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Functions in Oligopolistic-Oligopsonistic Markets**

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Abstract

In this study, we regard the oligopolistic-oligopsonistic markets within the framework of a “double auction” in which both buyers and sellers make bids. To this end, we introduce games where declarations of supply and demand functions (which need not be true) are treated as strategic variables of producers and consumers, respectively, rather than just as “binding commitments” on the part of these parties. Whether firms produce with positive or zero marginal cost, the number of agents on each side of the market, whether consumers act as a union or not and time structure of the moves lead to different games. Existence of symmetric equilibria of each of these games is established. Most of them are shown to be unique. The equilibrium outcomes of these are compared with the standard Cournot outcome as well as among themselves regarding the market price, total quantity produced, individual consumer’s surplus, individual firms’ profit and social welfare they lead to. To allow the consumers to behave strategically along with the producers, naturally makes the former better off and the latter worse off, while the net effect of this on total social welfare turns out to be case-contingent.

Key Words: buyer power, demand function equilibria, double auction, oligopoly, oligopsony, supply function equilibria.

JEL Classifications: C71, L1, L51

1. Introduction

As is well known, in perfectly competitive markets theory consumers and firms are assumed to occur in large numbers. Competitive firms can not affect the market price, nor can the consumers. Market price is determined by the intersection of aggregate demand and aggregate supply. No strategic role is attributed to either consumers or firms, for the impacts of individual agents’ actions upon supply and demand are so negligible that they go unnoticed by the market. On the other hand, oligopolistic market theory deals with market interactions of a small number of firms. The literature on the game theoretic analysis of oligopolistic markets mostly attributes a strategic role to firms but not to consumers and justifies this by the asymmetry in the sizes of both parties. Since consumers are assumed to occur in large numbers, each individual consumer

remains negligible, and so, their existence can only be traced in the market demand which is regarded as a binding commitment on the part of the whole consumer body, whereas firms, given market demand, enter to competition among each other by utilizing strategic variables which vary from quantity, price, supply function to mark-up over average cost.

However, we also observe that there are markets in which a small number of buyers interact. In fact there are even cases where a monopsonist prevails on the demand side or where consumers are not uniform, but highly differentiated regarding the size of their demands. Whenever big firms are demanding a particular good as an intermediate good beyond individuals' consumption demand, they may have the power to affect the market price, also known as buyer power. Food industry (see Sexton and Lavoie, 2001) and energy sectors (see Bolle, 2001) are typical examples of this. Another example is auctions on government bonds in which a certain number of large banks are allowed to participate. In weapons industry, the government stands as a monopsonist and a small number of firms are awarded contracts. Finally, a small number of players and clubs negotiate over contracts in variety of sports. Thus, in markets similar to the above it is natural to ascribe a strategic role to the demand side as well. Furthermore, such a consideration allows to analyze the welfare effects of a possible organized behavior on the part of buyers. To this end, here we model games where buyers act as active players by declaring demand functions (which need not be true, but become a binding commitment once declared) as strategies along with seller firms whose strategies are supply functions.

The roots of an approach which ascribes to also the consumers a strategic role by allowing them to manipulate demand functions can be traced back in the literature as well and will be discussed in the following. Grossman (1981) by introducing supply schedules as firms'

strategic variable obtains competitive equilibria as a Nash equilibria outcomes of supply functions under some restrictions on supply functions. Although Grossman (1981) assumes a deterministic industry demand and allows only firms to behave strategically, as a remark he mentions about the possible roles of consumers such as behaving monopsonistically, misrepresenting individual demands in various contexts.

Koray and Sertel (1989) and Klemperer and Meyer (1989) are two responses to Grossman (1981). For an oligopoly, Koray and Sertel (1989) suggest a franchise bidding mechanism similar to Loeb and Magat (1979). Koray and Sertel show that there is a Nash equilibrium in bids which maximizes social welfare. When industry is uniform, it turns out that every Nash equilibrium in bids leads to the social optimum. Thus, their results give the supply function equilibrium theorem in Grossman (1981) as corollary. In Koray and Sertel's (1989) consumers are not considered as players. Whether to attribute consumers a strategic role would lead to social optimum remains as open question. On the other hand, Klemperer and Meyer (1989) introduces uncertainty in industry demand. The justification of firm's commitment to supply functions turns out to be better adaptation to the uncertainty. When industry demand is subject to exogenous random shock, firms set price for each realization of random shock and so they achieve ex-post optimal adjustment to the shock. This adjustment reduces the set of equilibria in supply functions even to a unique equilibrium under appropriate assumptions. The Klemperer and Meyer's solution concept has been applied to strategic trade policy (see Laussel 1992), in the analysis of electricity spot market (see Bolle, 1992; Green, 1996; Bolle, 2001; Rudkevich, 2002) with some minor modifications depending on the problem at hand, and other studies such as Grant and Quiggin (1996) and Khün (1997).

Hurwicz (1977) is an early reference introducing the idea that consumers can

misrepresent their preferences. In an exchange economy with all goods are private if consumers are in finite numbers, he shows that when all other consumers stick to their true preferences and behave as price taker, it can be in his best interest of the remaining consumer to misrepresent his preferences. Therefore, he concludes that perfect competition may not be individually incentive compatible. Another reference in which demand functions are used as strategies is Wilson (1979) on share auctions. Binmore and Swierzbinski (2000) studies various auction formats in multi-unit auctions and can be seen as the extension of Wilson (1979) on share auctions to multi-unit case. Furthermore, the studies such as Engelbrecht-Wiggans, List, and Reiley (2006); Swinkels (2001); Rustichini, Satterthwaite, and Williams (1994) find strategic demand reduction in multi-unit auctions under incomplete information and show that this effect decreases as the number of bidders increase.

By considering consumers as players with demand functions together with the firms playing with supply functions, we think of oligopolistic markets from the point of view of “double auction” in which sellers make offers and buyers make bids. This approach is very much line with Sonnenschein (1985), who relates oligopoly theory to the auction theory and provides an example of a simple game in which both parties have strategic role. We model games distinguished by different institutional assumptions and study the implications of these models.

The plan of this study as follows: We proceed with the section The Model in which we introduce the model in general. Then the section Nash Games follows. There consumers either organized or unorganized play simultaneously with firms. Then, we introduce the Stackelberg Games: There firm is Stackelberg leader and consumers as organized are followers. Finally, we compare outcomes of these games among themselves and with the outcome of Cournot-Nash game. These constitute our four theorems. Note that the proofs are provided elsewhere (Bulut,

1999) and can also be requested from the authors. In the last section, we conclude.

2. The Model

We are in a market for a particular good in the economy. In this market there are n consumers and m firms, where n, m are positive integers. Consumers are identical with their demand functions and firms are identical with their cost functions. Each consumer has an affine demand function, $D(P) = a - bP$, where $a > 0$ and $b > 0$. We assume the slope parameter, b is known, whereas the intercept term, a is private to each consumer. Thus, consumers have an option to manipulate their intercept terms either individually or in an organized manner. If they are not organized, each consumer is picking a positive number, $\gamma > 0$ for his intercept term a and so giving a demand function, by aiming to maximize his consumer surplus. If they are organized, consumer union (CU) plays on behalf of consumers by manipulating the intercept term of the aggregate demand by aiming to maximize total consumers' surplus. Note that true aggregate demand is $AD(P) = na - nbP$ and CU is giving $AD(P) = \Gamma - nbP$, where $\Gamma > 0$.

We assume that the contract among consumers is the equal division of aggregate quantity demanded at the resulting equilibrium price. Since consumers are identical, this assumption is the most appropriate one. After division, each consumer can calculate his consumer surplus and compare with the one he obtains when they are not organized. On the other hand, each firm has a quadratic cost function, $C(q) = \alpha q^2$, where $\alpha \geq 0$. Though form of the cost function is known, cost parameter α is private to firms. Thus, firms have an option to misrepresent their cost parameters. Note that quadratic cost function implies linear marginal cost function

$MC(q) = 2\alpha q$, which in turn implies a supply function $q(P) = \frac{1}{2\alpha} P$ where slope parameter is

private to firms. Thus, each firm by picking a non-negative slope parameter $\beta \geq 0$, in fact by picking a non-negative number for cost parameter, is making a linear supply function commitment, $q(P) = \beta P$ where $\beta \geq 0$. Then, firms' strategies are their supply functions, in particular their slope terms. Given firms' supply function commitments and consumers' demand function commitments, we define outcome price, P , as the number such that aggregate supply equals to aggregate demand, i.e, P satisfies

$$\sum_{i=1}^m q_i(P) = \sum_{i=1}^n D_i(P)$$

Note that outcome price is a function of supply and demand functions and such a number exists and unique since supply functions are linear and demand functions are affine.

2.1. Nash Game

Consumers either organized or unorganized play Nash with firms. We are only interested in symmetric equilibria.

Definition 1. Let $(\gamma^*)_{i=1}^n$ be a list of strategies of consumers and $(\beta^*)_{i=1}^m$ be a list of strategies of firms. We say the list $((\gamma^*)_{i=1}^n, (\beta^*)_{i=1}^m)$ forms a Nash equilibrium in intercept terms of consumers and slope terms of firms when other agents stick to their strategies in the list, if for each consumer γ^* maximizes

$$CS_i = \int_0^{\gamma - bP(\gamma)} \left(\frac{a}{b} - \frac{1}{b}t \right) dt - P(\gamma)(\gamma - bP(\gamma))$$

with respect to for any positive intercept term, γ , where $P(\gamma)$ is the outcome price and

solved from (1) for each γ and for each firm β^* maximizes

$$\Pi_i = P(\beta)(\beta P(\beta)) - \alpha(\beta P(\beta))^2$$

with respect to for any non negative slope term, β , where $P(\beta)$ is the outcome price solved from (1).

When consumers are organized, we define Nash equilibrium as follows:

Definition 2. Let Γ^* be a particular strategy of consumer union and let $(\beta^*)_{i=1}^m$ be a list of firms' strategies. We say the list $(\Gamma^*, (\beta^*)_{i=1}^m)$ forms a Nash equilibrium in intercept term of aggregate demand and slope parameters of supply functions when other agents stick to their strategies in the list, if Γ^* maximizes total consumer surplus (TCS),

$$TCS = \int_0^{\Gamma - nbP(\Gamma)} \left(\frac{a}{b} - \frac{1}{bn}t \right) dt - P(\Gamma)(\Gamma - bnP(\Gamma))$$

with respect to for any non negative Γ , where $P(\Gamma)$ is solved from (1) as $\frac{\Gamma}{nb + m\beta^*}$, and

for each firm β^* maximizes

$$\Pi_i = P(\beta)(\beta P(\beta)) - \alpha(\beta P(\beta))^2$$

with respect to for any positive slope term β , where $P(\beta)$ is solved from (1) as

$$\frac{\Gamma}{nb + (m-1)\beta^* + \beta}.$$

Cournot-Nash Game

In Cournot-Nash game, consumers are not players. They submit their true demand functions.

Given aggregate demand, firms compete through declaring quantities. Typical firm's problem,

$$\max_{q_j} (q_j (\frac{a}{b} - \frac{1}{bn} (\sum_{i=1}^m q_i)) - \alpha q_j^2)$$

Now we proceed case by case and give equilibrium strategies:

Case 1: $\alpha = 0$, $n \geq 1$, $m \geq 2$: In this case firms produce with zero cost and there are at least one consumer and two firms.

Proposition 1. Let γ^* be equal to a and β^* be equal to ∞ . Now, the list $((\gamma^*)_{i=1}^n, (\beta^*)_{i=1}^m)$ forms a Nash equilibrium.

Proposition 2. Now, the list $((a)_{i=1}^n, (\infty)_{i=1}^m)$ forms a unique symmetric Nash equilibrium.

Lemma 1. Consider the Cournot- Nash game for this case. Now, for $q^* = \frac{an}{m+1}$ the list $(q^*)_{i=1}^m$ forms a unique symmetric Nash equilibrium.

Proposition 3. $P^{C-N} > P^{CPN}$, $Q^{C-N} < Q^{CPN}$, $CS^{C-N} < CS^{CPN}$, $\Pi^{C-N} > \Pi^{CPN}$ and $SW^{C-N} < SW^{CPN}$.

Proposition 4. The following is true: $\lim_{m \rightarrow \infty} P^{C-N} = P^{CPN}$, $\lim_{m \rightarrow \infty} CS^{C-N} = CS^{CPN}$, $\lim_{m \rightarrow \infty} \Pi^{C-N} = \Pi^{CPN}$, and $\lim_{m \rightarrow \infty} SW^{C-N} = SW^{CPN}$.

Corollary 1. The outcome of the game where consumers and firms play Nash is the same with the competitive equilibrium outcome for this case.

Case 2: $\alpha = 0$, $n \geq 1$, $m = 1$: For this case there is natural monopoly producing with zero cost and at least one consumer.

Proposition 5. Let γ^* be equal to $\frac{a(4n-2)}{(4n-1)}$ and β^* be equal to bn . Now, the list

$((\gamma^*)_{i=1}^n, \beta^*)$ forms a Nash equilibrium.

Proposition 6. Let γ^* and β^* as in Proposition 5. Now, the list $((\gamma^*)_{i=1}^n, \beta^*)$ forms a unique symmetric Nash equilibrium.

Proposition 7. Assume that consumers are organized. Let Γ^* be equal to $\frac{2an}{3}$ and β^* be equal to bn . Now, the list (Γ^*, β^*) forms a unique Nash equilibrium.

Lemma 2. Consider the Cournot-Nash Game. Now, $Q^M = \frac{an}{2}$ is the monopolist's profit maximizing level of output.

Proposition 8. The following is true: $P^M > P^{CPN} \geq P^{CUPN}$, $Q^M > Q^{CPN} \geq Q^{CUPN}$

$CS^{CUPN} \geq CS^{CPN} > CS^M$, $\Pi^M > \Pi^{CPN} \geq \Pi^{CUPN}$, and $SW^M > SW^{CPN} \geq SW^{CUPN}$. (where when

$n \geq 2$, inequalities are strict.)

Proposition 9. The following is true: $\lim_{n \rightarrow \infty} \gamma^* = a$,

$$\lim_{n \rightarrow \infty} Q^C = \lim_{n \rightarrow \infty} Q^{CPN} = \lim_{n \rightarrow \infty} Q^M = \lim_{n \rightarrow \infty} Q^{CUPN} = \infty$$

$$\lim_{n \rightarrow \infty} CS^{CPN} = \lim_{n \rightarrow \infty} CS^M = \frac{a^2}{8b} < \lim_{n \rightarrow \infty} CS^{CUPN} = \frac{a^2}{6b} < \lim_{n \rightarrow \infty} CS^C = \frac{a^2}{2b}$$

$$\lim_{n \rightarrow \infty} \Pi^C = 0 < \lim_{n \rightarrow \infty} \Pi^{CPN} = \lim_{n \rightarrow \infty} \Pi^M = \lim_{n \rightarrow \infty} \Pi^{CUPN} = \infty , \text{ and}$$

$$\lim_{n \rightarrow \infty} SW^C = \lim_{n \rightarrow \infty} SW^{CPN} = \lim_{n \rightarrow \infty} SW^M = \lim_{n \rightarrow \infty} SW^{CUPN} = \infty .$$

Case 3: $\alpha > 0$, $n \geq 1$, $m \geq 2$: In this case firms are producing with positive marginal cost and there is at least one consumer and more than one firm.

Proposition 10. Let γ^* be equal to $\frac{\frac{a}{b}\theta}{\frac{1}{b}\theta^2 + \frac{2}{nb+m\beta^*}\theta - \frac{(n-1)b}{(nb+m\beta^*)^2}}$ where

$$\theta = (1 - \frac{b}{nb+m\beta^*}) \text{ and } \beta^* \text{ be equal to } \frac{-(m-2-2\alpha nb) - \sqrt{\Delta}}{-4\alpha(m-1)} \text{ where}$$

$\Delta = (m-2-2\alpha nb)^2 + 8\alpha nb(m-1)$. Now, the list $((\gamma^*)_{i=1}^n, (\beta^*)_{i=1}^m)$ forms a Nash equilibrium.

Lemma 3. The following is true. $\gamma^* < a$, $\lim_{n \rightarrow \infty} \gamma^* = a$, $\lim_{n \rightarrow \infty} \beta^* = \frac{1}{2\alpha}$, $\lim_{n \rightarrow \infty} P^{CPN} = \frac{a}{b}$,

$$\lim_{n \rightarrow \infty} (\gamma^* - bP^{CPN}) = 0 , \lim_{n \rightarrow \infty} AD(P^{CPN}) = \frac{ma}{2\alpha b} , \lim_{m \rightarrow \infty} \gamma^* = a , \lim_{m \rightarrow \infty} \beta^* = \frac{1}{2\alpha} , \lim_{m \rightarrow \infty} P^{CPN} = 0 ,$$

$$\lim_{m \rightarrow \infty} (\gamma^* - bP^{CPN}) = a , \text{ and } \lim_{m \rightarrow \infty} AD(P^{CPN}) = na .$$

Lemma 4. The following is true: $\lim_{m \rightarrow \infty} CS^{CPN} = \lim_{m \rightarrow \infty} CS^{C-N} = \frac{a^2}{2b}$, $\lim_{m \rightarrow \infty} \Pi^{CPN} = \lim_{m \rightarrow \infty} \Pi^{C-N} = 0$,

$\lim_{m \rightarrow \infty} SW^{CPN} = \lim_{m \rightarrow \infty} SW^{C-N} = \frac{na^2}{2b}$, $\lim_{n \rightarrow \infty} CS^{CPN} = \lim_{n \rightarrow \infty} CS^{C-N} = 0$, $\lim_{n \rightarrow \infty} \Pi^{CPN} = \lim_{n \rightarrow \infty} \Pi^{C-N} = \frac{a^2}{4\alpha b}$, and

$\lim_{n \rightarrow \infty} SW^{CPN} = \lim_{n \rightarrow \infty} SW^{C-N} = \frac{ma^2}{4\alpha b}$.

Example 1: For $a = 1$, $b = 1$, $\alpha = 1$, $n = 1$, $m = 2$, the following is true: $P^{C-N} > P^{CPN}$, $Q^{C-N} > Q^{CPN}$, $CS^{CPN} > CS^{C-N}$, $\Pi^{C-N} > \Pi^{CPN}$ and $SW^{C-N} > SW^{CPN}$. One can put the values into the corresponding formulas given in Lemma 5 and verify the results.

The following example is to check if there exist more than one consumer whether relations above remains the same or not. It turns out that relations stay the same for this example.

Example 2. For $a = 1$, $b = 1$, $\alpha = 1$, $n = 2$, $m = 2$, the following is true: $P^{C-N} > P^{CPN}$, $Q^{C-N} > Q^{CPN}$, $CS^{CPN} > CS^{C-N}$, $\Pi^{C-N} > \Pi^{CPN}$ and $SW^{C-N} > SW^{CPN}$. The same procedure applies.

Case 4: $\alpha > 0$, $n \geq 2$, $m = 1$: For this case there are one firm producing with positive marginal cost and at least two consumers.

Proposition 11. Let γ^* be equal to $\frac{-2a(1+\alpha bn)(-1+2n-2\alpha bn+2\alpha bn^2)}{(1-4n+4\alpha bn-8\alpha bn^2+4\alpha^2 b^2 n^2-4\alpha^2 b^2 n^3)}$ and β^* be

equal to $\frac{nb}{1+\alpha bn}$. Now the list $((\gamma^*)_{i=1}^n, \beta^*)$ forms a Nash Equilibrium.

Proposition 12. Assume that consumers are organized. Let Γ^* be equal to $\frac{an(2+2\alpha an)}{(3+4\alpha an)}$ and

β^* be equal to $\frac{nb}{1+2\alpha nb}$. Now, the list (Γ^*, β^*) forms a unique symmetric Nash equilibrium.

Lemma 6. Consider the monopoly structure. Let q^* be equal to $\frac{an}{2(1+\alpha bn)}$. Now, q^* is the monopolist profit maximizing level of output.

Proposition 13. The following is true: $\lim_{n \rightarrow \infty} \gamma^* = a$, $\lim_{n \rightarrow \infty} \beta^* = \frac{1}{2\alpha}$,

$$\lim_{n \rightarrow \infty} P^{CPN} = \lim_{n \rightarrow \infty} P^M = \lim_{n \rightarrow \infty} P^C = \frac{a}{b} > \lim_{n \rightarrow \infty} P^{CUPN} = \frac{a}{2b}$$

$$\lim_{n \rightarrow \infty} Q^{CPN} = \lim_{n \rightarrow \infty} Q^M = \lim_{n \rightarrow \infty} Q^C = \frac{a}{2\alpha b} > \lim_{n \rightarrow \infty} Q^{CUPN} = \frac{a}{4\alpha b}$$

$$\lim_{n \rightarrow \infty} CS^{CPN} = \lim_{n \rightarrow \infty} CS^M = \lim_{n \rightarrow \infty} CS^C = \lim_{n \rightarrow \infty} CS^{CUPN} = 0, \quad \lim_{n \rightarrow \infty} TCS^{CUPN} = \frac{a^2}{8\alpha b^2},$$

$$\lim_{n \rightarrow \infty} \Pi^{CPN} = \lim_{n \rightarrow \infty} \Pi^M = \lim_{n \rightarrow \infty} \Pi^C = \frac{a^2}{4\alpha b^2} > \lim_{n \rightarrow \infty} \Pi^{CUPN} = \frac{a^2}{16\alpha b^2}$$

$$\lim_{n \rightarrow \infty} SW^{CPN} = \lim_{n \rightarrow \infty} SW^M = \lim_{n \rightarrow \infty} SW^C = \frac{a^2}{4\alpha b^2}, \quad \text{and} \quad \lim_{n \rightarrow \infty} SW^{CUPN} = \frac{3a^2}{16\alpha b^2}.$$

Case 5: $\alpha > 0$, $n = 1$, $m = 1$ In this case there is one consumer and one firm. The firm produces with positive cost.

Proposition 15. Let γ^* be equal to $\frac{a(2+2\alpha b)}{3+4\alpha b}$ and β^* be equal to $\frac{b}{(1+2\alpha b)}$. Now, the list

(γ^*, β^*) forms a unique Nash equilibrium.

2.2. Stackelberg Game

We can consider two games. In one of the games a firm is Stackelberg leader, whereas in the other consumers as organized are leaders. The latter reduces to Nash game because we observe that in all cases we cover so far, as long as there is symmetric bunch of strategies on the side of consumers, typical firm's problem is independent from the particular values of intercept terms. Thus, we proceed with the game in which the firm is Stackelberg leader and consumers are either organized or unorganized followers. Data and strategy sets are as defined in the Model section and we search for symmetric equilibria. The firm is leader so it moves first and announces a number for its slope term, which induces a game for consumers. Given the firm's strategy, consumers play Nash among themselves.

Definition 3. Let $\gamma^* > 0$ and $\beta^* \geq 0$ and $(\gamma^*)_{i=1}^n$ and (β^*) be a symmetric strategy profile of the consumers and the firm, respectively. We say the list $((\gamma^*)_{i=1}^n, (\beta^*))$ forms a Stackelberg equilibrium if $(\gamma^*)_{i=1}^n$ is the Nash equilibrium of the game induced by (β^*) and $((\gamma^*)_{i=1}^n, (\beta^*))$ is a Nash equilibrium of the whole game.

Now, we proceed case by case and give equilibrium strategies.

Case 5: $\alpha = 0$, $n = 1$, $m = 1$: In this case there is one consumer and one firm. The firm produces with zero cost.

Proposition 16. Assume that the firm is Stackelberg leader. Let γ^* be equal to $\frac{3a}{4}$ and β^* be equal to $2b$. Now, the list (γ^*, β^*) is the unique Stackelberg equilibrium.

Theorem 1. The following is true: $P^M > P^{CPN} > P^{FSL}$, $Q^M = Q^{FSL} > Q^{CPN}$, $CS^{FSL} > CS^{CPN} > CS^M$, $\Pi^M > \Pi^{FSL} > \Pi^{CPN}$, and $SW^M = SW^{FSL} > SW^{CPN}$.

Case 6: $\alpha = 0$, $n \geq 2$, $m = 1$: Only difference with the previous case is that there are more than one consumers. We require consumers to form a union, otherwise things are getting really complicated. As a new notation $FSL + CUF$ is the abbreviation for “Firm Stackelberg leader and CU is follower” and any variable takes it as superscript belongs to this game.

Proposition 17. Assume that consumers are organized and the firm is Stackelberg leader. Let Γ^* be equal to $\frac{3an}{4}$ and β^* be equal to $2bn$. Now, the list (Γ^*, β^*) forms a unique Stackelberg equilibrium.

Theorem 2. The following is true: $P^M > P^{CPN} > P^{CUPN} > P^{FSL+CUF}$, $Q^{FSL+CUF} = Q^M > Q^{CPN} > Q^{CUPN}$, $CS^{FSL+CUF} > CS^{CUPN} > CS^{CPN} > CS^M$, $\Pi^M > \Pi^{CPN} > \Pi^{FSL+CUF} > \Pi^{CUPN}$, and $SW^M = SW^{FSL+CUF} > SW^{CPN} > SW^{CUPN}$.

Case 7: $\alpha > 0$, $n = 1$, $m = 1$: For this case there is one consumer and one firm. The firm produces with positive marginal cost.

Proposition 18. Assume that firm is Stackelberg leader. Let γ^* be equal to $\frac{a(3+4\alpha b)}{4+8\alpha b}$ and

β^* be equal to $\frac{2b}{1+4\alpha b}$. Now, the list (γ^*, β^*) is a unique Stackelberg equilibrium.

Theorem 3. The following is true: $P^M > P^{CPN} > P^{FSL}$, $Q^M > Q^{FSL} > Q^{CPN}$,

$CS^{FSL} > CS^{CPN} > CS^M$, $\Pi^M > \Pi^{FSL} > \Pi^{CPN}$, and $SW^M > SW^{FSL} > SW^{CPN}$.

Case 8: $\alpha > 0$, $n \geq 2$, $m = 1$: In this case there are more than two consumers and one firm again. The firm produces with positive marginal cost.

Proposition 19. Assume that consumers are organized. Let Γ^* be equal to $\frac{an(3+4\alpha bn)}{(4+8\alpha bn)}$ and

β^* be equal to $\frac{2bn}{1+4\alpha bn}$. Now, the list (Γ^*, β^*) forms a unique Stackelberg equilibrium.

Theorem 4. The following is true: $P^M > P^{CPN} > P^{CPN} > P^{FSL}$,

if $ab \in \left(0, \frac{\sqrt{2}-1}{2n}\right)$: $Q^M > Q^{FSL+CUF} > Q^{CPN} > Q^{CUPN}$,

if $ab \in \left(\frac{\sqrt{2}-1}{2n}, \infty\right)$: $Q^M > Q^{CPN} > Q^{FSL+CUF} > Q^{CUPN}$,

if $ab = \frac{\sqrt{2}-1}{2n}$: $Q^M > Q^{CPN} = Q^{FSL+CUF} > Q^{CUPN}$, $CS^{FSL+CUF} > CS^{CUPN} > CS^{CPN} > CS^M$,

$\Pi^M > \Pi^{CPN} > \Pi^{FSL+CUF} > \Pi^{CUPN}$, and $SW^{CPN} > SW^M > SW^{FSL+CUF} > SW^{CUPN}$.

3. Conclusion

We have considered consumers as players along with firms in oligopolistic markets by defining games in which identical firms with quadratic cost functions restricted to announce linear supply functions and identical consumers with affine true demand functions are allowed to manipulate their intercept terms either in organized or unorganized manner. We have established the symmetric equilibria of both the Nash game and the Stackelberg game. In the latter a firm is assumed to be leader. When the consumers as organized are Stackelberg leaders, the game reduces to the Nash game. This is due to the assumption that consumers can only manipulate their intercept terms. We verified that when consumers' strategy sets are extended to include their slope parameters, firm's best response is a function of consumers' slope terms so when they are manipulated firm's strategy will change accordingly. When consumers are followers they are assumed to be organized. The case in which they are unorganized followers leads to the complications so we do not study here. For the Nash game, when there are at least two firms producing with positive marginal cost, we only make comparisons for particular set of values of parameters since outcome expressions are cumbersome. We support this piece of information with the limit results. Nevertheless, we cover fairly large class of cases to get an extensive idea on the possible effects of considering consumers as players along with firms and our findings are as follows:

Only for the case in which there are at least two firms producing with zero cost, telling the truth is the best response of each consumer to the others' strategy profile in which each of remaining consumers tells the truth and each firm announces its MC function. This Nash equilibrium is the unique symmetric one as we prove in Proposition 2. Moreover, the outcome of

this equilibrium leads to the social optimum. In all remaining cases that we cover, consumers misrepresent their intercept terms downwards as their best responses so declared demand function lies to the left of their true demand function at each price and firms announce a slope parameter which is less than the value in the supply function obtained from MC function as their best responses so declared supply function lies above of their MC functions at each quantity. We show that if either the number of consumers or firms gets larger and larger (whenever the case allows), they tell the truth in the limit.

One finding regarding the case of natural monopolist is that whenever forming union is meaningful, that is there are at least two consumers, it is strength for consumers so they increase their utility. Furthermore, if they form union in the Stackelberg game, they are better off compared to the Nash Game in which they form union. When there is one consumer, he or she is better off in Stackelberg game and the firm loses less compared to the Nash game. So Stackelberg equilibrium is an improvement for both parties in this case. When number of consumers more than one, this is not true anymore. Though consumers obtain the most compared to the other structures, the firm loses more compared to the Nash game in which consumers and the firm are players in this case. Thus, Stackelberg game outcome is an improvement for the consumers over the Nash game outcome.

The case of one firm (natural monopolist) is interesting for another reason. Since there is not any other firm providing the competitive behavior, franchise bidding procedure for monopoly due to Loeb and Magat (1979) is inapplicable. Only remaining option is to tax the monopolist. This policy might have deficiency since the cost parameter of the firm is private to itself. However, our results show that allowing consumers to manipulate their demand functions is equivalent to taxing the monopolist. Under these alternative institutional assumptions consumers

are better off and the firm is loser while the society gains or lose depending on the case. For example, for the case in which there are at least two consumers and one firm producing with positive cost there is welfare gain in the game all agents are players compared to the monopoly structure. Then total gain of consumers must be higher than the loss of the firm. This means consumers may even compensate the firm's loss and can be still better off than the initial monopoly structure while firm is made indifferent between two structures. However, there is much to gain for consumers in this case.

It is shown in Theorem 4 consumers obtain the most benefit under the Stackelberg game when CU is the follower. Thus, the designer can bargain to get the amount of tax that is equal to increase in total consumers' surplus when agents switch from monopoly structure to Stackelberg game. The firm does not lose the most profit in this game rather it is in the worst situation when it plays Nash with CU. From the relation about social welfare outcomes of both games we observe that the total loss of the firm is less than the total gain of consumers so there is profit arising from the design of the market. To sum up there are alternative structures and all of them transfer income from the monopolist to the consumers, which provides the designer of the market a bargaining power against the monopolist. When the firm produces with zero or positive cost and there is one consumer or when the firm produces with zero cost and there are at least two consumers, the same reasoning is valid.

In all cases, the distribution of total social welfare leads to the result that consumers become better off and so firms are losers compared to the Cournot-Nash game. Regarding the society's gain, we find that when there are at least two firms producing with zero cost, the Nash equilibrium in intercept terms of demand functions and slope parameters of supply functions yields the socially optimum outcome. For the positive marginal cost however, total social

welfare outcome of the game where consumers and firms play Nash turns out to be even less than the total social welfare obtained from the Cournot-Nash game. We note that as the number of consumers goes to infinity, the outcomes of Cournot-Nash game and the Nash game we define coincide in the limit.

Although for the case in which there is one firm producing with positive cost, there is some gain in total social welfare compared to the Cournot-Nash game, since when they are finite in numbers equilibrium do not occur on true demand function and IMC function, socially optimizing outcome is not obtained. Restricting the consumer to manipulate only his intercept term seems to be the reason. However, we verify (which is not shown here) that when the firm produces with positive cost, although consumers are allowed to manipulate their slope parameters as well, CU as the Stackelberg leader does not target the socially optimal outcome. What would happen if money transfers between the CU and the firm were allowed? This is an interesting case to study. We note that in the Stackelberg game in which CU (which is allowed to manipulate both intercept and slope parameters of the aggregate demand function) is the leader and one firm producing with zero cost is the follower the social optimal outcome is obtained in the limit of strategies of CU.

4. References:

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