2.114 Let T = {detects truth} and L = {detects lie}. The sample space is TT, TL, LT, LL. Since one suspect is guilty, assume the guilty suspect is questioned first:

a. $P(LL)$ = .95(.10) = 0.095 **b.** $P(LT)$ = ..95(.9) = 0.885
b. $P(TL)$ = .05(.10) = 0.005 **d.** 1 – (.05)(.90) = 0.955

2.115 By independence, $(.75)(.75)(.75)(.75) = (.75)^4$.

2.116 By the complement rule, P(system works) = 1 – P(system fails) = $1 - (.01)^3$.

2.117 **a.** From the description of the problem, there is a 50% chance a car will be rejected. To find the probability that three out of four will be rejected (i.e. the drivers chose team 2), note that there are $\binom{4}{3} = 4$ ways that three of the four cars are evaluated by team 2. Each one has probability (.5)(.5)(.5)(.5) of occurring, so the probability is $4(.5)^4 = 0.25$.

b. The probability that all four pass (i.e. all four are evaluated by team 1) is $(.5)^4 = 1/16$.

2.118 If the victim is to be saved, a proper donor must be found within eight minutes. The patient will be saved if the proper donor is found on the 1st, 2nd, 3rd, or 4th try. But, if the donor is found on the 2nd try, that implies he/she wasn't found on the 1st try. So, the probability of saving the patient is found by, letting A = {correct donor is found}:

$$P(\text{save}) = P(A) + P(\overline{A}A) + P(\overline{A}\overline{A}A) + P(\overline{A}\overline{A}\overline{A}A).$$

By independence, this is $.4 + .6(.4) + (.6)^2(.4) + (.6)^3(.4) = 0.8704$

2.119 **a.** Define the events: A: obtain a sum of 3 B: do not obtain a sum of 3 or 7
Since there are 36 possible rolls, $P(A) = 2/36$ and $P(B) = 28/36$. Obtaining a sum of 3 before a sum of 7 can happen on the 1st roll, the 2nd roll, the 3rd roll, etc. Using the events above, we can write these as A, BA, BBA, $BBBA$, etc. The probability of obtaining a sum of 3 before a sum of 7 is given by $P(A) + P(B)P(A) + [P(B)]^2P(A) + [P(B)]^3P(A) + \ldots$. (Here, we are using the fact that the rolls are independent.) This is an infinite sum, and it follows as a geometric series. Thus, $2/36 + (28/36)(2/36) + (28/36)^2(2/26) + \ldots = 1/4$.

b. Similar to part a. Define C: obtain a sum of 4 D: do not obtain a sum of 4 or 7
Then, $P(C) = 3/36$ and $P(D) = 27/36$. The probability of obtaining a 4 before a 7 is 1/3.

2.120 Denote the events G: good refrigerator D: defective refrigerator

a. If the last defective refrigerator is found on the 4th test, this means the first defective refrigerator was found on the 1st, 2nd, or 3rd test. So, the possibilities are DGGD, GDGD, and GGDD. So, the probability is $\left(\frac{2}{6}\right)\left(\frac{4}{5}\right)\left(\frac{3}{4}\right)\frac{1}{3}$. The probabilities associated with the other two events are identical to the first. So, the desired probability is $3\left(\frac{2}{6}\right)\left(\frac{4}{5}\right)\left(\frac{3}{4}\right)\frac{1}{3} = \frac{1}{5}$.

b. Here, the second defective refrigerator must be found on the 2nd, 3rd, or 4th test.
Define: A_1: second defective found on 2nd test
A_2: second defective found on 3rd test
A_3: second defective found on 4th test

2.137 Let A = {both balls are white}, and for i = 1, 2, … 5

A_i = both balls selected from bowl i are white. Then $\bigcup A_i = A$.

B_i = bowl i is selected. Then, $P(B_i) = .2$ for all i.

a. $P(A) = \sum P(A_i \mid B_i)P(B_i) = \frac{1}{5}\left[0 + \frac{2}{5}\left(\frac{1}{4}\right) + \frac{3}{5}\left(\frac{2}{4}\right) + \frac{4}{5}\left(\frac{3}{4}\right) + 1\right] = 2/5$.

b. Using Bayes' rule, $P(B_3|A) = \dfrac{\frac{3}{50}}{\frac{2}{50}} = 3/20$.

2.138 Define the events:

A: the player wins
B_i: a sum of i on first toss
C_k: obtain a sum of k before obtaining a 7

Now, $P(A) = \sum_{i=1}^{12} P(A \cap B_i)$. We have that $P(A \cap B_2) = P(A \cap B_3) = P(A \cap B_{12}) = 0$.
Also, $P(A \cap B_7) = P(B_7) = \frac{6}{36}$, $P(A \cap B_{11}) = P(B_{11}) = \frac{2}{36}$.
Now, $P(A \cap B_4) = P(C_4 \cap B_7) = P(C_4)P(B_7) = \frac{1}{3}\left(\frac{3}{36}\right) = \frac{3}{36}$ (using independence Ex. 119).
Similarly, $P(C_5) = P(C_9) = \frac{4}{10}$, $P(C_6) = P(C_8) = \frac{5}{11}$, and $P(C_{10}) = \frac{3}{9}$.
Thus, $P(A \cap B_5) = P(A \cap B_9) = \frac{2}{45}$, $P(A \cap B_6) = P(A \cap B_8) = \frac{25}{396}$, $P(A \cap B_{10}) = \frac{1}{36}$.
Putting all of this together, $P(A) = 0.493$.

2.139 From Ex. 1.112, $P(Y = 0) = (.02)^3$ and $P(Y = 3) = (.98)^3$. The event $Y = 1$ are the events FDF, DFF, and FFD, each having probability $(.02)^2(.98)$. So, $P(Y = 1) = 3(.02)^2(.98)$. Similarly, $P(Y = 2) = 3(.02)(.98)^2$.

2.140 The total number of ways to select 3 from 6 refrigerators is $\binom{6}{3} = 20$. The total number of ways to select y defectives and $3 - y$ nondefectives is $\binom{2}{y}\binom{4}{3-y}$, y = 0, 1, 2. So,

$P(Y = 0) = \dfrac{\binom{2}{0}\binom{4}{3}}{20} = 4/20$, $P(Y = 1) = 4/20$, and $P(Y = 2) = 12/20$.

2.141 The events $Y = 2$, $Y = 3$, and $Y = 4$ were found in Ex. 2.120 to have probabilities 1/15, 2/15, and 3/15 (respectively). The event $Y = 5$ can occur in four ways:

DGGGD *GDGGD* *GGDGD* *GGGDD*

Each of these possibilities has probability 1/15, so that $P(Y = 5) = 4/15$. By the complement rule, $P(Y = 6) = 5/15$.

b. The graph is omitted.
c. $P(Y > 1) = P(Y = 2) + P(Y = 3) = 0.00725$.

3.10 Denote R as the event a rental occurs on a given day and N denotes no rental. Thus, the sequence of interest is RR, RNR, $RNNR$, $RNNNR$, … . Consider the position immediately following the first R: it is filled by an R with probability .2 and by an N with probability .8. Thus, $P(Y = 0) = .2$, $P(Y = 1) = .8(.2) = .16$, $P(Y = 2) = .128$, … . In general,

$$P(Y = y) = .2(.8)^y, y = 0, 1, 2, \ldots .$$

3.11 There is a 1/3 chance a person has O^+ blood and 2/3 they do not. Similarly, there is a 1/15 chance a person has O^- blood and 14/15 chance they do not. Assuming the donors are randomly selected, if X = # of O^+ blood donors and Y = # of O^- blood donors, the probability distributions are

	0	1	2	3
$p(x)$	$(2/3)^3 = 8/27$	$3(2/3)^2(1/3) = 12/27$	$3(2/3)(1/3)^2 = 6/27$	$(1/3)^3 = 1/27$
$p(y)$	2744/3375	196/3375	14/3375	1/3375

Note that $Z = X + Y$ = # will type O blood. The probability a donor will have type O blood is 1/3 + 1/15 = 6/15 = 2/5. The probability distribution for Z is

	0	1	2	3
$p(z)$	$(2/5)^3 = 27/125$	$3(2/5)^2(3/5) = 54/27$	$3(2/5)(3/5)^2 = 36/125$	$(3/5)^3 = 27/125$

3.12 $E(Y) = 1(.4) + 2(.3) + 3(.2) + 4(.1) = 2.0$
$E(1/Y) = 1(.4) + 1/2(.3) + 1/3(.2) + 1/4(.1) = 0.6417$
$E(Y^2 - 1) = E(Y^2) - 1 = [1(.4) + 2^2(.3) + 3^2(.2) + 4^2(.1)] - 1 = 5 - 1 = 4.$
$V(Y) = E(Y^2) = [E(Y)]^2 = 5 - 22 = 1.$

3.13 $E(Y) = -1(1/2) + 1(1/4) + 2(1/4) = 1/4$
$E(Y^2) = (-1)^2(1/2) + 1^2(1/4) + 2^2(1/4) = 7/4$
$V(Y) = 7/4 - (1/4)^2 = 27/16.$
Let C = cost of play, then the net winnings is $Y - C$. If $E(Y - C) = 0$, $C = 1/4$.

3.14 **a.** $\mu = E(Y) = 3(.03) + 4(.05) + 5(.07) + \ldots + 13(.01) = 7.9$
b. $\sigma^2 = V(Y) = E(Y^2) - [E(Y)]^2 = 3^2(.03) + 4^2(.05) + 5^2(.07) + \ldots + 13^2(.01) - 7.9^2 = 67.14 - 62.41 = 4.73$. So, $\sigma = 2.17$.
c. $(\mu - 2\sigma, \mu + 2\sigma) = (3.56, 12.24)$. So, $P(3.56 < Y < 12.24) = P(4 \le Y \le 12) = .05 + .07 + .10 + .14 + .20 + .18 + .12 + .07 + .03 = 0.96$.

3.15 **a.** $p(0) = P(Y = 0) = (.48)^3 = .1106$, $p(1) = P(Y = 1) = 3(.48)^2(.52) = .3594$, $p(2) = P(Y = 2) = 3(.48)(.52)^2 = .3894$, $p(3) = P(Y = 3) = (.52)^3 = .1406$.
b. The graph is omitted.
c. $P(Y = 1) = .3594$.

Thus, $E(X_1) = E(X_2) = 2/3$. The expected profit for the owner of both firms is given by 90000(2/3 + 2/3) = $120,000.

3.26 The random variable Y = daily sales can have values \$0, \$50,000 and \$100,000.
If $Y = 0$, either the salesperson contacted only one customer and failed to make a sale or the salesperson contacted two customers and failed to make both sales. Thus $P(Y = 0) = 1/3(9/10) + 2/3(9/10)(9/10) = 252/300$.
If $Y = 2$, the salesperson contacted to customers and made both sales. So, $P(Y = 2) = 2/3(1/10)(1/10) = 2/300$.
Therefore, $P(Y = 1) = 1 - 252/300 - 2/300 = 46/300$.
Then, $E(Y) = 0(252/300) + 50000(46/300) + 100000(2/300) = 25000/3$ (or \$8333.33).
$V(Y)$ =380,561,111 and σ = \$19,507.98.

3.27 Let Y = the payout on an individual policy. Then, $P(Y = 85{,}000) = .001$, $P(Y = 42{,}500) = .01$, and $P(Y = 0) = .989$. Let C represent the premium the insurance company charges. Then, the company's net gain/loss is given by $C - Y$. If $E(C - Y) = 0$, $E(Y) = C$. Thus, $E(Y) = 85000(.001) + 42500(.01) + 0(.989) = 510 = C$.

3.28 Using the probability distribution found in Ex. 3.3, $E(Y) = 2(1/6) + 3(2/6) + 4(3/6) = 20/6$. The cost for testing and repairing is given by $2Y + 4$. So, $E(2Y + 4) = 2(20/6) + 4 = 64/6$.

3.29 $$\sum_{k=1}^{\infty} P(Y \ge k) = \sum_{k=1}^{\infty}\sum_{j=k}^{\infty} P(Y = k) = \sum_{k=1}^{\infty}\sum_{j=k}^{\infty} p(j) = \sum_{j=1}^{\infty}\sum_{k=1}^{j} p(j) = \sum_{j=1}^{\infty} j \cdot p(j) = \sum_{y=1}^{\infty} y \cdot p(y) = E(Y).$$

3.30 **a.** The mean of X will be larger than the mean of Y.
b. $E(X) = E(Y + 1) = E(Y) + 1 = \mu + 1$.
c. The variances of X and Y will be the same (the addition of 1 doesn't affect variability).
d. $V(X) = E[(X - E(X))^2] = E[(Y + 1 - \mu - 1)^2] = E[(Y - \mu)^2] = \sigma^2$.

3.31 **a.** The mean of W will be larger than the mean of Y if $\mu > 0$. If $\mu < 0$, the mean of W will be smaller than μ. If $\mu = 0$, the mean of W will equal μ.
b. $E(W) = E(2Y) = 2E(Y) = 2\mu$.
c. The variance of W will be larger than σ^2, since the spread of values of W has increased.
d. $V(X) = E[(X - E(X))^2] = E[(2Y - 2\mu)^2] = 4E[(Y - \mu)^2] = 4\sigma^2$.

3.32 **a.** The mean of W will be smaller than the mean of Y if $\mu > 0$. If $\mu < 0$, the mean of W will be larger than μ. If $\mu = 0$, the mean of W will equal μ.
b. $E(W) = E(Y/10) = (.1)E(Y) = (.1)\mu$.
c. The variance of W will be smaller than σ^2, since the spread of values of W has decreased.
d. $V(X) = E[(X - E(X))^2] = E[(.1Y - .1\mu)^2] = (.01)E[(Y - \mu)^2] = (.01)\sigma^2$.

3.59 If Y = # of defective motors, then Y is binomial with $n = 10$ and $p = .08$. Then, $E(Y) = .8$. The seller's expected next gain is $\$1000 - \$200E(Y) = \$840$.

3.60 Let Y = # of fish that survive. Then, Y is binomial with $n = 20$ and $p = .8$.
a. $P(Y = 14) = .109$.
b. $P(Y \geq 10) = .999$.
c. $P(Y \leq 16) = .589$.
d. $\mu = 20(.8) = 16$, $\sigma^2 = 20(.8)(.2) = 3.2$.

3.61 Let Y = # with Rh^+ blood. Then, Y is binomial with $n = 5$ and $p = .8$
a. $1 - P(Y = 5) = .672$.
b. $P(Y \leq 4) = .672$.
c. We need n for which $P(Y \geq 5) = 1 - P(Y \leq 4) > .9$. The smallest n is 8.

3.62 **a.** Assume independence of the three inspection events.
b. Let Y = # of plane with wing cracks that are detected. Then, Y is binomial with $n = 3$ and $p = .9(.8)(.5) = .36$. Then, $P(Y \geq 1) = 1 - P(Y = 0) = 0.737856$.

3.63 **a.** Found by pulling in the formula for $p(y)$ and $p(y - 1)$ and simplifying.

b. Note that $P(Y < 3) = P(Y \leq 2) = P(Y = 2) + P(Y = 1) + P(Y = 0)$. Now, $P(Y = 0) = (.96)^{90} = .0254$. Then, $P(Y = 1) = \frac{(90-1+1).04}{1(.96)}(.0254) = .0952$ and $P(Y = 2) = \frac{(90-2+1).04}{2(.96)}(.0952) = .1765$. Thus, $P(Y < 3) = .0254 + .0952 + .1765 = 0.2971$

c. $\frac{(n-y+1)}{yq} > 1$ is equivalent to $(n+1)p - yp > yq$ is equivalent to $(n+1)p > y$. The others are similar.

d. Since for $y \leq (n + 1)p$, then $p(y) \geq p(y - 1) > p(y - 2) > \ldots$. Also, for $y \geq (n + 1)p$, then $p(y) \geq p(y + 1) > p(y + 2) > \ldots$. It is clear that p(y) is maximized when y is a close to $(n + 1)p$ as possible.

3.64 To maximize the probability distribution as a function of p, consider taking the natural log (since ln() is a strictly increasing function, it will not change the maximum). By taking the first derivative of $\ln[p(y_0)]$ and setting it equal to 0, the maximum is found to be y_0/n.

3.65 **a.** $E(Y/n) = E(Y)/n = np/n = p$.
b. $V(Y/n) = V(Y)/n^2 = npq/n^2 = pq/n$. This quantity goes to zero as n goes to infinity.

3.66 **a.** $\sum_{y=1}^{\infty} q^{y-1}p = p\sum_{x=0}^{\infty} q^x = p\frac{1}{1-q} = 1$ (infinite sum of a geometric series)

b. $\frac{q^{y-1}p}{q^{y-2}p} = q$. The event $Y = 1$ has the highest probability for all p, $0 < p < 1$.

3.80 Let Y = # of tosses until the first 6 appears, so Y has a geometric distribution. Using the result from Ex. 3.77,

$$P(B \text{ tosses first } 6) = P(Y = 2, 4, 6, \ldots) = 1 - P(Y = 1, 3, 5, \ldots) = 1 - p\frac{1}{1-q^2}.$$

Since $p = 1/6$, $P(B$ tosses first 6$) = 5/11$. Then,

$$P(Y = 4 \mid B \text{ tosses the first } 6)\ \frac{P(Y=4)}{5/11} = \frac{(5/6)^2(1/6)}{5/11} = 275/1296.$$

3.81 With $p = 1/2$, then $\mu = 1/(1/2) = 2$.

3.82 With $p = .2$, then $\mu = 1/(.2) = 5$. The 5^{th} attempt is the expected first successful well.

3.83 Let Y = # of trials until the correct password is picked. Then, Y has a geometric distribution with $p = 1/n$. $P(Y = 6) = \frac{1}{n}\left(\frac{n-1}{n}\right)^5$.

3.84 $E(Y) = n$, $V(Y) = (1-\frac{1}{n})n^2 = n(n-1)$.

3.85 Note that $\frac{d^2}{dq^2}q^y = y(y-1)q^{y-2}$. Thus, $\frac{d^2}{dq^2}\sum_{y=2}^{\infty} q^y = \sum_{y=2}^{\infty} y(y-1)q^{y-2}$. Thus,

$$E[Y(Y-1)] = \sum_{y=1}^{\infty} y(y-1)q^{y-1} = pq\sum_{y=1}^{\infty} y(y-1)q^{y-2} = pq\frac{d^2}{dq^2}\sum_{y=2}^{\infty} q^y = pq\frac{d^2}{dq^2}\left\{\frac{1}{1-q} - 1 - q\right\} =$$

$$\frac{2pq}{(1-q)^3} = \frac{2q}{p^2}.$$ Use this with $V(Y) = E[Y(Y-1)] + E(Y) - [E(Y)]^2$.

3.86 $P(Y = y_0) = q^{y_0-1}p$. Like Ex. 3.64, maximize this probability by first taking the natural log.

3.87 $E(1/Y) = \sum_{y=1}^{\infty} \frac{1}{y}(1-p)^{y-1}p = \frac{p}{1-p}\sum_{y=1}^{\infty}\frac{(1-p)^y}{y} = -\frac{p\ln(p)}{1-p}$.

3.88 $P(Y^* = y) = P(Y = y+1) = q^{y+1-1}p = q^y p$, $y = 0, 1, 2, \ldots$.

3.89 $E(Y^*) = E(Y) - 1 = \frac{1}{p} - 1$. $V(Y^*) = V(Y - 1) = V(Y)$.

3.90 Let Y = # of employees tested until three positives are found. Then, Y is negative binomial with $r = 3$ and $p = .4$. $P(Y = 10) = \binom{9}{2}.4^3(.6)^7 = .06$.

3.91 The total cost is given by $20Y$. So, $E(20Y) = 20E(Y) = 20\frac{3}{.4} = \$50$. Similarly, $V(20Y) = 400V(Y) = 4500$.

3.101 **a.** Note that $P(Y = 11) = \binom{10}{4}p^5(1-p)^6$. Like Ex. 3.64 and 3.86, maximize this probability by first taking the natural log. The maximum is 5/11.
b. In general, the maximum is r/y_0.

3.102 Let Y = # of green marbles chosen in three draws. Then. $P(Y = 3) = \binom{5}{3}\Big/\binom{10}{3} = 1/12$.

3.103 Use the hypergeometric probability distribution with $N = 10$, $r = 4$, $n = 5$. $P(Y = 0) = \frac{1}{42}$.

3.104 Define the events: A: 1st four selected packets contain cocaine
B: 2nd two selected packets do not contain cocaine

Then, the desired probability is $P(A \cap B) = P(B|A)P(A)$. So,
$P(A) = \binom{15}{4}\Big/\binom{20}{4} = .2817$ and $P(B|A) = \binom{5}{2}\Big/\binom{16}{2} = .0833$. Thus,
$P(A \cap B) = .2817(.0833) = 0.0235$.

3.105 **a.** The random variable Y follows a hypergeometric distribution. The probability of being chosen on a trial is dependent on the outcome of previous trials.

b. $P(Y \geq 2) = P(Y = 2) + P(Y = 3) = \dfrac{\binom{5}{2}\binom{3}{1}}{\binom{8}{3}} + \dfrac{\binom{5}{3}}{\binom{8}{3}} = .5357 + .1786 = 0.7143$.

c. $\mu = 3(5/8) = 1.875$, $\sigma^2 = 3(5/8)(3/8)(5/7) = .5022$, so $\sigma = .7087$.

3.106 Using the results from Ex.103, $E(50Y) = 50E(Y) = 50[5\left(\frac{4}{10}\right)] = \$100$. Furthermore,
$V(50Y) = 2500V(Y) = 2500[5\frac{4}{10}\left(\frac{6}{10}\right)\left(\frac{5}{9}\right)] = 1666.67$.

3.107 The random variable Y follows a hypergeometric distribution with $N = 6$, $n = 2$, and $r = 4$.

3.108 Use the fact that P(at least one is defective) = 1 – P(none are defective). Then, we require P(none are defective) ≤ .2. If $n = 8$,

$$P(\text{none are defective}) = \left(\tfrac{17}{20}\right)\left(\tfrac{16}{19}\right)\left(\tfrac{15}{18}\right)\left(\tfrac{14}{17}\right)\left(\tfrac{13}{16}\right)\left(\tfrac{12}{15}\right)\left(\tfrac{11}{14}\right)\left(\tfrac{10}{13}\right) = 0.193.$$

3.109 Let Y = # of treated seeds selected.

a. $P(Y = 4) = \dfrac{\binom{5}{4}\binom{5}{0}}{\binom{10}{4}} = .0238$

b. $P(Y \leq 3) = 1 - P(Y = 4) = 1 - \dfrac{\binom{5}{4}\binom{5}{0}}{\binom{10}{4}} = 1 - .0238 = .9762$.

c. same answer as part (b) above.

3.122 Let Y = # of customers that arrive during the hour. Then, Y is Poisson with $\lambda = 7$.
a. $P(Y \le 3) = .0818$.
b. $P(Y \ge 2) = .9927$.
c. $P(Y = 5) = .1277$

3.123 If $p(0) = p(1)$, $e^{-\lambda} = \lambda e^{-\lambda}$. Thus, $\lambda = 1$. Therefore, $p(2) = \frac{1^2}{2!}e^{-1} = .1839$.

3.124 Using Table 3 in Appendix III, we find that if Y is Poisson with $\lambda = 6.6$, $P(Y \le 2) = .04$. Using this value of λ, $P(Y > 5) = 1 - P(Y \le 5) = 1 - .355 = .645$.

3.125 Let S = total service time = $10Y$. From Ex. 3.122, Y is Poisson with $\lambda = 7$. Therefore, $E(S) = 10E(Y) = 7$ and $V(S) = 100V(Y) = 700$. Also,
$P(S > 150) = P(Y > 15) = 1 - P(Y \le 15) = 1 - .998 = .002$, and unlikely event.

3.126 a. Let Y = # of customers that arrive in a given two–hour time. Then, Y has a Poisson distribution with $\lambda = 2(7) = 14$ and $P(Y = 2) = \frac{14^2}{2!}e^{-14}$.

b. The same answer as in part a. is found.

3.127 Let Y = # of typing errors per page. Then, Y is Poisson with $\lambda = 4$ and $P(Y \le 4) = .6288$.

3.128 Note that over a one–minute period, Y = # of cars that arrive at the toll booth is Poisson with $\lambda = 80/60 = 4/3$. Then, $P(Y \ge 1) = 1 - P(Y = 0) = 1 - e^{-4/3} = .7364$.

3.129 Following the above exercise, suppose the phone call is of length t, where t is in minutes. Then, Y = # of cars that arrive at the toll booth is Poisson with $\lambda = 4t/3$. Then, we must find the value of t such that

$$P(Y = 0) = 1 - e^{-4t/3} \ge .4.$$

Therefore, $t \le -\frac{3}{4}\ln(.6) = .383$ minutes, or about $.383(60) = 23$ seconds.

3.130 Define: Y_1 = # of cars through entrance I, Y_2 = # of cars through entrance II. Thus, Y_1 is Poisson with $\lambda = 3$ and Y_2 is Poisson with $\lambda = 4$.

Then, $P(\text{three cars arrive}) = P(Y_1 = 0, Y_2 = 3) + P(Y_1 = 1, Y_2 = 2) + P(Y_1 = 2, Y_2 = 1) + {} + P(Y_1 = 3, Y_2 = 0)$.

By independence, $P(\text{three cars arrive}) = P(Y_1 = 0)P(Y_2 = 3) + P(Y_1 = 1)P(Y_2 = 2) + P(Y_1 = 2)P(Y_2 = 1) + P(Y_1 = 3)P(Y_2 = 0)$.
Using Poisson probabilities, this is equal to 0.0521

3.131 Let the random variable Y = # of knots in the wood. Then, Y has a Poisson distribution with $\lambda = 1.5$ and $P(Y \le 1) = .5578$.

3.132 Let the random variable Y = # of cars entering the tunnel in a two–minute period. Then, Y has a Poisson distribution with $\lambda = 1$ and $P(Y > 3) = 1 - P(Y \le 3) = 0.01899$.

3.143 Since λ is a non–integer, $p(y)$ is maximized at $y = 5$.

3.144 Observe that with λ = 6, $p(5) = \frac{6^5 e^{-6}}{5!} = .1606$, $p(6) = \frac{6^6 e^{-6}}{6!} = .1606$.

3.145 Using the binomial theorem, $m(t) = E(e^{tY}) = \sum_{y=0}^{n} \binom{n}{y} (pe^t)^y q^{n-y} = (pe^t + q)^n$.

3.146 $\frac{d}{dt} m(t) = n(pe^t + q)^{n-1} pe^t$. At $t = 0$, this is $np = E(Y)$.

$\frac{d^2}{dt^2} m(t) = n(n-1)(pe^t + q)^{n-1}(pe^t)^2 + n(pe^t + q)^{n-1} pe^t$. At $t = 0$, this is $np^2(n-1) + np$.
Thus, $V(Y) = np^2(n-1) + np - (np)2 = np(1-p)$.

3.147 The moment–generating function is $m(t) = E(e^{tY}) = \sum_{y=1}^{n} pe^{ty} q^{y-1} = pe^t \sum_{y=0}^{\infty} (qe^t)^y = \frac{pe^t}{1 - qe^t}$.

3.148 $\frac{d}{dt} m(t) = \frac{pe^t}{(1 - qe^t)^2}$. At $t = 0$, this is $1/p = E(Y)$.

$\frac{d^2}{dt^2} m(t) = \frac{(1 - qe^t)^2 pe^t - 2pe^t(1 - qe^t)(-qe^t)}{(1 - qe^t)^4}$. At $t = 0$, this is $(1+q)/p^2$.
Thus, $V(Y) = (1+q)/p^2 - (1/p)^2 = q/p^2$.

3.149 This is the moment–generating function for the binomial with $n = 3$ and $p = .6$.

3.150 This is the moment–generating function for the geometric with $p = .3$.

3.151 This is the moment–generating function for the binomial with $n = 10$ and $p = .7$, so $P(Y \le 5) = .1503$.

3.152 This is the moment–generating function for the Poisson with λ = 6. So, μ = 6 and σ = $\sqrt{6} \approx 2.45$. So, $P(|Y - \mu| \le 2\sigma) = P(\mu - 2\sigma \le Y \le \mu + 2\sigma) = P(1.1 \le Y \le 10.9) = P(2 \le Y \le 10) = .940$.

3.153 **a.** Binomial with $n = 5$, $p = .1$
b. If $m(t)$ is multiplied top and bottom by ½, this is a geometric mgf with $p = ½$.
c. Poisson with λ = 2.

3.154 **a.** Binomial mean and variance: $\mu = 1.667$, $\sigma^2 = 1.111$.
b. Geometric mean and variance: $\mu = 2$, $\sigma^2 = 2$.
c. Poisson mean and variance: $\mu = 2$, $\sigma^2 = 2$.

4.9 **a.** Y is a discrete random variable because $F(y)$ is not a continuous function. Also, the set of possible values of Y represents a countable set.

b. These values are 2, 2.5, 4, 5.5, 6, and 7.

c. $p(2) = 1/8$, $p(2.5) = 3/16 - 1/8 = 1/16$, $p(4) = 1/2 - 3/16 = 5/16$, $p(5.5) = 5/8 - 1/2 = 1/8$, $p(6) = 11/16 - 5/8 = 1/16$, $p(7) = 1 - 11/16 = 5/16$.

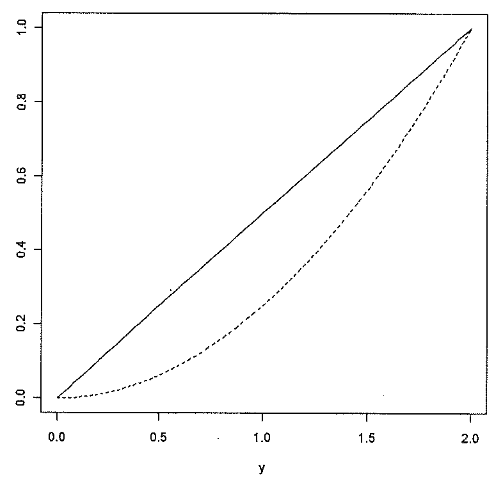
d. $P(Y \le \phi_{.5}) = F(\phi_{.5}) = .5$, so $\phi_{.5} = 4$.

4.10 **a.** $F(\phi_{.95}) = \int_0^{\phi_{.95}} 6y(1-y)dy = .95$, so $\phi_{.95} = 0.865$.

b. Since Y is a continuous random variable, $y_0 = \phi_{.95} = 0.865$.

4.11 **a.** $\int_0^2 cydy = \left[cy^2/2\right]_0^2 = 2c = 1$, so $c = 1/2$.

b. $F(y) = \int_{-\infty}^{y} f(t)dt = \int_0^y \frac{t}{2}dt = \frac{y^2}{4}$, $0 \le y \le 2$.



c. Solid line: $f(y)$; dashed line: $F(y)$

d. $P(1 \le Y \le 2) = F(2) - F(1) = 1 - .25 = .75$.

e. Note that $P(1 \le Y \le 2) = 1 - P(0 \le Y < 1)$. The region $(0 \le y < 1)$ forms a triangle (in the density graph above) with a base of 1 and a height of .5. So, $P(0 \le Y < 1) = \frac{1}{2}(1)(.5) = .25$ and $P(1 \le Y \le 2) = 1 - .25 = .75$.

4.27 First note that from Ex. 4.21, $E(Y) = .708$ and $V(Y) = .0487$. Then,
$E(W) = E(5 - .5Y) = 5 - .5E(Y) = 5 - .5(.708) = \$4.65$.
$V(W) = V(5 - .5Y) = .25V(Y) = .25(.0487) = .012$.

4.28 **a.** By using the methods learned in this chapter, $c = 105$.

b. $E(Y) = 105\int_0^1 y^3(1-y)^4\,dy = 3/8$.

4.29 $E(Y) = .5\int_{59}^{61} y\,dy = .5\frac{y^2}{2}\Big]_{59}^{61} = 60$, $E(Y^2) = .5\int_{59}^{61} y^2\,dy = .5\frac{y^3}{3}\Big]_{59}^{61} = 3600\frac{1}{3}$. Thus,
$V(Y) = 3600\frac{1}{3} - (60)^2 = \frac{1}{3}$.

4.30 **a.** $E(Y) = \int_0^1 2y^2\,dy = 2/3$, $E(Y^2) = \int_0^1 2y^3\,dy = 1/2$. Thus, $V(Y) = 1/2 - (2/3)^2 = 1/18$.

b. With $X = 200Y - 60$, $E(X) = 200(2/3) - 60 = 220/3$, $V(X) = 20000/9$.

c. Using Tchebysheff's theorem, a two standard deviation interval about the mean is given by $220/3 \pm 2\sqrt{20000/9}$ or (–20.948, 167.614).

4.31 $E(Y) = \int_2^6 y\left(\frac{3}{32}\right)(y-2)(6-y)\,dy = 4$.

4.32 **a.** $E(Y) = \frac{3}{64}\int_0^4 y^3(4-y)\,dy = \frac{3}{64}\left[y^4 - \frac{y^5}{5}\right]_0^4 = 2.4$. $V(Y) = .64$.

b. $E(200Y) = 200(2.4) = \$480$, $V(200Y) = 200^2(.64) = 25{,}600$.

c. $P(200Y > 600) = P(Y > 3) = \frac{3}{64}\int_3^4 y^2(4-y)\,dy = .2616$, or about 26% of the time the cost will exceed \$600 (fairly common).

4.33 **a.** $E(Y) = \frac{3}{8}\int_5^7 y(7-y)^2\,dy = \frac{3}{8}\left[\frac{49}{2}y^2 - \frac{14}{3}y^3 + \frac{y^4}{4}\right]_5^7 = 5.5$

$E(Y^2) = \frac{3}{8}\int_5^7 y^2(7-y)^2\,dy = \frac{3}{8}\left[\frac{49}{3}y^3 - \frac{14}{4}y^4 + \frac{y^5}{5}\right]_5^7 = 30.4$, so $V(Y) = .15$.

b. Using Tchebysheff's theorem, a two standard deviation interval about the mean is given by $5.5 \pm 2\sqrt{.15}$ or (4.725, 6.275). Since $Y \geq 5$, the interval is (5, 6.275).

c. $P(Y < 5.5) = \frac{3}{8}\int_5^{5.5}(7-y)^2\,dy = .5781$, or about 58% of the time (quite common).

4.42 The distribution function is $F(y) = \frac{y-\theta_1}{\theta_2-\theta_1}$, for $\theta_1 \le y \le \theta_2$. For $F(\phi_{.5}) = .5$, then $\phi_{.5} = \theta_1 + .5(\theta_2 - \theta_1) = .5(\theta_2 + \theta_1)$. This is also the mean if the distribution.

4.43 Let $A = \pi R^2$, where R has a uniform distribution on the interval (0, 1). Then,

$$E(A) = \pi E(R^2) = \pi\int_0^1 r^2\,dr = \frac{\pi}{3}$$

$$V(A) = \pi^2 V(R^2) = \pi^2[E(R^4) - \left(\frac{1}{3}\right)^2] = \pi^2\left[\int_0^1 r^4\,dr - \left(\frac{1}{3}\right)^2\right] = \pi^2\left[\frac{1}{5} - \left(\frac{1}{3}\right)^2\right] = \frac{4\pi^2}{45}.$$

4.44 **a.** Y has a uniform distribution (constant density function), so $k = 1/4$.

b. $$F(y) = \begin{cases} 0 & y < -2 \\ \int_{-2}^{y} \frac{1}{4}\,dy = \frac{y+2}{4} & -2 \le y \le 2 \\ 1 & y > 2 \end{cases}$$

4.45 Let Y = low bid (in thousands of dollars) on the next intrastate shipping contract. Then, Y is uniform on the interval (20, 25).
a. $P(Y < 22) = 2/5 = .4$
b. $P(Y > 24) = 1/5 = .2$.

4.46 Mean of the uniform: $(25 + 20)/2 = 22.5$.

4.47 The density for Y = delivery time is $f(y) = \frac{1}{4}$, $1 \le y \le 5$. Also, $E(Y) = 3$, $V(Y) = 4/3$.
a. $P(Y > 2) = 3/4$.
b. $E(C) = E(c_0 + c_1Y^2) = c_0 + c_1E(Y^2) = c_0 + c_1[V(Y) + (E(Y))^2] = c_0 + c_1[4/3 + 9]$

4.48 Let Y = location of the selected point. Then, Y has a uniform distribution on the interval (0, 500).
a. $P(475 \le Y \le 500) = 1/20$
b. $P(0 \le Y \le 25) = 1/20$
c. $P(0 < Y < 250) = 1/2$.

4.49 If Y has a uniform distribution on the interval (0, 1), then $P(Y > 1/4) = 3/4$.

4.50 Let Y = time when the phone call comes in. Then, Y has a uniform distribution on the interval (0, 5). The probability is $P(0 < Y < 1) + P(3 < Y < 4) = .4$.

4.51 Let Y = cycle time. Thus, Y has a uniform distribution on the interval (50, 70). Then,

$$P(Y > 65 \mid Y > 55) = P(Y > 65)/P(Y > 55) = .25/(.75) = 1/3.$$

4.52 Mean and variance of a uniform distribution: $\mu = 60$, $\sigma^2 = (70-50)^2/12 = 100/3$.