# International Business Cycles with Multiple-Input Investment Technologies:

# Appendices (Not for Publication)

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## A Data and estimation of shocks

**Consumption** Annual detailed consumption data is obtained from OECD National Accounts Statistics 1970-2004. The total consumption aggregate, as in Backus, Kehoe, and Kydland (1992) and Stockman and Tesar (1995), is the private final consumption of non-durables.

The traded aggregate of consumption includes Food and non-alcoholic beverages; Alcoholic beverages, tobacco and narcotics; Clothing and footwear; Furniture and furnishings, carpets and other floor coverings; Household textiles; Household appliances; Glassware, tableware and household utensils; Tools and equipment for house and garden; Medical products, appliances and equipment; Audio-visual, photographic and information processing equipment.

The non-traded consumption aggregate includes Housing, water, electricity, gas and other fuels; Goods and services for routine household maintenance; Out-patient services; Hospital services; Operation of personal transport equipment; Transport services; Communications; Other recreational items and equipment, gardens and pets; Recreational and cultural services; Newspapers, books and stationery; Package holidays; Education; Restaurants and hotels; Miscellaneous goods and services.

Yet another way to distinguish between traded and non-traded consumption is as in

Stockman and Tesar (1995): traded consumption is proxied by the private final consumption of non-durable goods, while the private final consumption of services is used for non-traded consumption.

The second moments of sectoral aggregates by both definitions however are close to each other.

**Gross Domestic Product** For sectoral traded and non-traded GDP value-added aggregation, 60-industry data provided by Groningen Growth Development Center (GGDC) is utilized. The industries are grouped according to the International Standard Industrial Classification (ISIC) revision 3, which is consistent with the decomposition of industries in most of the OECD statistics. The Traded aggregate of GDP includes Agriculture, Forestry, Fishing, Mining and quarrying, and Manufacturing. Manufacturing includes Food, drink & tobacco; Textiles; Clothing; Leather and footwear; Wood & products of wood and cork; Pulp, paper & paper products; Printing & publishing; Mineral oil refining, coke & nuclear fuel; Chemicals; Rubber & plastics; Non-metallic mineral products; Basic metals; Fabricated metal products; Mechanical engineering; Office machinery; Insulated wire; Other electrical machinery and apparatus; Electronic valves and tubes; Telecommunication equipment; Radio and television receivers; Scientific instruments; Other instruments; Motor vehicles; Building and repairing of ships and boats; Aircraft and spacecraft; Railroad equipment and transport equipment nec; Furniture, miscellaneous manufacturing; recycling.

The non-traded GDP aggregate includes Electricity, gas and water supply; Construction; Sale, Maintenance and repair of motor vehicles and motorcycles including retail sale of automotive fuel; Wholesale trade and commission trade, except of motor vehicles and motorcycles; Retail trade, except of motor vehicles and motorcycles; Repair of personal and household goods; Hotels & catering; Inland transport; Water transport; Air transport; Supporting and auxiliary transport activities; activities of travel agencies; Communications; Financial intermediation, except insurance and pension funding; Insurance and pension funding, except compulsory social security; Activities auxiliary to financial intermediation; Real estate activities; Renting of machinery and equipment; Computer and related activities; Research and development; Legal, technical and advertising; Other business activities; Public administration and defence including compulsory social security; Education; Health and social work; Other community, social and personal services; Private households with employed persons.

The industrywise data includes value added at current prices as well as the industrywise (price) deflator growth rates. The sectoral aggregation is then obtained by using Tornqvist aggregation method which is described below. Let $p_{it}$ and $v_{it}$ denote the price level and the *real* value-added in the $i$th industry in period $t$. Note that the data provides $p_{it}v_{it}$, that is, the value added at current prices, and $\triangle \ln p_{it}$, that is, the change in the sectoral nominal prices, for each industry. Let $p_t^X$, $X = T, N$, denote the sectoral deflator for sector $X$. Then the changes in sectoral deflators are given by

$$\triangle \ln p_t^X = \sum_{i \in X} s_{it} \triangle \ln p_{it}, \quad X = T, N, \tag{1}$$

where $s_{it}$ is the weight on the deflator of $i$th industry. These shares however are averaged over two consecutive periods to adjust for any structural changes:

$$s_{it} = \frac{1}{2} \left( \frac{p_{it}v_{it}}{\sum_{i \in X} p_{it}v_{it}} + \frac{p_{it-1}v_{it-1}}{\sum_{i \in X} p_{it-1}v_{it-1}} \right). \tag{2}$$

The change in real sectoral quantities, denoted by $v_t^X$ for $X = T, N$, is given by the change in the sectoral value-added at current prices minus the change in the sectoral deflator. That is,

$$\triangle \ln v_t^X = \triangle \ln \left( \sum_{i \in X} p_{it}v_{it} \right) - \triangle \ln p_t^X, \quad X = T, N, \tag{3}$$

where $\triangle \ln p_t^X$ is obtained from Eqs. (1) and (2) above. Setting $v_{baseyear} = 100$ and using Eq. (3) the complete volume time series for each sector is obtained in a straightforward manner.

**Solow residuals** This exercise is based on the industrywise output and hours data from 60-Industry database provided by GGDC (1979-2003) and industrywise gross capital stock data from OECD structural analysis network (STAN); the latter is available only for a limited number of countries: within G-7 there is no capital stock data available for the US and Japan; for Germany it is only partially (1991-2002) available. The sectoral capital stock data for the US however is available from the Bureau of Labor Statistics (BLS) which decomposes assets into Farm, Manufacturing, and Non-farm Non-manufacturing sectors. While assets relating to farm and manufacturing constitute the traded sector capital, nonfarm nonmanufacturing capital stock corresponds to the non-traded sector. Given the data (limitations), the data counterparts for obtaining Solow residuals of the two model countries are chosen as the US as the first country and an aggregate of four countries in G-7, namely, Canada, France, Italy, and United Kingdom (henceforth, G4), for which the gross capital stock data is available, as the second country.

The aggregate G4 sectoral output indices are constructed by aggregating the sectoral output volume indices by using the current GDP in PPP dollars as their respective weights. The GGDC output data is in Euros whereas OECD provides PPP exchange rates in ECU/dollar. Therefore, appropriate ECU/Euro conversion rates, obtained from Eurostat, are used.

STAN gross capital stock data is available in constant prices and in national currencies. The industrial decomposition of gross capital stock into traded and non-traded output follows that of the value-added GDP aggregation. GDP deflators are used to convert all countries' capital stock data to a common base year and then they are aggregated by converting all asset values to dollars by using PPP based exchange rates. The aggregation of hours involves simple addition.

Having obtained output, capital stock, and hours data for US as well as G4 aggregate the Solow residuals are easy to compute. Two time-series of sectoral Solow residuals each for US and G4 are then utilized for estimating the AR(1) productivity shock process. The estimation involves using a seemingly unrelated regression procedure. Additionally, symmetry restrictions are imposed to obtain symmetric autoregression coefficients across countries.

Alternatively, each country for which capital stock data is available, that is, G4 and Germany, is paired against US and AR(1) processes are estimated pairwise following the procedure described above. An average of pairwise estimates is then used for simulating the model economy.

# B Solution to the planner's problem and steady state allocations

Below, we solve the planner's problem in order to derive symmetric steady states. The planner's problem is to maximize $W_1+W_2$, where $W_1$ and $W_2$ denote the present discounted value of the representative agents utility in country 1 and 2 respectively, where

$$W \equiv \sum_{t=0}^{\infty} \beta^t U\left(c^T\left(c_x, c_m\right), c_n, \left(1-h_x-h_n\right)\right).$$

The set of constraints each country faces is

$$\begin{aligned} c_{xt} + \sum_{i\equiv x,n,d} x_{it} + e_{xt} &= F^x\left(\lambda_{xt}, k_{xt}, h_x\right), &(4)\\ c_{nt} + \sum_{i\equiv x,n,d} n_{it} &= F^n\left(\lambda_{nt}, k_{nt}, h_{nt}\right), &(5)\\ \left(1-\delta_x\right) k_{xt} + s_{xt} &= k_{xt+1}, &(6)\\ \left(1-\delta_n\right) k_{nt} + s_{nt} &= k_{nt+1}, &(7) \end{aligned}$$

and the distribution constraint:

$$\psi_x \; c_{xt} + \psi_m \; c_{mt} = s_{dt}. \qquad (8)$$

Notice that at a symmetric steady state $e_x = e_m^*$. Since the main interest here is to derive equations that obtain steady states, it suffices, as will become clear below, to consider only country 1's component of the Lagrangian of the social planner. Letting the planner's Lagrangian be $U = U_1 + U_2$. where $U_1$ denotes country 1's component, after appropriate substitutions, $U_1$ can be written as (the treatment of $U_2$ is implicit):

$$
\begin{aligned}
U_1 &= \sum_{t=0}^{\infty} \beta^t U \left( \begin{array}{c} \left[ F^x\left(\lambda_{xt}, k_{xt}, h_{xt}\right) - \sum_{i \equiv x,n,d} x_{it} - e_{xt} \right]^{\nu} \left( e^*_{mt} - \sum_{i \equiv x,n,d} m_{it} \right)^{1-\nu}, \\ F^n\left(\lambda_{nt}, k_{nt}, h_n\right) - \sum_{i \equiv x,n,d} n_{it}, \\ \left(1 - h_{xt} - h_{nt}\right) \end{array} \right) + \\
& \sum_{t=0}^{\infty} \beta^t \; q_{xt} \left( \left(1 - \delta_x\right) k_{xt} + s_{xt} - k_{xt+1} \right) \\
& \sum_{t=0}^{\infty} \beta^t \; q_{nt} \left( \left(1 - \delta_n\right) k_{nt} + s_{nt} - k_{nt+1} \right) + \\
& \sum_{t=0}^{\infty} \beta^t \; q_{dt} \left( \begin{array}{c} s_{dt} - \psi_x \left( F^x\left(\lambda_{xt}, k_{xt}, h_x\right) - \sum_{i \equiv x,n,d} x_{it} - e_{xt} \right) \\ -\psi_m \left( e^*_{mt} - \sum_{i \equiv x,n,d} m_{it} \right) \end{array} \right),
\end{aligned}
$$

where $q$s denote lagrangian multipliers on their respective constraints. The planner's decision variables are for country 1 are: $h_x, h_n$, and $j_i$ for $j = x, m, n$ and $i = x, n, d$; $e_x$ and $e^*_m$

are decisions that affect both countries. Using the functional forms for $s_I$s, the FOCs are:

$$x_{xt} \ : \ \nu\ U_{1t}\ \left(\frac{c_{mt}}{c_{xt}}\right)^{1-\nu} = q_{xt}\ \alpha_x\ \frac{s_{xt}}{x_{xt}} + \psi_x\ q_{dt}\ , \tag{9}$$

$$m_{xt} \ : \ (1-\nu)\ U_{1t}\ \left(\frac{c_{mt}}{c_{xt}}\right)^{-\nu} = q_{xt}\ \epsilon_x \frac{s_{xt}}{m_{xt}} + \psi_m\ q_{dt}\ , \tag{10}$$

$$n_{xt} \ : \ U_{2t} = q_{xt}\,(1-\alpha_x-\epsilon_x)\,\frac{s_{xt}}{n_{xt}}, \tag{11}$$

$$x_{nt} \ : \ \nu\ U_{1t}\ \left(\frac{c_{mt}}{c_{xt}}\right)^{1-\nu} = q_{nt}\ \alpha_n\ \frac{s_{nt}}{x_{nt}} + \psi_x\ q_{dt}, \tag{12}$$

$$m_{nt} \ : \ (1-\nu)\ U_{1t}\ \left(\frac{c_{mt}}{c_{xt}}\right)^{-\nu} = q_{nt}\ \epsilon_n\ \frac{s_{nt}}{m_{nt}} + \psi_m\ q_{dt}, \tag{13}$$

$$n_{nt} \ : \ U_{2t} = q_{nt}\ \ (1-\alpha_n-\epsilon_n)\,\frac{s_{nt}}{n_{nt}}, \tag{14}$$

$$x_{dt} \ : \ \nu\ U_{1t}\ \left(\frac{c_{mt}}{c_{xt}}\right)^{1-\nu} = q_{dt}\ \alpha_d\ \frac{s_{dt}}{x_{dt}} + \psi_x\ q_{dt}\ , \tag{15}$$

$$m_{dt} \ : \ (1-\nu)\ U_{1t}\ \left(\frac{c_{mt}}{c_{xt}}\right)^{-\nu} = q_{dt}\ \epsilon_d\ \frac{s_{dt}}{m_{dt}} + \psi_m\ q_{dt}\ , \tag{16}$$

$$n_{dt} \ : \ U_{2t} = q_{dt}\ \ (1-\alpha_d-\epsilon_d)\,\frac{s_{dt}}{n_{dt}}. \tag{17}$$

The FOC with respect to hours worked are:

$$h_x \ : \ \left[\nu\ U_{1t}\ \left(\frac{c_{mt}}{c_{xt}}\right)^{1-\nu} - \psi_x\ q_{dt}\right] F^x_{ht} = U_{3t}, \tag{18}$$

$$h_n \ : \ U_{2t}\ F^n_{ht} = U_{3t}. \tag{19}$$

The FOCs with respect to $t+1$ capital stocks yield

$$\beta\ q_{xt+1}\,(1-\delta_x) - q_{xt} + \beta\ \nu\ U_{1,t+1}\left(\frac{c_{mt+1}}{c_{xt+1}}\right)^{1-\nu} F^x_{k,t+1} - \beta\ q_{dt+1}\psi_x\ F^x_{k,t+1} \ = \ 0, \tag{20}$$

$$\beta\ q_{nt+1}\,(1-\delta_n) - q_{nt} + \beta\ U_{2,t+1}\ F^n_{k,t+1} \ = \ 0. \tag{21}$$

The FOC with respect to $e_{xt}$ is

$$\nu\ U_{1t}\ \left(\frac{c_{mt}}{c_{xt}}\right)^{1-\nu} - \psi_x\ q_{dt} = (1-\nu)\ U^*_{1t}\ \left(\frac{c^*_{xt}}{c^*_{mt}}\right)^{-\nu} - \psi_m\ q^*_{dt} \tag{22}$$

Equations (9) - (22), (4) - (8) along with $c^*_x = c_m, c^*_m = c_x$, $q_d = q^*_d$ and $e^*_m = e_x$ (by symmetry) are sufficient to obtain the steady state values for all variables of country 1; by symmetry, country 2's values are identical.

## B.1 Simplification

Equations (20) and (21) give

$$\nu\, U_1 \left(\frac{c_m}{c_x}\right)^{1-\nu} - q_d\psi_x = \frac{q_x}{F_k^x}\left(r+\delta_x\right), \tag{23}$$

$$U_2 = \frac{q_n}{F_k^n}\left(r+\delta_n\right). \tag{24}$$

Next, (22) becomes

$$\nu\, U_1 \left(\frac{c_m}{c_x}\right)^{1-\nu}\left(1-\frac{1-\nu}{\nu}\frac{c_x}{c_m}\right) = \left(\psi_x - \psi_m\right) q_d. \tag{25}$$

Using (25), (23) and (24), steady state versions of equations (9) - (17) are

$$x_x \;:\; F_k^x\, \alpha_x\, \frac{s_x}{x_x} = r+\delta_x, \tag{26}$$

$$m_x \;:\; F_k^x\, \epsilon_x\, \frac{s_x}{m_x} = r+\delta_x, \tag{27}$$

$$n_x \;:\; F_k^n\, \frac{q_x}{q_n}\,\left(1-\alpha_x-\epsilon_x\right)\frac{s_x}{n_x} = r+\delta_n, \tag{28}$$

$$x_n \;:\; F_k^x\, \frac{q_n}{q_x}\,\alpha_n\, \frac{s_n}{x_n} = r+\delta_x, \tag{29}$$

$$m_n \;:\; F_k^x\, \frac{q_n}{q_x}\,\epsilon_n\, \frac{s_n}{m_n} = r+\delta_x, \tag{30}$$

$$n_n \;:\; F_k^n\, \left(1-\alpha_n-\epsilon_n\right)\frac{s_n}{n_n} = r+\delta_n, \tag{31}$$

$$x_d \;:\; F_k^x\, \frac{q_d}{q_x}\,\alpha_d\, \frac{s_d}{x_d} = r+\delta_x, \tag{32}$$

$$m_d \;:\; F_k^x\, \frac{q_d}{q_x}\,\epsilon_d\, \frac{s_d}{m_d} = r+\delta_x, \tag{33}$$

$$n_d \;:\; F_k^n\, \frac{q_d}{q_n}\,\left(1-\alpha_d-\epsilon_d\right)\frac{s_d}{n_d} = r+\delta_n. \tag{34}$$

## B.2 Steady state values

Below we stack all equations that will be solved simultaneously for obtaining the steady state values. Using (23) and (24) with functional forms for $F^i$s, (18) and (19) gets

$$
\begin{aligned}
h_x &: q_x (r+\delta_x) \frac{1-\theta_x}{\theta_x} \frac{k_x}{h_x} = U_3, \\
h_n &: q_n \ (r+\delta_n) \frac{1-\theta_n}{\theta_n} \frac{k_n}{h_n} = U_3.
\end{aligned}
$$

The expressions for $q_x$ and $q_n$ directly follow from (23) and (24):

$$
\begin{aligned}
q_k &= \frac{\nu \ U_1 \left(\frac{c_m}{c_x}\right)^{1-\nu} - \psi_x \ q_d}{r+\delta_x} F_k^x, \\
q_n &= \frac{F_k^n \ U_2}{r+\delta_n}.
\end{aligned}
$$

Further, in steady state:

$$
k_x = \frac{s_x}{\delta_x}, \text{ and } \quad k_n = \frac{s_n}{\delta_n}.
$$

Include (25):

$$
\nu \ U_1 \ \left(\frac{c_m}{c_x}\right)^{1-\nu} \left(1 - \frac{1-\nu}{\nu} \frac{c_x}{c_m}\right) = (\psi_x \ - \psi_m) \, q_d
$$

The distribution constraint:

$$
\psi_x c_x + \psi_m c_m = s_d
$$

Resource constraints:

$$
\begin{aligned}
F^x (\lambda_x, k_x, h_x) &= c_x + c_m + \sum_{i=x,n,d} x_i + \sum_{i=x,n,d} m_i \\
F^n (\lambda_n, k_n, h_n) &= c_n + \sum_{i=x,n,d} n_i
\end{aligned}
$$

Next 9 equations follow from (26) - (34):

$$
\begin{aligned}
\frac{x_i}{m_i} &= \frac{\alpha_i}{1-\alpha_i} \qquad i = x, n, d \\
x_x &: F_k^x \; \alpha_x \; \frac{s_x}{x_x} = r + \delta_x \\
n_x &: F_k^n \; \frac{q_x}{q_n} \; (1 - \alpha_x - \epsilon_x) \; \frac{s_x}{n_x} = r + \delta_n \\
x_n &: F_k^x \; \frac{q_n}{q_x} \; \alpha_n \; \frac{s_n}{x_n} = r + \delta_x \\
n_n &: F_k^n \; (1 - \alpha_n - \epsilon_n) \frac{s_n}{n_n} = r + \delta_n \\
x_d &: F_k^x \; \frac{q_d}{q_x} \; \alpha_d \; \frac{s_d}{x_d} = r + \delta_x \\
n_d &: F_k^n \; \frac{q_d}{q_n} (1 - \alpha_d - \epsilon_d) \frac{s_d}{n_d} = r + \delta_n
\end{aligned}
$$

There are 19 equations in 19 unknowns: $h_x, h_n,$ and $j_i$ for $i = x, m, n$ and $j = x, n, d$; $k_n, k_x, c_x, c_m, c_n,$ $q_x, q_n,$ and $q_d$.

# C Relative prices in a competitive equilibrium

In a decentralized competitive equilibrium, the two investment goods, the distribution services as well as the three final goods have their own price in each country. Moreover, there is a distinction between the price of exportable and importable goods at the producer level and their corresponding prices at the consumer level. The price of exportable goods at the producer level in country 1 is the price of importable goods at the producer level in country 2 and viceversa. As the quantitative analysis below refers to some of these prices, particularly the consumer price index and the real exchange rate, these prices are described below.

The price of exportables and importables at the producer level are $\bar{p}_x$ and $\bar{p}_m$; at the retail level, after utilizing distribution services, the respective consumer prices are $p_x = \bar{p}_x + \psi_x \; p_d$ and $p_m = \bar{p}_m + \psi_x \; p_d$, where $p_d$ is the price of distribution services; $p_n$ is the price of nontradable goods. The price of investment goods to be employed in the tradable sector is $p_x^k$ and the price of investment goods to be used in the nontradable sector is $p_n^k$.

The consumer price index in country 1 is defined in the standard way: it is the minimum expense required to obtain $C(c_x, x_m, c_n) = 1$ given $p_x$, $p_m$, and $p_n$. Let $P$ denote the consumer price index in country 1; by assuming that the nominal exchange rate is equal to 1, the real exchange rate is RER$= P^*/P$.

The expression for relative prices are as derived below. First, the relative price of non-traded goods is

$$\frac{p_n}{\bar{p}_x} = \frac{F_h^x}{F_h^n} = \frac{1-\theta_x}{1-\theta_n}\frac{y_x}{y_n}\frac{h_n}{h_x}, \tag{35}$$

since wages across the two sectors must be equalized. Similarly, as intermediate capital goods producers and distribution services utilize exports as well as imports, their prices must follow

$$\frac{\bar{p}_m}{\bar{p}_x} = \frac{\alpha_j}{\epsilon_j}\frac{x_j}{m_j} \text{ for } j = x, n, d \tag{36}$$

The marginal value of a unit of exportable good in the distribution sector must equal its price. Therefore, the relative price of distribution services is

$$\frac{p_d}{\bar{p}_x} = \left(\frac{ds_d}{dx_d}\right)^{-1} = \left((1-\alpha_d-\epsilon_d)\frac{s_d}{x_d}\right)^{-1}, \tag{37}$$

The retail prices of exportables and importables are obtained in a straightforward manner:

$$\begin{aligned} p_x &= \bar{p}_x + \psi_x\, p_d \\ p_m &= \bar{p}_m + \psi_m\, p_d \end{aligned}$$

Finally, the Consumer Price Index is derived using the consumption index and the relative prices of its components as:

$$\begin{aligned} P &= \left[\omega^{\frac{1}{1+\rho}}\left(\frac{p_x^{\nu}p_m^{1-\nu}}{\nu^{\nu}(1-\nu)^{1-\nu}}\right)^{\frac{\rho}{1+\rho}} + (1-\omega)^{\frac{1}{1+\rho}}\; p_n^{\frac{\rho}{1+\rho}}\right]^{\frac{1+\rho}{\rho}} \\ \frac{P}{\bar{p}_x} &= \left[\omega^{\frac{1}{1+\rho}}\left(\frac{\left(1+\psi_x\frac{p_d}{\bar{p}_x}\right)^{\nu}\left(\frac{\bar{p}_m}{\bar{p}_x}+\psi_m\,\frac{p_d}{\bar{p}_x}\right)^{1-\nu}}{\nu^{\nu}(1-\nu)^{1-\nu}}\right)^{\frac{\rho}{1+\rho}} + (1-\omega)^{\frac{1}{1+\rho}}\;\left(\frac{p_n}{\bar{p}_x}\right)^{\frac{\rho}{1+\rho}}\right]^{\frac{1+\rho}{\rho}} \end{aligned} \tag{38}$$

Given these expressions the real exchange rate $\frac{P^*}{P}$ can be readily obtained.