(m) \\ &\quad + (1-\frac{\lambda_t}{2})\frac{2\phi}{(1+m\phi)^2}\begin{bmatrix} -[\frac{\tau q\theta(1-\alpha_l)}{\epsilon_0\bar{c}}] \\ +\frac{\phi}{(1+m\phi)}[1 - \frac{\tau(1-q\theta m)(1-\alpha_l)}{\epsilon_0\bar{c}}] \end{bmatrix} \\ &< \frac{\lambda_t}{2}(1 - F''(m)) - (1-\frac{\lambda_t}{2})C''(m) + (1-\frac{\lambda_t}{2})\frac{2\phi^2}{(1+m\phi)^3} < 0 \end{aligned}$$

when $F''(m) > 1$ and $C''(m) > 2\phi^2$, while:

$$\frac{\partial^2 W}{\partial \alpha_l^2} = -1 < 0 \text{ and } \frac{\partial^2 W}{\partial m \partial \alpha_l} = -\frac{\tau^2}{\bar{c}\epsilon_0}\frac{q_t\theta + \phi}{(\phi m + 1)^2} < 0 \quad \text{(A.61)}$$

Therefore the Hessian of $W(m, \alpha_l, \lambda_t)$ is given by:

$$\begin{aligned}
\Delta &= \frac{\partial^2 W}{\partial m^2} \cdot \frac{\partial^2 W}{\partial \alpha_l^2} - \left(\frac{\partial^2 W}{\partial m \partial \alpha_l}\right)^2 \\
&= \left[\frac{\lambda_t}{2}(F''(m) - 1) + (1 - \frac{\lambda_t}{2}) \left[\begin{array}{c} C''(m) + \frac{2\phi}{(1+m\phi)^2}\{[\frac{\tau q\theta(1-\alpha_l)}{\epsilon_0 \bar{c}}] \\ -\frac{\phi}{(1+m\phi)}[1 - \frac{\tau(1-q\theta m)(1-\alpha_l)}{\epsilon_0 \bar{c}}]\} \end{array} \right]\right] \\
&\quad - \frac{\tau^4}{(\bar{c}\epsilon_0)^2} \frac{(q_t\theta + \phi)^2}{(\phi m + 1)^4} \\
&> \left[\frac{\lambda_t}{2}(F''(m) - 1) + (1 - \frac{\lambda_t}{2}) \left[C''(m) - \frac{2\phi^2}{(1+m\phi)^3}\right]\right] \\
&\quad - \frac{\tau^4}{(\bar{c}\epsilon_0)^2} \frac{(q_t\theta + \phi)^2}{(\phi m + 1)^4}
\end{aligned}$$

and $\Delta > 0$ when $F''(m) > 1 + \frac{(\theta+\phi)^2}{(\bar{c}\epsilon_0)^2}$ and $C''(m) > 2\phi^2 + \frac{(\theta+\phi)^2}{(\bar{c}\epsilon_0)^2}$. Therefore $W(m, \alpha_l, \lambda_t)$ is concave in m, α_l when $C(.)$, and $F(m)$ are sufficiently convex. (ie. when $F''(m) > 1 + \frac{(\theta+\phi)^2}{(\bar{c}\epsilon_0)^2}$ and $C''(m) > 2\phi^2 + \frac{(\theta+\phi)^2}{(\bar{c}\epsilon_0)^2}$) **QED**.

Now consider $(\widetilde{m}^0(q_t), \widetilde{\alpha}_l^0(q_t)) = \arg\max_{m,\alpha_l} W(m, \alpha_l, 0)$ and $\widetilde{m}^1 = \arg\max_{m,\alpha_l} W(m, \alpha_l, 1)$. $\widetilde{m}^0$ respectively the optimal level of religious infrastructure of (A.57)when the secular elite (and the ruler) have full political power (ie. $\lambda = 0$), and when the society is in a religious state (the religious clerics weight is $\lambda = 1$). It is reasonable to make the following assumption:[39]

Assumption **M**: $\widetilde{m}^0(q_t) < \widetilde{m}^1$ for all $q_t \in [0, 1]$

namely that the clerics group always wish to have a higher level of religious infrastructures than the secular fraction of society (ruler and secular elite). We have then the following result:

Lemma 9 *Under assumption M, $\tilde{m}(\lambda_t, q_t)$ is increasing in λ_t and q_t.and $\widetilde{\alpha}_l(\lambda_t, q_t)$ is decreasing in λ_t and q_t.*

[39] A sufficient condition for assumption M to be satisfied is :

$$\frac{\tau\theta}{\epsilon_0 \bar{c}} < C'(\widetilde{m}^1)$$

where $\widetilde{m}^1$ is determined by the condition $\widetilde{m}^1 = \Phi'(\widetilde{m}^1)$.

Proof: Partial differentiation yields:

$$\frac{\partial W}{\partial m\partial\lambda} = \frac{m-F'(m)}{2}+\frac{C'(m)}{2} \tag{A.62}$$

$$-\frac{1}{2}\left[\begin{array}{c}\frac{-\phi}{(1+m\phi)^2}[1-\frac{\tau(1-q\theta m)(1-\alpha_l)}{\epsilon_0\overline{c}}] \\ +\frac{1}{1+m\phi}\frac{\tau q\theta(1-\alpha_l)}{\epsilon_0\overline{c}}\end{array}\right] \tag{A.63}$$

$$\frac{\partial W}{\partial m\partial q} = (1-\frac{\lambda_t}{2})\frac{1}{1+m\phi}\frac{\tau\theta(1-\alpha_l)}{\epsilon_0\overline{c}}\{\frac{1}{(1+m\phi)}\}>0$$

and

$$\frac{\partial^2 W}{\partial\alpha_l\partial\lambda}=0 \text{ and } \frac{\partial^2 W}{\partial\alpha_l\partial q}=-\frac{\tau^2\theta m}{\overline{c}\epsilon_0(1+\phi m)}<0 \tag{A.64}$$

Substitution of the FOC (A.60) into (A.62),one obtains when evaluated at the optimal point $\tilde{m},\tilde{\alpha}_l$:

$$\left(\frac{\partial W}{\partial m\partial\lambda}\right)=\frac{1}{(1-\frac{\lambda_t}{2})}(\tilde{m}-F'(\tilde{m})) \tag{A.65}$$

which is positive as long as $\tilde{m}\left(\lambda_t,q_t\right)\leq\widetilde{m}^1$. Moreover differentiation of the FOC in (A.60), provides

$$\begin{aligned}\left(\begin{array}{c}d\tilde{m}\\ d\tilde{\alpha}_l\end{array}\right) &= \frac{1}{\Delta}\left(\begin{array}{cc}\frac{\partial^2 W}{\partial\alpha_l^2} & -\frac{\partial^2 W}{\partial m\partial\alpha_l}\\ -\frac{\partial^2 W}{\partial m\partial\alpha_l} & \frac{\partial^2 W}{\partial m^2}\end{array}\right)\left(\begin{array}{c}-\frac{\partial^2 W}{\partial m\partial\lambda}d\lambda_t-\frac{\partial^2 W}{\partial m\partial q}dq_t\\ -\frac{\partial^2 W}{\partial\alpha_l\partial\lambda}d\lambda_t-\frac{\partial^2 W}{\partial\alpha_l\partial q}dq_t\end{array}\right)\\ &= \frac{1}{\Delta}\left(\begin{array}{c}-\frac{\partial^2 W}{\partial\alpha_l^2}\frac{\partial^2 W}{\partial m\partial\lambda}d\lambda_t+\left(-\frac{\partial^2 W}{\partial\alpha_l^2}\frac{\partial^2 W}{\partial m\partial q}+\frac{\partial^2 W}{\partial m\partial\alpha_l}\frac{\partial^2 W}{\partial\alpha_l\partial q}\right)dq_t\\ \frac{\partial^2 W}{\partial m\partial\alpha_l}\frac{\partial^2 W}{\partial m\partial\lambda}d\lambda_t+\left(-\frac{\partial^2 W}{\partial m^2}\frac{\partial^2 W}{\partial\alpha_l\partial q}+\frac{\partial^2 W}{\partial m\partial\alpha_l}\frac{\partial^2 W}{\partial m\partial q}\right)dq_t\end{array}\right)\end{aligned}$$

with all derivatives evaluated at $\tilde{m},\tilde{\alpha}_l$. Hence using (A.61), (A.62) and (A.64), one gets

$$\frac{\partial\tilde{m}}{\partial\lambda_t}=\frac{1}{\Delta}\cdot\frac{\partial^2 W}{\partial m\partial\lambda}$$

the sign of which is the same as the sign of $\frac{\partial^2 W}{\partial m\partial\lambda}$. Now under assumption M, one can see from (A.65) that $\tilde{m}\left(\lambda_t,q_t\right)$ is increasing in λ_t as long as $\tilde{m}\left(\lambda_t,q_t\right)<\widetilde{m}^1$. Note first that $\tilde{m}\left(1,q_t\right)=\widetilde{m}^1$. Suppose then that there exists a value $\lambda<1$ such that $\tilde{m}\left(\lambda,q_t\right)=\widetilde{m}^1$. From (A.60), and noting that

$$W\left(m,\alpha_l,\lambda\right)=\lambda W\left(m,\alpha_l,1\right)+(1-\lambda)W\left(m,\alpha_l,0\right)$$

at this point $\tilde{m}(\lambda, q_t)$, $\widetilde{\alpha}_l(\lambda, q_t)$, one should have

$$\left(\frac{\partial W}{\partial m}\right)_{\tilde{m}^1, \tilde{\alpha}_l^1} = \lambda \frac{\partial W(m, \alpha_l, 1)}{\partial m} + (1-\lambda) \frac{\partial W(m, \alpha_l, 0)}{\partial m} = 0$$

But $\tilde{m}(\lambda, q_t) = \widetilde{m}^1 = \arg\max_{m,\alpha_l} W(m, \alpha_l, 1)$, implies that $\left(\frac{\partial W(m,\alpha_l,1)}{\partial m}\right) = 0$ at such point. Hence to satisfy the previous equation, we should also have $\frac{\partial W(m,\alpha_l,0)}{\partial m} = 0$, which in turn implies that $\tilde{m}(\lambda, q_t) = \widetilde{m}^0(q_t)$, a contradiction with assumption M. From this we conclude that $\tilde{m}(\lambda, q_t) < \widetilde{m}^1$ for all $\lambda < 1$ or $\tilde{m}(\lambda, q_t) > \widetilde{m}^1$ for all $\lambda < 1$. The only case consistent with assumption M is obviously that $\tilde{m}(\lambda, q_t) < \widetilde{m}^1$ for all $\lambda < 1$. From this we conclude that under assumption M, $\frac{\partial^2 W}{\partial m \partial \lambda}$ evaluated at $\tilde{m}(\lambda, q_t)$, $\widetilde{\alpha}_l(\lambda, q_t)$ is positive and therefore $\frac{\partial \tilde{m}}{\partial \lambda_t} > 0$ (ie. religious infrastructures $\tilde{m}(\lambda_t, q_t)$ is increasing in the clerics' political weight λ_t).

Similarly, using (A.61), (A.62) and (A.64), we have:

$$\frac{\partial \tilde{\alpha}_l}{\partial \lambda_t} = \frac{1}{\Delta} \cdot \underbrace{\frac{\partial^2 W}{\partial m \partial \alpha_l}}_{-} \frac{\partial^2 W}{\partial m \partial \lambda}$$

Hence $\frac{\partial \tilde{\alpha}_l}{\partial \lambda_t} < 0$ under assumption M (ie. the tax enforcement effort of the secular elite $\widetilde{\alpha}_l(\lambda_t, q_t)$ is decreasing in the clerics' weight λ_t).

Finally, substituting (A.61), (A.62) and (A.64),we obtain

$$\frac{\partial \tilde{m}}{\partial q_t} = \frac{1}{\Delta} \left(\underbrace{\frac{\partial^2 W}{\partial m \partial q}}_{+} + \underbrace{\frac{\partial^2 W}{\partial m \partial \alpha_l}}_{-} \cdot \underbrace{\frac{\partial^2 W}{\partial \alpha_l \partial q}}_{-} \right) > 0$$

$$\frac{\partial \tilde{\alpha}_l}{\partial q_t} = \frac{1}{\Delta} \left(-\underbrace{\frac{\partial^2 W}{\partial m^2}}_{+} \cdot \underbrace{\frac{\partial^2 W}{\partial \alpha_l \partial q}}_{-} + \underbrace{\frac{\partial^2 W}{\partial m \partial \alpha_l}}_{-} \cdot \underbrace{\frac{\partial^2 W}{\partial m \partial q}}_{+} \right) < 0$$

QED.

In order to simplify the problem, we make the following assumption on the higher bound $\overline{\alpha}_l$:

$$\textbf{Assumption A: } \overline{\alpha}_l < \frac{\tau^2}{1+\phi m(1)} \frac{1-\theta m(1)}{\epsilon_0 \overline{c}}$$

Before going further with the proof, we establish this intermediary result:

Lemma 10 *Under Assumption A, $\alpha_l(\lambda, \beta = 0) = \overline{\alpha}_l$ for any $(\lambda, q) \in [0,1]^2$.*

Proof: In order to prove Lemma 10, we need to write the first-order derivative of the utility of the secular elites with respect to α_l is:

$$\frac{\partial U_l}{\partial \alpha_l} = -\alpha_l + (1-\beta)\frac{\tau^2(1-q\theta m)}{(1+\phi m)\epsilon_0 \overline{c}}. \tag{A.66}$$

Hence, when $\beta = 0$, under Assumption A, $\frac{\partial U_l}{\partial \alpha_l} > 0$ for any $\alpha_l \in [0, \overline{\alpha}_l]$ and for any $(\lambda, q) \in [0,1]^2$, so $\alpha_l(\lambda, \beta = 0, q) = \overline{\alpha}_l$ for any $(\lambda, q) \in [0,1]^2$. This concludes the proof of the Lemma. **QED.**

Since $(\tilde{\alpha}_l(\lambda_t, q_t), \tilde{m}(\lambda_t, q_t))$ maximizes the social welfare when the externalities are internalized, $(\lambda_{t+1}, \beta_{t+1})$ solves the optimization problem (15) when:

$$\begin{cases} \tilde{m}(\lambda_t, q_t) = m(\lambda_{t+1}), \text{ and} \\ \tilde{\alpha}_l(\lambda_t, q_t) = \alpha_l(\lambda_{t+1}, \beta_{t+1}, q_t) \end{cases} \tag{A.67}$$

Indeed, when the clerics and the ruler have power λ_{t+1} and β_{t+1}, institutions are designed for $t+1$ so as to induce a choice $m(\lambda_{t+1})$ and $\alpha_l(\lambda_{t+1}, \beta_{t+1}, q_t)$ in that period that maximizes the social welfare under the institutional framework of period t. It remains to be proven that the solution $(\lambda_{t+1}, \beta_{t+1})$ of the system (A.67) is unique. Consider the following system with two unknown variables x and y:

$$\begin{cases} \tilde{m}(\lambda_t, q_t) = m(x), \text{ and} \\ \tilde{\alpha}_l(\lambda_t, q_t) = \alpha_l(x, y, q_t), \end{cases} \tag{A.68}$$

Consider first the case where an interior solution exists. Since the function $m(.)$ is increasing in its argument, from Lemma 4, there exists a unique value $x(\lambda_t, q_t) \in [0,1]$ such that $\tilde{m}(\lambda_t, q_t) = m(x)$. Substituting $x(\lambda_t, q_t)$ in the second equation, we find:

$$\tilde{\alpha}_l(\lambda_t, q_t) = \alpha_l(x(\lambda_t, q_t), y, q_t), \tag{A.69}$$

By definition, $\tilde{\alpha}_l(\lambda_t, q_t) \in [0, \overline{\alpha}_l]$. Furthermore, as $\alpha_l(x(\lambda_t, q_t), y, q_t)$ is decreasing in y from Lemma 5, under Assumption A, $\alpha_l(x(\lambda_t, q_t), 1, q_t) = 0 \leq \alpha_l(x(\lambda_t, q_t), y, q_t) \leq \alpha_l(x(\lambda_t, q_t), 0, q_t) = \overline{\alpha}_l$. Hence, applying the theorem of intermediate values, there exists a single vector $(x(\lambda_t, q_t), y(\lambda_t, q_t)) \in [0,1]^2$ such that (A.67) holds. We have demonstrated that the system (A.67) admits a unique interior solution, when this solution exists.

An interior solution does not always exists, as it can be that $\tilde{m}(\lambda_t, q_t) > m(\lambda_{t+1})$ or $\tilde{m}(\lambda_t, q_t) < m(\lambda_{t+1})$ for any $\lambda_{t+1} \in [0,1]$. In these two cases, there is a single solution $(\lambda_{t+1}, \beta_{t+1})$ to the optimization problem (15), which is the unique vector such that $(m(\lambda_{t+1})$, $\alpha_l(\lambda_{t+1}, \beta_{t+1}, q_t))$ maximizes (A.57). Indeed, when $\tilde{m}(\lambda_t, q_t) > m(\lambda_{t+1})$, then $\lambda_{t+1} = 1$, and β_{t+1} solves

$$\tilde{\alpha}_l(\lambda_t, q_t) = \alpha_l(1, \beta_{t+1}, q_t) \tag{A.70}$$

for $\beta_{t+1} \in [0,1]$. As $\alpha_l(1, \beta_{t+1}, q_t)$ is decreasing in β_{t+1} from Lemma 5, under Assumption **??**, $\alpha_l(1, 1, q_t) = 0 \leq \alpha_l(1, \beta_{t+1}, q_t) \leq \alpha_l(1, 0, q_t) = \overline{\alpha}_l$. Applying the theorem of intermediate values, there exists a single $\beta_{t+1} \in [0,1]$ such that $\tilde{\alpha}_l(\lambda_t, q_t) = \alpha_l(1, \beta_{t+1}, q_t)$

The reasoning is similar when $\tilde{m}(\lambda_t, q_t) < m(\lambda_{t+1})$ for any $\lambda_{t+1} \in [0,1]$: $\lambda_{t+1} = 0$ and there is a unique solution $\beta_{t+1} \in [0,1]$ to the equation $\tilde{\alpha}_l(\lambda_t, q_t) = \alpha_l(0, \beta_{t+1}, q_t)$. From this we conclude that the optimization problem (15) admits a unique solution $(\lambda_{t+1}, \beta_{t+1})$.

We are now going to demonstrate that there exists a threshold $\overline{q}_d(\lambda_t)$ such that if $q_t > \overline{q}_d(\lambda_t)$, then $\lambda_{t+1} > \lambda_t$. Otherwise, $\lambda_{t+1} \leq \lambda_t$. In order to demonstrate this claim, we will show the following intermediary result:

Lemma 11 *$\lambda_{t+1} > \lambda_t$ if and only if $\tilde{m}(\lambda_t, q_t) > m(\lambda_t)$.*

Proof: Indeed, $\tilde{m}(\lambda_t, q_t) > m(\lambda_t)$ means that in (A.57), the ruler would want to commit to a provision level $\tilde{m}(\lambda_t, q_t)$ strictly above what is provided in equilibrium. Since $m(.)$ is increasing in λ (Lemma 4), we deduce that when the political weight λ_{t+1}, that decentralizes $\tilde{m}(\lambda_t, q_t)$ is such that $\tilde{m}(\lambda_t, q_t) = m(\lambda_{t+1})$, one has that $\lambda_{t+1} > \lambda_t$. A similar reasoning can be applied for the corners when $\lambda_{t+1} = 1$ when $\tilde{m}(\lambda_t, q_t) > m(1)$ or $\lambda_{t+1} = 0$ when $\tilde{m}(\lambda_t, q_t) < m(0)$. **QED.**

Lemma 12 *$\tilde{m}(\lambda_t, q_t) > m(\lambda_t)$ if and only if $q > \overline{q}_d(\lambda_t)$, with $\overline{q_d}(\lambda_t)$ is defined as the threshold the value of $q \in [0,1]$ such that*

$$q = \max\left[\min\left[\frac{\phi}{\theta}\{\frac{1}{\tau(1 - \tilde{\alpha}_l(\lambda_t, q))} - \frac{1}{\epsilon_0 \overline{c}}\}, 1\right], 0\right]. \tag{A.71}$$

Proof: Given that $\tilde{m}(\lambda_t, q_t)$ is increasing in q_t, the condition $\tilde{m}(\lambda_t, q_t) > m(\lambda_t)$ is equivalent to $q_t > \overline{q}_d(\lambda_t) \in [0,1]$ with $\overline{q}_d(\lambda_t)$ defined such

$$\begin{aligned}
\tilde{m}(\lambda_t, \overline{q}_d(\lambda_t)) &= m(\lambda_t) \text{ when } \tilde{m}(\lambda_t, 0) \leq m(\lambda_t) \leq \tilde{m}(\lambda_t, 1) \\
\overline{q}_d(\lambda_t) &= 0 \quad \text{when } \tilde{m}(\lambda_t, 0) > m(\lambda_t) \\
\overline{q}_d(\lambda_t) &= 1 \quad \text{when } \tilde{m}(\lambda_t, 1) < m(\lambda_t)
\end{aligned}$$

More specifically, the first-order condition associated with the determination of $m(\lambda)$ is:

$$\frac{\lambda}{2}(m - F'(m)) - (1 - \frac{\lambda}{2})C'(m) = 0. \tag{A.72}$$

The first-order condition for the determination of $\tilde{m}(\lambda, q)$ writes as

$$\frac{\lambda}{2}(m - F'(m)) - (1 - \frac{\lambda}{2})C'(m) + (1 - \frac{\lambda}{2})\left[\begin{array}{c} \frac{-\phi}{(1+m\phi)^2}[1 - \frac{\tau(1-q\theta m)(1-\tilde{\alpha}_l(\lambda,q))}{\epsilon_0 \overline{c}}] \\ +\frac{1}{1+m\phi}\frac{\tau q\theta(1-\tilde{\alpha}_l(\lambda,q))}{\epsilon_0 \overline{c}} \end{array}\right] = 0. \tag{A.73}$$

Given the two FOCs above, we deduce that $\tilde{m}(\lambda_t, q_t) > m(\lambda_t)$ if and only if:

$$(1 - \frac{\lambda_t}{2})\{\frac{-\phi}{(1+m\phi)^2}[1 - \frac{\tau(1 - q_t\theta m)(1 - \tilde{\alpha}_l(\lambda_t, q_t))}{\epsilon_0 \overline{c}}] + \frac{1}{1+m\phi}\frac{\tau q_t \theta(1 - \tilde{\alpha}_l(\lambda_t, q_t))}{\epsilon_0 \overline{c}}\} > 0, \tag{A.74}$$

or

$$\begin{aligned}
\phi[1 - \frac{\tau(1 - q_t\theta m)(1 - \tilde{\alpha}_l(\lambda_t, q_t))}{\epsilon_0 \overline{c}}] &< (1 + m\phi)\frac{\tau q_t \theta(1 - \tilde{\alpha}_l(\lambda_t, q_t))}{\epsilon_0 \overline{c}} \\
\phi\left[1 - \frac{\tau(1 - \tilde{\alpha}_l(\lambda_t, q_t))}{\epsilon_0 \overline{c}}\right] &< \frac{\tau q_t \theta(1 - \tilde{\alpha}_l(\lambda_t, q_t))}{\epsilon_0 \overline{c}} \\
\frac{\phi}{\theta}\left[\frac{\epsilon_0 \overline{c}}{\tau(1 - \tilde{\alpha}_l(\lambda_t, q_t))} - 1\right] &< q_t
\end{aligned}$$

which rewrites

$$q_t > \frac{\phi}{\theta}\{\frac{\epsilon_0 \overline{c}}{\tau(1 - \tilde{\alpha}_l(\lambda_t, q_t))} - 1\}. \tag{A.75}$$

Denote $\Sigma(\lambda, q)$ the function

$$\Sigma(\lambda, q) = q - \frac{\phi}{\theta}\{\frac{\epsilon_0 \overline{c}}{\tau(1 - \tilde{\alpha}_l(\lambda, q))} - 1\}$$

Given that $\tilde{\alpha}_l(\lambda, q)$ is a decreasing function of q, $\Sigma(\lambda, q)$ is an increasing function of q. Now condition (A.75)is equivalent to $q_t > \overline{q_d}(\lambda_t)$ with

$$\begin{aligned}
\overline{q_d}(\lambda_t) &= 0 \text{ when } \Sigma(\lambda_t, 0) = -\frac{\phi}{\theta}\{\frac{\epsilon_0 \overline{c}}{\tau(1-\tilde{\alpha}_l(\lambda_t, 0))} - 1\} > 0 \\
\overline{q_d}(\lambda_t) &= 1 \text{ when } \Sigma(\lambda_t, 1) = 1 - \frac{\phi}{\theta}\{\frac{\epsilon_0 \overline{c}}{\tau(1-\tilde{\alpha}_l(\lambda_t, 1)} - 1\} < 0 \\
\overline{q_d}(\lambda_t) &= q \in (0,1) \text{ such that } \Sigma(\lambda_t, q) = 0 \text{ otherwise}
\end{aligned}$$

Compactly, $\overline{q_d}(\lambda_t)$ is defined as the threshold the value of $q \in [0,1]$ such that

$$q = \max\left[\min\left[\frac{\phi}{\theta}\{\frac{\epsilon_0 \overline{c}}{\tau(1-\tilde{\alpha}_l(\lambda_t, q))} - 1\}, 1\right], 0\right]. \tag{A.76}$$

and $\tilde{m}(\lambda_t) > m(\lambda_t)$ if and only if $q > \overline{q}_d(\lambda_t)$. We deduce that $\overline{q}_d(\lambda_t)$ is increasing in ϕ and decreasing in θ and λ_t. Combining the results established in Lemma 11 and Lemma 12, we get that $\lambda_{t+1} > \lambda_t$ if and only if $q > \overline{q}_d(\lambda_t)$. **QED.**

Finally, we demonstrate that there exists a threshold $\tilde{q}_d(\lambda_t, \beta_t)$ such that if $q_t > \tilde{q}_d(\lambda_t, \beta_t)$, then $\beta_{t+1} > \beta_t$. Otherwise, $\beta_{t+1} \leq \beta_t$. In order to demonstrate this claim, we proceed in two steps. First, we show the following result:

Lemma 13 *$\beta_{t+1} > \beta_t$ if and only if $\tilde{\alpha}_l(\lambda_t, \beta_t) < \alpha_l(\lambda_{t+1}, \beta_t)$, with*

$$\lambda_{t+1} = \begin{cases} \lambda \ s.t \ m(\lambda) = \tilde{m}(\lambda_t) & \text{if } \tilde{m}(\lambda_t) \in (m(0), m(1)) \\ 1 & \text{if } \tilde{m}(\lambda_t) > m(1) \\ 0 & \text{if } \tilde{m}(\lambda_t) < m(0). \end{cases} \tag{A.77}$$

Proof: Indeed, $\tilde{\alpha}_l(\lambda_t, q_t) < \alpha_l(\lambda_{t+1}, \beta_t, q_t)$ means that – given that the clerics have an optimal weight λ_{t+1} – if the ruler could, he would wish the secular elite to provide a lower enforcement effort. Since $\alpha_l(\lambda_{t+1}, q_t, .)$ is a decreasing function of β_t, the ruler increases his own political weight β_t, so that the secular elite provides less effort: $\beta_{t+1} > \beta_t$. **QED.**

Lemma 14 *There exists a threshold $\tilde{q}_d(\lambda_t, \beta_t) \in [0,1]$ such that $\tilde{\alpha}_l(\lambda_t, q_t) < \alpha_l(\lambda_{t+1}, \beta_t, q_t)$ if and only if $q > \tilde{q}_d(\lambda_t, \beta_t)$, with $\tilde{q}_d(\lambda_t, 1) = 1$ and λ_{t+1} given in (A.77).*

Proof: The first-order condition associated with the determination of $\tilde{\alpha}_l(\lambda_t, q_t)$ is:

$$-\tilde{\alpha}_l(\lambda_t, q_t) - \rho + \frac{\tau^2(1 - q_t\theta\alpha_c(\ \tilde{m}(\lambda_t, q_t)))}{\bar{c}\epsilon_0(1 + \phi\alpha_c(\ \tilde{m}(\lambda_t, q_t))} = 0 \quad \text{(A.78)}$$

Given that $\tilde{m}(\lambda_t, q_t) = m(\lambda_{t+1})$, this rewrites as

$$-\tilde{\alpha}_l(\lambda_t, q_t) - \rho + \frac{\tau^2(1 - q_t\theta\alpha_c(m(\lambda_{t+1}))}{\bar{c}\epsilon_0(1 + \phi\alpha_c(m(\lambda_{t+1}))} = 0 \quad \text{(A.79)}$$

The first-order condition associated with the determination of $\alpha_l(\lambda_{t+1}, \beta_t, q_t)$ is:

$$-\alpha_l(\lambda_{t+1}, \beta_t, q_t) + (1 - \beta_t)\frac{\tau^2(1 - q_t\theta\alpha_c(m(\lambda_{t+1}))}{\bar{c}\epsilon_0(1 + \phi\alpha_c(m(\lambda_{t+1}))} = 0 \quad \text{(A.80)}$$

Hence, the inequality $\tilde{\alpha}_l(\lambda_t, q_t) < \alpha_l(\lambda_{t+1}, \beta_t, q_t)$ is verified when

$$\rho > \beta_t\frac{\tau^2(1 - q_t\theta\alpha_c(\tilde{m}(\lambda_t, q_t))}{\bar{c}\epsilon_0(1 + \phi\alpha_c(\tilde{m}(\lambda_t, q_t))}, \quad \text{(A.81)}$$

Now the RHS of (A.81) is decreasing in q_t as $\tilde{m}(\lambda_t, q_t)$ is an increasing function of q_t so there exists a unique threshold $\tilde{q}_d(\lambda_t, \beta_t)$ such that if $q > \tilde{q}_d(\lambda_t, \beta_t)$, then (A.81) is satisfied. Otherwise, it is not satisfied. Moreover given that the RHS of (A.81) is decreasing in λ_t (as $\tilde{m}(\lambda_t, q_t)$ and $\alpha_c(\tilde{m}(\lambda_t, q_t)$ are increasing in λ_t), and increasing in β_t, it follows that the threshold $\tilde{q}_d(\lambda_t, \beta_t)$ is decreasing in λ_t and increasing in β_t. **QED.**

Combining the results established in Lemmas 12 and 14, we have demonstrated that $\beta_{t+1} > \beta_t$ if and only if $q > \tilde{q}_d(\lambda_t, \beta_t)$.

Summarizing, we have demonstrated the followings:

- The optimization problem (15) admits a unique solution $(\lambda_{t+1}, \beta_{t+1}) \in [0, 1]^2$.
- there exists a threshold $\tilde{q}_d(\lambda_t, \beta_t)$ such that if $q > \tilde{q}_d(\lambda_t, \beta_t)$ then $\beta_{t+1} > \beta_t$. Otherwise, $\beta_{t+1} \leq \beta_t$.
- There exists a threshold $\overline{q}_d(\lambda_t)$ such that if $q_t > \overline{q}_d(\lambda_t)$, then $\lambda_{t+1} > \lambda_t$. Otherwise, $\lambda_{t+1} \leq \lambda_t$.
- $\tilde{q}_d(\lambda_t, \beta_t)$ is decreasing in λ_t and increasing in β_t. and $\overline{q}_d(\lambda_t)$ is decreasing in λ_t.

Finally, $\tilde{q}_d(\lambda_t, 1) = 1$ because in equilibrium, the secular elite provides no effort, $\alpha_l(\lambda_t, 0, q_t) = 0$ and have zero utility. Hence, an epsilon increase in their political weight $1-\beta_t$ will increase the social welfare by increasing both the utility of the ruler, and of the secular elite. This concludes the proof of Proposition 4. **QED.**

A.10 Proof of Proposition 5

As in the proof of Proposition 2, we first deduce from the maximization program (9) that $d^*_{Re} = D_{Re}((1-q_t)\Delta V_{Re}, m)$ with $D_{Re}(0,m) = 0$, and $D_{Re}(\cdot,\cdot)$ an increasing function of both arguments $(1-q_t)\Delta V_{Re}$ and m. Also from (10) $d^*_S = D_S(q_t \Delta V_S)$ is an increasing function of $q_t \Delta V_S$.

Parents do not know the realization of their children's capacity c to escape taxation when cultural transmission occurs. Consequently, the paternalistic motives have to be amended to involve expectations of the induced utilities with respect such capacity c. More precisely we have:

$$\begin{cases} V_{ReRe}(\lambda,\beta,q) = \frac{(1-\tau_{Re})}{1+\phi\alpha_c(\lambda)} \int_{\tau_{Re}/\epsilon}^{\overline{c}} \frac{dc}{\overline{c}} + \int_0^{\tau_{Re}/\epsilon} \frac{(1-c\epsilon)}{1+\phi\alpha_c(\lambda)} \frac{dc}{\overline{c}} \\ V_{Re\ S}(\lambda,\beta,q) = \frac{(1-\tau_{Re})}{(1+\phi\alpha_c(\lambda))} \int_{\tau/\epsilon}^{\overline{c}} \frac{dc}{\overline{c}} + \int_0^{\tau/\epsilon} \frac{(1-c\epsilon)}{1+\phi\alpha_c(\lambda)} \frac{dc}{\overline{c}}, \end{cases} \tag{A.82}$$

with $\epsilon = \epsilon_0/(1-\alpha_l(\lambda,\beta,q))$. Hence,

$$\Delta V_{Re}(\lambda,\beta,q) = \frac{(\tau\theta\alpha_c(\lambda))^2(1-\alpha_l(\lambda,q,\beta))}{2\overline{c}\epsilon_0(1+\phi\alpha_c(\lambda))}. \tag{A.83}$$

Similarly, we find that

$$\Delta V_S(\lambda,\beta,q) = \Delta V_{Re}(\lambda,\beta,q) = \Delta V(\lambda,\beta,q) = \frac{(\tau\theta\alpha_c(\lambda))^2(1-\alpha_l(\lambda,\beta,q))}{2\overline{c}\epsilon_0(1+\phi\alpha_c(\lambda))}. \tag{A.84}$$

Again the result that $\Delta V_s(\lambda,\beta,q) = \Delta V_{re}(\lambda,\beta,q)$ follows from the quadratic specification of the expected payoff functions. Note as well that because $\alpha_l(\lambda,\beta,q)$ depends on q (ie. is a decreasing function in q), $\Delta V(\lambda,\beta,q)$ also depends on q and is an increasing function of q

Now, the cultural dynamics write as

$$q_{t+1} - q_t = q_t(1-q_t)D(\lambda_t,\beta_t,q_t). \tag{A.85}$$

with

$$D(\lambda_t, \beta_t, q_t) = d^*_{Re} - d^*_S = D_{Re}\left[(1-q_t)\Delta V(\lambda_t, \beta_t, q_t), m(\lambda_t)\right] - D_S\left[q_t \Delta V(\lambda_t, \beta_t, q_t)\right]$$

can be interpreted as the relative "cultural fitness" of the religious trait in the population. Again simple inspection shows

$$D(\lambda_t, \beta_t, 0) = D_{Re}\left[\Delta V(\lambda_t, \beta_t, 0), m(\lambda_t)\right] > 0$$

and

$$D(\lambda_t, \beta_t, 1) = -D_S\left[\Delta V(\lambda_t, \beta_t, 1)\right] < 0$$

From this it follows that there exists a threshold $q^*_d(\lambda_t, \beta_t) \in (0, 1)$ such that

$$D(\lambda_t, \beta_t, q^*_d(\lambda_t, \beta_t)) = 0 \tag{A.86}$$

Compared to the benchmark model, $D(\lambda_t, \beta_t, q_t)$ may not be always decreasing function in q_t, as $\Delta V(\lambda_t, \beta_t, q_t)$ is increasing in q_t and the uniqueness of the threshold $q^*_d(\lambda_t, \beta_t)$ is not necessarily ensured. When however $(1-q)\Delta V(\lambda, \beta, q)$ is a decreasing function of q,[40] simple inspection shows that $D(\lambda_t, \beta_t, q_t)$ is a decreasing function of q_t and that $q_{t+1} < q_t$ if and only if $q_t > q^*_d(\lambda_t, \beta_t)$, as stated in proposition 5.

In such a case, defining again the sensitivity of parents' socialization to paternalistic motives by the following elasticities:

$$\epsilon_{Re} = \frac{\partial D_{Re}(x, y)}{\partial x} \cdot \frac{x}{D_{Re}} \text{ and } \epsilon_S = \frac{\partial D_S}{\partial z} \cdot \frac{z}{D_{Re}}\Bigg|$$

evaluated respectively at $x = (1-q)\Delta V(\lambda, \beta, q)$ and $y = m(\lambda)$, and $z = q\Delta V(\lambda, \beta, q)$, we obtain

$$\frac{\partial q^*_d(\lambda_t, \beta_t)}{\partial \lambda} = \frac{\left[\epsilon_{Re} - \epsilon_S\right] d^*\left(\lambda_t, \beta_t\right) \cdot \frac{\Delta V'_\lambda(\lambda_t, \beta_t)}{\Delta V(\lambda_t, \beta_t)} + \frac{\partial D_{Re}}{\partial m} \cdot m'(\lambda_t)}{-\frac{\partial D}{\partial q}(\lambda_t, \beta_t, q^*_d(\lambda_t, \beta_t))} \tag{A.87}$$

with $d^*\left(\lambda_t, \beta_t\right) = d^*_{Re}\left((\lambda_t, \beta_t, q^*_d(\lambda_t, \beta_t)\right) = d^*_S\left((\lambda_t, \beta_t, q^*_d(\lambda_t, \beta_t)\right)$, the equilibrium commun socialization rate at the threshold $q^*_d(\lambda_t, \beta_t)$. Again the numerator is composed of two terms reflecting the two channels through which the institutional environment λ_t affects cultural

[40] This is ensured when $1 > \frac{\tau^2}{\bar{c}\epsilon_0} \max\left(\frac{\theta}{\phi}, 1\right)$

transmission. The first term $K(\lambda_t) = [\epsilon_{Re} - \epsilon_S]\, d^*(\lambda_t, \beta_t) \cdot \frac{\Delta V'_\lambda(\lambda_t,\beta_t)}{\Delta V(\lambda_t,\beta_t)}$ is the paternalistic motive channel. As $\Delta V'_\lambda(\lambda_t, \beta_t) > 0$, the sign of $K(\lambda_t)$ depends on the relative sensitivity of parents' socialization to paternalistic motives. It is positive when $\epsilon_{Re} > \epsilon_S$, namely when the socialization rate of religious parents d^*_{Re} is more sensitive to paternalistic motives than the one of secular parents d^*_S. The second positive term $\frac{\partial D_{Re}}{\partial m} \cdot m'(\lambda_t)$ reflects the positive effect of promoting religious infrastructures as complementary inputs in the transmission process of the religious trait.

As in the benchmark model, it follows again that when religious parents' socialization efforts are more sensitive to paternalistic motives than secular parents (ie. $\epsilon_{Re} > \epsilon_S$) , and (or) when religious infrastructures are strong enough complementary inputs to socialization to the religious trait, then the numerator of (A.87) is positive and $q^*_d(\lambda_t, \beta_t)$ is increasing in λ_t.

- **Example with constant elasticity socialization cost functions**

Consider the following socialization cost functions:

$$\begin{cases} h_{Re}(d) = \frac{d^{1+\eta_{re}}}{1+\eta_{re}} \cdot \frac{1}{\chi(m)} \text{ and} \\ h_s(d) = \frac{d^{1+\eta_s}}{1+\eta_s}, \end{cases} \tag{A.88}$$

with $\eta_s \geq \eta_{re} > 0$ and $\chi'(m) > 0$**.** The optimal socialization efforts are such that:

$$\begin{cases} d^*_{Re}(q_t, \lambda_t) = ((1 - q_t)\Delta V(\lambda_t, \beta_t, q_t))^{\frac{1}{\eta_{re}}} \cdot [\chi(\lambda_t)]^{\frac{1}{\eta_{re}}} \\ d^*_S(q_t, \lambda_t) = (q_t \Delta V(\lambda_t, \beta_t, q_t))^{\frac{1}{\eta_s}}. \end{cases} \tag{A.89}$$

and in this constant elasticity specification $\epsilon_{Re} - \epsilon_S = \frac{1}{\eta_{re}} - \frac{1}{\eta_s} \geq 0$. Cultural dynamics are described as:

$$q_{t+1} - q_t = q_t(1 - q_t)\{((1 - q_t)\Delta V(\lambda_t, \beta_t, q_t))^{\frac{1}{\eta_{re}}} \cdot [\chi(\lambda_t)]^{\frac{1}{\eta_{re}}} - (q_t \Delta V(\lambda_t, \beta_t, q_t))^{\frac{1}{\eta_s}}\}, \tag{A.90}$$

which admits two unstable steady states $q = 0$ and $q = 1$, and in general a unique interior attractor, which we denote $q^*_d(\lambda_t, \beta_t)$ such that:

$$\frac{q^*_d(\lambda_t, \beta_t)^{\frac{1}{\eta_s}}}{(1 - q^*_d(\lambda_t, \beta_t))^{\frac{1}{\eta_{re}}}} = [\frac{(\tau\theta\alpha_c(\lambda_t))^2(1 - \alpha_l(\lambda_t, \beta_t, q^*_d(\lambda_t, \beta_t))}{2\overline{c}\epsilon_0(1 + \phi\alpha_c(\lambda_t))}]^{\frac{\eta_S - \eta_{re}}{\eta_S \eta_{re}}} \cdot [\chi(\lambda_t)]^{\frac{1}{\eta_{re}}} \tag{A.91}$$

From the last equation, and given that $\eta_S > \eta_{re}$, we deduce that $q_d^*(\lambda_t, \beta_t)$ is increasing in θ, λ_t and β_t and decreasing in ϕ. This concludes the proof of Proposition 5.

- **Joint dynamics with $q_d^*(\lambda_t, \beta_t)$ independent from β_t.**

Consider the case where the socialization cost functions of religious and secular parents are given by the following form

$$h_{Re}(d,m) = \frac{d^{1+\eta}}{1+\eta}\frac{1}{\chi(m)},\, h_s(d) = \frac{d^{1+\eta}}{1+\eta} \quad \text{with } \eta > 0$$

from (A.91), it is immediate that the threshold $q_d^*(\lambda_t, \beta_t)$ is given by:

$$q_d^*(\lambda_t, \beta_t) = q_d^*(\lambda_t) = \frac{[\chi(\lambda_t)]}{1 + [\chi(\lambda_t)]}$$

and is therefore independent from β_t. In such a case the dynamics of λ_t and q_t are such that: $\lambda_{t+1} > \lambda_t$ if and only if $q_t > \overline{q}_d(\lambda_t)$, and $q_{t+1} > q_t$ if and only if $q_t < q_d^*(\lambda_t)$ They are then decoupled from the dynamics of β_t and follow the same pattern as in the benchmark model. Consequently, depending on the initial conditions (λ_0, q_0), (λ_t, q_t) converge towards *a religious regime* $(1, q_d^*(1))$ or a *secular regime* $(0, q_d^*(0))$. Associated to these dynamics, the dynamics of political centralization then converges towards strong state centralization with $\beta_1^* = \widetilde{\beta}_d(1, q_d^*(1))$, or weak state centralization $\beta_0^* = \widetilde{\beta}_d(0, q_d^*(0)) < \beta_1^*$. **QED.**