

# Sustainable Fiscal Policy with Rising Public Debt-to-GDP Ratios \*

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## Abstract

In financial and economic policy circles concerned with public debt in developing countries, a rising debt-GDP ratio is interpreted as a signal of overborrowing, warning of debt defaults if strong fiscal corrections are not adopted in time. This paper shows why this interpretation is incorrect by building a simple model of fiscal policy in which upward-sloping debt paths are observed even though the probability of default is “almost surely” equal to zero.

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# 1 Motivation

In light of the international records reported by Reinhart et al. (2003), it is clear that sovereign defaults are far from being exceptional events of fiscal policy. Yet, economists in financial and economic policy circles concerned with public debt in developing countries are still finding it hard to specify when a country starts to transit into an overborrowing that calls for drastic fiscal policy corrections. Moreover, the empirical evidence on sovereign default is, at best, mixed when it comes to recognize such overborrowing: the median public debt-GDP ratio among countries that defaulted on their public debt during the last 30 years is about 50 percent; 35 (30) percent of the defaults happened at debt ratios of less (more) than 40 (80) percent (IMF, 2003a, Ch.III).

The lack of overborrowing benchmarks leads policymakers and debt analysts to actively search for the aspects of public debt that could early warn about debt unsustainability, i.e., the inability of a government to repay its debt without undergoing severe adjustments in its expenditures or taxes. Applied researchers emphasize, for instance, that the benchmark level of debt for developing countries should not exceed the average value of public debt among industrialized countries (see, for instance, Reissen, 1989), while others emphasize that this benchmark is strongly affected by the countries past repayment behaviors (Reinhart et al., 2003). Following the tradition started with Blanchard (1990) and Blanchard et al. (1990), one of the most common practices points out that upward-sloping time paths of public debt signal fiscal policy unsustainability. IMF (2003b) clearly explains that the slope of the debt path is a key component of a framework elaborated by the IMF and the World Bank for assessing debt sustainability, and the same IMF document states that investment houses and risk-rating agencies also base their fiscal sustainability analyses on the short-term dynamics of public debt. Recent applications of this approach can be seen for instance in Celasun et al. (2006) and the references therein.

By building a model in which rising debt paths are consistent with an almost surely nil probability of default, this paper shows why it is misleading to assess fiscal sustainability by focusing on the slopes of the paths of public debt. The model is designed and calibrated

so that the government seeks to smooth the provision of public goods and transfers without ever borrowing more than what it is payable in the worst feasible fiscal scenario. As by construction the fiscal policy in the model never leads to public debt default, the model becomes an appropriate tool for discussing the circumstances under which public debt-to-GDP ratios could increase even after ruling out the possibility of default.

## 2 The Basics

Debt sustainability analyses focus on the government budget constraint, which takes the following form in a non-monetary economy:

$$b_t = b_{t-1}\mathcal{R}_t + g_t - \tau_t \tag{1}$$

where all variables except the growth-adjusted interest rate  $\mathcal{R}_t \equiv R_t/\Gamma_t$  are measured as GDP ratios; here,  $R_t$  and  $\Gamma_t$  are, respectively, the gross rates of interest and GDP growth;  $\tau_t$  represents fiscal revenues and  $g_t$  non-interest fiscal outlays; and  $b_t$  is the stock of one-period, public debt issued by the government at the end of period  $t$ . Hereafter,  $R_t$ ,  $\Gamma_t$ , and  $\tau_t$  are assumed to be exogenous and stochastic with their dynamics governed by a Markov chain  $\mathcal{Q}(q_{t+1}; q_t)$ , where  $q_t \equiv [\tau_t, R_t, \Gamma_t]'$ . Observe that, once the dynamics of  $q_t$  is specified, the dynamics of  $b_t$  depends on the realizations of  $q_t$  and the government's decisions on  $g_t$ .

Seeking to identify unsustainable debt ratios, Mendoza and Oviedo (2006a, 2006b) adopt the notion of the natural debt limit (NDL) of the precautionary-savings literature (Aiyagari, 1994; see also Aiyagari, et al. 2002) to pinpoint the maximum sustainable level of public debt that a country can support. Levels of debt that exceed the NDL are inconsistent with solvency (i.e., debt repayment) in all states of nature. In Mendoza and Oviedo (2006a) the NDL on public debt,  $\phi$ , depends on the “worst” state of the Markov chain  $\mathcal{Q}$ , as well as the minimum level of  $g_t$ , say  $g^{\min}$ , considered as socially or politically tolerable; concretely, the NDL arises from solving equation (1) for the maximum level of debt consistent with  $g^{\min}$

and the worst state in the Markov chain  $\mathcal{Q}$ :

$$\phi \equiv \min \{(\mathcal{R}_t - 1)^{-1}(\tau_t - g^{\min})\}; \quad (2)$$

By never borrowing more than  $\phi$  a country guarantees its ability to pay even under the most adverse scenario. If borrowing more than  $\phi$  is allowed, there are feasible scenarios in which the country is unable to repay even after setting  $g_t = g^{\min}$  forever.

### 3 A Model of Sustainable Fiscal Policy

Consider a government that seeks to provide a smooth path of fiscal outlays without incurring too large a debt or primary fiscal deficits  $d_t \equiv g_t - \tau_t$ , while ensuring the provision of the minimum level of outlays  $g^{\min}$ . This can be the goal of a government interested in meeting policy goals like those of the Maastricht fiscal convergence criteria, which require the primary deficit and the debt ratio to not exceed 3 and 60 percent of GDP, respectively.

Assume that the policy goals are summarized by the following infinite-horizon loss function:

$$\mathcal{L}_0 = \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \{ -\log(g_t) + \omega [\iota(d_t \geq \bar{d}) + \iota(b_t \geq \bar{b})] \}, \quad (3)$$

subject to: (i) the budget constraint (1); (ii)  $b_t \leq \phi$ , according to the definition of  $\phi$  given in (2) for a given  $g^{\min}$ ; (iii) the Markov chain  $\mathcal{Q}(q_{t+1}; q_t)$ ; and (iv)  $b_{-1}$  given. In (3),  $\mathbb{E}_0$  is the conditional expectation operator and  $\beta \in (0, 1)$  a discount factor;  $\iota$  is an indicator function that takes the value 1 (0) if the condition stated in its argument does (does not) occur. The parameters  $\bar{d}$  and  $\bar{b}$  are “upper desirable limits” on  $d_t$  and  $b_t$ , externally imposed to the fiscal authority, and  $\omega \geq 0$  is a penalty parameter capturing the adverse political consequences of exceeding these limits. Notice that under the current formulation of the loss function, exceeding the two upper limits  $\bar{b}$  and  $\bar{d}$  is considered as more costly than exceeding just one of these limits.

The term  $-\log(g_t)$  captures the perceived benefits from government outlays and induces an outlay-smoothing effect similar to the one that helps Mendoza and Oviedo (2006a)

to explain the procyclicality of fiscal policy in developing countries. As in social choice theory, concave preferences over  $g_t$  reflect either that opportunistic policymakers benefit from rents that are proportional to fiscal expenditures or that benevolent policymakers internalize the welfare derived by residents from the consumption of public goods. The smoothing goal is pursued weighting the smoothing benefit with the current and future fiscal penalties that could arise when the government incurs in deficits and issues debt so as to avoid sharp adjustments in  $g_t$ .

## Calibration

The foregoing model is calibrated to the 1985-2006, Costa Rican fiscal variables of the central government. The starting point is  $\hat{q}_t$ , the empirical counterpart of  $q \equiv [\tau_t, R_t, \Gamma_t]'$ . Data from the World Bank ( $\hat{\Gamma}$ ), the Costa Rican Ministry of Finance (GDP,  $\hat{g}$  and  $\hat{R}$ , the latter computed as the effective interest rate payed by the national government, namely, interest payments<sub>*t*</sub>/public debt<sub>*t*</sub>), and the Central Bank of Costa Rica ( $\hat{\tau}$ ) are used to obtain the 8 realizations of  $\hat{q}_t$ . Then, a VAR(1) system  $\hat{q}_t = A + B\hat{q}_{t-1} + e_t$  is estimated, where  $A$  and  $B$  are the  $3 \times 3$  matrices of coefficients of the VAR system and  $e_t$  is a spherical error vector for which  $E[e_t, e_t'] = \Sigma$ . Estimation results (i.e.  $\hat{A}$ ,  $\hat{B}$ , and  $\hat{\Sigma}$ ) are passed to the Tauchen's (1991) algorithm to obtain the Markov chain of eight states  $\mathcal{Q}(q_{t+1}; q_t)$  that represent the statistical properties of  $q_t$  in actual data and that is shown in the appendix. Table 1 shows the statistical moments of  $\tau_t$  and  $\mathcal{R}_t$  implied by the Markov chain.<sup>1</sup>

To guarantee that the unconditional mean values of  $b_t$  and  $g_t$  match their empirical counterparts,  $\beta$  is set equal to 0.981 and the mean value of  $R_t$  is set to 1.0656, a value commonly used in models of international finance. As for the parameters  $\bar{b}$  and  $\bar{d}$ , it is assumed that the political constraints imposed on the fiscal authority require observing limits on the fiscal deficit and the stock of public debt similar to those stated in the Maastricht criteria. Therefore,  $\bar{d} = 0.03$ , and  $\bar{b} = 0.40$ , the latter arising after considering that the size of the Costarrican central government is about 2/3 of the total size of the government.

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<sup>1</sup>The Costa Rican data, as well the VAR estimation results, are available from the author upon request.

The minimum politically and socially tolerable level of fiscal outlays  $g^{\min}$  is set to 0.0933, equal to 2 standard deviations below the mean in Costarrican data and lower than any of the considered values of  $g_t$  in the data set. This value of  $g^{\min}$ , along with the states of the Markov chain, imply that the NDL  $\phi = 0.4621$ . Finally,  $\omega = 0.010982$  is obtained iteratively as the maximum value of the penalty parameter that reduces to zero (up to 15 significant digits) the probabilities that  $\Pr[d_t > \bar{d}]$  and/or  $\Pr[b_t > \bar{b}]$ , so  $b_t \ll \bar{b}$  and  $d_t \ll \bar{d}$  almost surely  $\forall t$ . As  $\bar{b} < \phi$ , the economy is always able to honor its public debt and consequently, the fiscal policy is sustainable.

## 4 Rising Debt Ratios with Sustainable Fiscal Policy

The constrained loss-minimization problem is solved by formulating a discrete-state dynamic programming problem which in turn is solved by value function iteration. The state space is made of 500 equidistant values of  $b \in [0.1, \phi]$  and the 8 triples of  $q$  of the Markov chain. The controls are  $g_t$  and  $b_t$ ; the pay-off function is the negative of the instantaneous loss function in (3); last, the budget constraint (1) and the transition probability matrix of the Markov chain  $\mathcal{Q}$  are the states' laws of motion.

Let call  $\tilde{g}(b, q)$  and  $\tilde{b}(b, q)$  to the optimal, recursive fiscal outlays and debt policy functions that determine the optimal values of  $g_t$  and  $b_t$  given the state of the economy represented by the values of  $b_{t-1}$  and  $q_t$ . And let  $\pi^\infty$  represent the limiting distribution of the states  $(b, q)$ . Table 1 shows the moment statistics implied by  $\pi^\infty$  and Figure 1a shows the induced marginal limiting distribution of  $b_t$ . The figure shows that  $\Pr[b_t > \bar{b}] = 0$ , and therefore  $\Pr[b_t > \phi] = 0$ . Furthermore, notice that the mean ratio of public debt at 33.3% of GDP is different from the NDL, something that shows that the latter is just the upper bound of all sustainable stocks of debt. Thus, by the definition of the NDL, the fiscal policy represented by  $g = \tilde{g}(b, q)$  and  $b' = \tilde{b}(b, q)$  poses no fiscal risk: the government is always able to repay its debt!

Can this policy generate short to midterm increasing public debt paths? To align model's results with the language used to discuss fiscal sustainability in financial and policy

circles, let us use two computable objects to answer this question:<sup>2</sup> (a) the “forecasting functions” of  $b_t$ , which represent the model implied conditional mean of the debt ratio given an initial state  $(b, q)$ ; and (b) a fan chart, which was recently introduced in the literature by Celasun et al. (2006) to represent the results of a large number of simulations of equation (1). Typically, the simulations consider a reaction function of the primary balance with respect to the debt ratio and a distribution of shocks affecting that ratio, both estimated from data of the recent history of the examined country or of a set of similar countries. The starting point of the simulations commonly is the most recently observed debt ratio. The reaction functions are the empirical counterpart of  $\tilde{g}(b, q)$  and  $\tilde{b}(b, q)$  in the model.

Starting with the first exogenous state and five initial debt ratios ranging between 0.15 and 0.4 ( $=\bar{b}$ ), Figure 2a shows the corresponding forecasting functions. When  $b_{-1} = 0.15$  ( $b_{-1} = 0.4$ ), the model predicts a time-increasing (time-decreasing) debt ratio. More generally, as the forecasting functions represent conditional means, all these functions converge to the same unconditional mean of  $b_t$ ,  $\mu(b) = 0.33$ , shown in Table 1. Therefore, any initial debt ratio such that  $b_{-1} < \mu(b)$  generates time-increasing debt ratios. Increasing or decreasing ratios, however, cannot be interpreted as signals of fiscal policy unsoundness for it has been shown that debt default is an impossible event in the modelled economy.

The fan chart in Figure 2b arises from 1000 simulations of  $\mathcal{Q}$  and the use of the optimal policy functions to describe the dynamics of the endogenous state variables. The initial debt ratio is  $b_{-1}$  is ten percentage points below  $\mu(b)$ . From lighter to darker, the colored areas show deciles at 20, 40, 60 and 80 percent confidence intervals around the median projection; it can be inferred from the chart, for instance, that the probability of observing a rising debt ratio is approximately equal to 90%. However, a highly likely increasing debt ratio, again, cannot be interpreted as a signal of overborrowing.

The uninformativeness of the slope of the time-path of public debt to signal unsustainability reasoned from the design of the model in combination with either Figure 2a or 2b is not a result particular to the loss function specified in (3). Other fiscal policies that

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<sup>2</sup>It should be clear, however, that the two objects convey the same information since both are derived from the optimal policy functions  $\tilde{g}$  and  $\tilde{b}$ .

generates a distribution of  $b_t$  whose support is a subset of  $(-\infty, \phi]$  could lead to the same conclusion.

## 5 Sources of Increasing Debt Ratios

To serve as examples of events that could raise public debt ratios without necessarily derailing the sustainability of the fiscal policy, this section discusses a fiscal and a macroeconomic event that lead to increasing debt ratios without threatening the fiscal solvency. In both cases it is assumed that the initial debt ratio is equal to its long-run mean  $\mu(b_t)$  specified in Table 1, so that no medium-run increment in  $b_t$  would be expected in absence of a perturbing event.

### 5.1 A Growth Slowdown Period

Calvo et al. (1996), Neumeyer and Perri (2005), and Uribe and Yue (2006), among others, have documented that the liquidity of international financial markets affects the rate of economic growth in developing countries. Section’s 3 model can be used to quantify the effect on the debt ratio of an interest-rate-driven growth slowdown. The Markov chain in the appendix shows that the state number 3 represents the most adverse scenario that combines the highest interest rate and the second-to-lowest growth rate. Furthermore, and coincidentally with what is observed in practice, state number 3 is the one with the lowest realization of the fiscal revenue. To assess the effect of this adverse scenario on the debt ratio, the model of Section 3 is simulated assuming that “nature” draws the worst state of the chain for a number of periods.

As shown in Figure 3a, if the debt ratio is equal to its long-run mean of 33.3% in year 1, two years later it is equal to 37.8%, and by the end of year 5 it becomes equal to 39.95%, almost reaching  $\bar{b}$ . An external observer just focusing on this 4-year, 6.5-percentage-point increase in the debt ratio might recommend to shorten the fiscal deficits by raising tax rates and/or lowering government outlays although none of the policies are necessary in the context of the foregoing model.

## 5.2 Relaxing the Strictness of the Fiscal Policy Criteria

If the fiscal authorities discover that the penalty for violating the deficit and debt limits are much less severe than previously expected or that the fiscal criteria are not going to be strictly enforced, the use of debt markets would allow the authorities to provide a smoother path of fiscal outlays  $g_t$ , even without threatening the fiscal solvency.

To model the relaxing of the strictness of constraints on the fiscal policy, assume that the penalty parameter  $\omega$  falls to zero. The new marginal distribution of the stock of public debt is shown in Figure 1b where it can be seen that, again by construction, the stock of debt never rises above its natural debt limit, so the optimal fiscal policy cannot be characterized as unsustainable. Notwithstanding this, the unconditional mean debt ratio  $\mu(b)$  steps up to 38.8% from the 33.4% reported in Table 1.

In the short run, the chances of observing a significant increment in the debt ratio depend on how fast this ratio converges to the new long-run mean. The central, light region of the fan chart in Figure 3b shows that the most likely scenario is precisely one of rising debt ratios although this cannot be associated to a fiscal sustainability problem.

## 6 Conclusion

This paper has shown that rising paths of public debt-to-GDP ratios cannot be interpreted as a signal of overborrowing that calls for immediate fiscal policy corrections. If an analyst or country external reviewer assesses the sustainability of a country's fiscal policy based on the projected dynamics of the debt ratio, he might well advise adopting unnecessary, welfare-reducing fiscal adjustments despite fiscal solvency of the country is unquestionable.

## Appendix: The Empirical Markov Chain

The Markov chain representing the shocks affecting the model of Section 3 has the following states  $q = [\tau, R, \Gamma]$  and transition probability matrix  $\mathcal{P}$ :

$$q = \begin{pmatrix} 0.1241 & 1.0438 & 1.0236 \\ 0.1393 & 1.0331 & 1.0304 \\ 0.1241 & 1.1027 & 1.0257 \\ 0.1393 & 1.0920 & 1.0324 \\ 0.1241 & 1.0438 & 1.0695 \\ 0.1393 & 1.0331 & 1.0763 \\ 0.1241 & 1.1027 & 1.0715 \\ 0.1393 & 1.0920 & 1.0783 \end{pmatrix}$$

$$\mathcal{P} = \begin{pmatrix} 0.15361 & 0.05648 & 0.06335 & 0.02329 & 0.36407 & 0.13385 & 0.15014 & 0.05520 \\ 0.03170 & 0.22805 & 0.01835 & 0.13200 & 0.04560 & 0.32804 & 0.02639 & 0.18988 \\ 0.35718 & 0.07139 & 0.24233 & 0.04843 & 0.13936 & 0.02785 & 0.09455 & 0.01890 \\ 0.08433 & 0.32982 & 0.08030 & 0.31407 & 0.01997 & 0.07811 & 0.01902 & 0.07438 \\ 0.07438 & 0.01902 & 0.07811 & 0.01997 & 0.31407 & 0.08030 & 0.32982 & 0.08433 \\ 0.01890 & 0.09455 & 0.02785 & 0.13936 & 0.04843 & 0.24233 & 0.07139 & 0.35718 \\ 0.18988 & 0.02639 & 0.32804 & 0.04560 & 0.13200 & 0.01835 & 0.22805 & 0.03170 \\ 0.05520 & 0.15014 & 0.13385 & 0.36407 & 0.02329 & 0.06335 & 0.05648 & 0.15361 \end{pmatrix}$$

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Table 1: Unconditional Moments Implied by the Limiting Distribution of the State Variables

Variable ( $x$ ) (GDP ratios, except $\mathcal{R}_t$ )	$\mu(x)$ $\times 100$	$cv(x)$ $\times 100$	$\rho(x)$	$\rho(x, y)$	
				$y = \tau$	$y = \mathcal{R}$
Fiscal revenue ( $\tau_t$ )	13.6	5.6	0.62	1.00	-0.19
Growth adjusted int. rate ( $\mathcal{R}_t$ )	1.017	3.6	0.21	-0.19	1.00
Public debt ( $b_t$ )	33.3	12.5	0.96	-0.25	0.02
Non-interest outlays ( $g_t$ )	13.0	5.1	0.74	0.53	-0.42
Primary balance ( $-d_t$ )	0.6	119.0	0.54	0.59	0.20

Notes:  $\mu(x)$ ,  $cv(x)$ , and  $\rho(x)$ , stand for the mean, coefficient of variation (the ratio of the standard deviation to the mean), and autocorrelation of  $x$ .  $\rho(y, x)$  is the contemporaneous cross correlation between  $y$  and  $x$ .

Figure 1: Limiting Marginal Distribution of the Public Debt-to-GDP Ratio

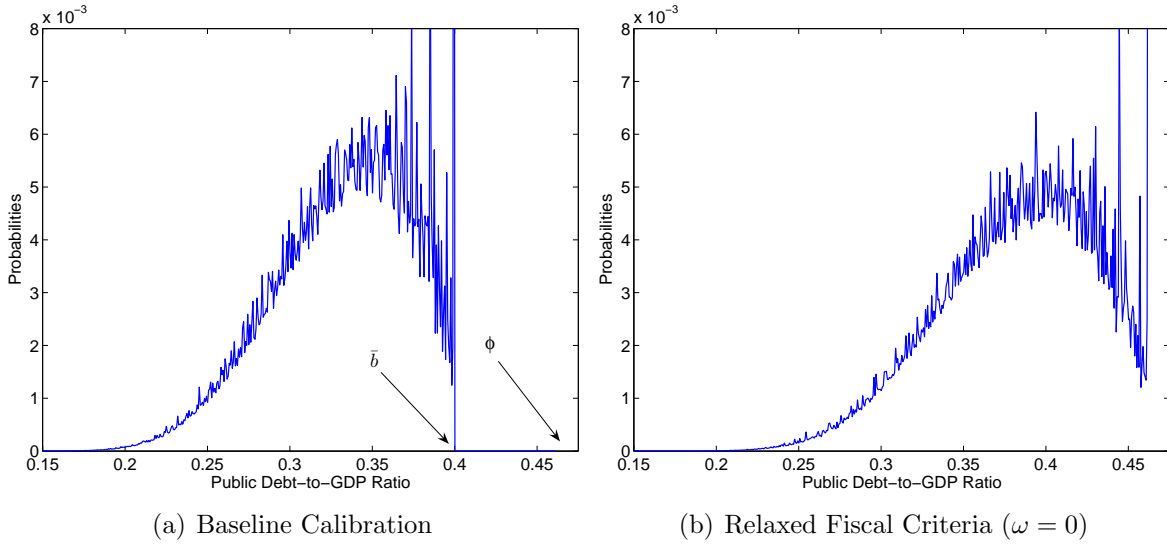


Figure 2: Public Debt-GDP Ratio: Forecasting Functions and a Fan Chart

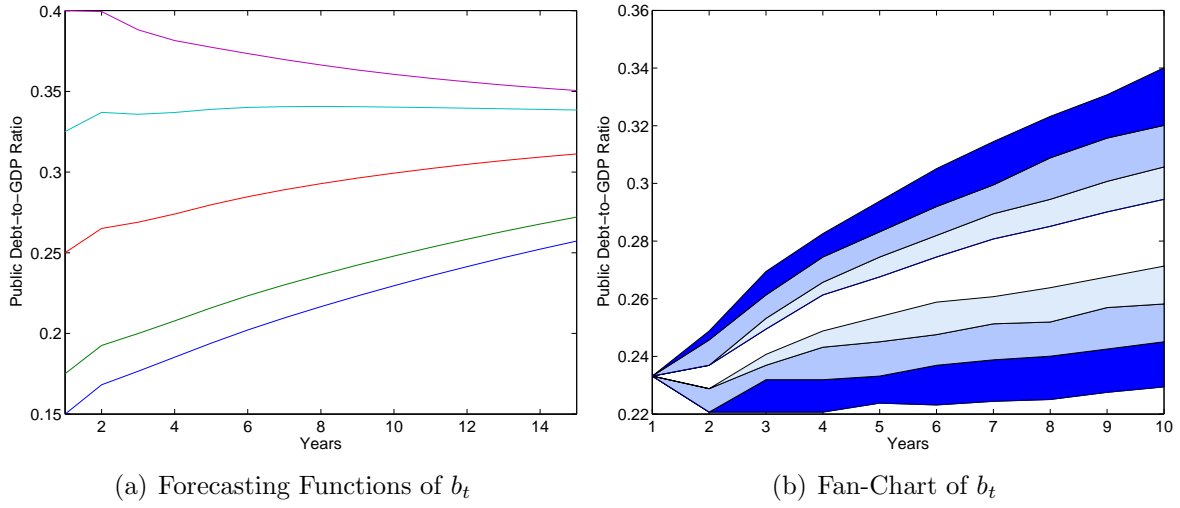


Figure 3: Policy and External Shocks that can Raise the Debt Ratio without Harming Fiscal Sustainability

