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Notes on the Lucas Critique, Time Inconsistency, and Related Issues

1 Preliminary Remarks on Government Policy Rules

Consider an economy that exists over time periods $t = 1, 2, \dots$, where period t is the interval from t to $t + 1$. Let I_{t-1} denote the information set available to government at the beginning of period t (i.e., at the end of period $t - 1$). Let h_t denote a possible policy choice vector for government in period t , where the components of h_t might include tax rates, the money growth rate, etc.

Definition: Let I denote the collection of all possible government information sets, and let H denote the collection of all possible policy choice vectors. Then a sequence (h_1, h_2, \dots) of government policy choice vectors is said to be chosen in accordance with a (*deterministic*) *policy rule* if there exists some well-defined function $g: I \rightarrow H$ mapping information sets into policy choice vectors such that

$$h_t = g(I_{t-1}) \quad , \quad t = 1, 2, \dots \quad . \quad (1)$$

The policy rule $g: I \rightarrow H$ is said to be a *feedback policy rule* if it depends nontrivially on the information sets, i.e., if there exist information sets I^* and I^{**} in I such that $g(I^*)$ is not equal to $g(I^{**})$. Otherwise it is said to be a *fixed policy rule*. If government implements a sequence (h_1, h_2, \dots) of policy choice vectors that does *not* satisfy (1) for any well-defined function $g: I \rightarrow H$, then government is said to be engaging in *discretionary policy choice*.

Example: Optimal Policy Rule Selection via a Social Welfare Function

The following example illustrates the way in which government policy choice problems were commonly represented in articles stressing microfoundations of macro prior to the rational expectations debate beginning in the early nineteen seventies, and are still represented in many microfoundations of macro articles today. The framework, originally due to Nobel Laureate Jan Tinbergen (1967), will hereafter be referred to as the *Tinbergen framework* for government policy choice.

For each $t = 1, \dots, T$, let x_{t-1} be a vector describing the state of an economy at the beginning of period t , where x_0 is exogenously given in period 1. Also, let $I_{t-1} = (x_{t-1}, I^o) \subseteq I$ denote the information set available to government at the beginning of period t , where the contents of I^o are clarified below. Finally, let $H(I_{t-1}) \subseteq H$ denote the collection of policy choice vectors h_t in H that are feasible for government to choose in period t , conditional on government's information set I_{t-1} .

Assume the motion of the state of the economy over time is described by a system of state equations of the form

$$x_t = S(h_t, v_t, x_{t-1}) \quad , \quad t = 1, \dots, T \quad , \quad (2)$$

where v_t denotes a vector of time-t stochastic shock terms and $v = (v_1, \dots, v_T)$ is determined in accordance with a probability distribution function $P(v)$. For example, the period-t state equation (2) might represent the period-t equations for a stochastic variant of a dynamic IS-LM model conditioned on a vector h_t of government fiscal and monetary policy variables.

Suppose that social welfare for the economy over periods 1 through T is measured in period 1 by a utilitarian social welfare function

$$W(h, v, x) = \sum_{t=1}^T \beta^t U_t(h_t, v_t, x_{t-1}) \quad , \quad (3)$$

where $h = (h_1, \dots, h_T)$, $x = (x_0, \dots, x_{T-1})$, $U_t(\cdot)$ denotes the utility function of the representative consumer in period t , and β denotes the discount rate for the representative

consumer.

Call $g:I \rightarrow H$ a *feasible* policy rule for government if g maps I_{t-1} into $H(I_{t-1})$ for each $t = 1, \dots, T$. Suppose the problem facing government at the beginning of period 1 is to choose a feasible policy rule for generating $h = (h_1, \dots, h_T)$ over periods $1, \dots, T$ that maximizes expected social welfare subject to (2), conditional on the period-1 information set $I_0 = (x_0, I^o)$, where I^o contains the structure of the state equations (2), the form of the probability distribution function $P(v)$, and the form of the the social welfare function $W(h, v, x)$. Formally, then, government's optimization problem in period 1 takes the following form:

Maximize expected social welfare

$$E [W(h, v, x)|I_0] = \int W(h, v, x)P(dv) \quad (4)$$

with respect to the choice of $h = (h_1, \dots, h_T)$ subject to

$$x_t = S(h_t, v_t, x_{t-1}) \quad , \quad t = 1, \dots, T \quad ; \quad (5)$$

$$h_t \in H(I_{t-1}) \quad , \quad t = 1, \dots, T \quad ; \quad (6)$$

$$I_{t-1} = (x_{t-1}, I^o) \quad , \quad t = 1, \dots, T \quad ; \quad (7)$$

$$x_0 = \bar{x}_0 \quad . \quad (8)$$

By construction, the solution of this stochastic optimization problem is a feasible policy rule $g^*:I \rightarrow H$ that determines how a particular policy vector h_t should be chosen from $H(I_{t-1})$ in each period $t = 1, \dots, T$, given the dynamic state equations (5) that reveal how a choice of h_t in period t affects all current and future states of the economy. Such recursively structured problems are often solved numerically by means of “dynamic programming” methods; see Ljungqvist and Sargent (2004).

In this engineering view of government policy optimization, the economy is a machine. Once the econometrician discovers the true structure of the machine as represented by the

state function $S(h_t, v_t, x_{t-1})$ and the shock probability distribution $P(v)$, and a social consensus is reached concerning the appropriate specification for the social welfare function $W(h, v, x)$ and the feasible policy choice sets $H(I_{t-1})$, the government can optimally control the machine by pressing the right buttons (selecting the right policy rule g).

2 The Lucas Critique

Definition: Estimated functional forms obtained for an economic model are said to be *deep* (or *structural*) if they are invariant to announced or perceived changes in government policy rules.¹ In particular, when unknown functional forms are replaced by functions that are assumed known up to unknown parameter values, the estimated parameter values are said to be *deep* when they are invariant to announced or perceived changes in government policy rules.

The Lucas Critique: Estimated functional forms obtained for macroeconomic models in the Keynesian tradition (e.g. dynamic IS-LM models) are *not* deep because these models do not correctly take into account the dependence of private agent behavior on perceived or anticipated government policy rules for generating current and future values for government policy variables. While such models may be useful for forecasting future states of the economy conditional on a *given* government policy rule, they are fatally flawed for the purpose of analyzing the effects on the economy of *changes* in government policy rules. Specifically, they can seriously mislead policy makers with regard to the effectiveness and desirability of their policy rule choices.

The best known source for the Lucas Critique is Lucas (1976). In this study, Lucas criticizes government policy optimization frameworks, such as the Tinbergen framework

¹Since the term “structural” already has many other meanings in economics and econometrics, henceforth only the term “deep” will be used.

illustrated above, for not taking into account the degree to which estimated functional forms fail to be deep. Note that the Lucas Critique does not presume that agents have rational expectations, although Lucas (1976) motivates this critique using only rational expectations contexts. Rather, the Lucas Critique points out a problem that can occur whenever private agent behavior depends to *some* degree on government policy rules and this dependence is not taken into account.

For example, to carry out real-world policy analysis within a Tinbergen framework, an econometrician would first need to determine the form of the state function $S(h_t, v_t, x_{t-1})$ on the basis of past observations. In traditional econometric practice (i.e. econometric practice prior to 1976), this estimation would have proceeded without any attention being paid to the particular government policy *rules* in effect at the time these observations were made that were used to generate the observed settings for policy variables.² Once the form of this state function was estimated, it would then have been incorporated into the Tinbergen framework as if it were deep. That is, it would have been treated as a constant of the model as alternative choices of policy rules $g:I \rightarrow H$ were considered for government.

Lucas points out that there are good theoretical reasons to suppose that the structural form of state functions such as $S(h_t, v_t, x_{t-1})$ may actually vary systematically in period t in response to a change in the government policy *rule* perceived to be in effect in period t , even if this change only affects future yet-to-be-realized variables. Specifically, Lucas argues that the structure of the state equations will depend on the form of the government policy rule if such equations incorporate behavioral equations reflecting the behavior of “rational” agents who condition their behavior on an understanding of the implications of this policy rule for the choice of policy variables over future as well as current periods. For example, the response of a consumer today to a certain percentage cut in her income tax rate may

²Note, here, the critical distinction between government policy *variables* such as the components of h_t and government policy *rules* of the form $g:I \rightarrow H$.

very well depend on her perception of what changes government also intends to implement with regard to future income tax rates, i.e. on her perception of what change government has implemented in the income tax *rule* it will use to generate these future income tax rates.

3 Illustration of the Lucas Critique

Suppose a representative consumer's optimal investment demand function requires that her investment demands $\{k_t : t = 1, 2, \dots\}$ satisfy³

$$k_t = \lambda k_{t-1} - \alpha \sum_{j=0}^{\infty} \delta^j E_t \tau_{t+j} , \quad (9)$$

where $E_t \tau_{t+j}$ is the tax rate that the consumer in period t expects will prevail at time $t + j$, and α , λ , δ , and k_0 are exogenous parameters satisfying $0 < \alpha$, $0 < \lambda < 1$, $0 < \delta < 1$, and $0 < k_0$. Suppose that government has a tax policy rule of the form

$$\tau_{t+1} = \theta \tau_t , \quad t \geq 1 , \quad (10)$$

characterized by a parameter θ satisfying $-1 \leq \theta \leq 1$, where τ_1 is exogenously given at the beginning of period $t=1$. A change in government's tax policy rule can thus be identified with a change in the value of the parameter θ .

Assume the consumer has strong-form RE, implying in particular that the consumer knows the structural form of the tax policy rule (10), the structural form of her optimal investment demand function (9), and the values of all exogenous variables. Since the model is deterministic, this implies that the consumer's subjective expected future tax rates appearing in (9) take the perfect foresight RE form

$$E_t \tau_{t+j} = (\theta)^j \tau_t = \tau_{t+j} . \quad (11)$$

Consequently, her optimal investment demand function (9) reduces to

$$k_t = \lambda k_{t-1} - \gamma \tau_t , \quad (12)$$

³This example is adapted from Sargent (1986, pages 4-6).

where

$$\gamma = \alpha \left[\frac{1}{1 - \delta\theta} \right] . \quad (13)$$

Note the similarity of the form of (12) to the form of (6). However, in (12) the parameter γ is not simply an exogenous constant. Rather, as seen in (13), γ depends on the government's choice of its tax policy *rule* (10) as characterized by the parameter θ .

In traditional econometric practice, the investment demand function (12) for period t would typically have been estimated by determining best-fit values $\hat{\lambda}$ and $\hat{\gamma}$ for the parameters λ and γ by regressing the consumer's past investment demands on one-period lagged investment demand and the prevailing tax rate, without regard for the particular tax policy rule in effect. If, for example, the government had consistently imposed a tax policy rule with $\theta = 1$ prior to time period t , the estimated value for γ would have the form

$$\hat{\gamma} = \alpha \left[\frac{1}{1 - \delta} \right] . \quad (14)$$

In short, the traditional econometrician would first have estimated an investment demand function of the form

$$k_t = \hat{\lambda}k_{t-1} - \hat{\gamma}\tau_t \quad (15)$$

on the basis of past observations. The estimated function (15) would then have been used by the econometrician in time period t to predict the effects on future investment demand of a prospective change in θ (i.e., in the government's tax policy rule) as follows: (a) plug the changed value of θ into the tax policy rule (10); (b) use this modified rule to generate a sequence of future tax rates $\{\tau_{t+j} : j = 1, 2, \dots\}$; and (c) plug this sequence of tax rates into the estimated investment demand function (15) for successive time periods to sequentially generate predictions for the consumer's future investment demands $\{k_{t+j} : j = 1, 2, \dots\}$.

Imagine, however, that the change in tax policy rule under consideration by government for imposition in period t is a change from $\theta = 1$ to a permanent new setting of $\theta = -1$. In

this case, under strong-form RE, the optimal investment demand function for the consumer in period t would abruptly switch to the form

$$k_t = \lambda k_{t-1} - \gamma^* \tau_t , \tag{16}$$

where

$$\gamma^* = \alpha \left[\frac{1}{1 + \delta} \right] . \tag{17}$$

Consequently, all of government's investment predictions obtained using $\hat{\gamma}$ in (14) would be invalid.

Econometrically, in a later time period $t+T$ this problem would show up as an occurrence of a “structural break” in the estimate for γ at time t . That is, if the sampling period were broken up into two subsamples, one spanning periods prior to t , and one spanning periods after t , it would be seen that the best-fit estimates for γ over these two subsamples were statistically different from each other. The point stressed by the Lucas Critique, however, is that such problems can be foreseen and prevented by using an appropriate microfoundations analysis to identify how the actions of rational agents will depend not just on government policy *variables* but also on government policy *rules*.

A conceptual difficulty with this analysis, however, is that a “sudden change in policy rule” is not really well defined under strong-form RE. For example, in the previous illustration, it is first assumed that the consumer expects that government will maintain forever its originally announced policy rule with $\theta=1$. Then, after suddenly realizing that the government has switched in period t to a new policy rule with $\theta = -1$, the consumer switches to believing that government will now forever maintain this new policy rule. If the consumer indeed has strong-form RE, however, the consumer will be able to foresee and plan for the possibility that government will switch its policy rule. More precisely, if some event could cause government to switch to a new policy rule, the consumer will be aware of this possibility since, by definition, she has full structural knowledge of the economy in which she

resides. Indeed, it could reasonably be argued that the event-triggered “switch” is actually not a switch at all; it’s just part of some more comprehensive information-conditioned government policy rule known to the consumer. See Hoover (1988, Chapter 8, pp. 195-197) for further discussion of this issue.

4 Extensions of the Lucas Critique

The introduction of government as an optimizing agent in macroeconomic models in which optimizing private-sector agents engage in sophisticated forms of expectation formation in effect turns the macroeconomy into a complex game among multiple strategic agents. This raises a host of challenging concerns related to the Lucas Critique, both theoretical and practical. Three of these concerns are as follows:

1. Rules vs. Discretionary Behavior:

The basic case against discretionary policy for government is that it can destabilize the structure of the economy. Only when changes in policy take the form of fully discussed and understood changes in policy *rules* will the resulting changes in the structure of the economy be econometrically predictable.

More precisely, rules can be learned in a way that discretionary behavior cannot; for, by definition, discretionary behavior changes in ways unsystematically related to the state of the economy. When private agents have no stable basis for predicting government behavior, the expectations of the private agents are likely to be volatile and the impact of government behavior on the actions of the private agents might then be hard to predict. To form coherent expectations, private sector agents must have some idea how government is actually selecting its policy variables. In this case there is a chance that the relationship between policy choices and expectations can be

discovered econometrically.⁴

2. Criticism of the “Stackelberg game” approach to macro policy models:

The Tinbergen framework (4) through (6) traditionally used for macro policy analysis essentially views the economy as an asymmetrical “Stackelberg game” between an active government leader and a passive collection of private-sector followers. Specifically, private-sector agents are assumed to respond passively (non-strategically) to any sequence of policy actions announced by government for current and future time periods, ignoring how this sequence is actually generated. Government first estimates the response of the economy to each feasible policy action sequence and then chooses a policy rule so that the resulting sequence of policy actions generated under this rule is optimal in terms of some specified social welfare function. The argument for this solution concept is that government is a very large agent whereas the private sector consists of relatively small and disorganized agents who take policy actions as given in determining their optimal choices.

Critics of this Stackelberg-game model of a macroeconomy argue that government is not the only large actor on the economic stage. Large firms, political parties, etc., may all have enough power to influence government’s behavior in their own behalf. In such cases, it is misleading to assume that all private sector agents optimize taking government’s policy actions as given. A related criticism is that government may face the problem of re-election, in which case it becomes to some extent a creature of the private sector agents that it seeks to control.

Even if all private sector agents are modeled as price takers who do not see the true structure of their economy and who do not perceive any strategic interaction

⁴As Lucas has frequently stressed, the U.S. Constitution (Article I, Section 9) explicitly requires transparency in U.S. Congressional behavior as follows: “No Money shall be drawn from the Treasury, but in Consequence of Appropriations made by Law; and a regular Statement and Account of the Receipts and Expenditures of all public Money shall be published from time to time.”

effects with other private agents, these agents cannot help but see that government policy actions (e.g., taxes) affect their budget constraints. This perceived “behavioral externality” could then lead sophisticated private agents to behave strategically in an attempt to affect government’s policy action choices.

For example, if private agents believe government is selecting its policy actions in accordance with a feedback policy rule, they might attempt to affect various state variables in government’s information sets to ensure policy actions more favorable to themselves. In this case private sector agents cannot be viewed as passive followers who simply condition their actions on announced government policy actions.

3. Credible Commitments and Time Inconsistency:

How can government make private sector agents believe that the policy rule it announces today will actually be carried out over future time periods? Indeed, under what circumstances *will* government actually carry out its announced policy rule as time proceeds and the future becomes the present?

This section elaborates more fully on each of the above three points. See, also, Schelling (1982), Hoover (1988, Chapter 8), and Mishkin (1996).

The above three points provide empirical arguments cautioning against the desirability and/or feasibility of government behaving as a “Stackelberg game” leader in relationship to private sector “followers.” However, the *logical* coherency of this Stackelberg view of government behavior can also be questioned.

Unless government can make a once-and-for-all self-binding commitment to the optimal policy rule it chooses at time 1, private sector agents will presumably expect government to be able to re-optimize this rule at later dates. In this case, private sector agents might realize that, in some future time period $t > 1$, government could have an incentive to exhibit *time inconsistency*, i.e., to deviate in period t from the optimal policy rule it chose in period

1. This is called the *time inconsistency problem*, first brought to the attention of economists by Kydland and Prescott (1977).

The reason that time inconsistency can arise is that, from government's point of view, it might be optimal to use its announced government policy rule in period 1 covering current and future time periods to encourage private sector agents to commit to certain actions over the near term. Once private sector agents have committed to these actions, however, government might then find it optimal to shift to a new policy rule.

Interestingly, time inconsistency can arise even in the absence of external shocks to the economy and even if government is benevolently attempting to maximize the expected utility of a representative consumer who always fully believes everything government announces. For example, suppose that the representative consumer does not take into account the manner in which his own actions impinge on government's future budget sets, an "externality" that prevents the applicability of the basic First Welfare Theorem guaranteeing every competitive equilibrium is Pareto optimal. In this case, as shown in Tesfation (1986), a continually active government intervention requiring successive time inconsistency (deviations from previously announced policies) will generally be needed to ensure that the representative consumer follows a Pareto optimal path over time.

More realistically, however, if private sector agents realize that government might deviate in future time periods from its currently announced policy rule, government might lose its credibility. That is, private sector agents might *not* condition their choices on government's announced policy rule choice for period 1 but might instead consider the likelihood of possible deviations from this policy rule. Moreover, private sector agents would also presumably realize that any policy rule chosen by a re-optimizing government in some future time period t will depend to some extent on their choices in periods $s < t$. Consequently, government is not in the pure leadership position assumed by the Tinbergen framework.

On what basis do people assess the credibility of government? Schelling (1982) addresses this issue in some detail.

First, there is the credibility of *facts*. Actually in economics there are few real facts. There are interpretations, hypotheses, principles, and predictions (theories coupled with initial conditions that generate predicted future outcomes).

The most government can commit to is a program, an input, not a result. The promised results are only as credible as the commitment to the program *and* the theory upon which the predicted results are based. Translating even a confidently shared expectation of government action into a shared expectation of results requires that a decisive subset of economic agents confidently share a theory, identical to the government's theory, of how program inputs relate to outcomes. Without the shared theory, only the inputs can be credible.

Yet precise predictions about the effects of government policies depend on knowing more about the way things work than anyone at the current time actually knows. Even if government and private sector agents share the same theory with regard to the mapping of actions to outcomes, what happens if this theory is not correct? Will a government stick with its currently announced policy rule if it observes deviations between actual and predicted outcomes?

After facts (principles, predictions...) we come to intentions. Can a government persuade private sector agents that it will do what it says, that it will not change its mind? Does government have the incentive to deliver what is promised? Does it have the authority? Does it have the votes? Will it survive the next election?

How does government try to make its actions credible? One technique is to delegate authority to someone whose motives already enjoy credibility. One example observed in many countries is the deliberate surrender of authority and responsibility for monetary policy to the authorities of an independent central banking system (e.g. the Federal Reserve Board in the U.S.). Another tactic of governments is to arrange for small-scale demonstrations of

their commitment – e.g., President Reagan in the early nineteen eighties demonstrating his commitment to a strong anti-inflation (and anti-union) stance by permitting the firing of striking union air controllers who were demanding higher wages and better working conditions. Even more effective is some form of institutionalized enforceable means for government to bind itself to its announced policy rules, a belief that has fueled repeated attempts in the U.S. Congress to enact some type of balanced budget legislation.

Another force that helps to ensure government credibility is reputational effects. Governments recognize that time inconsistent behavior on their part will ultimately lead to a loss of credibility (reputation) that might imply a more unfavorable menu of policy rule options for them in the future as people become more cynical about government's intentions.

This raises further subtle and interesting issues, however. Why do governments often fail in their attempts to make people believe things?

In this regard, it is important to note an important distinction forcefully stressed by Schelling (1982): namely, the distinction between trying to get people to believe a proposition that is *unconditionally true* and trying to get people to believe a proposition that *would* be true if (and perhaps only if) enough people believed it were true. In the latter case, if government attempts to make people believe an event will happen, but not enough people are convinced, then the event will not happen and this will tend to confirm people's disbelief in government's announcements.

Here are some examples of self-confirming expectations.

- Bank SpendThrift is solvent
- The stock market is not about to crash.
- Energy prices will rise (so it is important to hoard fuel now to avoid paying high prices later).

- Political Party X is going to win the next election (so it is futile for individual members of Party Y to vote and it is futile for individuals to give money to Party Y candidates).

Conversely, one must also consider the Cassandra effect.⁵ If government warns that some dire event is about to occur, and enough people believe government, then steps might be taken that prevent the dire event from occurring. This may also lead people to question government's credibility.

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⁵Cassandra was a prophetess in Greek legend who correctly foretold evil events but was fated never to be believed.

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