

# The Role of Education in Development\*

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First Version: September 2006

This Version: February 2007

## Abstract

Most education around the globe is public. Moreover, invest rates in education as well as schooling attainment differ substantially across countries. We construct a general equilibrium life-cycle model that is consistent with these facts. We provide simple analytical solutions for the optimal educational choices, which may entail pure public provision of education, and their general equilibrium effects. We calibrate the model to fit cross-country differences in demographic and educational variables. The model is able to replicate a number of key regularities in the data beyond the matching targets. We use the model to identify and quantify sources of world income differences, and find that demographic factors, in particular mortality rates, explain most of the differences. We also use the model to study the role of public education, and the effects of the HIV/AIDS pandemic in development.

*Keywords:* schooling, life expectancy, mortality rates, cross-country income differences.

*JEL Classification:* I22, J24, O11.

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\*We thank Mark Bills, Daniele Coen-Pirani, David N. DeJong, Pohan Fong, Paul Gomme, Tatiana Koreshkova and participants in seminars at the University of Rochester, Concordia University, and University of Delaware for their valuable comments.

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# 1 Introduction

The provision of education around the world has three salient features. First, public education is the predominant form of education. It accounts on average for 85% and 78% of primary and secondary enrollment respectively, a fact illustrated in Figure 1 for a set of 91 countries. Second, richer countries invest significantly more resources in education per pupil than poor countries. For example, the US invests 200 times more resources per pupil a year than the average of the 5 poorest countries in our sample. Figure 2 illustrates an aspect of this fact. It portrays the investment rate in education, defined as the ratio of public expenditures in education per pupil to per-capita GDP, against per-capita GDP. This investment rate differs substantially across countries and increases with income. The third feature is well-known: schooling attainment also increases with income and differs substantially across countries (Barro and Lee, 2000).

The purpose of this paper is to construct a general equilibrium model of education and human capital accumulation that is consistent with the set of facts mentioned above. In particular, we study the role public education and investments in education, as well as years of schooling, in the formation of human capital and in the wealth of nations. Our analysis has two main distinctive features. First, in our model young individuals face credit market frictions and the government alleviates them by providing public education. This is a departure from the frictionless Ben-Porath model that has been the focus of recent research on human capital formation, including Manuelli and Seshadri (MS, 2005), Hendricks (2005), and Hugget, Ventura and Yaron (2006), among others. Second, we calibrate the key parameters of the human capital production function by targeting moments of the cross-country distribution of schooling and returns to schooling. Competing papers instead only employ U.S. evidence to calibrate key parameters. Thus, our calibration strategy avoids issues that arise due to the special nature of the U.S. educational system.

We devise a simple model that incorporates key features of the data regarding education and development, and yet has simple closed-form solutions. This allows us to identify the mechanisms at work and properly quantify the contribution of different fundamentals, such as TFP, public education expenditures, mortality rates and taxes in explaining income differences across countries.

We show that the model performs well in other dimensions beyond the matching targets. For example, although we only target the average return to schooling in the calibration, the model replicates the overall distribution of returns better than alternative well-known models. Also, our model replicates the prevalence of public education around the world even though private education is also available in the model. Overall, we conclude that our model provides a plausible quantitative theory of schooling, human capital, and incomes.

We then employ the model for several purposes. First, we compute the long-run elasticity of output to TFP. As argued by MS, this elasticity increases substantially in models with endogenous human capital and implies that given TFP differences result in larger income differences. We find an elasticity of 2.4 in our model which is substantially lower than the one found by MS of 9. On the other hand, our estimate is close to the 2.8 value found by Erosa, Koreshkova and Restuccia (EKR, 2006) although using a different model. Our estimate implies that smaller but still important TFP differences are required to explain income differences across countries compared to models of exogenous human capital. In contrast, MS' large elasticity implies that no TFP differences are required to explain income differences.<sup>1</sup>

Second, we construct human capital stocks for a set of 91 countries and compare the results to existing alternatives. We find that our estimates are more disperse and significantly lower for poor countries than the ones estimated using Mincer style equations. The reason is that Mincer equations abstract from expenditures in education while our human capital production function includes both years of schooling and expenditures as inputs. Since public expenditures per pupil are positively correlated with per capita income, including differences in education expenditures across countries makes human capital much lower in poorer countries. These findings are qualitatively similar to those of MS and EKR, and quantitatively close to those of Córdoba and Ripoll (CR, 2004).

Our human capital estimates are significantly different from Hendricks (2002) who finds relative minor differences in human capital across countries using earnings of immigrants in the US.

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<sup>1</sup>This comparison is between the complete models that include, among others things, variation in demographics and in the price of capital.

However, we show that both estimates can be conciliated if immigrants are positively self-selected to some minor extent. For example, we find that a typical Mexican immigrant would need close to 50% more human capital than a non-immigrant with same schooling for both estimates to be identical. Our estimates are roughly consistent with those of CR who use a completely different methodology based on country specific rural-urban wage gaps to estimate human capitals.

Third, we assess the sources of cross-country income differences according to the model. A simple variance decomposition exercise shows that the role of TFP in explaining steady state income differences decreases significantly compared to models with exogenous human capital, from 60% to 34%. In addition, counterfactual exercises show that eliminating TFP differences would reduce the world variance of log incomes by 48%, while eliminating differences in mortality rates would reduce this variance by 51%. Finally, eliminating differences in the provision of public education would do so by 18%. We thus find that TFP still plays a major role in explaining income differences, but that mortality differences are at least as important.

Finally, we use the model to assess the effects of the HIV/AIDS pandemic on long run development. This pandemic has increased mortality rates dramatically particularly in Sub-Saharan Africa. Contrary to some claims in the literature, we find that the long run consequences of the pandemic are devastating for development. For example, the model predicts that if rates of infection persist at the current levels, the log variance of income would increase in 28%.

Our paper is related to the recent quantitative work by MS and EKR regarding human capital differences across countries using microfounded models of human capital formation. The first paper derives the implication of a frictionless Ben-Porath model for human capital differences, while the second builds upon a heterogenous agent model in the Bewley tradition. MS consider pure private education, a feature inconsistent with Figure 1. Our model is similar to MS in its use of a representative agent model and its treatment of demographic factors, but we focus on the role of public education and credit market frictions in the formation of human capital, and our calibration strategy is very different. While MS calibrate the key parameters of the human capital technology to match life-cycle earning in the US, we calibrate these parameters to match features

of the cross-country distribution of schooling and returns to schooling. We arrive to substantially different quantitative results.

On the other hand, our model is similar to EKR in the treatment of public education and in some aggregate results. The models, however, are very different. In their model, TFP is the only source of variation explaining income and schooling differences, while mortality rates, as well as TFP, play the key role in our model. EKR do not consider the role of mortality rates in generating differences in human capital accumulation across countries. In contrast, MS, Bils and Klenow (2000) and Ferreira and Pessoa (2003) find, as we do, that these differences in mortality are important. Finally, EKR calibrate their model to match moments of the cross sectional distribution of earnings and schooling in the US, while we target the cross-country distribution of schooling and returns.

Finally, in our model permanent TFP differences do not affect years of schooling while they do in MS and EKR. This feature of their models provides an additional channel for TFP to affect income. However, it also implies that their models do not possess a balanced growth path, while ours does. To the extent that a balanced growth path is regarded as a desirable property of a growth model, our model provides a plausible theory of human capital and income differences. A further advantage of our modeling approach is that we are able to obtain closed-form solutions for most of our results.

The remaining of the paper is organized as follows. Section 2 lays down the model. Section 3 calibrates it and provides overidentifying tests. Section 4 reports human capital estimates. Section 5 presents the implications of the model for cross-country income differences as well as some counterfactual exercises. Section 6 reports the results for HIV/AIDS pandemic. Section 7 concludes.

## 2 The Model

Consider an economy composed of  $I$  generations of individuals, competitive firms hiring labor and renting capital to produce final goods, and competitive mutual funds managing individuals savings and the capital stock of the economy. Denote  $N_i$ , for  $i = \{1, \dots, I\}$ , the population size of generation

$i$ , and  $N$  the total population. Schooling-age individuals ( $i = 1$ ), or "children" for short<sup>2</sup>, can work or go to school while adults ( $i = \{2, \dots, I\}$ ) only work. A period in the model last for  $T$  years, and individuals live for a maximum of  $I \cdot T$  years.

**Demographics** Of any given cohort only a fraction  $\pi_i$  survive to age  $i$ . We define  $\pi_1 = 1$  and  $\pi_{I+1} = 0$ . Life expectancy at birth is given by  $T \cdot \sum_{i=1}^I \pi_i$ . The population size of children satisfy  $N_1 = fN_2$ , where  $f$  is the fertility rate of young adults ( $i = 2$ ). Only young adults have children. We focus on a steady state demographic situation in which total population is growing at a constant rate. Since  $N'_1 = fN'_2 = f\pi_2N_1$ , where ( $'$ ) denotes next period, then the common growth rate for all groups along a steady state is:

$$\frac{N'}{N} = f\pi_2. \quad (1)$$

Moreover, in steady state the age composition of the population is stable and described by (see details in the Appendix):

$$n_i \equiv \frac{N_i}{N} = \frac{\pi_i \cdot (f\pi_2)^{-i}}{\sum_{j=1}^I \pi_j \cdot (f\pi_2)^{-j}}. \quad (2)$$

**The Production of Human Capital** Children's human capital,  $h_1$ , is assumed to be a fraction  $\theta_1$  of the *average* parental human capital,  $\bar{h}_p$ . Young adults' human capital,  $h_2$ , depends on human capital investments undertaken during childhood:

$$h_2 = z(\chi + s)^{\gamma_1} x^{\gamma_2} \quad (3)$$

where  $[z, \gamma_1, \gamma_2] > 0$  are technological parameters,  $\chi$  are pre-school years,  $s$  are years of formal schooling,  $x$  are educational expenditures in the form of final goods. Human capital for later periods of life are assumed to be a proportion  $\theta_i \geq 0$ ,  $i \in \{3, \dots, I\}$ , of  $h_2$ . Thus, human capital investments during childhood determine the whole path of human capitals during the individual's life. The parameters  $\theta_i$  allow to capture the life-cycle profile of earnings while  $h_2$  determines its level. For example,  $\theta_i = 1$  represents a flat life-cycle earning profile at the level  $h_2$ . For completeness,

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<sup>2</sup>In the calibration below, "children" are individuals between the ages of 6 to 26 years.

define  $\theta_2 \equiv 1$ .

**Individuals' Problem** Individuals maximize their lifetime welfare as described by:

$$\sum_{i=1}^I \pi_i \rho^{i-1} \ln c_i$$

where  $c_i$  is consumption at age  $i$ , and  $\rho$  is a discount factor. Moreover, individuals face the following resource and technological restrictions during their life time:

$$\bar{w}h_1(1 - s/T) = c_1 + e, \quad e \geq 0, \quad s \in [0, T]. \quad (4)$$

$$\bar{w}h_i + (1 + \bar{r})a_{i-1} = c_i + a_i, \quad a_1 = a_I = 0, \quad i = 2, \dots, I. \quad (5)$$

$$h_1 = \theta_1 \bar{h}_p, \quad (6)$$

$$h_2 = z(\chi + s)^{\gamma_1} (p + e)^{\gamma_2}, \quad (7)$$

$$h_i = \theta_i h_2 \text{ for } i = 3, \dots, I. \quad (8)$$

where  $\bar{w} = (1 - \tau)w$  is the after-tax wage,  $\tau$  is a proportional tax on income,  $a_i$  are savings at age  $i$ ,  $\bar{r}$  is the net return on savings,  $e$  are private expenses in education, and  $p$  are public expenditures in education per child. Individuals maximize over  $\{c_i, a_i, h_i\}_{i=1}^I, s, e\}$ .

The first restriction states that the after-tax labor income during the first period of life can be used to consume or to invest in education. In particular, children cannot borrow. Equations in (5) are the set of budget constraints during adulthood. The remaining equation describe the accumulation of human capital. Notice that children receive public subsidies for education,  $p$ , which enter directly in the constraint (7).

Although parents in our life-cycle formulation are not altruistic, they do help their children in two ways. First, they provide them with an earning potential during their schooling-age years that is tied to their own earning potential. This is a form of “unintended” bequest that allows children

to consume and invest in education. Moreover, it implies that individuals in richer economies can consume and invest more in education than individuals in poorer economies. Second, parents also help children by financing the public education system through proportional income taxes. This form of social “bequest” reinforces the ability of children in richer economies to afford more consumption and investment.

**Firms** Output is produced by a representative competitive firm operating a Cobb-Douglas technology:

$$Y = AK^\alpha H^{1-\alpha}, \quad 0 < \alpha < 1,$$

where  $Y$  denotes output,  $K$  represent physical capital services, and  $H$  represents human capital services. The firm hires labor and capital in competitive markets at the rates  $w$  per unit of human capital and  $r$  per unit of capital. Profit maximization entails the following conditions:

$$w = (1 - \alpha) \frac{Y}{H}, \quad r = \alpha \frac{Y}{K}. \quad (9)$$

**Mutual Funds** Individuals deposit their savings in mutual funds (MFs). MFs own the capital stock of the economy, and rent it to firms at the rate  $r$ . MFs operate a constant returns to scale technology that transform  $q$  units of output into 1 unit of capital. Thus,  $q$  is the price of capital in terms of final goods, the numeraire. MFs are competitive, pay proportional taxes  $\tau$  on earned income, and returns  $\bar{r}$  to the surviving depositors. Without loss of generality, consider a single MF that holds all the capital stock in the economy,  $K' = \sum_{i=1}^I N_i a_i / q$ . Free entry guarantees zero profits so that:

$$((1 - \delta)q + r(1 - \tau))K' = (1 + \bar{r}) \sum_{i=1}^I \frac{\pi_{i+1}}{\pi_i} N_i a_i,$$

where  $\delta$  is the depreciation rate of capital and  $\pi_{i+1}/\pi_i$  is the fraction of the population of age  $i$  that reaches age  $i + 1$ . Thus:

$$1 + \bar{r} = (r(1 - \tau)/q + 1 - \delta) \frac{\sum_{i=1}^I N_i a_i}{\sum_{i=1}^I \frac{\pi_{i+1}}{\pi_i} N_i a_i}.$$

Note that for the case of constant survival probability,  $\pi_i = \pi_{i-1}$ , this equation becomes the more standard expression  $1 + \bar{r} = (r(1 - \tau)/q + 1 - \delta)/\pi$ .

**Government** The government collects proportional taxes on capital and labor income to finance government expenditures,  $G$ , that is,  $G = \tau(wH + rK)$ . Using (9) this equation can be written as  $G = \tau Y$ . Moreover, a fraction  $0 < \varepsilon < 1$  of  $G$  is allocated to public education:  $N_1 p = \varepsilon G$ . Thus:

$$p = \varepsilon \tau Y / N_1. \quad (10)$$

**Resource Constraints** Gross output, which includes undepreciated capital, can be used for consumption by all individuals ( $C$ ), private education, government services, or to accumulate capital:

$$Y + (1 - \delta)qK = C + N_1 e + G + qK'.$$

Next period capital stock,  $K'$ , is the sum of individuals' savings divided by  $q$ . Finally, aggregate human capital,  $H$ , is the sum of individuals' human capital available for production.

**Definition of Equilibrium** Let  $y = Y/N$ ,  $k = K/N$ , and  $h = H/N$ . The following is our definition of equilibrium:

**Definition** A stationary competitive equilibrium for given taxes  $\tau$  are prices  $\bar{w}$  and  $\bar{r}$ , quantities

$\left\{ \{c_i^*, a_i^*, h_i^*\}_{i=1}^I, s^*, e^* \right\}$ ,  $\bar{h}_p$ ,  $k$ ,  $y$ , and  $h$ , and subsidies  $p$  such that (i) given  $\bar{h}_p$ , prices, taxes, and subsidies,  $\left\{ \{c_i^*, a_i^*, h_i^*\}_{i=1}^I, s^*, e^* \right\}$  solves the individual problem; (ii) given  $y$ ,  $k$  and  $h$ ,  $\bar{w}$  and  $\bar{r}$  satisfy  $\bar{w} = (1 - \tau)(1 - \alpha)y/h$ ,  $1 + \bar{r} = (\alpha \frac{y}{k}(1 - \tau)/q + 1 - \delta) \left( \sum_{i=1}^I N_i a_i^* / \sum_{i=1}^I \frac{\pi_{i+1}}{\pi_i} N_i a_i^* \right)$ ; (iii)  $\bar{h}_p = h_2^*$ ,  $h = (1 - s^*/T)n_1 h_1^* + \sum_2 n_i h_i^*$ ,  $qk = \sum_{i=1}^I n_i a_i^*$ ,  $y = Ak^\alpha h^{1-\alpha}$ ,  $p = \varepsilon \tau y / n_1$ .

Note that in a stationary equilibrium:

$$h = \left( (1 - s^*/T) n_1 \theta_1 + \sum_2 \theta_i n_i \right) h_2^*, \text{ and} \quad (11)$$

$$h_2^* = z (\chi + s^*)^{\gamma_1} (p + e^*)^{\gamma_2}. \quad (12)$$

## 2.1 Characterization of the Equilibrium

**Individuals' Problem** We now characterize the solution to the individuals' problem given prices and policies. Optimal choices may imply corner solutions. We denote “public education” the case in which  $e^* = 0$  and  $s^* > 0$ , “no formal education” the case in which  $s^* = 0$  and  $e^* > 0$ , “mixed education” the case  $e^* > 0$  and  $s^* > 0$ , and “no education” the case  $e^* = 0$  and  $s^* = 0$ .

The following proposition describes optimal educational choices. All proofs are in the Appendix.

First define:

$$\tilde{\rho} \equiv \sum_{i=2}^I \rho^{i-1} \pi_i; \quad \bar{m} \equiv \frac{\tilde{\rho} \gamma_2 (1 + \chi/T)}{1 + \tilde{\rho} \gamma_1}; \quad \underline{m} \equiv \frac{(1 + \tilde{\rho} \gamma_2) \chi/T}{\tilde{\rho} \gamma_1} - 1$$

The following assumption facilitates the presentation of the results (reduces the number of cases to present), but plays no essential role in the derivations or in the quantitative work.

**Assumption 1**  $\tilde{\rho} \gamma \psi \geq \chi/T$ .

This assumption guarantees that pre-school is not too important as to make formal schooling unattractive. It is easy to check that under Assumption 1  $\bar{m} > \underline{m} > 0$ .

**Proposition 1** Given prices and policies, the optimal solution for schooling years,  $s^*$ , and private expenditures in education,  $e^*$ , are:

$$s^* = \begin{cases} 0 & \text{for } \underline{m} > \frac{p}{wh_1} \\ \frac{\tilde{\rho} \gamma_1 T}{1 + \tilde{\rho} (\gamma_1 + \gamma_2)} \left[ \frac{p}{wh_1} - \underline{m} \right] & \text{for } \bar{m} \geq \frac{p}{wh_1} \geq \underline{m} \\ \frac{\tilde{\rho} \gamma_1 T - \chi}{1 + \tilde{\rho} \gamma_1} & \text{for } \frac{p}{wh_1} > \bar{m} \end{cases},$$

$$\frac{e^*}{\bar{w}h} = \begin{cases} \frac{\tilde{\rho}\gamma_2 - p/\bar{w}h_1}{1 + \tilde{\rho}\gamma_2} & \text{for } \underline{m} > \frac{p}{\bar{w}h_1} \\ \frac{1 + \tilde{\rho}\gamma_1}{1 + \tilde{\rho}(\gamma_1 + \gamma_2)} \left[ \bar{m} - \frac{p}{\bar{w}h_1} \right] & \text{for } \bar{m} \geq \frac{p}{\bar{w}h_1} \geq \underline{m} \\ 0 & \text{for } \frac{p}{\bar{w}h_1} > \bar{m} \end{cases} .$$

Proposition 1 is illustrated in Figure 3.<sup>3</sup> It typifies educational choices in three categories depending on the ratio of public subsidies to potential earnings during childhood,  $p/(\bar{w}h_1)$ . Informally, the proposition shows that lack of public funding may prevent any formal schooling to take place, and too much public funding may crowd-out completely private investments in education.

More formally, according to Proposition 1 the “public education” case arises when  $p > \bar{m}\bar{w}h_1$ . In this situation, individuals do not invest resources but only time in formal education. Since the threshold level depends on labor income during childhood, the model predicts that poorer individuals would be more likely to choose only public education than richer individuals. The “mixed education” case occurs when the amount of public funding falls within an intermediate range ( $\bar{m} \geq p/(\bar{w}h_1) \geq \underline{m}$ ). Finally, the case of “no formal education” occurs when the public provision of education is below certain threshold ( $p/(\bar{w}h_1) < \underline{m}$ ). In this case, insufficient funding discourages formal schooling. Under Assumption 1, the case “no education” cannot arise.

Proposition 1 also has the following implications for the role of public education on educational outcomes. First, additional public funding increases years of schooling only if  $\bar{m} \geq p/(\bar{w}h_1) \geq \underline{m}$ . Otherwise, years of schooling do not respond to changes in public funding. An interpretation of this result is that increasing public funding of education does not affect years of schooling of individuals already in public schools, although it enhances their human capital. Second, private funding responds less than one to one to changes in public funding. Specifically,  $|\partial e/\partial p| < 1$  whenever  $e > 0$ . Thus, additional public funding crowds out private funding but not completely, despite public and private funding being perfect substitutes in the production function. The reason is that public funding relaxes borrowing constraints which hinder private funding in the first place.

<sup>3</sup>A useful result to check the continuity of the choices at the threshold levels is to note that:

$$\bar{m} - \underline{m} = \frac{1 + \tilde{\rho}\psi(\gamma_1 + \gamma_2)}{(1 + \tilde{\rho}\psi\gamma_2)\tilde{\rho}\psi\gamma_1} (\tilde{\rho}\psi\gamma_1 - \chi/T).$$

Proposition 1 also highlights the critical role of demographic factors in educational choices. It states that individuals in economies with lower mortality rate, larger  $\tilde{\rho}$ , would choose more years of schooling and larger private funding of education for any given size of the public sector. As we see below, this mechanism turns out to explain most of the differential in schooling across countries. Moreover, Proposition 1 also states that  $z$ , the learning ability parameter, does not affect the choice of schooling years or educational expenditures. Our model predicts that children do not self-select into a pure public, private, or mixed educational regime based on their learning ability. In particular, high ability children do not invest more years or resources in education than low ability children.

Finally, Proposition 1 states that private funding of education depends on potential earning levels ( $\bar{w}h_1$ ) but years of schooling depends on the *ratio* of public education spending to earnings ( $p/(\bar{w}h_1)$ ). An important implication of this result, to be described below in detail, is that TFP differences would induce differences in educational expenses but not in years of schooling. In this regard, our model has different implications from the models analyzed by MS and EKR, models in which years of schooling depend on TFP levels.

**Education in General Equilibrium** The following proposition characterizes the educational choices in general equilibrium. For this purpose, the ratio  $p/(\bar{w}h_1)$  can be solved from equations (10), (9) and (11) as:

$$\frac{p}{\bar{w}h_1} = \frac{\varepsilon\tau(1 - s/T + \sum_2 \theta_i n_i / (\theta_1 n_1))}{(1 - \tau)(1 - \alpha)}. \quad (13)$$

Note that  $\varepsilon\tau$  is the share of public education in output (see equation 10). Substituting this result into Proposition 1 yields Proposition 2. Proofs are in the Appendix. First define:

$$\bar{s}_H \equiv \frac{(1 - \tau)(1 - \alpha)\bar{m}}{(1 + \chi/T + \bar{m}) / (1 + \tilde{\rho}) + \sum_2 \theta_i n_i / (\theta_1 n_1)}; \underline{s}_H \equiv \frac{(1 - \tau)(1 - \alpha)\underline{m}}{1 + \sum_2 \theta_i n_i / (\theta_1 n_1)}$$

**Proposition 2** The optimal solution for schooling years,  $s^*$ , and the share of educational expen-

ditures in output,  $s_H^*$ , are given by:

$$s^* = \begin{cases} 0 & \text{for } \underline{s_H} > \varepsilon\tau \\ \frac{(\varepsilon\tau - \underline{s_H})\tilde{\rho}\gamma_1 T \bar{m}/\underline{s_H}}{1 + \tilde{\rho}(\gamma_1 + \gamma_2 + \varepsilon\tau\gamma_1/((1-\tau)(1-\alpha)))} & \text{for } \bar{s_H} \geq \varepsilon\tau \geq \underline{s_H} \\ \frac{\tilde{\rho}\gamma_1 T - \chi}{1 + \tilde{\rho}\gamma_1} & \text{for } \varepsilon\tau > \bar{s_H} \end{cases} ,$$

$$s_H^* = \begin{cases} \varepsilon\tau + \frac{(1-\tau)(1-\alpha)}{1 + \sum_2 \theta_i n_i / (\theta_1 n_1)} \frac{\tilde{\rho}\gamma_2 - \varepsilon\tau \bar{m}/\underline{s_H}}{1 + \tilde{\rho}\gamma_2} & \text{for } \underline{s_H} > \varepsilon\tau \\ \varepsilon\tau + \frac{(1-\tau)(1-\alpha)}{1 - s^*/T + \sum_2 \theta_i n_i / (\theta_1 n_1)} \frac{(\bar{s_H} - \varepsilon\tau)(1 + \tilde{\rho}\gamma_1)\bar{m}/\bar{s_H}}{1 + \tilde{\rho}(\gamma_1 + \gamma_2 + \varepsilon\tau\gamma_1/((1-\tau)(1-\alpha)))} & \text{for } \bar{s_H} \geq \varepsilon\tau \geq \underline{s_H} \\ \varepsilon\tau & \text{for } \varepsilon\tau > \bar{s_H} \end{cases} .$$

Proposition 2 is analogous to Proposition 1. It characterizes educational choices in terms of the size of public education. If the public provision of education is too large no private funds are invested in education, while if it is too small then no schooling takes place. Proposition 2 also states that years of schooling depend on demographic parameters and on public policy parameters but it is independent of the TFP level  $A$ . As a result, differences in TFP in our model are not amplified through years of schooling, as is the case in MS and EKR. Due to this feature our model possesses a balanced growth path, while MS's and EKR's do not. On the other hand, in all three models TFP differences produce differences in the “quality of schooling”: larger TFP increases human capital through its effect on educational expenditures.

Similarly, the share of educational expenditures in output depends on demographic and policy parameters, but it is independent of TFP. For the cases in which  $s_H^* > \varepsilon\tau$ ,  $s_H^*$  decreases with  $\varepsilon\tau$  but less than one-to-one. This is because lower taxes increase private wealth and therefore savings in the form of private educational investments, which partly offsets the fall in public education.

**Production in General Equilibrium** Given the general equilibrium results for educational variables,  $s^*$  and  $s_H^*$ , provided in Proposition 2, we can now derive some implications of the model for aggregate output, and role of different parameters in explaining output differences across countries.

The following Proposition provides the solution for the equilibrium level of output. First, define

$$\begin{aligned}\Theta_j(\bar{r}) &\equiv \frac{\pi_j \rho^{j-1} (1 + \bar{r})^j}{\tilde{\rho}} \sum_{i=2}^I \frac{\theta_i}{(1 + \bar{r})^i} \text{ for } j \geq 2; \\ \Phi(\bar{r}) &\equiv \sum_{i=2}^I n_i \sum_{j=2}^i (1 + \bar{r})^{i-j} (\theta_j - \Theta_j(\bar{r})); \text{ and} \\ \Phi_a(\bar{r}) &\equiv \sum_{i=2}^I \frac{\pi_{i+1}}{\pi_i} n_i \sum_{j=2}^i (1 + \bar{r})^{i-j} (\theta_j - \Theta_j(\bar{r}))\end{aligned}$$

**Proposition 3** The equilibrium level of output is given by:

$$y = A^{\frac{1}{(1-\alpha)(1-\gamma_2)}} \left(\frac{k}{y}\right)^{\frac{\alpha}{(1-\alpha)(1-\gamma_2)}} \left(\frac{h}{y^{\gamma_2}}\right)^{\frac{1}{1-\gamma_2}} \quad (14)$$

where

$$\frac{h}{y^{\gamma_2}} = \left( (1 - s^*/T) n_1 \theta_1 + \sum_2 \theta_i n_i \right) z (\chi + s^*)^{\gamma_1} (s_H^*/n_1)^{\gamma_2}, \quad (15)$$

$$\frac{k}{y} = \frac{(1 - \alpha)(1 - \tau)/q}{(1 - s^*/T) n_1 \theta_1 + \sum_2 \theta_i n_i} \Phi(\bar{r}), \quad (16)$$

$$\bar{r} = \left( 1 - \delta + \alpha(1 - \tau) \frac{y}{k} / q \right) \frac{\Phi(\bar{r})}{\Phi_a(\bar{r})} - 1 \quad (17)$$

Equation (14) in Proposition 3 provides the determination of aggregate and per capita output in three components. The first component,  $A^{\frac{1}{(1-\alpha)(1-\gamma_2)}} = A^{1 + \frac{\alpha}{1-\alpha} + \frac{\gamma_2}{(1-\alpha)(1-\gamma_2)}}$ , collects the direct and indirect effects of TFP on steady state output. The elasticity of output with respect to TFP,  $1 + \alpha/(1 - \alpha) + \gamma_2/[(1 - \alpha)(1 - \gamma_2)]$ , can be itself decomposed in three parts. The first part captures the direct effect of TFP on output; the second part captures the indirect effect of TFP on output through its effect on capital accumulation; and the third part captures the indirect effect of TFP on output through its effect on human capital accumulation. This last effect has been recently stressed in the works of MS and EKR and its quantitative significance depends only the size of  $\gamma_2$ . If  $\gamma_2 = 0$ , then equation (14) adopts the more standard form used in the works of Hall and Jones (HJ, 1999) and Klenow and Rodriguez-Clare (KRC, 1997), among others.

The second component collects the direct and indirect effects of the capital-output ratio in output. According to equation (14), the elasticity of output with respect to the capital-output ratio,  $\alpha/[(1-\alpha)(1-\gamma_2)]$ , is also affected by  $\gamma_2$ . This elasticity can be written as  $\alpha/(1-\alpha) + \gamma_2/[(1-\alpha)(1-\gamma_2)]$ . The second term captures the effect that a higher capital-output has on the accumulation of human capital and on output.

The third component,  $(h/y^{\gamma_2})^{\frac{1}{1-\gamma_2}}$ , is related to the accumulation of human capital. In a standard formulation with exogenous human capital ( $\gamma_2 = 0$ ), this component is just the stock of human capital. If human capital is endogenous, this component captures the effect of educational decisions, which themselves are function of demographic and policy parameters, but independent of TFP, as shown in equation (15).

Equations (16) and (17) jointly determine the capital output ratio,  $k/y$ , and the net returns  $\bar{r}$ . Both variables depend on demographic factors  $(\pi_i, n_i)$ , years of schooling  $s^*$ , and taxes  $\tau$ . Importantly, they are independent of TFP, particularly because  $s^*$  is independent of TFP as stated in Proposition 2. The positive dependence of  $k/y$  years schooling is due to the fact that a larger fraction of students reduces labor supply, increases wages for all, in particular for adults, and therefore their savings. Thus, the effect of schooling on the term  $k/y$  captures an extensive margin, as more schooling reduces labor supply, while its effect on  $h/y^{\gamma_2}$  captures an intensive margin, as schooling increases adult's human capital.

**Mincerian Returns** Psacharopoulos and Patrinos (2002) have collected Mincerian returns for a fairly large set of countries. Matching these returns provides a natural test for any quantitative model of schooling. Mincer returns, or simply returns to schooling, are obtained from a cross-section of individuals in each country using a OLS regressions of the type:

$$\ln(d_i) = a + bs_i + cX_i + v_i \tag{18}$$

where  $d_i$  are earnings of individual  $i$ ,  $b$  is known as the Mincerian return, and  $X_i$  is a vector of controls. In order to use these estimates, we need to derive the theoretical counterpart of equation

(18) in our model. This requires to reformulate the model so that individuals in a given cohort are heterogenous in terms of earnings and schooling. Suppose only for this section that each cohort is composed of a cross-section of individuals that differ both in their learning ability,  $z_i$ , and in their discount parameter  $\tilde{\rho}_i$ . Yearly earnings during young adulthood,  $d_i$ , are given in our model by  $d_i = w(z_i(\chi + s_i)^{\gamma_1}(p + e_i)^{\gamma_2})/T$ . Alternatively, using Proposition 1:

$$d_i = \begin{cases} w \left( z_i \chi^{\gamma_1} \left( \frac{\gamma_2 \bar{w} h_1}{\gamma_1 T} \right)^{\gamma_2} \right) / T & \text{for } \underline{m} \geq \frac{p}{wh_1} \\ w \left( z_i (\chi + s_i)^{\gamma_1 + \gamma_2} \left( \frac{\gamma_2 \bar{w} h}{\gamma_1 T} \right)^{\gamma_2} \right) / T & \text{for } \bar{m} \geq \frac{p}{wh_1} \geq \underline{m} \\ w \left( z_i (\chi + s_i)^{\gamma_1} p^{\gamma_2} \right) / T & \text{for } \frac{p}{wh_1} \geq \bar{m}. \end{cases}$$

Collecting terms one can write this equation more compactly as:

$$\ln d_i = \begin{cases} a_1 + \gamma_1 \ln(\chi + 0) + v_{1i} & \text{for } \underline{m} \geq \frac{p}{wh_1} \\ a_2 + (\gamma_1 + \gamma_2) \ln(\chi + s_i) + v_{2i} & \text{for } \bar{m} \geq \frac{p}{wh_1} \geq \underline{m} \\ a_3 + \gamma_1 \ln(\chi + s_i) + v_{3i} & \text{for } \frac{p}{wh_1} \geq \bar{m}. \end{cases}$$

where the terms  $[a_1, a_2, a_3]$  collect all constant terms, and the variables  $v_{ji}$ ,  $j = \{1, 2, 3\}$ , collect the terms associated to  $z_i$ . A linear approximation of the schooling term on the right hand side of this equation around the steady state produces:

$$\ln d_i \simeq \begin{cases} \bar{a}_1 + v_{1i} & \text{for } \underline{m} \geq \frac{p}{wh_1} \\ \bar{a}_2 + \frac{\gamma_1 + \gamma_2}{\chi + s^*} s_i + v_{2i} & \text{for } \bar{m} \geq \frac{p}{wh_1} \geq \underline{m} \\ \bar{a}_3 + \frac{\gamma_1}{\chi + s^*} s_i + v_{3i} & \text{for } \frac{p}{wh_1} \geq \bar{m}. \end{cases} \quad (19)$$

Since according to Proposition 1,  $cov(s, v) = 0$ , then this expression provides a theoretical counterpart of equation (18) in our model. We use this expression below to assess the model's implication for Mincer returns.

### 3 Data and Empirical Implementation

#### 3.1 Calibration

We now calibrate the model to match certain features of the cross-country data. For this purpose we assume that the parameters  $\{A_j, q_j, \pi_{2j}, \dots, \pi_{Ij}, f_j, \tau_j, \varepsilon_j\}$  vary across countries while the parameters  $[\alpha, \delta, \theta_1, \dots, \theta_I, T, \gamma, \chi, z, \rho]$  are identical across countries. Individuals in the model are born at age 6 which roughly corresponds to the beginning of the schooling age in the data. Consistently, we set  $\chi = 6$ . Moreover, we consider 5 generations ( $I = 5$ ), and a time period length  $T = 20$ . In this specification, formal schooling takes place between the ages of 6 to 26, and individuals live up a maximum of 106 years. Parameter  $\alpha$  is set to 1/3 as in Mankiw, Romer and Weil (MRW, 1992), HJ, KRC, Bils and Klenow (BK, 2000), and others. Parameter  $\delta$  is set such that the annual depreciation rate is 0.06, which is the value assumed by HJ to compute the capital stocks that we use. This requires to set  $\delta = 1 - (1 - 0.06)^{20}$ .

**Calibration of  $\pi_j$  and  $f_j$**  We estimate  $\pi_{ij}$  as follows. We take the US life table of total population from National Vital Statistics Report by the US Census Bureau and, for each country  $j$ , we adjust the US age-specific mortality rates to reproduce the life expectancy of country  $j$  as reported by the World Bank. Mortality rates for ages 0 to 5 are adjusted using the World Bank data on mortality rates under age 5. Mortality rates over ages 5 are multiplied by a factor which allows to exactly replicate country's  $j$  life expectancy.  $\{\pi_{ij}\}_{i=2}^I$  is computed, using the results of the table, as the number of survivors at ages 36, 56, 76, and 96 respectively divided by the number of survivors at age 6.

Fertility rates,  $f_j$ , are computed using equation (2). According to this equation, the share of population under 26 is given by  $n_1 = (f\pi_2)^{-1} / \left(\sum_{j=1}^I \pi_j \cdot (f\pi_2)^{-j}\right)$ . We use data from the World Population Prospects (2004) to compute  $n_1$  for each country.<sup>4</sup>

An important consideration is the year of the data used for these computations. Since the late 70's mortality rates have increased significantly, particularly in Sub-Saharan Africa, due to the

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<sup>4</sup>Given data restriction, our data on  $n_1$  actually corresponds to population under 24.

HIV/AIDS pandemic. For this reason we choose 1980 demographic variables, which are still not affected by the pandemic and are likely more relevant to explain economic performance up to the mid 90's when the pandemic became more pronounced. We also conduct experiments below to assess the long run consequences of the HIV/AIDS pandemic.

Figure 4 shows the demographic data and the calibrated values of fertility and survival probabilities. A salient feature of the data is that survival rates are very different across countries and particularly low for very poor countries. The Figure also shows the estimated fertility rates relative to the US. They also decrease significantly with income levels, exhibit substantial difference across countries, and are particularly large for very poor countries.

**Calibration of  $\tau$  and  $\varepsilon_j$**  In the model,  $G = \tau Y$ . Accordingly, we set  $\tau$  equal to the share of government expenditures in GDP in 1995 from the Penn World Tables. Furthermore, we compute  $\varepsilon_j$  for each country as  $\varepsilon_j = s_{Hj}^p / \tau_j$ , where  $s_{Hj}^p$  is the share of public expenditures in education to GDP for 1995 according to UNESCO. Figure 5 shows the resulting tax rate,  $\tau_j$ , and Figure 2 shows subsidy per pupil as a fraction of per capita GDP  $p/y = s_{Hj}^p / n_1$ . While taxes decrease slightly with income, subsidies clearly increase with income specially because the fraction of schooling age children is systematically larger in poor countries than in rich countries.

**Calibration of  $\theta$**  We calibrate  $\theta = [\theta_1, ..\theta_5]$  to reproduce the life-cycle pattern of earnings. For this purpose, we employ the estimates of BK. Using cross-country evidence on earnings, they estimate that earnings at age  $i$  satisfy:

$$w(i) = a(s) \cdot e^{m(ex)},$$

where  $ex$  stands for years of experience and  $m(ex) = 0.0512 \cdot ex - 0.0071 \cdot (ex)^2$ . Assuming 0 years of experience for children, 20 for young adults, 40 for adults, and 60 for older adults, and no earnings for elders, we estimate  $\theta$  as  $\theta = [\theta_1 = e^{m(0)-m(20)}, \theta_2 = 1, \theta_3 = e^{m(40)-m(20)}, \theta_4 = e^{m(60)-m(20)}, \theta_5 = 0]$  which produces  $\theta = [0.48, 1, 1.19, 0.80, 0]$ .

**Calibration of  $\gamma_1$ ,  $\gamma_2$  and  $\rho$**  To calibrate the key parameters  $\gamma_1$ ,  $\gamma_2$ , and  $\rho$  we target moments of the cross-country distributions of returns to schooling and the average years of schooling. Returns to schooling are obtained from Psacharopoulos and Patrinos (2002), while average years of schooling in each country are obtained from Barro and Lee for the year 1995. Following the derivations in Section 3, we compute returns to schooling as  $r_s = (\gamma_1 + \gamma_2)/(\chi + s)$  for a mixed educational system,  $r_s = \gamma_1/(\chi + s)$  for a pure public system,  $r_s = (\gamma_1 + \gamma_2)/\chi$  for a pre-school only system, and  $r_s = \gamma_1/\chi$  for a no-schooling regime. The equilibrium educational system – whether mixed, public, or other – is obtained using the model (more precisely, Proposition 2) for different potential values of  $\gamma_1$ ,  $\gamma_2$ , and  $\rho$ . In these calculations, the values of other parameters are set at their calibrated values already described. Importantly, the calibration of  $\gamma_1$ ,  $\gamma_2$ , and  $\rho$  is independent of the parameter values described below ( $A_j$  and  $q_j$ ) and therefore robust to alternative specification of those parameters.

**Table 1: Calibration of  $\gamma_1, \gamma_2$  and  $\rho$**

	Data	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\gamma_2$	-	0.100	0.200	0.250	0.300	0.350	0.400	0.450	0.500
$\gamma_1$	-	1.080	1.035	0.969	0.893	0.803	0.722	0.637	0.579
$\rho$	-	0.970	0.972	0.975	0.978	0.984	0.990	0.999	1.008
$\bar{s}$	5.840	5.844	5.832	5.836	5.832	5.837	5.834	5.845	5.839
$\bar{r}_s$	0.097	0.097	0.097	0.097	0.097	0.097	0.098	0.097	0.097
$s_{std}$	2.890	0.986	1.042	1.124	1.239	1.403	1.554	<b>1.658</b>	1.633
$\bar{r}_{sstd}$	0.042	0.025	0.029	0.031	<b>0.033</b>	0.032	0.030	0.026	0.025

Table 1 reports statistics related to schooling and returns to schooling from the model and the data for different possible choices of  $\gamma_1$ ,  $\gamma_2$  and  $\rho$ . In these experiments,  $\gamma_2$  is allowed to vary (between 0 to 0.5) and  $\gamma_1$  and  $\rho$  are chosen to match the average years of schooling ( $\bar{s}$ ) and the average returns to schooling ( $\bar{r}_s$ ). The table illustrates different trade-offs in the choice of  $\gamma_2$ . For example,  $\gamma_2 = 0.30$  produces the best results in terms of the standard deviation of returns to schooling (0.042 in the data vs 0.033 in the model), while  $\gamma_2 = 0.45$  produces the best results in

terms of matching the standard deviation of schooling (2.89 vs 1.658). In both cases, however, the model falls short of explaining all variation in returns and schooling. We choose the following parameters:  $\gamma_2 = 0.40$ ,  $\gamma_1 = 0.722$  and  $\rho = 0.99$ .

**Calibration of  $q_j$**  Given parameters  $(\alpha, \delta, T, \rho, \theta, \tau_j, \pi_{ij}, f_j)$  already obtained, and years of schooling,  $s_j$ , equations (16) and (17) can be used to solve for  $q_j k_j / y_j$  and  $\bar{r}_j$  for each country. On the other hand, HJ provide capital-output ratios for a group of countries for 1996. They compute capital stocks using annual investments valued at common international prices rather than at domestic prices. Therefore, their capital-output ratio corresponds to that of  $k/y$  instead of  $qk/y$ , which allows domestic prices of capital,  $q_j$ , to vary across countries. Given  $q_j k_j / y_j$  predicted from the model and  $k_j / y_j$  from the data,  $q_j$  can be obtained.

### 3.2 Assessment

We now assess the ability of the model to fit evidence beyond the matching targets.

**Schooling** Panel A of Figure 6 compares schooling attainments in the data and in the model. Although we only target the mean of schooling, the model is able to replicate the entire distribution of schooling fairly well. The correlation between schooling in the data and in the model is 86%. Years of schooling in the data range from a minimum of 0.69 to a maximum of 12.18, and in the model they range between 1.73 and 8.55. Finally, the correlation between schooling and relative incomes is 85.3% in the data, versus 82.8% in the model.

**Returns to Schooling** Panel B of Figure 6 and Table 2 compare returns to schooling in the data and the model, as well as the predictions of two alternative models. The first model is that of HJ who estimate returns to schooling as 13.4% for first four years of schooling, 10.1% for the next four years, and 6.8% for the remaining years. The second model is one of the versions in BK—their intermediate version with moderate decreasing returns to schooling. In this version, returns to schooling satisfy the expression  $r_s = 0.18 \cdot s^{-0.28}$ .

**Table 2: Returns to Schooling**

	Data	Model	HJ Model	BK Model
$mean(r_s)$	<b>0.097</b>	<b>0.097</b>	0.101	0.1167
$stdev(r_s)$	0.042	0.030	0.0256	0.023
$correl(r_s, r_{smodel})$	-	0.345	0.345	0.302
$correl(y, r_s)$	-0.366	-0.773	-0.792	-0.680
$min(r_s)$	0.027	0.041	0.068	0.089
$max(r_s)$	0.288	0.168	0.134	0.200

Our model is able to reproduce the overall pattern of decreasing returns as income level increases, and 71% of the dispersion of returns. Similarly to the other two models, our model overpredicts the correlation between returns and relative incomes and underpredicts the dispersion of returns. However, it is encouraging that our model is comparable to alternative models despite the fact that our calibration only targets the mean of returns, while the competing models target the overall pattern of returns.

**School Regime and School Expenditures** Panel C of Figure 6 shows the predicted equilibrium educational system –mixed, public, etc.– for countries according to their relative income levels, and Panel D shows observed public, and both predicted private and total shares of educational expenditures in GDP. The model predicts that most countries (82 out of 91) adopt a mixed system, and 9 countries adopt a pure public system. For a subsample of 53 countries for which information on private education expenditures is available from the World Bank, the model predicts that a significant fraction of total education expenditures, 65.4% on average, corresponds to public education. This share is 77.5% in the data. For OECD countries, public education is on average 85.7% of total education expenditures, and the model predicts it is 83%. For the US, public education is 74.8% of total spending in education, and it is 87% in the model.

Finally, for the same subsample of 53 countries, the correlation between total education spending per pupil as a fraction of GDP per capita and GDP per capita relative to the US is 62.7% in the data, and 75.6% in the model. For the case of private education spending per pupil as a fraction

of GDP this correlation is  $-22.7\%$  in the data and  $-38.4\%$  in the model.

## 4 Human Capital Stocks

### 4.1 A Cross-Section

In this section we use the model to estimate human capital stocks for a set of 91 countries and compare the results to some existing alternatives. According to equations (11) and (12), human capital stocks are given by:

$$h_j = \left( (1 - s_j/T) n_{1j} \theta_1 + \sum_2 \theta_{ij} n_{ij} \right) z (\chi + s_j)^\gamma (p_j + e_j^*)^{1-\gamma}.$$

We use this formula to compute human capital stocks relative to the US, which eliminates the need to estimate  $z$ , a parameter assumed to be identical across countries. Notice that once the technological parameters are obtained, this formula could map observables  $n_{ij}$ ,  $s_j$  and  $p_j + e_j$  into human capitals. However, due to the lack of reliable data on private education expenditures we utilize the model's predictions on total expenditures to compute human capital stocks. We also report the results when only available information on public expenditures is employed.

Figure 7 and Table 3 characterize six different estimates of human capital stocks:

- (1) those from our model, denoted CR;
- (2) our model but using only public expenditures of education (CR-P);
- (3) Hendricks' (2002) estimates based on immigrants' earnings in the US and no self-selection<sup>5</sup>;
- (4) a version of BK that uses the formula  $h = e^{f(s)+0.0512 \cdot (age-s-6)-0.00071 \cdot (age-s-6)^2}$ , where  $f(s) = (0.18/0.72) s^{0.72}$ . These estimates are similar to those obtained by HJ;
- (5) MRW's estimates based on schooling (here we use primary plus secondary schooling as argued by KRC);

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<sup>5</sup>Using Hendricks' notation, we compute human capital in country  $c$  as  $w_c^\eta/w_c^k$  using the information in his Table 1. This measure of human capital includes both measured and unmeasured skills. See discussion in Section III.A in Hendricks (2002).

(6) Cordoba and Ripoll’s (CR, 2004) estimates based on rural-urban wage gaps.

Panel A of Figure 7 and the first two columns of Table 3 characterize the human capital estimates obtained with our model using either total educational expenditures or public expenditures only. Both estimates are highly correlated between them (correlation of 0.98) and with relative incomes. Moreover, relative human capitals are systematically higher than relative incomes: while the average relative income is 0.31 of the US, the average relative human capital is 0.42 or 0.39 of the US, depending on what expenditures are used. Although the overall properties of both estimates are similar, there are important country specific differences particularly for high income countries. For example, Hong Kong’s and Singapore’s relative human capital fall substantially when only public expenses are considered. Given that according to UNESCO, 93% of primary and 89% of secondary enrollment in Hong Kong is in private institutions, it seems clear that some significant imputation for private expenditures is required.<sup>6</sup> For this reason, we favor our human capital estimates that include imputed values for private expenditures in education.

**Table 3: Estimates of Average Human Capital Relative to the US**

	CR	CR-P	Hendricks	BK	MRW	CR2004
	(1)	(2)	(3)	(4)	(5)	(6)
<i>mean</i>	0.42	0.38	0.75	0.58	0.48	0.51
<i>stdev</i>	0.29	0.31	0.18	0.18	0.24	0.24
<i>min</i>	0.09	0.05	0.48	0.30	0.06	0.07
<i>max</i>	1.15	1.21	1.11	1.00	1	1.02
<i>corre(h, y)</i>	0.98	0.95	0.88	0.87	0.85	0.83

Panels B and C of Figure 7 and columns 3 to 5 of Table 3 compare our human capital estimates to those of BK, Hendricks (2002) and MRW. They are similar for high income countries but differ substantially for low income countries. Thus, while BK and Hendricks’ estimates imply relative minor differences in human capital between rich and poor countries, our estimates, as well as MRW’s estimates, imply substantial differences. The means of the estimates are 0.75 for Hendricks, 0.58

<sup>6</sup>Unfortunately, UNESCO does not report enrollment for Singapore or Taiwan.

for BK, 0.48 for MRW, and 0.42 for our model. Moreover, our estimates exhibit significant more dispersion than the alternatives.

A further comparison is to the estimates of Cordoba and Ripoll (2004). They construct human capital stocks for a set of countries weighting rural and urban estimates. They estimate urban stocks using a standard Mincer approach (as in HJ) while rural stocks are Mincer type estimates adjusted to account for the observed rural-urban wage gap in each country. Panel D of Figure 7 and the last column of Table 3 compares our estimates to those of CR (2004). The 2004 estimates are midway between our current estimates and those of BK, but both estimates provide a substantial revision of human capital stocks relative to previous estimates, and they are at some extent similar to those obtained by just taking average years of schooling relative to the US.

Finally, we can compare our human capital stocks with those of MS. Since they report average human capital relative to the US by income deciles, we construct equivalent measures for our estimates in Table 4. As shown in the table, our human capital estimates are consistently below MS's, proportionally more so for those countries below the 50th percentile. For instance, the average country in the 50th – 60th percentile has 60% of US's human capital according to MS, while it has only 31% of US's human capital according to our estimates. The only exception are those countries in the lowest decile: they have 10% of US's human capital according to our estimates, and 8% according to MS's.

Table 4 also reports BK's human capital relative to the US. We use BK's estimates to compute the implied "quality" differentials in human capital across deciles for both our model and MS's (see last two columns). In particular, the ratio between our human capital and BK's can be interpreted as the relative human capital quality. As shown in Table 4, quality differences exists at all income deciles in our estimates, while in MS they are only relevant for countries below the 40th percentile. Specifically, while the average country in the 40th – 50th percentile range has about the same human capital quality of the US according to MS, it has around 1/2 of the quality according to our estimates. As noted above, the exception is in the lowest decile. Our human capital estimates imply that a country in the lowest decile of the income distribution has a human capital quality

around 1/3 of the US, while it is around 1/4 according to MS. In conclusion, our human capital estimates imply quantitatively important adjustments, proportionally more so for countries below the 50th percentile of the world income distribution.<sup>7</sup>

**Table 4: Estimates of Average Human Capital Relative to the US by Decile**

Decile	CR	MS	BK	Quality index	
	(1)	(2)	(3)	CR	MS
90 – 100	0.97	0.95	0.90	1.08	1.06
80 – 90	0.80	0.88	0.76	1.05	1.16
70 – 80	0.63	0.79	0.71	0.89	1.11
60 – 70	0.43	0.71	0.65	0.66	1.09
50 – 60	0.31	0.60	0.56	0.55	1.07
40 – 50	0.27	0.50	0.51	0.53	0.98
30 – 40	0.23	0.43	0.48	0.48	0.90
20 – 30	0.18	0.32	0.43	0.42	0.74
10 – 20	0.13	0.20	0.38	0.34	0.53
0 – 10	0.10	0.08	0.33	0.30	0.24

## 4.2 Self-Selection of Immigrants

To assess how plausible our estimates are, remember that Hendricks’ estimates reported in Table 3 are not adjusted for possible self-selection of immigrants, while BK’s estimates are not adjusted for “quality” differences. Can the differences between Hendricks’s estimates and ours be explained by self-selection? Figure 8 Panel A plots the ratio of Hendricks’ estimates to our estimates for the subset of 62 common countries (under the name CR). This ratio spans from 0.93 to 4.25 and has an average of 1.90. This is a much lower degree of self-selection of what Hendricks (2002) argued was implausible. In particular, Hendricks argued against the degree of self-selection implied in his Figure 2, which spans from 1 to 12. Such large degree of self-selection would explain all

<sup>7</sup>At this point we are unable to compare our human capital estimates to those of EKR, since they do not report their estimates.

cross-country income differences without TFP differences. In contrast, our series require Mexican immigrants, the main source of immigration to the US, to have only around 50% more human capital than non-immigrants which is just a small fraction of the income gap between the two countries (of 4.36 times).

Our series imply that on average immigrants have 90% more human capital than non-immigrants, while the income gap on average is 10.27 times. Moreover, our 2004 estimates imply even much lower degrees of self-selection than our current estimates, as shown in Figure 8 Panel B (33% on average and 19% for Mexican immigrants).

**Table 5: Selection of Immigrants**

	Upper Bound	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\gamma_2$	-	0.10	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$\gamma_1$	-	1.08	1.03	0.96	0.89	0.80	0.72	0.63	0.57
$\rho$	-	0.97	0.97	0.97	0.97	0.98	0.99	0.99	1.00
$\overline{ss}$	3.070*	1.39	1.68	1.78	1.84	1.84	1.89	2.02	2.17
$ss_{\max}$	13.06*	2.27	3.24	3.57	3.81	3.94	4.24	4.74	5.35
$ss_{\min}$	0.690*	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46
$ss_{mex}$	2.060	1.21	1.46	1.55	1.54	1.48	1.48	1.54	1.62
$\overline{pss}$	0.810*	0.64	0.70	0.71	0.72	0.71	0.72	0.73	0.75
$pss_{\max}$	0.999*	0.89	0.96	0.97	0.97	0.97	0.98	0.99	0.99
$pss_{\min}$	0.320*	0.49	0.47	0.45	0.44	0.42	0.44	0.47	0.47
$pss_{mex}$	0.760*	0.57	0.64	0.66	0.66	0.64	0.64	0.66	0.67

$ss$  = self-selection (Hendricks human capital/our human capital),  $\overline{ss}$  = average self-selection,  $pss$  = percentile position of immigrant,  $\overline{pss}$  = average percentile.

Following Hendricks (2002, pg 208-9), we can also estimate the position of immigrants in the earnings distribution of the source country implied by our model by assuming that earnings are lognormal distributed, and using the Klaus Deininger and Lyn Squire's (1996) Gini coefficients

for income.<sup>8</sup> Figure 8B describes these results. The series denoted Hendricks is the upper bound he provided. Our series imply much lower degree of self-selection than the upper bound for most countries. Table 5 reports additional exercises for the different parametrization described in Table 1. Thus, for example, our benchmark parametrization ( $\gamma_2 = 0.4$ ) implies that the typical Mexican immigrant is drawn from the 65 percentile of the earnings distribution. This is consistent with the findings of Chiquiar and Hanson (2005), who find intermediate positive self-selection of Mexican immigrants. They find that the typical immigrant is drawn from the 72 percentile. Table 5 also illustrates that  $\gamma_2 = 0.35$  or  $\gamma_3 = 0.4$  produce the best results in terms of intermediate self-selection, which lends further (and strong) support to our benchmark calibration.

## 5 The Sources of Income Differences

**Development Accounting and the TFP Elasticity of Income** A key parameter in our model is  $\gamma_2$ . Our assessment of the previous section lends support to our estimate of  $\gamma_2$  as it gives rise to plausible predictions in terms of schooling, returns to schooling, and human capital estimates. This parameter controls, among other things, the extent up to which TFP differences explain cross country income differences. As stated in Equation (14), the elasticity of output to TFP in our model is given by  $\frac{1}{1-\alpha} \frac{1}{1-\gamma_2} = (1.5) \times (1.6) = 2.4$ . This is a significantly larger elasticity than what is obtained in model with exogenous human capital (of only 1.5). This elasticity is remarkably similar to the EKR estimate of 2.8 despite the substantially different modeling approaches. A crucial difference is that our model explains schooling differences mostly in terms of demographic differences, while EKR abstract from demographic differences and rely on TFP differences only.

On the other hand, our elasticity is much lower than the MS estimate of around 9. Their large elasticity implies that almost no differences in TFP are required to explain income differences, while important TFP differences are still required in our model. There are two differences in our modelling approaches that help explain the different results. First, TFP differences affect the steady

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<sup>8</sup>The Gini coefficient of a log-normal distribution satisfies  $G = 2\Phi\left(\frac{\sigma}{\sqrt{2}}\right) - 1$  where  $\Phi$  is the standard normal distribution. Thus,  $\sigma = \sqrt{2}\Phi^{-1}\left(\frac{1+G}{2}\right)$ .

state levels of both quality and quantity of schooling ( $e$  and  $s$ ) in MS while it only affects quality of schooling in our model. An implication is that our model possess a balanced growth path but MS does not. Second, our calibration targets cross-country differences in schooling and returns to schooling. They instead target the earning gains between ages 25 to 50 in the US. Thus, they assume that the technology of learning on the job is the same as the technology of learning in school. We instead formulate and calibrate two different technologies and tie the estimation of  $\gamma_2$  to schooling attainments and returns to schooling rather than to the life-cycle earning profile.

We now use our model to revisit the question of the sources of cross-country differences. The first question we address is the relative contribution of TFP differences versus other sources in explaining cross-country income differences. For this purpose we follow KRC and write equation (14) as:

$$Y = A_1 \cdot X_1$$

where  $A_1 \equiv A^{\frac{1}{(1-\alpha)(1-\gamma_2)}}$  and  $X_1 \equiv (K/Y)^{\frac{\alpha}{(1-\alpha)(1-\gamma_2)}} (H/Y^{\gamma_2})^{\frac{1}{(1-\gamma_2)}}$ .  $A_1$  captures the role of TFP in output, and  $X_1$  captures the role other variables such as demographics, public education, and taxes. We also follow KRC in using a variance decomposition. We thus define the contribution of  $A_1$  and  $X_1$  as  $CA_1 = cov(\ln A_1, \ln Y)/Var(\ln Y)$  and  $CX_1 = cov(\ln X_1, \ln Y)/var(\ln Y)$  respectively.

**Table 6: Development Accounting**

Value of $\gamma_2$	$CA_1$	$CX_1$		
	Total	$K/Y$	$H/Y$	Total
0.0	60%	15.4%	23.8%	40%
0.1	52%	17.2%	29.8%	48%
0.2	45%	19.3%	35.4%	55%
0.3	40%	22.1%	37.0%	60%
0.4	38%	25.7%	36.0%	62%
0.5	30%	30.9%	38.7%	70%

The first row of Table 6 reports the development accounting results in a standard model with

exogenous human capital. In particular we use the BK human capital estimates described above and assume  $\gamma_2 = 0$ . According to this model, most income differences are due to TFP differences (60%), a result consistent with the findings of KRC, HJ, and others. The second row of Table 6 reports the results obtained in our model. Perhaps surprisingly, the role of TFP decreases significantly, from 60% to 34%, when human capital is endogenous. It may seem surprising that the TFP elasticity of income increases but its total contribution decreases. However, as Proposition 3 states, not only the TFP elasticity increases but also the elasticity of the capital-output ratio increases. The overall effect is an increase in the role of other factors and a reduction in the role of TFP in explaining income differences. These results contrast to those of MS who find that, once variation in demographics and the price of capital are taken into account, no differences in TFP are needed to explain income differences. We find that TFP differences are still substantial, although not the main source of income differences.

**Counterfactuals** We now use the full model to assess the sources of cross-country income differences. According to our model, countries differ in their incomes due to differences in fundamentals  $F_j = [A_j, \pi_j, f_j, \tau, \varepsilon_j, q_j]$ . Denote  $F_{ji}$  the  $i$  element of this vector. A way to assess the contribution of a fundamental in explaining income differences is to equate the fundamental to its US value and compute the resulting reduction in income dispersion. More precisely, define:

$$\Phi_i = 1 - \frac{\text{var}(\ln y_c(F_{ji} = F_{us,i}))}{\text{var}(\ln y)},$$

where  $y_c(F_{ji} = F_{us,i})$  is the vector of counterfactual levels of outputs obtained when the parameter  $F_{ji}$  is equated to its US value,  $F_{us,i}$ . Thus, for example  $\Phi_A = 1$  would mean that by equating all TFP's income differences would be eliminated. The results are surprising. The single most important fundamental is the survival rate followed by TFP. According to the model, equating mortality rates across countries would reduce the variance of log incomes in 51%, while equating TFP would reduce that variance in 48%. The role of public education is also important. Equating the share of education in the government expenditures would reduce income variance in 18%.

## 6 The Effects of the HIV/AIDS Pandemic in Development

As documented in the previous section, according to our model, mortality rates are a major determinant of income differences. Although a general pattern of convergence in demographic variables took place until recently, this pattern changed substantially with the HIV/AIDS pandemic. Panel A in Figure 9 illustrates the changes in life expectancy between 1980 and 2003. Life expectancy fell in many countries with already short life expectancy in 1980, while it increased in most countries with already large life expectancy. The most dramatic cases are Botswana, Zimbabwe, Lesotho, Zambia, and South Africa where life expectancy fell in 20, 16, 15, 13 and 11 years respectively between 1980 to 2003.

A number of papers have tried to assess the short and long run effects of the AIDS pandemic in development within a general equilibrium framework.<sup>9</sup> Young (2005) provides an assessment for South Africa under the assumptions that the epidemic disappears completely within a 50 years period, that savings rate rates remain unchanged, and that student time is the only input in the production of human capital. He finds that per capita income will increase for generations following the end of the pandemic. In contrast, Arndt and Lewis (2000) find significant negative effects even in the short run, and using a very simple model of human capital accumulation. Ferreira and Pessoa (2003) and Corrigan, Glomm and Mendez (2004) study the effects of a more permanent change in mortality rates and find that the epidemic has significant negative effects. Their human capital models assume that student time is the only input the production of human capital.

We now use our model to assess the long-run development consequences of the HIV/AIDS pandemic. For this purpose, we follow the process described in Section 3 to compute survival probabilities for ages 6 and above using 2003 statistics on life expectancy at birth and mortality rates under age 5. We feed the model with the new survival probabilities and calculate the predicted levels of output, capital, and human capital. Results are shown in Figure 9. The model predicts a 28% increase in the variance of log incomes mostly explained by substantial set backs in Africa.

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<sup>9</sup>Econometric exercises include Bloom and Mahal (1997), who find no major effect, and Ukpolo (2004) who finds major effects.

Both physical and human capital fall substantially, although physical capital reacts more strongly.

**Table 7: Long-run Effects of the AIDS Pandemic**

	Ferreira-Pessoa				Cordoba-Ripoll					
	<i>Ratio of</i>		<i>Ratio of</i>		<i>Change</i>			<i>Fraction Old Adults</i>		
	y	s	y	s	h	k	$\Delta$ LE	$\Delta$ s	$\pi_4(1980)$	$\pi_4(2003)$
Botswana	0.56	0.29	0.24	0.18	0.40	0.09	-20.08	-3.88	0.17	0.00
Zimbabwe	0.65	0.38	0.28	0.35	0.43	0.12	-16.62	-2.87	0.11	0.00
Zambia	0.67	0.49	0.35	0.57	0.49	0.17	-14.18	-2.41	0.06	0.00
Lesotho	0.74	0.54	0.26	0.04	0.41	0.10	-17.58	-4.08	0.09	0.00
South Africa	0.74	0.57	0.44	0.76	0.59	0.23	-12.41	-1.90	0.15	0.00
Kenya	0.79	0.62	0.51	0.61	0.65	0.05	-9.51	-1.37	0.11	0.01
Malawi	0.82	0.66	0.45	0.31	0.56	0.28	-9.45	-1.80	0.02	0.00
Algeria	1.16	1.34	1.29	1.20	1.17	1.57	+8.74	+0.77	0.25	0.51
Tunisia	1.18	1.38	1.27	1.20	1.16	1.54	+8.92	+0.73	0.31	0.58
Egypt	1.2	1.38	1.39	1.21	1.22	1.80	+9.63	+0.91	0.18	0.46

Table 7 reports our predictions and compares them to those of Ferreira and Pessoa (2003).<sup>10</sup> Our predictions for schooling are somehow similar (and reasonable given the reductions in life expectancy) but our predictions for output are very different, mainly due to dramatic falls in capital stocks. The reason is that when individuals die younger, since they are at the upward sloping part of their earning schedule, savings for retirement are dramatically reduced.<sup>11</sup>

<sup>10</sup>Results are not strictly comparable. Ferreira and Pessoa use 1985 demographics in their benchmark and 1999 demographics in the experiment while we use 1980 versus 2003 demographics.

<sup>11</sup>As observed in Table 7, our results are the opposite to those of Young (2005). Part of the difference is that he assumes that the pandemic will completely and linearly disappear within the next 50 years. We instead assume a more permanent effect. In addition, Young does not consider the effects on savings, but only on human capital and fertility.

## 7 Concluding Comments

Using a life-cycle model with public and private spending in schooling, we find that differences in mortality rates are at least as important as TFP differences in accounting for the variance of log incomes. In particular, elimination of TFP differences would reduce the variance of log income in 48%, while elimination of mortality differences would do so by 51%. In addition, the equilibrium of our model implies a set of human capital estimates for all countries in the sample. We find that these estimates suggest a larger dispersion than standard Mincer-equation estimates. In particular, human capital in poorer countries is substantially lower.

In an apparent contradiction to our results, Acemoglu and Johnson (2006) find that increases in life-expectancy since 1950 did not have much effect on output growth. However, we view our paper and theirs as complementary. First, the reduction in mortality that they refer to is mostly children mortality. In our model, this would be the same as an increase in fertility rates and therefore should actually decrease output. The reason is that having more children around dilutes public expenditures in education and can actually reduce human capital per worker rather than increase it, depending on the response of years of schooling. Second, since different from Acemoglu and Johnson we mostly consider adult mortality, our model is suitable for analyzing events such as the HIV/AIDS pandemic, while theirs is not.

In our model, differences in life expectancy and mortality rates across countries are taken as exogenous. If these variables were mainly determined by income, our variance decomposition exercises would not be precise. However, evidence provided by Soares (2005, 2006) indicates that during the period 1960-1995 demographic changes occurred independently of changes in income. Moreover, Soares (2005) documents that a large part of the mortality changes during the twentieth century were unrelated to economic development per se. In fact, the large changes in mortality observed in the developing world were mainly due to the absorption of previously available knowledge, and so they were exogenous to these countries. In light of this, our results mainly suggest that understanding the origin of differences in mortality rates is an important item in the research agenda in cross-country income inequality.

APPENDIX

**Demographics** Note that by definition of  $\pi_i$  one has that  $N_{i,t} = \pi_i N_{1,t-(i-1)}$ . Thus,

$$N_t = \sum_{i=1}^I N_{it} = N_{1t} \sum_{i=1}^I \frac{N_{it}}{N_{1t}} = N_{1t} \sum_{i=1}^I \pi_i \frac{N_{1,t-(i-1)}}{N_{1t}}.$$

In addition, from (1),  $N_{1t} = (f\pi_2)^{i-1} N_{1,t-(i-1)}$ . Thus,

$$\frac{N_{1,t}}{N_t} = \frac{1}{\sum_{i=1}^I \frac{\pi_i}{(f\pi_2)^{i-1}}}.$$

Finally,

$$\frac{N_{it}}{N_t} = \frac{N_{1t}}{N_t} \frac{N_{it}}{N_{1t}} = \frac{N_{1t}}{N_t} \frac{\pi_i N_{1,t-(i-1)}}{N_{1t}} = \frac{N_{1t}}{N_t} \frac{\pi_i}{(f\pi_2)^{i-1}}.$$

These two equation can be written as in (2).

**Solution to the Individual Problem** Note that the set of budget constraints (5) can be written as:

$$c_1 + e = \bar{w}h_1 (1 - s/T); \quad \sum_{i=2}^I \frac{c_i}{(1 + \bar{r})^{i-2}} = \sum_{i=2}^I \frac{\bar{w}h_i}{(1 + \bar{r})^{i-2}} = \Theta \bar{w}h_2 \quad (20)$$

where  $\Theta(\bar{r}) \equiv \sum_{i=2}^I \frac{\theta_i}{(1 + \bar{r})^{i-2}} = 1 + \sum_{i=3}^I \frac{\theta_i}{(1 + \bar{r})^{i-2}} > 1$ . The individual problem can thus be written as:

$$\begin{aligned} & \max_{s, e, \{c_i\}_{i>2}^I} + \ln(\bar{w}h_1 (1 - s/T) - e) + \sum_{i>2}^I \pi_i \rho^{i-1} \ln(c_i) \\ & + \rho\pi_2 \ln \left[ \Theta \bar{w}z(\chi + s)^{\gamma_1} (p + e)^{\gamma_2} - \sum_{i>2}^I \frac{c_i}{(1 + \bar{r})^{i-2}} \right] \\ & + \mu s + \lambda e \end{aligned}$$

where  $\mu$  and  $\lambda$  are Lagrange multipliers. An optimal solution must satisfy:

$$s : \bar{w}h_1/T = \rho\pi_2 \frac{c_1}{c_2} \gamma_1 \Theta \bar{w}h_2 \frac{1}{\chi + s} + \mu \quad (21)$$

$$e : 1 = \rho\pi_2 \frac{c_1}{c_2} \gamma_2 \Theta \bar{w}h_2 \frac{1}{p + e} + \lambda \quad (22)$$

$$c_i : \rho\pi_2 \frac{1}{c_2} \frac{1}{(1 + \bar{r})^{i-2}} = \pi_i \rho^{i-1} \frac{1}{c_i} \text{ for } i = 3, \dots, I. \quad (23)$$

The last equation produces:

$$c_i = \frac{\pi_i}{\pi_2} [\rho(1+\bar{r})]^{i-2} c_2 \text{ for } i = 3, \dots, I \quad (24)$$

Using this result and (20) one obtains:

$$\begin{aligned} c_2 &= \Theta \bar{w} h_2 - \sum_{i=3}^I \frac{c_i}{(1+\bar{r})^{i-2}} = \Theta \bar{w} h_2 - c_2 \sum_{i=3}^I \frac{\pi_i}{\pi_2} \rho^{i-2} \\ &= \frac{\Theta \bar{w} h_2}{1 + \sum_{i=3}^I \frac{\pi_i}{\pi_2} \rho^{i-2}}. \end{aligned} \quad (25)$$

Then, one can use (24) and (25) to write  $c_j$  more generally as:

$$c_j = \Theta_j(\bar{r}) \bar{w} h_2 \text{ for } j = 2, \dots, I.$$

where

$$\Theta_j(\bar{r}) \equiv \frac{\frac{\pi_j}{\pi_2} [\rho(1+\bar{r})]^{j-2} \Theta}{1 + \sum_{i=3}^I \frac{\pi_i}{\pi_2} \rho^{i-2}} = \frac{\sum_{i=2}^I \theta_i (1+\bar{r})^{j-i}}{\sum_{i=2}^I \frac{\pi_i}{\pi_j} \rho^{i-j}}.$$

Furthermore, one can obtain  $a_i$  as:

$$a_i = \bar{w} h_i + (1+\bar{r})a_{i-1} - c_i$$

so that

$$\begin{aligned} a_1 &= 0, \\ a_2 &= (1 - \Theta_2) \bar{w} h_2, \\ &\dots \\ a_i &= (\theta_i - \Theta_i) \bar{w} h_2 + (1+\bar{r}) a_{i-1}, \text{ for } i = 3, \dots, I. \end{aligned}$$

Alternatively

$$\begin{aligned} a_3 &= [(\theta_3 - \Theta_3) + (1+\bar{r})(1 - \Theta_2)] \bar{w} h_2 \\ a_4 &= [(\theta_4 - \Theta_4) + (1+\bar{r})(\theta_3 - \Theta_3) + (1+\bar{r})^2(1 - \Theta_2)] \bar{w} h_2 \\ a_i &= \bar{w} h_2 \sum_{j=2}^i (1+\bar{r})^{i-j} (\theta_j - \Theta_j) \text{ for } i = 2, \dots, I. \end{aligned}$$

Moreover,

$$qK = \sum_{i=1}^I n_i a_i = \bar{w} h_2 \sum_{i=2}^I n_i \sum_{j=2}^i (1+\bar{r})^{i-j} (\theta_j - \Theta_j(\bar{r})). \quad (26)$$

Define  $\Phi(\bar{r}) \equiv \sum_{i=2}^I n_i \sum_{j=2}^i (1 + \bar{r})^{i-j} (\theta_j - \Theta_j(\bar{r}))$ . Then

$$qK = \bar{w}h_2\Phi(\bar{r}) \quad (27)$$

Furthermore, using (9) and (11):

$$\frac{qK}{Y} = \frac{(1 - \alpha)(1 - \tau_w)}{(1 - s/T)n_1\theta_1 + \sum_2 \theta_i n_i} \Phi(\bar{r}) \quad (28)$$

Moreover,

$$\begin{aligned} \frac{\sum_{i=1}^I N_i a_i}{\sum_{i=1}^I \frac{\pi_{i+1}}{\pi_i} N_i a_i} &= \frac{\sum_{i=2}^I n_i \bar{w}h_2 \sum_{j=2}^i (1 + \bar{r})^{i-j} (\theta_j - \Theta_j)}{\sum_{i=2}^I \frac{\pi_{i+1}}{\pi_i} n_i \bar{w}h_2 \sum_{j=2}^i (1 + \bar{r})^{i-j} (\theta_j - \Theta_j)} \\ &= \frac{\sum_{i=2}^I n_i \sum_{j=2}^i (1 + \bar{r})^{i-j} (\theta_j - \Theta_j)}{\sum_{i=2}^I \frac{\pi_{i+1}}{\pi_i} n_i \sum_{j=2}^i (1 + \bar{r})^{i-j} (\theta_j - \Theta_j)} \end{aligned}$$

**Interior Solution:**  $s^* \geq 0$  and  $e^* \geq 0$  Consider first interior solutions ( $\mu = 0$  and  $\lambda = 0$ ). From (21) and (22) one obtains:

$$e = \frac{\gamma_2 \bar{w}h_1}{\gamma_1 T} (\chi + s) - p. \quad (29)$$

Moreover, from (22), (25) and (4) it follows that:

$$\begin{aligned} p + e &= \rho\pi_2\gamma_2 \frac{c_1}{c_2} \Theta \bar{w}h_2 \\ &= \rho\pi_2\gamma_2 \frac{\bar{w}h_1(1 - s/T) - e}{c_2} \Theta \bar{w}h_2 \\ &= \rho\pi_2 \left( 1 + \sum_{i=3}^I \frac{\pi_i}{\pi_2} \rho^{i-2} \right) \gamma_2 (\bar{w}h_1(1 - s/T) - e) \\ &= \tilde{\rho}\gamma_2 (\bar{w}h_1(1 - s/T) - e - p + p), \end{aligned}$$

or

$$p + e = \frac{\tilde{\rho}\gamma_2\psi\bar{w}h_1}{1 + \tilde{\rho}\gamma_2\psi} \left( 1 - s/T + \frac{p}{\bar{w}h_1} \right) \quad (30)$$

where  $\tilde{\rho} \equiv \sum_{i=2}^I \rho^{i-1} \pi_i$ . Equating this expression to (29) and solving for  $s$  produces the optimal level of schooling in an interior solution:

$$s^* = \frac{\tilde{\rho}\gamma_1\psi T}{1 + \tilde{\rho}(\gamma_2 + \gamma_1)\psi} \left[ \frac{p}{\bar{w}h_1} - \underline{m} \right] \text{ if } \frac{p}{\bar{w}h_1} \geq \underline{m}. \quad (31)$$

The last condition guarantees that  $s^* \geq 0$ . In addition, substituting this result into (29) produces the optimal level of private expenditures for an interior solution:

$$e^* = \frac{1 + \tilde{\rho}\psi\gamma_1}{1 + \tilde{\rho}(\gamma_1 + \gamma_2)\psi} \left[ \bar{m} - \frac{p}{\bar{w}h_1} \right] \bar{w}h_1 \text{ if } \bar{m} \geq \frac{p}{\bar{w}h_1}. \quad (32)$$

The last condition guarantees that  $e^* \geq 0$ . Furthermore, from (7) and (29):

$$\begin{aligned} h_2 &= z(\chi + s)^{\gamma_1} (p + e)^{\gamma_2} \\ &= z(\chi + s) \left( \frac{\gamma_2 \bar{w} h_1}{\gamma_1 T} \right)^{\gamma_2} \\ &= z(\chi + s) \left( \frac{1 - \gamma \bar{w} \theta_1}{\gamma T} \right)^{\gamma_2} h_p^{\gamma_2} \end{aligned}$$

**Corner Solution:**  $e = 0$  and  $s \geq 0$  Next, consider corner solutions of the form  $e = 0$  and  $s \geq 0$ . From equations (21), (25) and (4) one obtains:

$$\frac{\chi + s}{T} = \rho \pi_2 \gamma_1 \Theta \frac{c_1 h_2}{c_2 h_1} = \rho \pi_2 \gamma_1 \Theta \frac{\bar{w} h_1 (1 - s/T) h_2}{\frac{\Theta \bar{w} h_2}{1 + \sum_{i=3}^I \frac{\pi_i}{\pi_2} \rho^{i-2}}} h_1 = \tilde{\rho} \gamma_1 (1 - s/T)$$

and solving for  $s$  gives:

$$s^* = \frac{\tilde{\rho} \gamma_1 T - \chi}{1 + \tilde{\rho} \gamma_1}. \quad (33)$$

Furthermore,  $e = 0$  is optimal if  $\lambda \geq 0$  or

$$\begin{aligned} p &\geq \rho \pi_2 \gamma_2 \frac{c_1}{c_2} \Theta \bar{w} h_2 = \rho \pi_2 \gamma_2 \frac{\bar{w} h_1 (1 - s^*/T)}{\frac{\Theta \bar{w} h_2}{1 + \sum_{i=3}^I \frac{\pi_i}{\pi_2} \rho^{i-2}}} \Theta \bar{w} h_2 \\ &= \tilde{\rho} \gamma_2 \bar{w} h_1 \left( 1 - \frac{\tilde{\rho} \gamma_1 - \chi/T}{1 + \tilde{\rho} \gamma_1} \right) = \tilde{\rho} \gamma_2 \bar{w} h_1 \frac{1 + \chi/T}{1 + \tilde{\rho} \gamma_1} \end{aligned}$$

or equivalently  $\frac{p}{\bar{w} h_1} \geq \bar{m}$ .

**Corner Solution:**  $s^* = 0$  and  $e^* \geq 0$  Now consider a corner solution of the form  $s = 0$  and  $e \geq 0$ . In this case, equation (30) is still valid because it does not use (21). Imposing  $s = 0$  into this equation and solving for  $e$  produces:

$$e^* = \frac{\tilde{\rho} \gamma_2 \bar{w} h_1 - p}{1 + \tilde{\rho} \gamma_2} \text{ if } \frac{p}{\bar{w} h_1} \leq \tilde{\rho} \gamma_2.$$

One also needs to check that  $\mu \geq 0$ . From (21) and (25) this is the case if:

$$\begin{aligned} \bar{w} h_1 / T &\geq \rho \pi_2 \gamma_1 \frac{c_1}{c_2} \Theta \bar{w} h_2 \frac{1}{\chi} = \rho \pi_2 \gamma_1 \psi \frac{\bar{w} h_1 - e^*}{\frac{\Theta \bar{w} h_2}{1 + \sum_{i=3}^I \frac{\pi_i}{\pi_2} \rho^{i-2}}} \Theta \bar{w} h_2 \frac{1}{\chi} \\ &= \tilde{\rho} \gamma_1 \left( \bar{w} h_1 - \frac{\tilde{\rho} \gamma_2 \bar{w} h_1 - p}{1 + \tilde{\rho} \gamma_2} \right) \frac{1}{\chi} \\ &= \tilde{\rho} \gamma_1 \left( \frac{\bar{w} h_1 + p}{1 + \tilde{\rho} \gamma_2} \right) \frac{1}{\chi} \end{aligned}$$

This inequality can be rewritten as  $\underline{m} \geq \frac{p}{\bar{w}h}$ . Thus, this case requires  $\underline{m} \geq \frac{p}{\bar{w}h}$  and  $\rho\gamma_2\psi \geq \frac{p}{\bar{w}h}$ . However, one can check that the first inequality implies the second under Assumption 1.

**Solution to the general equilibrium problem** >From (9), (11) and (6) one has that:

$$\bar{w}h_1 = (1 - \tau_w)(1 - \alpha) \frac{y}{h} h_1 = \frac{(1 - \tau_w)(1 - \alpha) y/n_1}{1 - s/T + \sum_2 \frac{\theta_i n_i}{\theta_1 n_1}} \quad (34)$$

Define

$$\hat{m} \equiv \frac{\bar{m}}{\frac{1 + \chi/T + \bar{m}}{1 + \tilde{\rho}\psi} + \sum_2 \frac{\theta_i n_i}{\theta_1 n_1}}$$

and

$$\omega = \frac{\epsilon\tau}{(1 - \tau_w)(1 - \alpha)}$$

**Mixed Education** Equation (13) states that  $\frac{p}{\bar{w}h_1} = \omega \left( 1 - s^*/T + \sum_2 \frac{\theta_i n_i}{\theta_1 n_1} \right)$ . For the case of mixed education we then have that:

$$\frac{p}{\bar{w}h_1} = \omega \left( 1 + \sum_2 \frac{\theta_i n_i}{\theta_1 n_1} - \frac{\tilde{\rho}\gamma_1}{1 + \tilde{\rho}(\gamma_1 + \gamma_2)} \left[ \frac{p}{\bar{w}h_1} - \underline{m} \right] \right)$$

Solving for  $\frac{p}{\bar{w}h}$  produces:

$$\begin{aligned} \frac{p}{\bar{w}h_1} &= \frac{1 + \tilde{\rho}(\gamma_1 + \gamma_2)}{1 + \tilde{\rho}(\gamma_1\omega + \gamma_1 + \gamma_2)} \omega \left( 1 + \sum_2 \frac{\theta_i n_i}{\theta_1 n_1} + \frac{\tilde{\rho}\gamma_1 \underline{m}}{1 + \tilde{\rho}(\gamma_1 + \gamma_2)} \right) \\ \bar{m} &\equiv \frac{\tilde{\rho}\gamma_2(1 + \chi/T)}{1 + \tilde{\rho}\gamma_1}; \quad \underline{m} = \frac{(1 + \tilde{\rho}\gamma_2)\chi/T}{\tilde{\rho}\gamma_1} - 1 \end{aligned}$$

Alternatively, one can use the definitions of  $\underline{m}$  and  $\bar{m}$  to rewrite this expression as:

$$\frac{p}{\bar{w}h_1} = \frac{1 + \tilde{\rho}(\gamma_1 + \gamma_2)}{1 + \tilde{\rho}(\gamma_1\omega + \gamma_1 + \gamma_2)} \omega \left( \sum_2 \frac{\theta_i n_i}{\theta_1 n_1} + \frac{1 + \chi/T + (1 + \tilde{\rho}\gamma_1)\bar{m}}{1 + \tilde{\rho}(\gamma_1 + \gamma_2)} \right)$$

Substituting the first expression into (31) and the second expression into (32), using (34) and simplifying produces:

$$\begin{aligned} s^* &= \frac{\tilde{\rho}\gamma_1 T \left[ \omega \left( 1 + \sum_2 \frac{\theta_i n_i}{\theta_1 n_1} \right) - \underline{m} \right]}{1 + \tilde{\rho}(\gamma_1\omega + \gamma_1 + \gamma_2)}; \\ e^* &= \frac{\bar{w}h_1(1 + \tilde{\rho}\gamma_1)}{1 + \tilde{\rho}(\gamma_1\omega + \gamma_1 + \gamma_2)} \left[ \bar{m} - \omega \left( \frac{1 + \chi/T + \bar{m}}{1 + \tilde{\rho}} + \sum_2 \frac{\theta_i n_i}{\theta_1 n_1} \right) \right] \end{aligned}$$

or defining  $s_H \equiv \frac{N_1(p+e)}{Y} = \varepsilon\tau + \frac{\frac{1}{3}e}{Y}$ , which uses (10), one obtains

$$s_H = \varepsilon\tau + \frac{(1 - \tau_w)(1 - \alpha)}{1 - s/T + \sum_2 \frac{\theta_i n_i}{\theta_1 n_1}} \frac{(1 + \tilde{\rho}\gamma_1) \frac{\bar{m}}{m} [\hat{m} - \omega]}{1 + \tilde{\rho}(\gamma_1\omega + \gamma_1 + \gamma_2)}$$

**Corner Solution:**  $e^* = 0$  and  $s^* \geq 0$  The solution for  $s^*$  is given by (33) and  $s_H = \varepsilon\tau$ .

**Corner Solution:**  $s^* = 0$  and  $e^* \geq 0$ . For this case (13) produces

$$\frac{p}{\bar{w}h_1} = \omega \left( 1 + \sum_2 \frac{\theta_i n_i}{\theta_1 n_1} \right)$$

and from Proposition 1

$$\frac{e^*}{\bar{w}h_1} = \frac{\tilde{\rho}\gamma_2 - \omega \left( 1 + \sum_2 \frac{\theta_i n_i}{\theta_1 n_1} \right)}{1 + \tilde{\rho}\gamma_2}.$$

Moreover, using (34)

$$\frac{N_1 e^*}{Y} = \bar{w}h \frac{e^*}{\bar{w}h_1} \frac{N_1}{Y} = \frac{(1 - \tau_w)(1 - \alpha)}{1 + \sum_2 \frac{\theta_i n_i}{\theta_1 n_1}} \frac{e^*}{\bar{w}h_1}$$

so that

$$s_H = \varepsilon\tau + \frac{(1 - \tau_w)(1 - \alpha)}{1 + \sum_2 \frac{\theta_i n_i}{\theta_1 n_1}} \frac{\tilde{\rho}\gamma_2\psi - \omega \left( 1 + \sum_2 \frac{\theta_i n_i}{\theta_1 n_1} \right)}{1 + \tilde{\rho}\gamma_2\psi}.$$

**Other General Equilibrium Results** Per-capita output can be written as:

$$\begin{aligned} y &= Ak^\alpha (h)^{1-\alpha} = A \left( \frac{k}{y} \right)^\alpha \left( \frac{h}{y^{\gamma_2}} \right)^{1-\alpha} y^{\alpha+\gamma_2(1-\alpha)} \\ &= A^{\frac{1}{(1-\alpha)(1-\gamma_2)}} \left( \frac{k}{y} \right)^{\frac{\alpha}{(1-\alpha)(1-\gamma_2)}} \left( \frac{h}{y^{\gamma_2}} \right)^{\frac{1}{1-\gamma_2}} \end{aligned}$$

Moreover, notice that in a stationary equilibrium  $h$  satisfies, using (11) and (12):

$$\begin{aligned} h &= \left( (1 - s^*/T) n_1 \theta_1 + \sum_2 \theta_i n_i \right) z (\chi + s^*)^{\gamma_1} (p + e)^{\gamma_2} \\ &= \left( (1 - s^*/T) n_1 \theta_1 + \sum_2 \theta_i n_i \right) z (\chi + s^*)^{\gamma_1} \left( \frac{n_1 (p + e) y}{y n_1} \right)^{\gamma_2} \\ &= \left( (1 - s^*/T) n_1 \theta_1 + \sum_2 \theta_i n_i \right) z (\chi + s^*)^{\gamma_1} (s_H^*/n_1)^{\gamma_2} y^{\gamma_2} \end{aligned}$$

so that

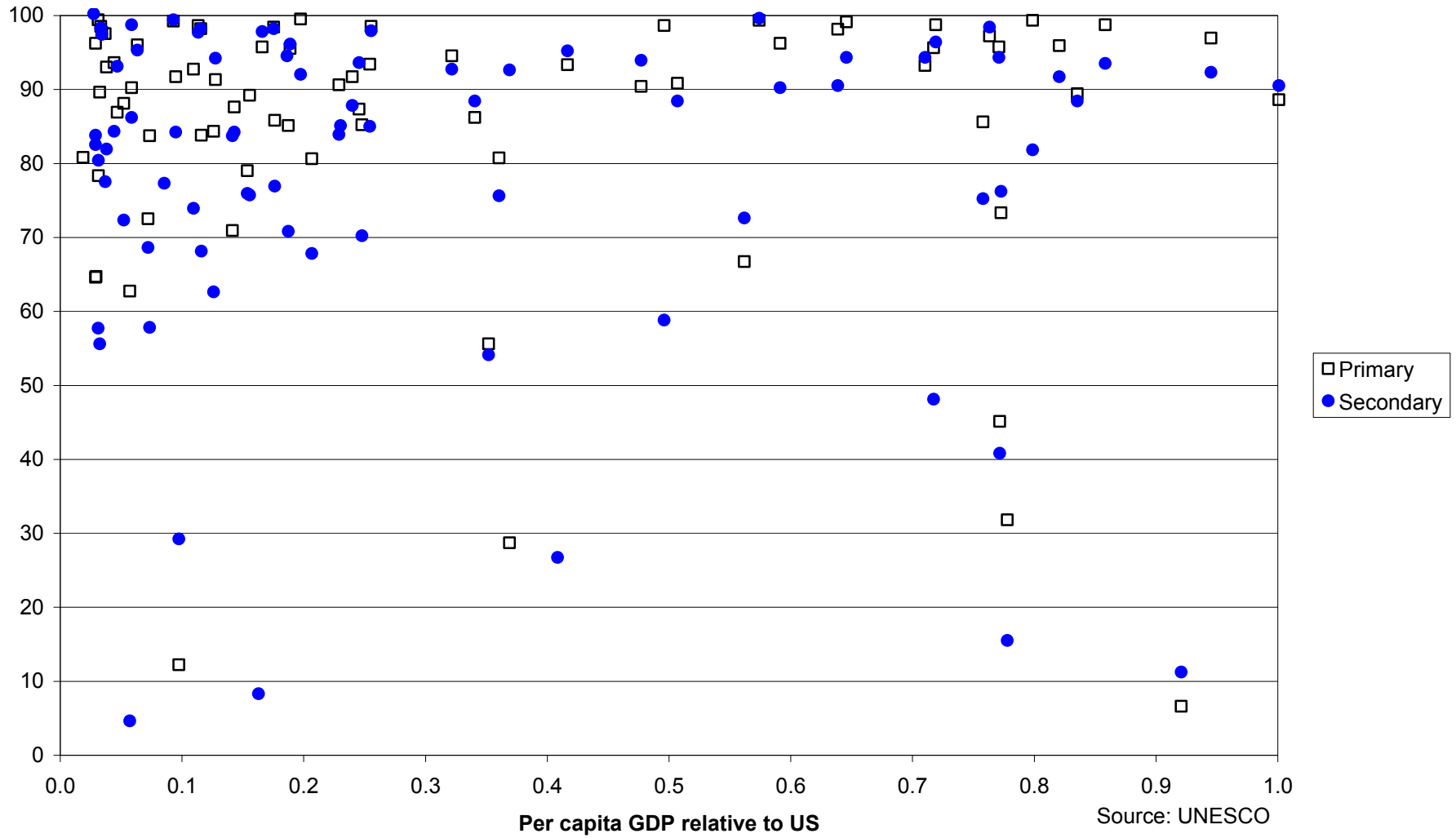
$$\frac{h}{y^{\gamma_2}} = \left( (1 - s^*/T) n_1 \theta_1 + \sum_2 \theta_i n_i \right) z (\chi + s^*)^{\gamma_1} (s_H^*/n_1)^{\gamma_2}.$$

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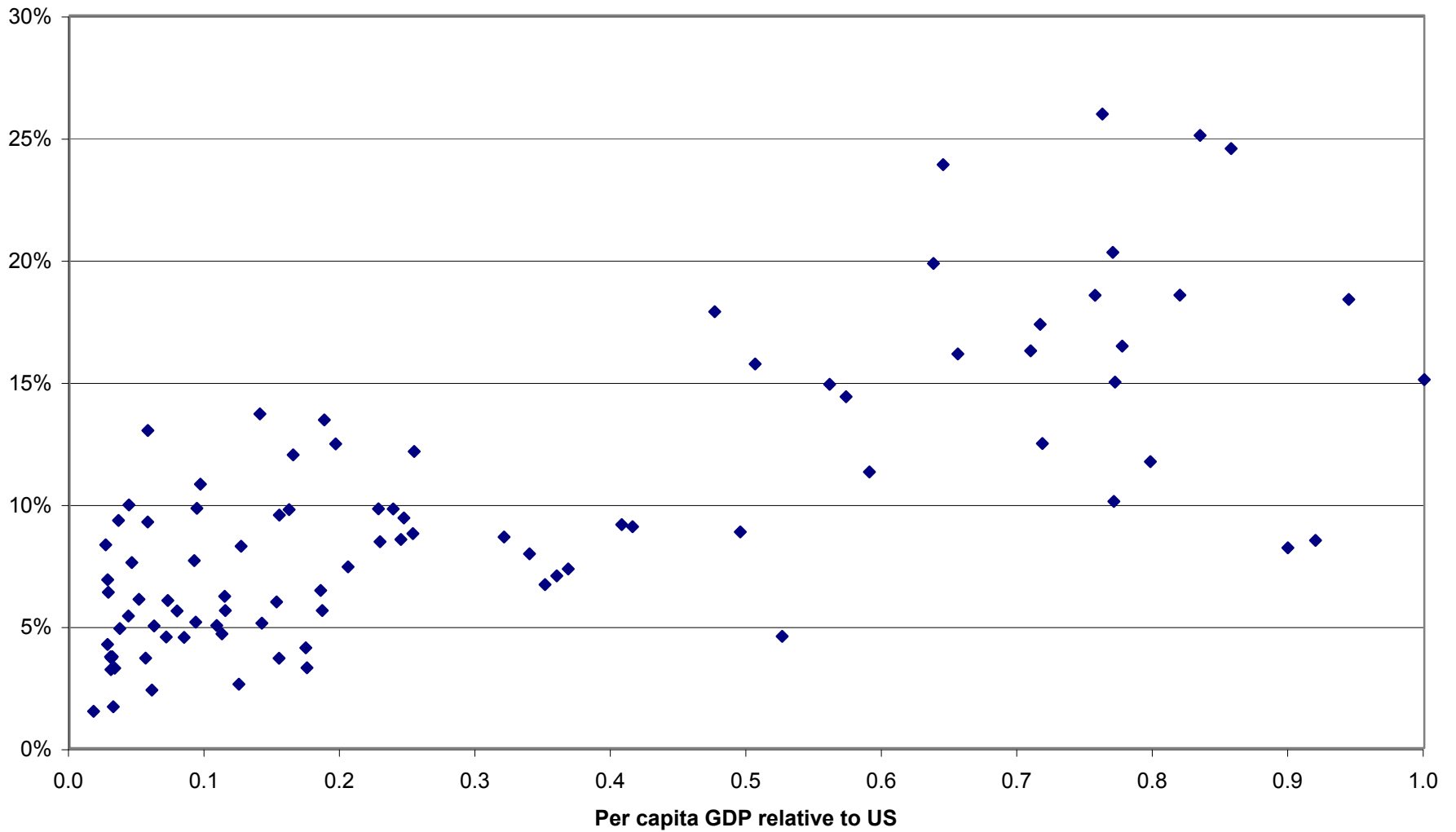
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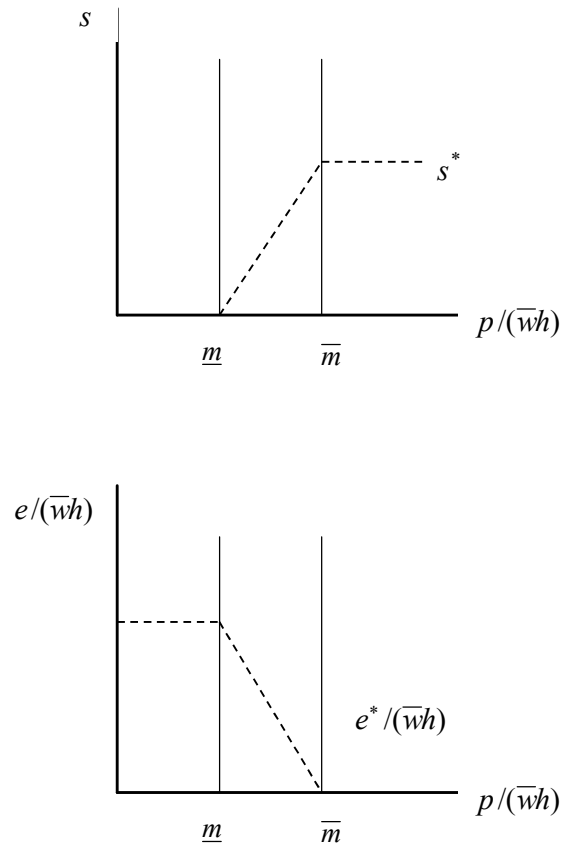
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**Figure 1**  
**% Enrollment in Public Schools**



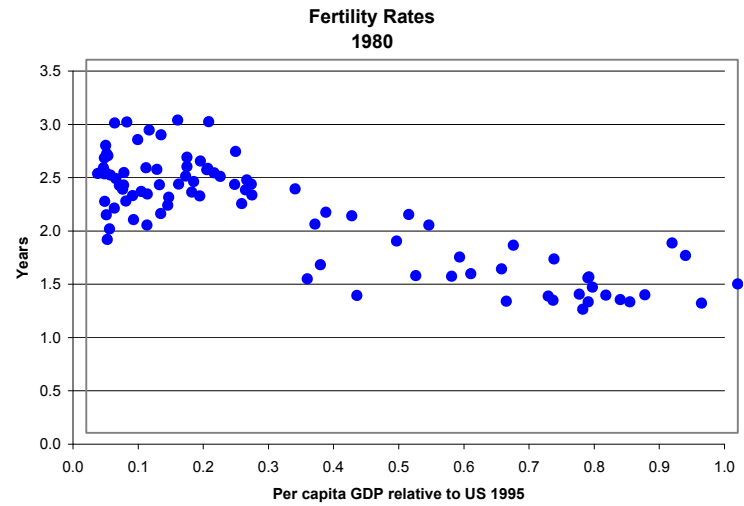
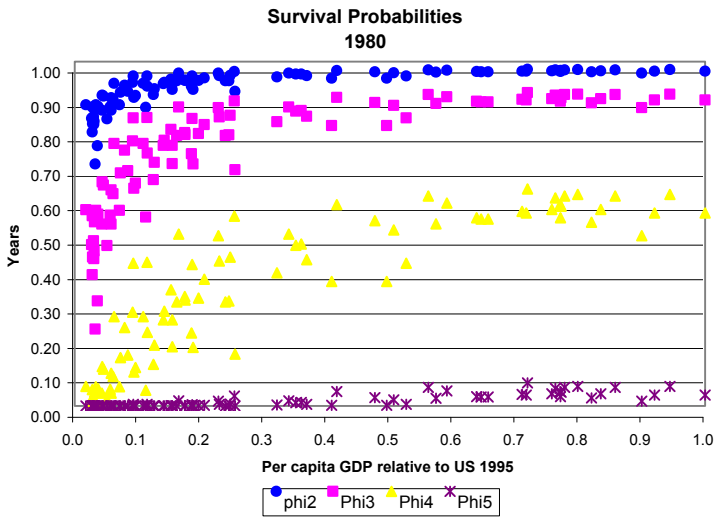
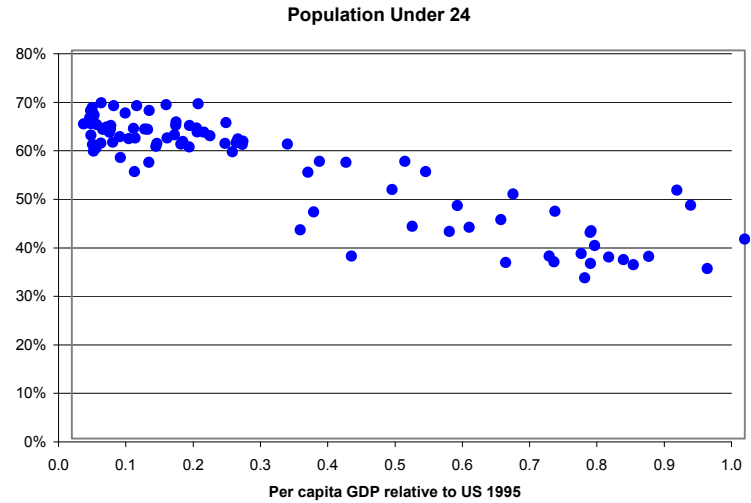
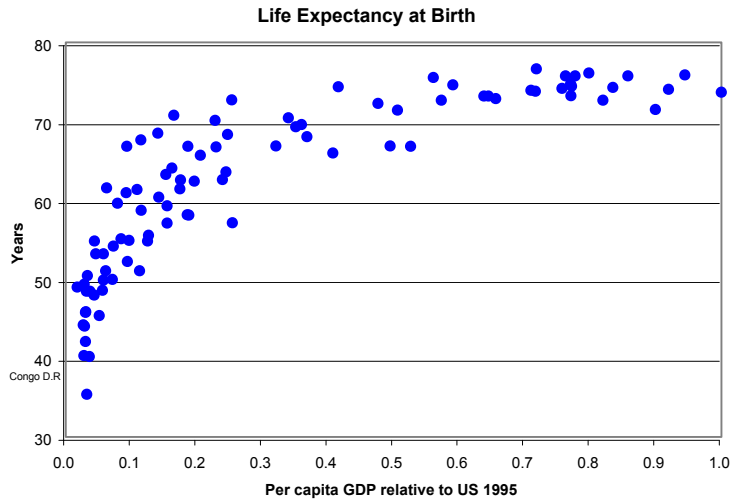
**Figure 2. Public Education Expenditures per Pupil  
as a Percentage of GDP per capita**



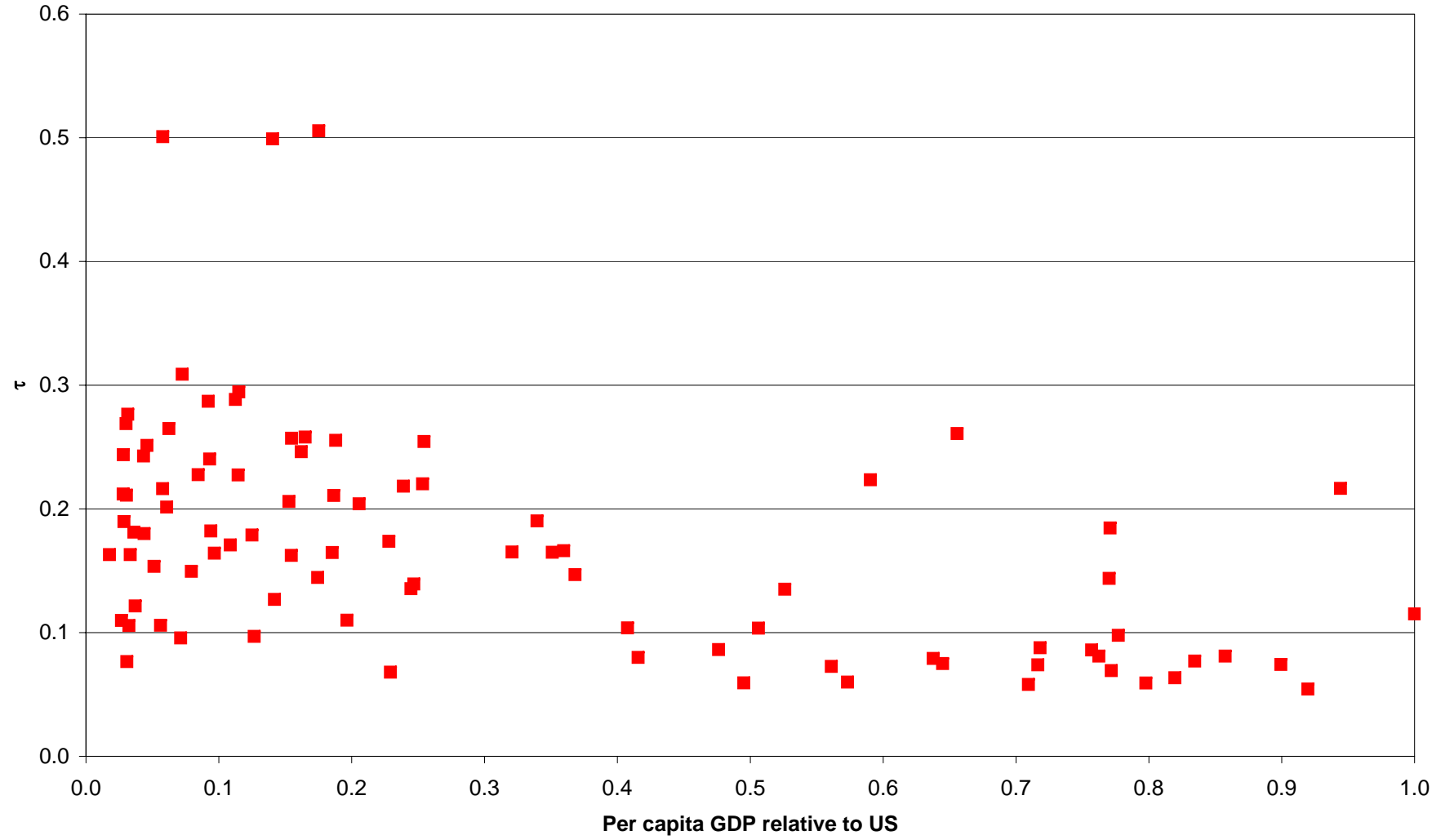


**Figure 3. Optimal Educational Choices**

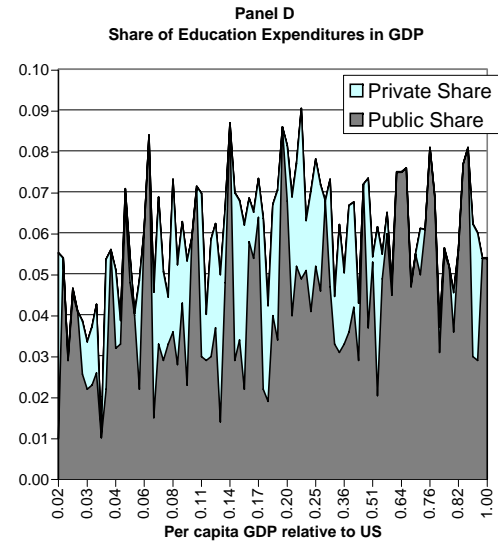
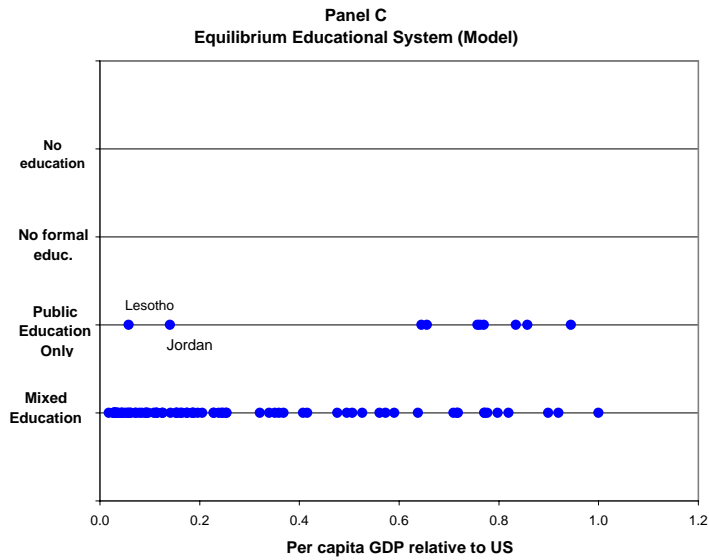
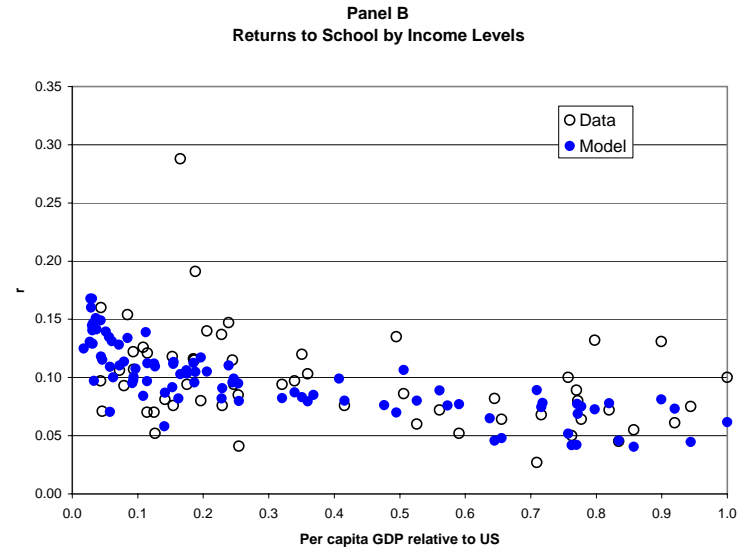
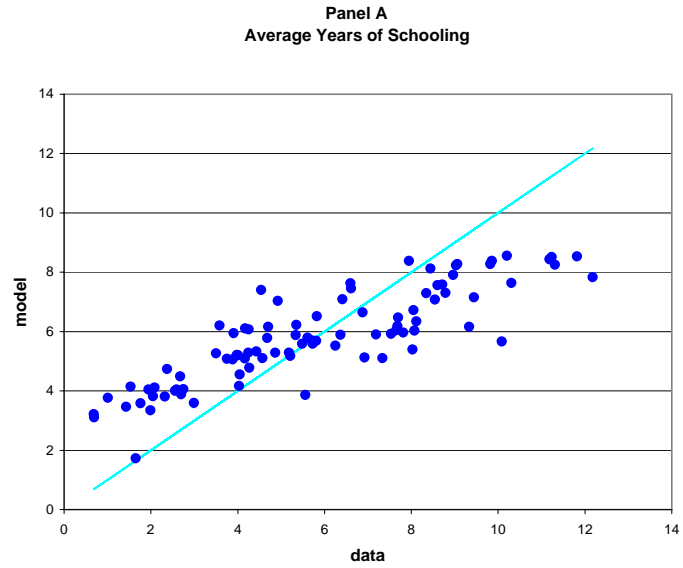
**Figure 4**  
**Demographics**



**Figure 5**  
**Capital Taxes and Labor Taxes ( $\tau$ )**



**Figure 6**  
**Educational Variables: Model Vs Data**



**Figure 7**  
**Human Capital Stocks Estimates**

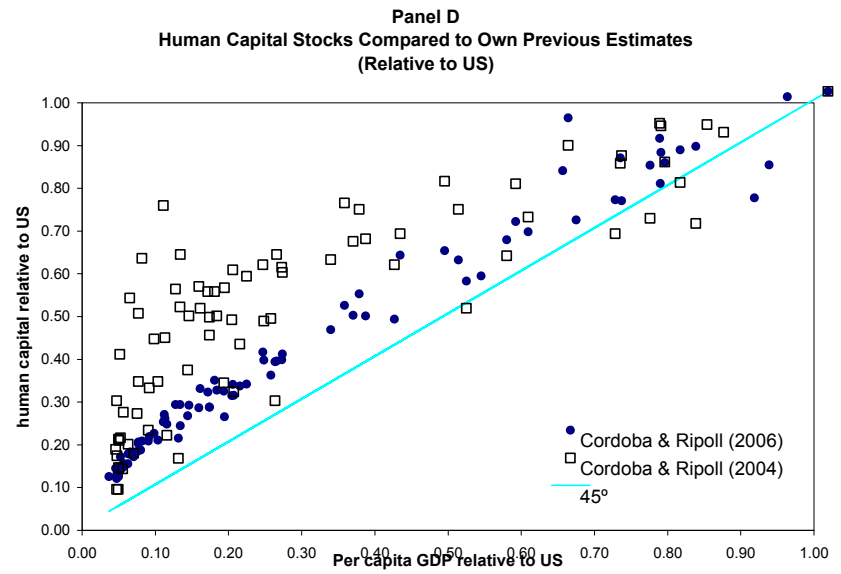
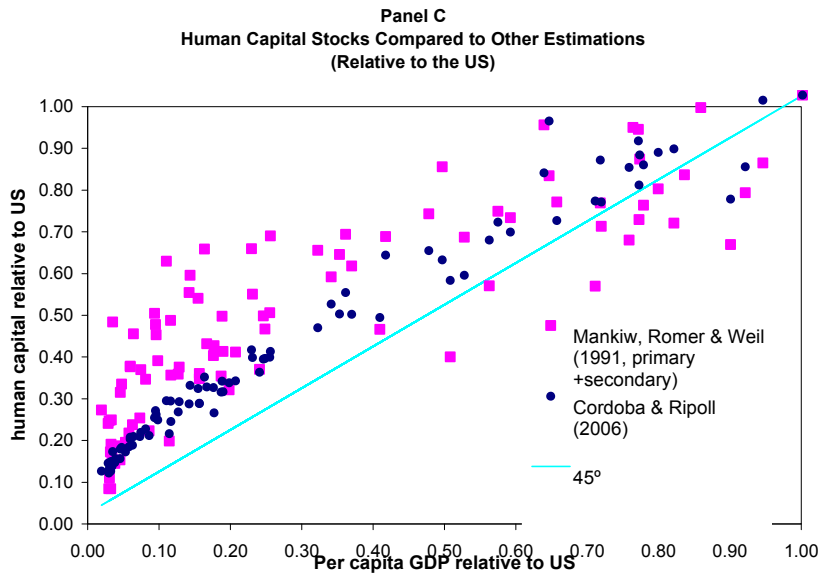
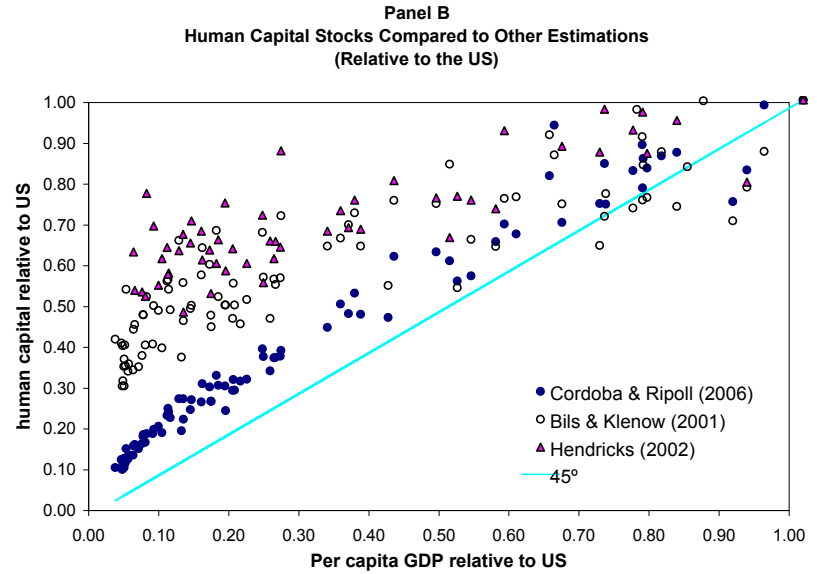
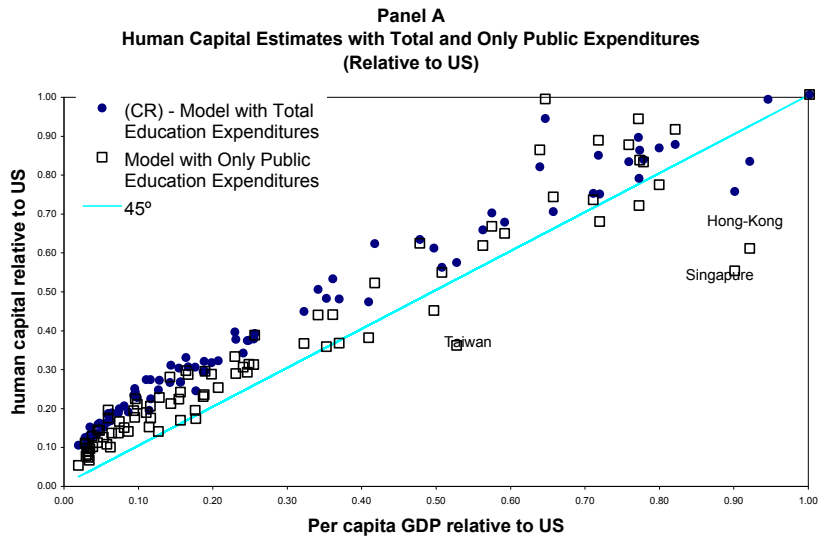
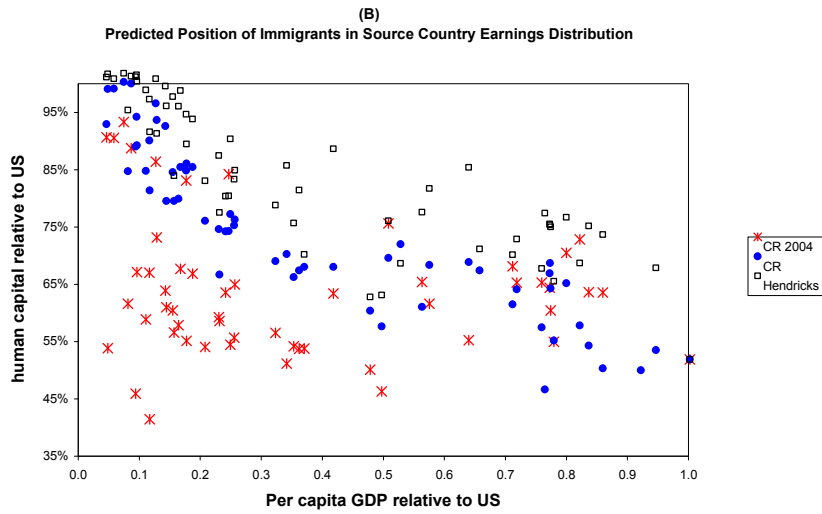
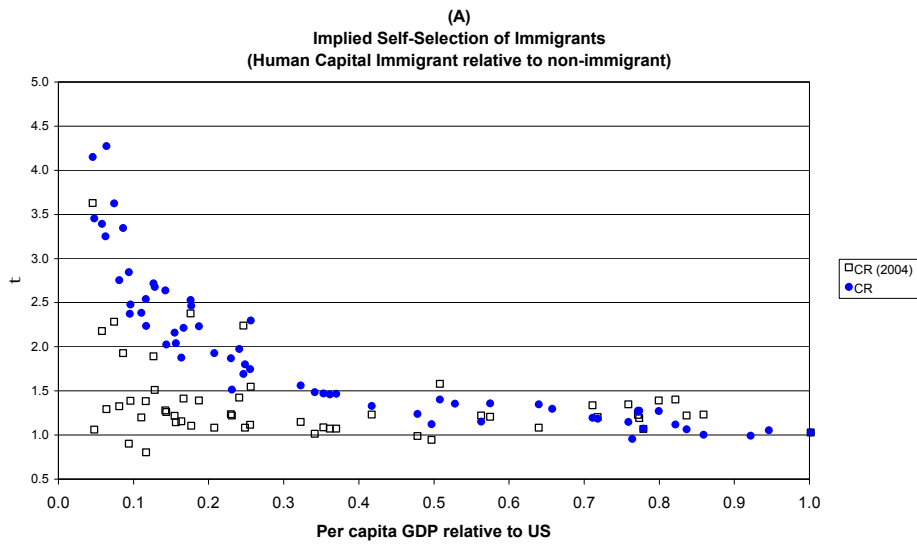


Figure 8  
Self-Selection of Immigrants



**Figure 9**  
**Predicted Long Run Effects of the HIV/AIDS Pandemic**

