HW 8.

Total: 60 pts.

2.120 6 points

a. ~~Denote the events G: good refrigerator~~

~~D: defective refrigerator~~

The total number of ways in arranging the 6 friges are P_6^6.

If the last defective frige is the 4th one. then there are C_3^1 methods to arrange the first defective frige, and C_2^1 methods to choose the first one. And P_4^4 methods in arranging the rest 4 good friges. therefore the total methods where the last defective friges is the 4th one equal to $C_3^1 C_2^1 P_4^4$.

The prob. that the last defective frige is found on the 4th test is

$$\frac{C_3^1 C_2^1 P_4^4}{P_6^6} = \frac{3\times 2}{5\times 6} = \frac{1}{5}$$ (2 pts)

b. This means that both defective friges are found during the 1st four tests. The total method is: $C_4^2 P_2^2 P_4^4$.

The prob. that this occurs is $\frac{C_4^2 P_2^2 P_4^4}{P_6^6} = \frac{[(4\times 3)/(1\times 2)]\cdot(1\times 2)}{5\times 6} = \frac{2}{5}$

(2 pts)

c. The prob. is $\frac{C_2^1 \, P_3^3}{P_4^4} = \frac{2}{4} = \frac{1}{2}$ (2 pts)

2. 137. 6 points Denote the events:

A_i: both balls selected from bowl i are white

B_i: bowl i is selected.

A: both balls selected are white.

Then

a. $P(A) = \sum_{i=1}^{5} P(A_i | B_i) P(B_i) = \frac{1}{5}[0 + 1/C_5^2 + C_3^2/C_5^2 + C_4^2/C_5^2 + C_5^2/C_5^2)$

$= \frac{1}{5}[\frac{1}{10} + \frac{3}{10} + \frac{3}{5} + 1] = \frac{2}{5}$ (3 pts)

b. Using Bayes' rule.

$P(B_3|A) = \frac{P(A \cap B_3)}{P(A)} = \frac{P(A|B_3)P(B_3)}{P(A)} = \frac{\frac{3}{10} \cdot \frac{1}{5}}{\frac{2}{5}} = \frac{3}{20}$. (3 pts)

3. 10. 6 points The prob. that one expensive equipment is leased is: $\frac{1}{5}$ (2 pts)
Use R denote rental occurs and N denote not occur. then the sequence of interest is RR ($Y=0$), RNR ($Y=1$). RNNR ($Y=2$) and so on.

Then $\text{prob}(Y=0) = \frac{1}{5}$

$\text{prob}(Y=1) = \frac{4}{5} \cdot \frac{1}{5}$

$\text{prob}(Y=2) = (\frac{4}{5})^2 \frac{1}{5}$ (2 pts)

$\vdots$

Thus $\text{prob}(Y=N) = (\frac{4}{5})^N \frac{1}{5}$ for $N = 0, 1, 2, \ldots$. (2 pts)

3.32 4 points 1 pt for each part.

a. The mean of U is greater than μ if $\mu < 0$, is less than μ if $\mu > 0$. is equal to μ if $\mu = 0$.

b. $E(U) = E(Y/10) = \frac{1}{10}E(Y) = \frac{\mu}{10}$.

c. The variance of U is smaller than σ^2, as the spread of U has decreased.

d. $V(U) = E[U - E(U)]^2 = E[(\frac{1}{10}Y - \frac{1}{10}\mu)^2] = E[\frac{1}{100}(Y-\mu)^2]$

$= 0.01\, E[(Y-\mu)^2] = 0.01\, V(Y) = 0.01\,\sigma^2$

6 points

3.61. Denote Y as # of volunteers with Rh factor in their blood. Y is binomial distributed with $n=5$ and $p=0.8$

a. prob (at least one does not have Rh factor)

$= 1 - P(Y=5) = 1 - C_5^5 (0.8)^5 (0.2)^0 = 0.672$ (2 pt)

b. $P(Y \le 4) = 1 - P(Y=5) = 0.672$ (2 pt)

c. We need $P(Y \ge 5) \ge 90\%$

That is $1 - P(Y \le 4) \ge 90\%$. $P(Y \le 4) \le 0.1$

Thus $1 - P(Y=5) = 1 - C_n^5 (0.8)^5 (0.2)^{n-5} \le 0.1$

$C_n^5 (0.8)^5 (0.2)^{n-5} \ge 0.9$

The smallest n that satisfy this condition is 8. (2 pt)

3 points

3.81. Let Y denote the # of times to toss to obtain the 1st head. then Y is a random variable with geometric distribution.

$E(Y) = \frac{1}{p} = \frac{1}{0.5} = 2$, thus the # of times I expected to toss is 2.

3 points

3.102 Let Y denote the # of green marbles drawn.

$$P(Y=3) = \frac{C_5^3}{C_{10}^3} = \frac{1}{12}$$

8 points

3.126.a. Let Y denote the # of arrivals in a two-hour period. Then Y has a possion dist. with $\lambda = 2\times 7 = 14$.

$$P(Y=2) = \frac{14^2}{2!} e^{-14} = 98 e^{-14}$$ (4 pts)

b. Let X denote the # of arrivals in a one-hour period. X has a possion dist. with $\lambda = 7$.

$$P(X=1)P(X=1) + P(X=2)P(X=0) + P(X=0)P(X=2)$$

$$= \left(\frac{7^1}{1!}e^{-7}\right)^2 + 2\left(\frac{7^2}{2!}e^{-7}\right)\left(\frac{7^0}{0!}e^{-7}\right)$$

$$= 49e^{-14} + 49e^{-14} = 98e^{-14}$$ (4 pts)

147. 4 points

$$p(y) = q^{y-1} p \quad \text{for } y = 1, 2, \ldots \infty$$

$$m(t) = E(e^{tY}) = \sum_{y=1}^{\infty} e^{ty} p(y)$$

$$= \sum_{y=1}^{\infty} e^{ty} q^{y-1} p = pe^t \sum_{y=1}^{\infty} (e^t q)^{y-1} = pe^t \frac{1}{1-e^t q}$$

148. 4 points

$$\frac{d}{dt} m(t) = \frac{pe^t(1-qe^t) + qe^t \cdot pe^t}{(1-qe^t)^2} = \frac{pe^t}{(1-qe^t)^2}$$

At $t=0$, this is $\frac{p}{(1-q)^2} = \frac{1}{p} = E(Y)$ (2 pts)

$$\frac{d^2}{dt^2} m(t) = \frac{pe^t(1-qe^t)^2 + 2(1-qe^t)qe^t pe^t}{(1-qe^t)^4}$$

$$= \frac{pe^t(1-qe^t)[1-qe^t+2qe^t]}{(1-qe^t)^4} = \frac{pe^t(1+qe^t)}{(1-qe^t)^3}$$

At $t=0$, this is $\frac{p(1+q)}{(1-q)^3} = \frac{1+q}{p^2} = E(Y^2)$

$$V(Y) = E(Y^2) - [E(Y)]^2 = \frac{1+q}{p^2} - \frac{1}{p^2} = \frac{q}{p^2}$$ (2 pts)

Chapter 4.

5 points 1 pt for each part.

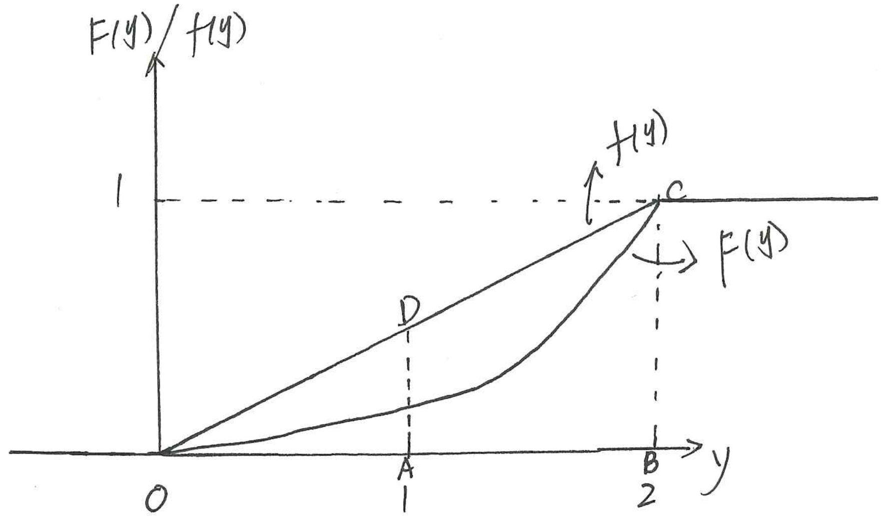
11. a. $\int_{-\infty}^{\infty} f(y)dy = 1 \Rightarrow \int_0^2 cy\,dy = 1$

$$\Rightarrow c\frac{y^2}{2}\Big|_0^2 = 2c = 1 \Rightarrow c = \frac{1}{2}$$

b. $F(y) = \int_{-\infty}^{y} f(t)\,dt = \int_0^y \frac{t}{2}\,dt = \frac{t^2}{4}\Big|_0^y = \frac{y^2}{4} \quad 0 \le y \le 2$

$F(y) = 0$ if $y < 0$ and $F(y) = 1$ if $y > 2$.

c.



d. $P(1 \le Y \le 2) = F(2) - F(1) = 1 - \frac{1}{4} = \frac{3}{4}$

e. $P(1 \le Y \le 2)$ equals the area ABCD, which is:

$\frac{f(1)+f(2)}{2} \cdot 1 = (\frac{1}{2} + 1)/2 = \frac{3}{4}$

3 points, 1 pt for each part.

30. a. $E(Y) = \int_{-\infty}^{\infty} y f(y)\,dy = \int_0^1 y \cdot 2y\,dy = \frac{2}{3} y^3 \Big|_0^1 = \frac{2}{3}$

$E(Y^2) = \int_0^1 y^2 \cdot 2y\,dy = \frac{2}{4} y^4 \Big|_0^1 = \frac{1}{2}$

$V(Y) = E(Y^2) - [E(Y)]^2 = \frac{1}{2} - (\frac{2}{3})^2 = \frac{1}{18}$.

b. $E(X) = E(200Y - 60) = 200E(Y) - 60 = 200 \cdot \frac{2}{3} - 60 = \frac{220}{3}$

$V(X) = V(200Y - 60) = 200^2 V(Y) = \frac{200^2}{18} = \frac{20000}{9}$

c. A two standard deviation interval of the mean is given by $\frac{220}{3} + 2\sqrt{\frac{20000}{9}}$, or $(-20.948, 167.614)$

2 points, 1 pt each.

44.

a. $\int_{-\infty}^{\infty} f(y)\,dy = 1 \Rightarrow \int_{-2}^{2} k\,dy = 1 \Rightarrow 4k = 1 \Rightarrow k = \frac{1}{4}$

b. $F(y) = \int_{-2}^{y} \frac{1}{4}\,dt = \frac{t}{4}\Big|_{-2}^{y} = \frac{y+2}{4}$ for $-2 \le y \le 2$.

$F(y) = 0$ if $y < -2$

$F(y) = 1$ if $y > 2$.