Public Debt, Fiscal Solvency, and Macroeconomic Uncertainty in Latin America: The Cases of Brazil, Colombia, Costa Rica, and Mexico*

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Abstract

The ratios of public debt as a share of GDP of Brazil, Colombia, and Mexico were 12 percentage points higher on average during the period 1996-2005 than in the period 1990-1995. Costa Rica's debt ratio remained stable but at a high level near 50 percent. Is there reason to be concerned for the solvency of the public sector in these economies? We provide an answer to this question based on the quantitative predictions of a variant of the framework proposed by Mendoza and Oviedo (2006). This methodology yields forward-looking estimates of debt ratios that are consistent with fiscal solvency for a government that faces revenue uncertainty and can issue only non-state-contingent debt. In this environment, aversion to a collapse in outlays leads the government to respect a "natural debt limit" equal to the annuity value of the primary balance in a "fiscal crisis." A fiscal crisis occurs after a long sequence of adverse revenue shocks and public outlays adjust to their tolerable minimum. The debt limit also represents a credible commitment to remain able to repay even in a fiscal crisis. The debt limit is not, in general, the same as the sustainable debt, which is driven by the probabilistic dynamics of the primary balance. The results of a baseline scenario question the sustainability of current debt ratios in Brazil and Colombia, while those in Costa Rica and Mexico are inside the limits consistent with fiscal solvency. In contrast, current debt ratios are found to be unsustainable in all four countries for plausible changes to lower average growth rates or higher real interest rates. Moreover, sustainable debt ratios fall sharply when default risk is taken into account.

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1 Introduction

Comparing recent figures with those at the beginning of the 1990s reveals that public debt-to-GDP ratios have been rising steadily in Latin America during the last 15 years (see Figure 1). In particular, the Colombian and Brazilian debt ratios increased by 15.7 and 11.5 percentage points; in Mexico, before a recent decrease, the debt ratio had risen from 41 percent in 1990 to 50 percent in 2003; and the Costa Rican public debt has been fluctuating around 50 percentage points of GDP for more than 10 years. Given that growing public debt has traditionally been an indicator of financial weakness and vulnerability to economic crisis in the region, there is concern for assessing whether the observed high levels of debt are in line with the solvency of the public sector or should be taken as a warning signal that requires policy intervention.

The goal of this paper is to assess the consistency of public debt ratios in Brazil, Colombia, Costa Rica and Mexico with the conditions required to maintain fiscal solvency. The debt dynamics in these countries are also compared with the recent polar experiences in Chile and Argentina, two Latin American countries that had approximately the same debt-to-GDP ratio at the beginning of the 1990s and that ended in opposite extremes by the mid 2000s (see Figure 1b); whereas the Chilean stock of public debt fell down to zero, the Argentine debt ratio leapt above 100 percent before the country defaulted on its debt in 2001.

The fiscal solvency assessment conducted in this paper is based on the framework proposed by Mendoza and Oviedo (2006). In particular, this paper applies a variant of their model that abstracts from the behavior of the private sector to focus on a simplified version of the government's problem characterized by exogenous government-expenditure rules. This methodology produces estimates of the short- and long-run dynamics of public debt ratios in a setup in which public revenues are subject to random shocks and the government aims to maintain its outlays relatively smooth. The government is handicapped in its efforts to play this insurer's role because it can only issue non-state contingent debt.

Mendoza and Oviedo (2006) show that in this environment with incomplete contingent-claims markets, a government averse to a collapse in its public outlays and facing revenue uncertainty will impose on itself a "natural debt limit" (NDL) determined by the growth-adjusted annuity value of the primary balance in a state of "fiscal crisis." The state of fiscal crisis is the state at which a country arrives after experiencing a sufficiently long sequence of adverse shocks to public revenues on one side, and after adjusting the fiscal outlays to a minimum admissible level on the other.

An important implication of the NDL is that it allows the government to offer its creditors a credible commitment to remain able to repay "almost surely" at all times, even during fiscal crises. This commitment is not an ad-hoc assumption but an implication of the assumptions that (a) the government is averse to suffering a collapse of its outlays, (b) public revenues are stochastic, and (c) markets of contingent claims are incomplete. However, the commitment is in terms of an "ability-to-pay criterion," and as such it does not rule out default scenarios that may result from "willingness-to-pay" or strategic reasons.

The NDL sets the upper bound for public debt but is not, in general, the same as the "sustainable" or equilibrium level of debt. The model does not require public debt to remain constant at the level of the NDL. Indeed, in Mendoza and Oviedo's (2006) model, while the government's NDL is equal 132 percent of GDP, the long-run average of the debt ratio is equal to 53 percent of GDP. In the short-run, the dynamics of the distribution of public debt are driven by the government budget constraint and depends on the initial debt and revenue conditions, the probabilistic process driving revenues, and the policy rules governing public outlays.

Under the limiting assumption made here that the government keeps an invariant level of non-interest fiscal outlays except when it faces the state of fiscal crisis, in the long run, the government can end up paying off all its debt or hitting the debt limit. Which long-run equilibrium will be reached depends on the alternative sequences of realizations of public revenues and the initial fiscal conditions including the initial stock of public debt. On the contrary, the general equilibrium framework in Mendoza and Oviedo (2006) features a unique, invariant long-run distribution of public debt, as the government uses its access to debt markets to optimize the use of its outlays over time.

The results of the fiscal-solvency assessment of this paper suggest that current debt ratios in Brazil and Colombia are near the natural debt limits that would be consistent with fiscal solvency only if one assumes perceived commitments to large reductions in non-interest outlays during a fiscal crisis. For instance, Brazil and Colombia should be able to cut their outlays by 6.2 and 10.6 percentage points of GDP. The difficulty of observing such large outlays reductions can be illustrated considering the recent Argentine experience in which avoiding defaulting on the sovereign debt would have required outlays cutbacks of at least 5.2 percentage points of GDP. Mexico and Costa Rica were also near their debt limits in 1998 and 2003, respectively. However, the reductions in outlays that would have been needed to keep the governments in these two countries solvent if revenues had continued to suffer adverse shocks were smaller (at 2 and 3.7 percentage points of GDP, respectively).

The above results are very sensitive to the underlying assumptions regarding the long-run real interest and growth rates. Current debt ratios in all four examined countries are found to be unsustainable for plausible reductions in growth rates to the averages of the last 25 years, instead of the average of the last 45 years used in the baseline scenario. Similarly, the current debt ratios are found to be unsustainable if the long-run real interest rate is set at 8 percent instead of the 5 percent value of the baseline estimates.

The Mendoza-Oviedo framework was designed as a forward-looking policy tool that intentionally sets aside the default risk. This was done because the framework is intended to produce the sustainable debt ratio that a government that does not consider the option of defaulting on its obligations could support. This is in line with the assumptions of the traditional approaches to assess debt sustainability based on deterministic steady-state estimates or empirical applications of the intertemporal government budget constraint. However, while abstracting from sovereign default serves to set the ideal benchmark in a forward-looking policy analysis, the strategy does have the drawback that it does not take into account how default risk considerations could affect sustainable debt dynamics. To address this issue, this paper studies how estimates of natural debt limits and simulated debt dynamics vary when the basic Mendoza-Oviedo model is modified to incorporate exogenous default risk. Introducing default risk results in marked reductions in the levels of natural debt limits.

The paper also compares the results of the Mendoza-Oviedo model with those produced by the conventional methodology based on calculations of steady-state debt ratios (or “Blanchard ratios”). In countries with large average primary surpluses such as Costa Rica and Mexico, the Blanchard ratio yields higher debt ratios than the natural debt limits of the Mendoza-Oviedo model. This shows that assessments of sustainable debt based on steady-state calculations that use averages of revenues and outlays and fail to take into account aggregate, non-insurable fiscal shocks can lead countries to borrow more than what is consistent with fiscal solvency. In contrast, in countries with small average primary surpluses, the Blanchard ratios would yield negligible debt ratios. The Mendoza-Oviedo model can explain high levels of debt when the governments can credibly commit to large enough cuts in outlays in a state of fiscal crisis. To be credible, however, these cuts must not represent unusually large deviations relative to the historical mean.

The rest of the paper is organized as follows. Section 2 is a short survey of the existing methods for calculating public debt ratios consistent with fiscal solvency. Section 3 summarizes the one-good variant of the Mendoza-Oviedo model. Section 4 applies the model to the cases of Brazil, Colombia, Costa Rica and Mexico and discusses the results. This Section includes sensitivity analysis and an extension to incorporate exogenous default risk. Section 5 reflects on important caveats of the analysis and provides general conclusions.

2 Computing Public Debt Ratios Consistent with Fiscal Solvency: A Survey

Developing effective tools for determining whether a stock of debt is "sustainable" or not, in the sense of being consistent with the fiscal solvency conditions implied by current and future patterns of government revenue and outlays, has proven a difficult task. The first problem that studies in this area face is how to give operational content to the notion of fiscal sustainability. There is a tendency to associate the notion of unsustainable public debt with failure to satisfy the government budget constraint, or with the government holding a negative net-worth position.

From an analytical standpoint, however, focusing on either the budget constraint or on the net-worth position of the government can be misleading. This is because the "true" government budget constraint, interpreted as an accounting identity relating the overall public sector borrowing requirement to all sources and uses of government revenue, must always hold. Thus, an analysis that shows that a given stock of public debt fails a "particular" definition of the budget constraint is ultimately reflecting the failure to incorporate into the analysis important features of the actual fiscal situation of the country under study. How this failure translates into a judgment about the sustainability of public debt depends on assessments (typically implicit in the analysis) about the macroeconomic outcomes associated with the different mechanisms open to maintain fiscal balance.

Arguments about sustainability are therefore implicitly arguments about the pros and cons of these alternative mechanisms, not about whether the government's intertemporal budget constraint holds. Consider a basic example. The canonical long-run analysis of public debt sustainability considers long-run, average levels of public revenue and expenditures, and views as the sustainable debt-output ratio the annuity value of a long-run target of the primary balance-output ratio. If a government has a large stock of contingent liabilities because of the high risk of a banking crisis, the stock of public debt may be judged to be unsustainable because, once these contingent liabilities are added, the debt-output ratio exceeds the long-run indicator of sustainability. However, the government budget constraint must hold, and thus if a banking crisis does occur, the government will ultimately have to adjust the primary balance or rely on other "sources" of financing such as the inflation tax or a debt default. Adjustments via the primary balance are generally judged as consistent with this canonical view of sustainability, while adjustments via inflation or default would not because these would be viewed as alternatives inferior to adjustment of the primary balance in terms of social welfare.

Beyond the problem of defining an operational concept of public debt sustainability, there are also problems in the design of methods for calculating sustainable debt levels. These difficulties reflect the gap between the aspects of fiscal policy emphasized in the different methods and those aspects that seem empirically relevant for explaining the actual fiscal position. The literature on methods for assessing public debt sustainability reflects the evolution of ideas on these issues. The lines below review the main features of the different methods. The intent is not to conduct a comprehensive survey of the literature but to highlight the central differences among the existing methods.[1]

The starting point of most of the existing methods for calculating sustainable public debt-output ratios is the period budget constraint of the government. This constraint is merely an accounting identity that relates all the flows of government receipts and payments to the change in public debt:

$$B_{t+1} = B_t(1 + r_t) - (\mathcal{T}_t - G_t) \tag{1}$$

where B_{t+1} is the stock of public debt issued by the end of period t; B_t is maturing public debt on which the government pays principal and the real interest rate r_t; $\mathcal{T}_t$ is total real government revenue; and G_t represents current, real, non-interest government outlays, so that $\mathcal{T}_t - G_t$ is the primary fiscal balance.

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[1] For literature surveys see Chalk and Hemming (2000), or IMF (2002) and (2003).

Long-Run Methods

The canonical long-run approach to debt sustainability is based on steady-state, perfect-foresight considerations that transform the government's accounting identity (1) into an equation that maps the long-run primary fiscal balance as a share of output into a "sustainable" debt-to-output ratio that remains constant over time (see Buiter,1985; Blanchard, 1990; and Blanchard, Chouraqui, Hagemann and Sartor, 1990). In particular, when γ is the long-run rate of output growth, some basic algebraic manipulation of the accounting identity in (1) yields:

$$b = \frac{\tau - g}{r - \gamma} \tag{2}$$

where b is the long-run debt-to-GDP ratio, τ and g are the long-run GDP shares of current revenue and outlays, and r is the steady-state real interest rate. Condition (2) can be read as an indicator of fiscal policy action (i.e., of the "permanent" primary balance-output ratio that needs to be achieved by means of revenue or expenditure policies so as to stabilize a given debt-output ratio), or as an indicator of a "sustainable" debt ratio (i.e., the target debt-output ratio implied by a given projection of the long-run primary balance-output ratio).

Intertemporal Methods

An important shortcoming of the long-run approach is that it fails to recognize that the "long run" is a theoretical construct. In the short run governments face a budget constraint that does not reduce to the simplistic formula of the long-run analysis. There can be temporarily high debt ratios, or temporarily large primary deficits, that are consistent with government solvency, and indeed incurring in such temporarily high debt or deficits could be optimal from a tax-smoothing perspective. To force a country into the straight jacket of keeping its public debt-output ratio no larger than the level that corresponds to the long-run stationary state can therefore be a serious mistake.

The realization of these flaws in the long-run calculations led to the development of intertemporal-budget-constraint methods that shifted the focus from analyzing directly the debt-output ratio to studying the time-series properties of the fiscal balance, so as to test whether these properties are consistent with the conditions required to satisfy the government's intertemporal budget constraint. This intertemporal constraint serves as a means to link the short-run dynamics of debt and the primary balance with the long-run solvency constraint of the government.

In their original form (see Hamilton and Flavin, 1986) the intertemporal methods aimed to test whether the data rejected the hypothesis that the condition ruling out Ponzi games on public debt holds. This condition states that at any date t the discounted value of the stock of public debt $t+j$ periods into the future should vanish as j goes to infinity: $\lim_{j\to\infty}\Pi_{k=0}^{j}(1+r_{t+k})^{-1}B_{t+1+j}=0$. In other words, in the long run the stock of debt cannot grow faster than the gross interest rate. If this no-Ponzi-game (NPG) condition holds, the forward solution of (1) implies that the intertemporal government budget constraint holds. Namely, the present value of the primary fiscal balance is equal to the value of the existing stock of debt, and hence the existing public debt or public debt-output ratio is deemed "sustainable."

A number of articles tried different variations of this test by testing for stationarity and co-integration in the time series of the primary balance and public debt, and produced different results using U.S. data (Chalk and Hemming, 2000, review this literature). These intertemporal-budget-constraint methods have also introduced elements of uncertainty into public debt sustainability analysis, but mostly in an indirect manner as sources of statistical error in hypothesis testing, or by testing the NPG condition in expected value or as an orthogonality condition.[2]

Bohn (1998) provided an alternative interpretation of intertemporal methods that reduces to testing if the primary balance responds positively to increases in public debt. In particular, *if the primary balance-output ratio and the debt-output ratio are stationary*, the following

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[2] The orthogonality condition considers that, at equilibrium, the sequence of real interest rates used to discount the "terminal" debt stock must match the intertemporal marginal rate of substitution in private consumption. This requires assessing whether the following condition holds in the data:

$$\lim_{j\to\infty}\mathrm{E}_t\left[\beta^{t+1+j}\,\frac{u'(C_{t+1+j})}{u'(C_t)}B_{t+1+j}\right]=0$$

where E_t is the expectation operator that conditions on the information available at time t; $u'(C_t)$ is the marginal utility of consumption at time t; and β is the standard exponential discount factor.

regression can be used to test for sustainability:

$$s_t = \rho b_t + \alpha \cdot Z_t + \epsilon_t \tag{3}$$

where s_t is the ratio of the primary fiscal balance over GDP, ϵ_t is a well-behaved error term and Z_t is a vector of determinants of the primary balance other than the initial stock of public debt. In Bohn's case, these determinants included a measure of "abnormal" government expenditures associated to war episodes and the cyclical variations in U.S. GDP. Bohn found strong evidence in favor of $\rho > 0$ which indicates that, controlling for war-time spending and the business cycle, the debt-output ratio was mean-reverting in U.S. Moreover, a systematic and positive linear response of the primary balance to increases in the stock of debt is sufficient (albeit not necessary) to ensure that the intertemporal government budget constraint holds. The intuition is that if a "large negative shock" raises considerably the stock of public debt above its mean, then primary surpluses are going to eventually reverse that stock of debt to its mean level.

Chapter III of IMF (2003) applied Bohn's method to a panel of industrial and developing country data and found that the condition $\rho > 0$ held for some but not all developing countries. Moreover, the study found evidence of non-linearities in the relationship between debt and primary balances. This evidence indicates that countries that were able to sustain larger debt ratios in the data also displayed a stronger response of the primary balance to debt increases.

Recent Development: Probabilistic Methods and Methods with Financial Frictions

Recent developments in public debt sustainability analysis follow two strands. One emphasizes that governments, particularly in emerging markets, face significant sources of uncertainty as they try to assess the patterns of government revenue and expenditures, and hence the level of debt that they can afford to maintain. From the perspective of these probabilistic methods, measures of sustainability derived from the long-run approach or the intertemporal analysis are seen as inaccurate for governments that hold large stocks of debt and face large shocks to their revenues and expenditures. The key question here is not whether public debt is sustainable at some abstract steady state, or whether in a sample of a country's recent or historical past the NPG condition holds. The key question is whether the current debt-output ratio is sustainable given the current domestic and international economic environment and its future prospects.

The second strand aims to incorporate elements of the financial frictions literature applied to the recent emerging-markets crises. For example, public debt in many emerging markets displays a characteristic referred to as "liability dollarization" (i.e., debt is often denominated in foreign currency or indexed to the price level). As a result, abrupt changes in domestic relative prices that are common in the aftermath of a large devaluation, or a 'sudden stop' to net capital inflows, can alter dramatically standard long-run calculations of sustainable debt ratios and render levels of debt that looked sustainable in one situation unsustainable in another. Calvo, Izquierdo and Talvi (2003) evaluate these effects for the Argentine case and find that large changes in the relative price of nontradables alter significantly the assessments obtained with standard steady-state sustainability analysis.

The probabilistic methods for assessing fiscal sustainability propose alternative strategies for dealing with macroeconomic uncertainty. A method proposed at the IMF by Barnhill and Kopits (2003) incorporates uncertainty by adapting the value-at-risk (VaR) principles of the finance industry to debt instruments issued by governments. The aim of this approach is to model the probability of a negative net worth position for the government. The method requires estimates of the present values of the main elements of the balance sheet of the total consolidated public sector (financial assets and liabilities, expected revenues from sales of commodities or other goods and services, as well as any contingent assets and liabilities), and an estimate of the variance-covariance matrix of the variables that are viewed as determinants of those present values in reduced form. This information is then used to compute measures of dispersion relative to the present values of the different assets and liabilities that determine the value at risk (or exposure to negative net worth) of the government.

A second probabilistic method recently considered for country surveillance at the IMF (see IMF, 2003) modifies the long-run method to incorporate variations to the determinants of sustainable public debt in the right-hand-side of equation (2), and also examines short-term debt dynamics that result from different assumptions about the short-run path of the variables that enter the government budget constraint in deterministic form. For example, deterministic debt dynamics up to 10 periods into the future are computed for variations of the growth rate of output of two standard deviations relative to its mean.

The same IMF publication proposes a stochastic simulation approach that computes the probability density function of possible debt-output ratios. This stochastic simulation model, like the VaR approach, is based on a non-structural time-series analysis of the macroeconomic variables that drive the dynamics of public debt (particularly output growth, interest rates, and the primary balance). The difference is that the stochastic simulation model produces simulated probability distributions based on forward simulations of a vector-autoregression model that combines the determinants of debt dynamics as endogenous variables with a vector of exogenous variables. The distributions are then used to make assessments of sustainable debt in terms of the probability that the simulated debt ratios are greater or equal than a critical value.

Xu and Ghezzi (2003) developed a third probabilistic method to evaluate sustainable public debt. Their method computes "fair spreads" on public debt that reflect the default probabilities implied by a continuous-time stochastic model of the dynamics of treasury reserves in which exchange rates, interest rates, and the primary fiscal balance follow Brownian motion processes (so that they capture drift and volatility observed in the data). The analysis is similar to that of the first-generation models of balance-of-payments crises. Default occurs when treasury reserves are depleted, and thus debt is deemed unsustainable when the properties of the underlying Brownian motions are such that the expected value of treasury reserves declines to zero (which occurs at an exponential rate).

3 A Basic Version of the Mendoza-Oviedo Model

The probabilistic methods summarized in the last section make significant progress in incorporating macroeconomic uncertainty into debt sustainability analysis but they are largely based on non-structural econometric methods. In contrast, the Mendoza-Oviedo (MO) method aims to provide an explicit dynamic equilibrium model of the mechanism by which macroeconomic shocks affect government finances. The MO method also differs from the other probabilistic methods in that it models explicitly the nature of the government's forward-looking commitment to remain solvent, instead of focusing on computing estimates of exposure to negative net worth or depletion of treasury reserves. As explained below, the MO method determines sustainable debt ratios that respect a natural debt limit consistent with a credible commitment to repay similar in principle to the one implicit in the long-run and intertemporal methods.

The structural emphasis of the MO approach comes at the cost of the reduced flexibility and increased complexity of the numerical solution methods required to solve non-linear, dynamic stochastic equilibrium models with incomplete asset markets. At the same time, by proceeding in this manner the MO framework seeks to produce estimates of sustainable public debt that are robust to the Lucas critique. The non-structural or reduced-form tools used in the other probabilistic methods to model the dynamics of public debt are vulnerable to the policy instability problems resulting from the Lucas critique. This is not a serious limitation when these methods are used for an ex-post evaluation of how well past debt dynamics matched fiscal solvency conditions, but it can be a shortcoming for a forward-looking analysis that requires a framework for describing how equilibrium prices and allocations, and hence the ability of the government to raise revenue and service debt, adjust to alternative tax and expenditure policies or other changes in the environment.

The basic principles of the MO method are as follows. Assume that output follows a deterministic trend so that it grows at a constant, exogenous rate, γ, and the real interest rate, r, is constant. Public revenues follow an exogenous stochastic process and the government is averse to suffering a collapse in its outlays. Hence, it aims to keep its outlays smooth unless the loss of access to debt markets forces it to adjust these outlays to minimum tolerable levels. Domestic debt markets are incomplete so the government can only issue non-state-contingent debt. The government budget constraint in (1) can then be re-written as:

$$(1+\gamma)b_{t+1} = b_t(1+r) - (\tau_t - g_t) \tag{4}$$

where lowercase letters refer to ratios relative to GDP.

Since the government wants to rule out a collapse of its outlays below their tolerable minimum levels, it would not want to hold more debt than the amount it could service if the primary balance were to remain forever (or "almost surely" in the language of probability theory) at its lowest value, or "fiscal crisis" state. A state of fiscal crisis is defined as a situation reached after a "sufficiently" long sequence of the worst realization of public revenues and after public outlays have been adjusted to their tolerable minimum. This upper bound on debt is labeled the "Natural Debt Limit" (NDL), which is the term used in the precautionary-savings literature for an analogous debt limit that private agents impose on themselves when they can only use non-state-contingent assets to smooth consumption (see Aiyagari, 1994). The NDL is given by the growth-adjusted annuity value of the primary balance in the state of fiscal crisis.

The "history of events" leading to a fiscal crisis has non-zero probability (although it could be a very low probability) as long as that crisis state is an event within the support of the probability distribution of the primary balance, and as long as there are non-zero conditional probabilities of moving into this crisis state from other realizations of the primary balance. Inasmuch as the government internalizes that there is some probability that it could suffer a fiscal crisis in the future, the government must not hold more debt than it could service while paying for the crisis level of outlays.

Since the NDL is a time-invariant debt level that satisfies the government budget constraint with revenues and outlays set at their minimum, it follows that the NDL implies that the government remains able to service its debt even in a state of fiscal crisis. Thus, the NDL that a government imposes on itself to self-insure against the collapse of public outlays below its tolerable minimum also allows that government to offer lenders a credible commitment to remain able to repay its debt in all states of nature.

To turn the above notions of the NDL and their implied credible commitment to repay into operational concepts, one needs to be specific about the factors that determine the probabilistic dynamics of the components of the primary balance. On the revenue side, the probabilistic processes driving tax revenues reflect the uncertainty affecting tax rates and tax bases. These processes have one component that is the result of domestic policy variability and the endogenous response of the economy to this variability, and another component that is largely exogenous to the domestic economy (which typically results from the nontrivial effects of factors like fluctuations in commodity prices and commodity exports on government revenues). The version of the MO model used in this paper incorporates explicitly the second component.[3]

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[3] Note that the exogenous determinants of public revenue dynamics can be important even in economies

On the expenditure side, government expenditures adjust largely in response to policy decisions, but the manner in which they respond varies widely across countries.[4] In addition, the "adjustment" or minimum level of public outlays to which the government can commit to adjust in a fiscal crisis is particularly important for determining the NDL and the sustainable debt ratios in the MO model. Labeling the fiscal-crisis level (or lowest realization) of the government revenue-GDP ratio as $\tau^{\min}$ and the minimum level of the ratio of outlays to GDP that the government can commit to deliver as $g^{\min}$ (for $g^{\min} < \tau^{\min}$), it follows from the government budget constraint in (4) that the NDL is the value of b^* given by:

$$b_{t+1} \leq b^* \equiv \frac{\tau^{\min} - g^{\min}}{r - \gamma} \tag{5}$$

This NDL is lower for governments that have (a) higher variability in public revenues, (b) less flexibility to adjust public outlays, and (c) lower growth rates or higher real interest rates.

The NDL represents a credible commitment to repay in the sense that it ensures that the government remains able to repay even in a state of fiscal crisis for a given known stochastic process driving revenues and a given policy setting the minimum level of outlays. However, this should not be interpreted as suggesting that the need to respect the NDL rules out the possibility of sovereign default. Default triggered by "inability to pay" remains possible if there are large, unexpected shocks that drive revenues below what was perceived to be the value of $\tau^{\min}$ or if the government turns out to be unable to reduce outlays to $g^{\min}$ when a fiscal crisis hits. In addition, default triggered by "unwillingness to pay" remains possible since the NDL is only an ability to pay criterion that cannot rule out default for strategic reasons. Section 4 explores an extension of this framework that incorporates default risk into the basic MO setup.

Consider a government with exogenous, random fiscal revenues (say, for example, oil export revenues) and an ad-hoc smoothing policy rule for government expenditures, such that $g_t = g$ (for $g \geq g^{\min}$) as long as $b_{t+1} \geq b^*$, otherwise g_t adjusts to satisfy condition (5). By (4) and (5), if at a particular date the current debt ratio is below b^* and the realization

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that have successfully diversified their exports away from primary commodities. In Mexico, for example, oil exports are less than 15 percent of total exports but oil-related revenues still represent more than 1/3 of public sector revenue.

[4] For instance, it is known that whereas government spending is counter-cyclical in industrial countries, it tends to be acyclical or slightly procyclical in developing countries; see for example Gavin and Perotti (1997) and Talvi and Végh (2005).

of the revenue-output ratio is $\tau^{\min}$, the government finances g by increasing b_{t+1}. In contrast, if at some date the current debt ratio is at b^* and the realization of revenues is $\tau^{\min}$, (4) and (5) imply that $g_t = g^{\min}$. In a simple example with zero initial debt, it is straightforward to show that if the government keeps drawing the minimum realization of public revenue, it will take the T periods to hit the NDL, where T solves the following equation:

$$\left(\frac{R}{\gamma}\right)^T = \frac{g - g^{\min}}{g - \tau^{\min}} \tag{6}$$

This result indicates that, in the worst-case scenario in which revenues remain "almost surely" at their minimum, the government can access the debt market to keep the ratio of public outlays at the level g for a longer period of time the larger is the excess of "normal" government outlays over minimum government outlays relative to the excess of normal outlays over the minimum level of revenues. Thus, the government uses debt to keep its outlays as smooth as possible given its capacity to service debt as determined by the volatility of its public revenues, reflected in the value of $\tau^{\min}$, and its ability to reduce public outlays in a fiscal crisis, reflected in the value of $g^{\min}$.

The key element of the expenditure policy is not the level of $g^{\min}$ per se but the credibility of the announcement that outlays would be so reduced during a fiscal crunch. The ability to sustain debt and the credibility of this announcement depend on each other because a government with a credible ex ante commitment to major expenditure cuts during a fiscal crisis can borrow more and access the debt market for longer time; hence, everything else the same, this government faces a lower probability to be called to act on its commitment. In a more general case in which public revenue is not an exogenous probabilistic process but it is in part the result of tax policies and their interaction with endogenous tax bases, the credibility argument extends to tax policy. Governments that can credibly commit to generate higher and less volatile tax revenue-output ratios will be able to sustain higher levels of debt, and to the extent that this helps the economy produce stable tax bases, it helps to support the credibility of the government's ability to raise revenue.

The condition defining the NDL in (5) has a similar form as the formula for calculating sustainable debt ratios under the long-run method: $b = (\tau - g)/(r - \gamma)$. However, the implications for assessing fiscal sustainability under the two methods are sharply different.

The long-run deterministic rule always identifies as sustainable debt-output ratios that are unsustainable once uncertainty on the determinants of the fiscal balance and the NDL are taken into account. This is because the long-run method ignores the role of the volatility of the elements of the fiscal balance; on the contrary, the MO model finds that, everything else the same, governments with less variability in tax revenues can sustain higher debt ratios.

Consider the case of two governments with identical long-run averages of tax revenue-output ratios at 20 percent. The tax revenue-output ratio of government A has a standard deviation of 1 percent relative to the mean, while that of Government B has a standard deviation of 5 percent relative to the mean. Assuming for simplicity that the distributions of tax revenue-output ratios are Markov processes with $\tau^{\min}$ set at two standard deviations below the mean, the probabilistic model would compute the natural debt limit for A using a value of $\tau^{\min}$ of 18 percent, while for B it would use 10 percent. The deterministic long-run method yields the same debt ratio for both governments and uses the common 20 percent average tax revenue-output ratio to compute it. In contrast, the MO method would find that debt ratio unsustainable for both governments and would produce a debt limit for B that is lower than that for A.

The two methods also differ on the role given to the limiting debt ratios. In the long-run analysis, the steady-state debt ratio is viewed either as a target ratio to which a government should be forced to move to, or as the anchor for a target primary balance-GDP ratio that should be achieved by means of a policy correction. In contrast, the NDL in the MO method only defines the maximum level of debt. Unless the NDL binds, that maximum is *not* the equilibrium or sustainable level of debt that should be issued by the government, although it plays a central role in determining both. Furthermore, according to the MO method, a country can have levels of debt much lower than the NDL and may take a very long time on average to enter a state of fiscal crisis or even never arrive at it.

The MO methodology models uncertainty in the form of discrete Markov processes. Given the information on the current stock of public debt, the current tax revenue-GDP ratio and the assumed behavioral rules for government outlays and statistical moments of the public revenue process, the model produces conditional one-period-ahead and unconditional long-run distributions of the debt-output ratio, as well as estimates of the average number of periods in which b^* is expected to be reached from any initial b_0. Depending on the nature of the random processes and policy rules of revenues and outlays, it may take a few quarters to hit the debt ceiling on average, or it may take an infinite number of quarters to do it.

4 Results of the MO Method for Brazil, Colombia, Costa Rica, and Mexico

This section applies the MO method to four Latin American countries: Brazil, Colombia, Costa Rica, and Mexico. The debt dynamics and their determinants in these countries are compared with those observed in two recent polar experiences in Argentina and Chile, characterized, respectively, by debt default and full repayment. Seeking to identify key parameters needed to simulate the debt dynamics and solve for the natural debt limits, the section begins with a brief review of the recent growth performance and evolution of the fiscal variables. The data sources are detailed in the Appendix.

Review of Growth Performance and Fiscal Dynamics

Over the last 25 years, the growth performance of the four countries examined in this study was weak. As shown in Table 1, average growth in GDP per capita for the period 1981-2005 was less than one-half of a percent in Brazil, 0.8 percent in Mexico, and around one-and-a-quarter percent in Colombia and Costa Rica. These countries grew at faster rates in the past. Taking averages starting in 1961, the smallest and largest average per-capita GDP growth rate were 1.86 percent (Colombia) and 2.3 percent (Brazil). As for the growth rate in the two reference countries, Table 1 shows that the average growth rate was higher (lower) in Chile (Argentina) than in the four examined countries.

Given the apparent structural breaks in the trend of GDP per capita, the public debt analysis below defines a baseline growth scenario that uses the 1961-2005 average growth rates, and compares the results with those of a scenario that views the growth slowdown of the last two decades as permanent by using 1981-2005 average growth rates.

Table 1 shows that among the four examined countries, the mean debt-to-GDP ratio for the full sample ranged from 37.5 percent in Colombia to 50.3 percent in Costa Rica.[5] In sharp contrast with these ratios, the average ratios in Chile and Argentina were 7.8 and 60.7 percent respectively. These full-sample averages, however, hide the evidence shown in Figure 1 that debt ratios have been in general growing rapidly, except in Costa Rica where the stock of debt has remained high (at around 50 percent). By splitting the sample to create averages for 1990-1995 and 1996-2005, one finds that the mean debt ratios of Mexico, Brazil, and Colombia increased by about 10, 12, and 14 percentage points between the first and the second periods. Furthermore, as shown in Figure 1, during the second period all the four examined countries (Brazil, Colombia, Costa Rica, and Mexico) displayed debt ratios around 55 percent at some point in time, as it happened in Argentina before defaulting on its debt. A key question to answer is whether a debt ratio around 55 percent is consistent with fiscal solvency given the pattern of growth, interest rates, and the fiscal-revenue volatilities faced by these countries.

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[5] Reliable cross-country estimates of public debt stocks at the general government level are hard to obtain. As detailed in the Appendix, we use statistics available from national sources and from the IMF, so it must

Real interest rates on public debt are hard to measure because public debt instruments differ in maturity, currency denomination, indexation factors, and residence of creditors. One proxy is the measure of sovereign risk proposed by Neumeyer and Perri (2005), which is the spread of the EMBI+ index relative to the U.S. T-Bill rate deflated by an estimate of expected inflation in the U.S. GDP deflator. The sample period of this measure is relatively short (starting in 1994) and biased because it includes mainly observations for a turbulent period in world capital markets. Thus, this measure of real interest rates on public debt shows substantial premia over the world's risk free rate and can be taken as an upper bound estimate of the interest rate. For instance, the averages for a quarterly sample from 1994:1 to 2002:2 are 12.9 percent for Brazil and 10.3 percent for Mexico. The lower bound would be the real interest rate on U.S. public debt. The 1981:1-2005:4 average of the U.S. 90 day T-bill rate deflated by observed U.S. CPI inflation is about 2.17 percent.

Given the above considerations about measurement of interest rates on public debt and the observations of the average U.S. T-bill rate of 2.17 percent and the Brazilian average real interest rate on foreign sovereign debt of 13 percent, two interest-rate scenarios are considered. In the baseline scenario, the real interest rate is set equal to 5 percent, which represents a small premium of about twice the U.S. T-bill rate. The alternative is a high-real-interest-rate scenario characterized by an 8 percent interest rate. Both scenarios remain relatively

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be noticed that the reported data may not be strictly comparable across countries.

optimistic about growth prospects, using the average growth rates of the period 1961-2005.

The measure of public revenues needed for conducting the debt-sustainability analysis is the total of all tax and non-tax government revenues excluding grants. Government expenditures should comprise total non-interest government outlays, including all expenditures and transfer payments and excluding all forms of debt service. Limitations of the existing international databases make it difficult to retrieve consistent measures of these variables that apply at the level of the entire non-financial public sector and, in the case of the outlays, that include the annuity values of all contingent liabilities resulting from obligations like banking- or pension-system bailouts. We put together estimates of the revenues and outlays ratios by combining data from national sources with IMF and World Bank data (see Appendix).

The average ratios of total public revenues to GDP during the period 1990-2005 in the four examined countries ranged from 21 percent in Mexico to 32 percent in Costa Rica (see Table 1). The volatilities of the public revenues were relatively low in Costa Rica and Mexico with coefficients of variation of the revenue-output ratios at 7 and 7.8 percent of the mean. At the other end, Colombia showed the highest coefficient of variation of public revenues at 18.5 percent. Comparing these figures with those of the two reference countries, one notes that in terms of averages, the Argentine public revenue ratios were lower than the ratios observed in the four examined countries; on the other hand, the coefficients of variation of the public revenue ratios in the four examined countries have largely exceeded the Chilean 4.2 percent ratio.

Turning to the other component of the primary fiscal balance, the average non-interest outlays-to-GDP ratio during the period 1990-2005 was relatively low in Mexico at 18.4 percent, and relatively high in the other three examined countries where the ratio ranged between 24.6 and 29.6 percent. The volatilities of these outlays ratios were lower in Mexico (7.2 percent) and Costa Rica (9 percent) than in Brazil (14.7 percent) and Colombia (23.3 percent). Interestingly, Chile, the reference country that payed off its debt, displayed the lowest average and the second-to-lowest volatility of the non-interest outlays ratio.

Natural Debt Limits: Baseline Scenario and Two Alternatives

Table 1 reports three sets of calculations of natural debt limits for Brazil, Colombia, Costa Rica and Mexico. The baseline scenario considers the 1961-2005 average growth rates of GDP per capita and a 5 percent real interest rate. The growth-slowdown (GS) scenario uses the 1981-2005 average growth rates and keeps the real interest rate at 5 percent. The high-real-interest-rate (HRIR) scenario uses a real interest rate of 8 percent and sets the growth rates equal to the 1961-2005 averages.

The baseline scenario differs from the other two because it is designed to produce a coefficient of fiscal adjustment that yields a NDL equal to the largest debt ratio observed in each of the four examined countries during the 1990-2005 period. Using the maximum observed debt ratio to define the NDL in the two reference countries is, however, less meaningful. For instance, the Argentine 164 percent ratio was clearly inconsistent with fiscal solvency and the high debt ratios in that country later unfolded into a debt crisis. Hence, instead of using the maximum historic debt ratios, the NDLs in Argentina and Chile are both set equal to the average of the maximums debt ratios in the four examined countries, which is equal to 55.7 percent.

Table 1 reports the coefficient of "implied fiscal adjustment". This coefficient indicates the number of standard deviations relative to the mean that non-interest outlays should be lowered so as to yield a debt limit equal to the NDL. The implied fiscal adjustment is calculated taken as given the data for means and coefficients of variation of revenues and outlays, the average growth rates, and the assumed real interest rate. The calculation uses floors of public revenues equal to two standard deviations below the corresponding means and solves for the minimum value of non-interest outlays consistent with the NDL according to the definition given in eq. (5). The table also shows the implied minimum ratio of outlays to GDP resulting from the coefficient of fiscal adjustment. The GS and HRIR scenario keep the same fiscal adjustment coefficient and just alter either the growth rate or the real interest rate.

The public debt-GDP ratios of the four countries under study peaked at similar levels during the 1990-2005 period (ranging from 54.5 percent in Costa Rica to 57.2 percent in Brazil). The coefficients of implied fiscal adjustment reported in Table 1 show that, in order to produce a NDL that can support the maximum observed level of debt, Brazil and Colombia need credible commitments to undertake large cuts in their outlays if they were to hit a fiscal crisis. For instance, the Brazilian adjustment measures 2.5 standard deviations below the mean of non-interest government outlays which is equivalent to asking a cutback in the outlays-GDP ratio of about 6.2 percentage points.

The Argentine numbers in Table 1 are illustrative of how the impossibility of implementing large cuts in fiscal outlays can lead to unsustainable levels of debt. With a NDL set at 55.7 percent, maintaining fiscal solvency in Argentina would have required fiscal-crisis cuts in non-interest outlays equal to 2.92 standard deviations away from its historical average outlays. This is equivalent to a 5.2 percentage point reduction in the country's outlays-GDP ratio. The Argentine experience is diametrically different from the Chilean. Chile does not require any cut in its expenditures to sustain the 55.7 percent assumed NDL; this is because even the two standard-deviation floor of public revenues is higher than the average non-interest outlays ratio.

When measured in number of standard deviations with respect to their historical means, the Colombian, Mexican, and Costa Rican outlays-cut commitments (ranging from 1.39 to 1.64) for a scenario of fiscal-crisis are less stringent than the Brazilian commitment. However, the cut that the Colombian fiscal authorities should be able to implement to maintain fiscal solvency in a fiscal-crisis scenario amounts to reduce the country's outlays-GDP ratio in 10.6 percentage points, which is an extraordinarily sever fiscal effort. Thus, the model is consistent with the data in predicting that Costa Rica and Mexico, the countries with lower public-revenue volatility, should be the ones that have a better chance of sustaining high debt ratios.

The potential dangers of using the Blanchard ratios for conducting debt sustainability analysis are illustrated in the baseline results. The Blanchard ratio, which would compute a sustainable debt ratio using the difference between the *average* public revenue and the *average* government outlays, yields debt ratios between 71.5 and 97.5 percent (see Table 1). These ratios largely exceed the NDLs produced by the MO model and make evident that the Blanchard ratios could be inconsistent with the notion of being able to honor the public debt in any conceivable history of the public finances.

Consider next the natural debt limits in the growth-slowdown (GS) and high-real-interest-rate scenarios (HRIR). If the growth slowdown of the last two decades persists, and even assuming that the coefficients of fiscal adjustment were to remain as high as estimated in the baseline scenario, the current debt ratios would exceed the natural debt limits of all four analyzed countries by large margins. The Argentine and Brazilian situations would be particularly compromised (even after the Argentine debt restructuring program) because the 2005 debt ratios of 73.3 and 51.5 percent would exceed the maximum debt ratios consistent with fully credible commitments to repay in the GS scenario by 27 percentage points (Argentina) and 18 percentage points (Brazil).

The HRIR scenario, in which for example a retrenchment of world capital markets or the pressure of large fiscal deficits in industrial countries push the real interest rate on emerging markets public debt to 8 percent, has even more damaging effects. In this case, even if the growth rates recover to the 1961-2005 averages and even with the large fiscal adjustment coefficients set in the baseline scenario, the natural debt limits of all four examined countries fall below 29 percent. Notice, however, that the prediction of the model is not that an increase of the interest rate to 8 percent would trigger immediate fiscal crises in all four countries. For a fiscal crisis to occur immediately, the increase in the interest rate would have to be once-and-for-all and permanent. A transitory hike in the real interest rate could be absorbed in an analogous manner as a transitory downturn in public revenues, and hence a fiscal crisis would only be triggered after a sufficiently long sequence of adverse shocks. This last observation highlights again the fact that the NDL is not (in general) the same as the sustainable or equilibrium level of debt, which is determined by the dynamics driven by the government budget constraint. We turn to study these dynamics next.

Debt Dynamics

The simulations of debt dynamics consider public debt-GDP ratios ranging from 0.10 to 0.50 and assume that if the government budget constraint yields a negative debt at any time, the corresponding fiscal surplus is rebated to the private sector as a lump-sum transfer. The dynamics of sustainable debt can be traced from any initial public debt ratio in this interval. However, one needs to be careful in studying the long-run dynamics of debt ratios because this basic version of the MO model features two long-run distributions of public debt, one converging to 0 and the other to the NDL. Which of these two distributions is attained in the long run depends on initial conditions.

The prediction that the long-run debt ratio is not determined within the model (i.e., that the long-run debt ratio depends on initial conditions) is not a peculiarity unique to the MO model. The classic tax-smoothing framework of Barro (1979) predicts a similar outcome for the debt dynamics, and the outcome is also in line with the findings on Ramsey optimal taxation problems in which smooth taxes are optimal taxes (see Chapter 14 of Ljungqvist and Sargent, 2004).

The stochastic processes of public revenues used in the simulations are characterized by time-invariant Markov chains. Each country-specific chain is defined by three objects: an n-element vector of realizations of the revenues, τ, an $n \times n$ transition probability matrix, $\mathbb{P}$, and a probability distribution for the initial value of the realization of revenues, π_0. The typical element of the transition probability matrix, $\mathbb{P}_{ij}$, indicates the probability of observing revenues $\tau = \tau_j$ in the next period given that revenues are $\tau = \tau_i$ in the current period. For each country, the vector of realizations of revenues has 5 elements (n=5). The lowest value of τ is set two standard deviations below the mean tax revenue in each of the four countries under analysis. We then use Tauchen's (1985) univariate quadrature method to set the rest of the elements of the vector of realizations τ and the transition probability matrix $\mathbb{P}$ so as to approximate the first-order autocorrelation and standard deviation of public revenues observed in the data.

The stochastic simulations require generating a T-period time series of realizations of revenues, i.e. $\tau_1, \tau_2, ...\tau_T$, drawn from the Markov vector τ. This time series is constructed using the matrix $\mathbb{P}$ and realizations of a uniform random variable $u \in [0, 1]$ as follows; if the tax revenue at time t is equal to the value of i-th element of vector τ, the tax revenue in period $t + 1$ is equal to the value of the j-th element of τ when the following condition holds:

$$\sum_{l=1}^{j} \mathbb{P}_{il} < u \leq \sum_{l=1}^{j+1} \mathbb{P}_{il}$$

and, it is equal to the value of the first element of τ if $u < \mathbb{P}_{i1}$.

Figure 2 illustrates the first application of the stochastic simulations; it shows the relative frequencies of fiscal crises in each examined country and in Argentina, for five starting values of the debt-GDP ratio ranging from 10 to 50 percent. A fiscal crisis occurs when the debt ratio in a given country hits the country's NDL and the government adjusts its non-interest outlays. The relative frequencies shown in the figure were computed simulating the basic MO model using the country-specific values of the non-interest outlays, the NDLs, and simulations of the fiscal-revenue processes. The simulated tax revenues correspond to the realizations of revenues drawn from the country-calibrated Markov chains discussed above. Ten thousand simulations of 50 observations are conducted for each starting debt ratio in each of the five countries referred in the figure. Thus, the reported relative frequencies show the probabilities of observing a fiscal crisis within the next 50 years in each country for each initial debt ratio.

Note that in all countries, the likelihood of a fiscal crisis increases with the initial debt ratio and it is equal to zero for the lowest considered initial debt ratio (10 percent) in most of the countries; this happens because it is more likely that a given realization of the primary balance falls short of the interest outlays when the debt ratio is high than when it is low. In contrast, when the debt ratio is low, the primary balance exceeds the value of the interest outlays for most of the realizations of the tax revenue and the government uses the overall budget surplus for debt buybacks.

According to the results shown in Figure 2, taking the 1990-2005 Argentine history of fiscal revenues into account and assuming that the non-interest outlays are not modified except that the country hits its NDL, the probability of observing a fiscal crisis is scarcely lower than 100 percent when the initial debt ratio is equal to 50 percent. The result is consistent with the recent Argentine experience in which the country was unable to honor its debt services. The result also illustrates the danger of facing a fiscal crisis when a country that holds a large debt ratio does not adjust its expenditures during non-crisis times. The Chilean case, not shown in the figure, is radically different from the Argentine. In Chile, if the non-interest outlays and fiscal revenues observed in the 1990-2005 period are observed in the future, fiscal solvency is guaranteed even at the 50 percent starting debt ratio.

For the initial 50 percent debt ratio, the likelihoods of observing a fiscal crisis in Brazil and Colombia are large at 79 and 84 percent, respectively. The probabilities of a fiscal crisis fall to 74 and 23 percent in Costa Rica and Mexico for the same initial debt ratio. Mexico is the country that displays the soundest fiscal policy; note that when the initial debt ratio is 40 percent, which is close to the 44 percent observed in Mexico in 2005 (see Figure 1), the probability that adverse sequences of fiscal-revenue shocks end up causing a fiscal crisis is barely higher than zero (0.15 percent). On the other hand, even the lowest initial debt ratios have high chances of producing a fiscal crisis in Colombia. This is indicative that if the recent evolution of the Colombian fiscal revenues were observed in the future, fiscal solvency could only be guaranteed by undergoing large cutbacks in non-interest outlays. In Brazil, only debt ratios below 40 percent guarantee that the likelihood of a fiscal crisis is below 40 percent.

In Figure 3 the focus changes to the most adverse fiscal scenarios that the countries examined in this study could face in the future. For each country and initial debt ratio, Figure 3 reports the minimum number of periods that it took to hit a fiscal crisis among the 10,000 conducted simulations. When the initial public debt-GDP ratio is equal to 10 percent, a fiscal crisis could be observed in 20 years in Brazil and in 8 years in Colombia but no single crisis could be observed in Costa Rica and Mexico. However, for the highest initial debt ratio (50 percent), it could only take 3 years in all countries to face a fiscal crisis, except in Mexico where it would take 4 years. When one thinks about the most adverse fiscal scenario that the countries in the region could face in the future, these results show the dangers implicit in the recent high debt ratios observed in Latin America.

The results in Figure 2 serve to illustrate how uncertainty affects the dynamics of public debt and the extent to which the maximum debt differs from possible equilibrium paths of public debt. The nonappearance of the bars corresponding to some initial debt ratios in Costa Rica and Mexico shows that the simulated debt ratios never reached the NDL in any single time period. Consider for example the initial debt ratio approximately equal to 30 percent in Costa Rica. Whereas the extreme adverse scenario calculations demonstrate that it is conceivable to observe a fiscal crisis within the next 3 years, only in approximately half of the simulations, the debt ratio hits its maximum level. Similarly, for the initial stock of debt equal to 0.5 times the Mexican GDP, the debt ratio never reached its limit in 7,721 out of 10,000 simulations.

Figure 4 shows a sample of simulated time series of the public debt ratio and illustrates further how much the NDL and the sustainable debt ratios differ. The figure shows 20 simulations of debt-output ratios for a unique starting ratio equal to 30 percent, using the parameters values calibrated to the Argentine economy. At each period t, a random draw of public revenues along with the t-stock of debt and the fiscal rules for public outlays are used to determine the value of the debt at time $t+1$. Notice that whereas for some paths the debt ratio increases rapidly until it reaches the NDL, for other paths it takes a long time to reach it and for other paths the debt goes to zero. As explained above, for a large range of initial values of public debt, the model predicts that the debt-to-GDP ratio will reach the debt limit while for some other initial values the debt ratio goes to zero. This implies that for starting values of the debt ratio above 0.30, the fraction of paths driving the debt to its maximum level increases and that for starting values below 0.30 that fraction decreases.

Default Risk

Up to this point the analysis followed the methodology proposed by Mendoza and Oviedo (2006) in which sovereign default was set aside to focus on modeling the optimal debt policy consistent with fiscal solvency and the desire to smooth public outlays. The only way in which default risk was taken into account was in setting the value of the constant real interest rate used to solve for the NDLs and to compute the debt dynamics. However, time-varying default risk premia are an important feature of public debt in emerging markets. It may make sense for a government to conduct a forward-looking debt sustainability exercise in which it is assumed that there is no default risk, or that default risk is time invariant, as a benchmark scenario, but it is important to study how the results vary when time-varying default risk is introduced.

One important limitation of the analysis of default risk is that existing theoretical models of optimal sovereign debt contracts face serious challenges in explaining observed debt ratios. The canonical model of Eaton and Gersovitz (1981) considers a risk-neutral lender and a risk-averse borrower that has the option of defaulting at the cost of facing permanent exclusion of the debt market. The lender is willing to take on the risk of default by charging a rate of interest that incorporates a premium consistent with the probability of repayment. There are well-known theoretical problems with this setup related to the classic Bulow-Rogoff critique showing that the threat of exclusion may not be credible because of the option to enter in deposit contracts with lenders. But even if the model were not affected by these problems, recent quantitative studies show that optimal contracts of sovereign debt in the Eaton-Gersovitz tradition support very small debt ratios of less than 10 percent of GDP (see Arellano, 2006). This is because the models yield probabilities of default that increase too rapidly at low levels of debt.

Faced with the difficulties in developing a complete theory of endogenous default risk, a pragmatic approach is followed next; the approach takes into account the same risk-neutral lender of the Eaton-Gersovitz model but incorporates an exogenous probability of repayment.

The arbitrage condition of the risk-neutral lender implies:

$$R(b_t) = \frac{R^w}{\lambda(b_t)} = \frac{R^w}{\exp(-ab_t)}; \qquad a > 0 \tag{7}$$

In this expression, R^w is the gross world risk-free real interest rate and $\lambda(b_t)$ is the probability of repayment (i.e., $1 - \lambda(b_t)$ is the probability of default). The repayment probability is modeled with an exponential probability distribution: $\exp(-ab_t)$, where the curvature parameter a determines the speed at which the repayment probability falls as debt increases.

The exponential formulation of default risk has the advantage that it is consistent with two key properties of the optimal default probability of the Eaton-Gersovitz contract: (a) the probability of default is increasing and convex on the level of debt and (b) the probability of default is zero if the stock of debt is zero. The formulation fails to reproduce the property of the Eaton-Gersovitz contract that the probability of default approaches 1 for a well-defined rationing level of debt at which debtors always find it preferable to default than to repay. In the exponential formulation the probability of default approaches 1 asymptotically as debt goes to infinity. However, the formulation still allows for values of a that would yield very large risk premia for high levels of debt.

We calibrate the value of a so that the arbitrage condition in (7) holds taking as given the EMBI+ country risk premium and the public debt ratio in Mexico in 1998, the year of Mexico's maximum debt ratio in the 1990-2005 sample. Mexico's debt ratio in 1998 was $b_t = 0.549$ and the real interest rate that the country faced on this debt, measured as the U.S. 90-day T-bill rate plus the EMBI+ spread, was $R(b_t) = 10.48$ percent. The average risk-free rate (i.e., the real U.S. 90-day T-bill rate) in 1998 was R^w =3.2 percent. Plugging these figures into (7), the equation holds for $a = 0.124$.

As shown below, default risk has two important implications for the analysis of sustainable debt based on the MO model. First, it lowers the levels of NDLs, since the rate of interest considered in Table 1 is lower than those resulting in the worst state of nature with default risk. Second, it alters the dynamics of public debt since the rate of interest now increases with the level of debt. These two effects result in lower NDLs, reduced levels of sustainable debt and faster convergence to states of fiscal crisis.

Table 2 shows the effects of introducing time-varying default risk in the calculations of the NDLs.[6] All the estimates shown in this table assume that the risk free rate is set at the 1981-2005 average of the real 90-day T-bill rate, which is equal to 2.17 percent, and that the curvature parameter of the probability of repayment is kept at $a = 0.124$.

The first panel of Table 2 shows how the benchmark estimates of the NDLs change when default risk is introduced. These benchmark estimates take the same growth rates and minimum levels of public revenues and outlays as in the benchmark scenario of Table 1. The resulting NDLs are significantly smaller (by 19.8 to 22.5 percentage points of GDP) than those in the benchmark case without default risk. Note that this sharp decline of the NDLs occurs despite the risk-free rate (at 2.17 percent) is below one half the long-run real interest rate used in the benchmark scenario of Table 1. The repayment probabilities near 96 percent and the default risk premia between 4.3 and 4.6 percent are similar across countries. The NDLs in this case ensure that governments would be able to repay even during a fiscal crisis, but they still may choose to default on debt ratios about 0.33 with 4 percent probability.

The second panel of Table 2 shows how NDLs change in the growth slowdown scenario. Again, relative to the growth slowdown scenario of Table 1, the risk-free rate is lowered from 5 to 2.17 percent and a time-varying default risk in introduced. To isolate the contribution of the latter, the third panel of Table 2 shows the NDLs obtained using the growth rates of the growth slowdown scenario but assuming that there is no default risk so that countries can borrow at the 2.17 percent risk-free rate. Since this rate is less than half the one used in Table 1, the resulting NDLs are high and above 100 percent of GDP for most countries.

Two comparisons are interesting to make using the second and third panels of Table 2. First, the fact that the NDLs of the growth slowdown scenario in Table 1 (ranging from 34 to 44 percent of GDP among the examined countries) are much smaller than those of the no-default-risk case in Panel 3 of Table 2 shows that the strategy of setting a long-run real interest rate of 5 percent as a proxy for default risk in the estimates of Table 1 was not a bad approximation. Second, the calculations of the NDLs of the second and third panels differ only because the second incorporates the time varying default risk premium (i.e., both have the same risk-free rate of 2.17 percent). Since the NDLs without time-varying default risk are several times larger than those with default risk, this comparison shows that default risk has major implications for estimates of NDLs.

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[6] Note that with default risk, the constant rate of interest in the denominator of the formula for the NDL is replaced with the interest rate including default risk defined in equation (7). Since the interest rate depends on the level of debt, the NDL is now the solution to a non-linear equation.

The last panel of Table 2 re-computes the required adjustment in outlays (i.e. the values of $g^{\min}$) needed to support the NDLs of the benchmark scenarios of Table 1 but now taking into account time varying default risk. The adjustments in outlays are significantly larger than those reported in Table 1. The required adjustment in outlays exceeds the two-standard deviation threshold for all countries and it is larger for Mexico than for the other examined countries (but still lower than for Argentina). Measured in terms of percentage points of GDP, the adjustments are 5.10 percent in Mexico; 6.5 percent in Costa Rica; and 10 and 13.5 percent in Brazil and Colombia. This ranking (along with the similarities with the Argentine results) suggests again that the debt positions of Brazil and Colombia are more difficult to reconcile with fiscal solvency considerations than those of Costa Rica and Mexico.

Figure 5 illustrates the implications of default risk for the dynamics of public debt reflected by the relative frequency of a fiscal crisis. The relative frequencies of fiscal crises increase for all countries and initial debt ratios when the interest rate incorporates a risk premium that responds positively to public indebtedness. In other words, considering risk-adjusted interest rates implies that countries hit more often their NDLs than when the interest rate is fixed. In Mexico, for example, whereas an initial debt ratio of 40 percent has a zero probability of leading to a fiscal crisis under a fixed interest rate, that probability rises to 56 percent after introducing default risk.

5 Conclusions, Caveats and Extensions

The application of the basic version of the MO model to the cases of Brazil, Colombia, Costa Rica, and Mexico shows that public debt ratios in these countries are already close to their natural debt limits; these are the debt limits that the countries should respect to preserve the credibility of their perceived commitment to remain able to repay their debts. The result arises after assuming a relatively optimistic scenario in which the growth slowdown of the last two decades is reversed to recover the average per-capita growth rate observed between 1961 and 2005, and the ex ante real interest rate remains at a low level of 5 percent. This baseline scenario is also optimistic in that it requires credible commitments to large cuts in government outlays which recent experience indicates are a low-probability event. Considering less optimistic scenarios in which growth continues at the trend of the last two decades or the real interest rate increases to 8 percent, current debt ratios would be inconsistent with a credible commitment to repay even with the same tough stance to cut outlays in a state of fiscal crisis assumed in the baseline scenario.

The model predicts that the long-run dynamics of the debt ratio are undetermined; more precisely, the model predicts that there is no unique, invariant limiting distribution for the debt output ratio. This result needs to be considered carefully. On one hand, note that the result does not require very strong assumptions: stochastic revenue, relatively inflexible outlays and some limit to debt-market access (whether a NDL or some ad-hoc debt limit). Also, the same outcome would result if outlays are given and instead one considers arguments for tax smoothing as in Barro (1979). If these are the maintained assumptions of public debt sustainability analysis, the stochastic simulations of debt ratios shown in Figures 2 to 4, together with the natural debt limits, summarize all relevant information for assessing whether observed public debt dynamics are sustainable.

On the other hand, the assumptions that lead to the result of an indeterminate long-run distribution of debt may be questionable. Instead of setting ad-hod rules, in the setup proposed by Mendoza and Oviedo (2006), the government chooses its outlays optimally. It is there assumed that the government is aimed to provide a smooth path of non-interest outlays as it seeks to provide social insurance in a world where only non-contingent markets exist to trade financial assets. In this setup, the government has a precautionary-savings motive that yields a unique, invariant limiting distribution of public debt. Furthermore, the role of the natural debt limit is clearer because the desire to respect it emerges not just from an assumed commitment to remain able to repay, but from the fact that otherwise the government is exposed to the risk of experiencing states in which its outlays can be very low, and the government is very averse to these states because of the constant-relative-risk-aversion nature of the utility function of public expenditures.

Most of the analysis in this paper is conducted giving a limited role to default risk by simply setting a long-run, time-invariant real interest rate with a premium above the risk-free rate. This was done following the approach of the MO model to provide a forward-looking tool to design fiscal programs with the explicit intention of preserving the government's ability to fulfill its financial obligations. However, default risk is an important feature of emerging markets of sovereign debt, and hence it is worth adding it to the analysis of debt sustainability.

Default risk was introduced by adopting an exogenous, exponential probability of repayment that varies with the level of debt. Lenders are assumed to be risk neutral and hence willing to take default risk by lending at a rate that incorporates the premium that equates the expected return of risky lending with the risk-free interest rate. Introducing this change into the the basic version of the MO model produces smaller debt limits and speeds up the dynamics that lead to states of fiscal crisis in which NDLs are reached. NDLs that completely ignore default risk support dynamic paths of sustainable debt with much higher debt ratios than those obtained when default risk is introduced. However, since the basic MO model approximates the long-run component of default risk by adding a constant premium above the risk-free interest rate, it yields results for debt dynamics that are a much closer approximation to those produced by the model with time-varying default risk than those of a model that ignores default risk completely.

The application of the MO model undertaken in this paper did not consider two other important elements of dynamics of public debt in emerging economies: the endogeneity of the tax bases and fiscal policy choices and the role of financial frictions like liability dollarization. The endogeneity of the tax bases can be incorporated into the structure of the MO model. This requires introducing the decisions of the private sector with regard to the variables that determine the allocations and prices that conform tax bases (such as labor supply, consumption, the current account and capital accumulation), something that could be explored in future research.

Appendix: Data Sources

This appendix details the data sources used in the paper. Per-capita output growth rates for all countries are from the World Development Indicators, a database maintained by the World Bank and available at *http://genderstats. worldbank.org/dataonline.* U.S. 90-day T-bill interest rates and the series of CPI are from the Federal Reserve Bank of St. Louis's databases.

Argentina. Data on public debt, output, and fiscal revenues and non-interest outlays for the period 1993-2005 were taken from the Economic Policy Office (Secretaría de Política Económica) of the Argentine Ministry of Economy and Production (Ministerio de Economía y Producción). See *http://www.mecon.gov.ar.* The data correspond to the national non-financial public sector and do not include state- and city-level data. For the period 1991-1992 the data on debt and fiscal ratios were taken from Tables 1 and 2 of Rozenwurcel (1994).

Brazil. Data on fiscal revenues and non-interest outlays refer to those of the national government (União), as published by the National Treasure (Tesouro Nacional) of the Brazilian Ministry of Finance (Ministério da Fazenda). See *http://www.tesouro.fazenda.gov.br.* Fiscal revenues in the paper are the sum of all sources of current revenue plus all capital revenues not associated with financial operations. Fiscal outlays are equal to the sum of current outlays plus capital expenditures not associated to financial operations. The Brazilian National Treasury reports revenues and outlays in reals (R$) of 2006 and the ratios to GDP are constructed using the series of GDP in R$ of 2005 reported by the Brazilian Central Bank (available at *http://www.bcb.gov.br*); before computing the ratios, the GDP series is updated to 2006 prices by using the actualization coefficient 1.0155298 (based on the price index IGP-DI) reported by the Brazilian National Treasury. Data on debt-to-GDP ratios are from the IMF, World Economic Outlook, September 2003 and IMF country reports.

Chile. Data on debt and fiscal revenues and non-interest outlays were taken from the Statistical Database of the Central Bank of Chile, available at *http://www.bcentral.cl.* Public debt refers to the net total financial debt of the central government. Output data (National Accounts, 1996 base year) were obtained from the same source although they are only available from 1996 on; GDP data for the period 1990-1995 were obtained from Correa, et al. (2002).

Colombia. The Colombian revenue and non-interest data were obtained from the Colombian Central Bank (Banco de la República) which can be accessed from *http://www.banrep.gov.co*. Data on fiscal revenues and outlays refer to the overall non-financial public sector. Public debt ratios for the period 1993-1995 are from IMF, World Economic Outlook, September 2003; data for the period 1996-2005 are from the Central Bank of Colombia and correspond to total net debt of the non-financial public sector minus the external assets of the National Treasury.

Costa Rica. The Costa Rican public revenues and non-interest outlays were obtained from the Costa Rican National Treasury (Tesorería Nacional) of the Ministry of Finance (Ministerio de Hacienda) and comprise the central government, the central bank, and all other organisms of the non-financial public sector. Revenue and outlays ratios were computed using the series of Costa Rican GDP provided by the same source. Deb-to-GDP ratios for the period 1992-2002 are from IMF, World Economic Outlook, September 2003; the 2003-2005 ratios were estimated using the Costa Rican National Treasury data.

Mexico. The Mexican debt ratios are from are from IMF, World Economic Outlook, September 2003, and from IMF country reports. The data on fiscal revenues and non-interest outlays as well as GDP data were obtained from Instituto Nacional de Estadística Geográfica e Informática, INEGI, (*http://www.inegi.gob.mx*).

Table 1: Fiscal Sector Statistics and Natural Debt Limits
1990-2005 (in percentages of GDP)

	Argentina	Brazil	Chile	Colombia	Costa Rica	Mexico
FISCAL SECTOR STATISTICS						
Public debt[1]						
average	60.74	43.04	7.86	37.47	50.28	46.06
maximum	164.44	57.18	30.31	56.17	54.50	54.90
year of maximum	2002	2003	1990	2003	2003	1998
Public revenue						
average	19.11	26.80	21.82	29.95	32.01	21.26
coeff. of variation	12.04	12.83	4.20	18.47	7.05	7.71
two-standard dev. Floor	14.51	19.92	19.99	18.89	27.49	17.99
Non-interest outlays						
average	17.57	24.62	15.99	27.71	29.60	18.37
coeff. of variation	10.18	14.66	7.48	23.28	9.01	7.20
Average primary balance	1.53	2.18	5.83	2.24	2.41	2.89
Implied fiscal adjustment[2]	2.92	2.55	-2.25	1.64	1.39	1.52
Implied minimum non-interest outlays	12.35	18.38	18.69	17.12	25.88	16.36
BENCHMARK NATURAL DEBT LIMITS AND BLANCHARD RATIOS						
(1961-2005 growth rates, 5% real interest rate)						
Growth rate	1.13	2.31	2.66	1.86	2.04	2.03
Natural debt limit[3]	55.69	57.18	55.69	56.17	54.50	54.90
Blanchard ratio	39.65	81.01	249.05	71.55	81.48	97.49
GROWTH SLOWDOWN SCENARIO						
(1981-2005 growth rates, 5% real interest rate, benchmark fiscal adjustment)						
Growth rate	0.49	0.45	3.45	1.22	1.33	0.81
Natural debt limit	46.61	33.83	82.28	42.03	43.93	38.91
Blanchard ratio	34.02	47.93	377.23	59.28	65.67	69.09
HIGH REAL INTEREST RATE SCENARIO						
(1961-2000 growth rates, 8% real interest rate, benchmark fiscal adjustment)						
Growth rate	1.13	2.31	2.66	1.86	2.04	2.03
Natural debt limit	31.36	27.04	24.42	28.70	27.05	27.29
Blanchard ratio	22.33	38.31	109.21	36.56	40.43	48.47

Notes: The source of data is detailed in the Appendix.

1. The debt ratios in Argentina, Colombia, and Costa Rica correspond to the periods 1991-2005, 1992-2005, and 1993-2005, respectively.
2. The implied fiscal adjustment is the number of standard deviations relative to the mean needed to obtain the benchmark natural debt limit. The minimum non-interest outlays are the values of these outlays consistent with the implied fiscal adjustment.
3. The natural debt limits of Brazil, Colombia, Costa Rica, and Mexico are equal to the largest public debt ratios observed in each country's data; for Argentina and Chile, the natural debt limit is the average natural debt limit in the other four countries.

Table 2: Natural Debt Limits with Default Risk

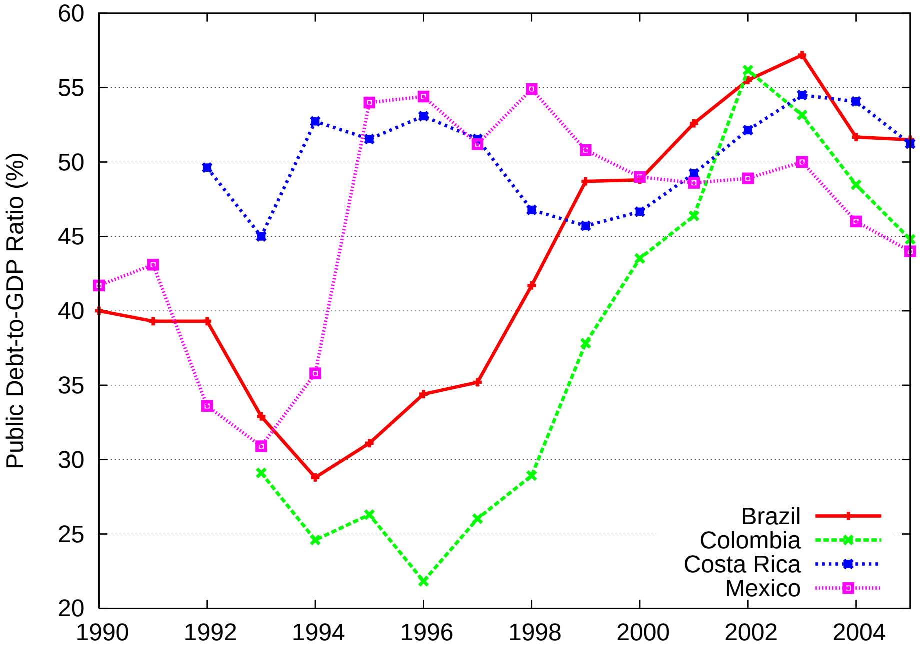
	Argentina	Brazil	Chile	Colombia	Costa Rica	Mexico
Benchmark NDLs with default risk[1]						
Natural debt limit	36.82	34.93	33.22	36.63	34.71	34.86
Probability of repayment	95.52	95.74	95.95	95.66	95.77	95.75
Default risk premium	4.79	4.54	4.31	4.63	4.51	4.53
NDLs in the growth slowdown scenario with default risk						
Natural debt limit	34.70	28.46	36.57	33.32	32.14	30.59
Probability of repayment	95.77	96.52	95.55	95.94	96.08	96.26
Default risk premium	4.51	3.69	4.76	4.33	4.17	4.17
NDLs in the growth slowdown scenario without default risk and risk free rate of 2.36%						
Natural debt limit	127.75	89.35	-	183.93	191.68	119.82
Required fiscal adjustment to support observed maximum debt ratios as NDLs[2]						
Natural debt limit	55.69	57.18	55.69	56.17	54.50	54.90
Probability of repayment	93.30	93.13	93.30	93.25	93.44	93.39
Default risk premium	7.34	7.54	7.34	7.40	7.17	7.23
Implied min. non-int. outlays	9.49	14.63	16.62	14.19	23.13	13.27
relative to average outlays	-8.09	-9.99	0.63	-13.51	-6.47	-5.10
in number of std. devs.	4.52	2.77	-0.53	2.10	2.43	3.85

Notes: Calculations done as described in the text, using a risk free rate of 2.17 percent, which is the 1981-2005 average of the inflation-adjusted 90-day US T-bill rate, and a curvature parameter for the risk function of $a = 0.124$. The value of a was calibrated to match the EMBI+ spread and the debt ratios observed in Mexico in 1998.

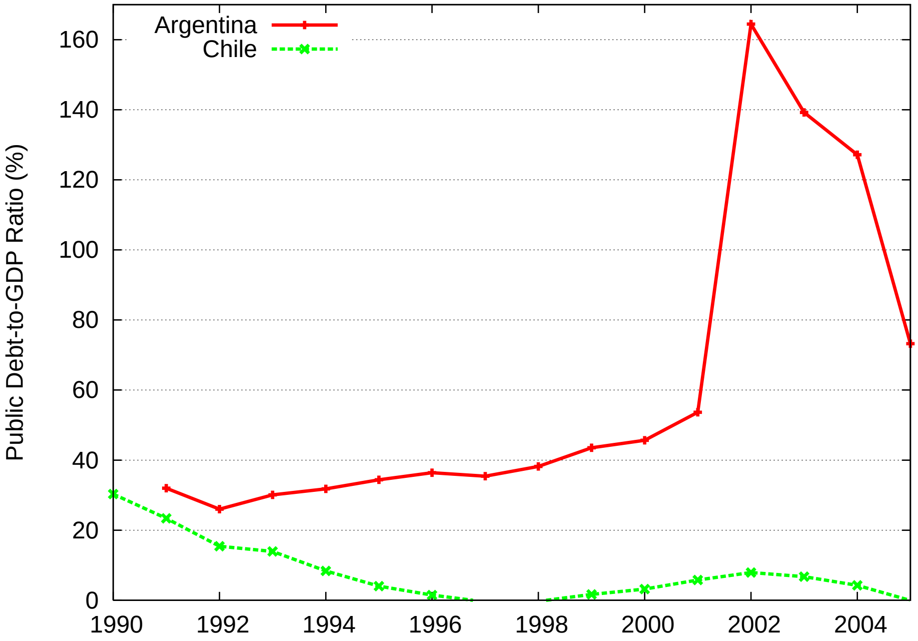
1. Based on the benchmark values of growth rates and minimum revenue and outlays shown in Table 1.

2. Values of minimum outlays required to support maximum debt ratios shown in Table 1 as NDLs in the setting with default risk, using growth rates from the benchmark scenario.

Figure 1: Public Debt-GDP Ratios in Seven Latin American Countries (In percentages))



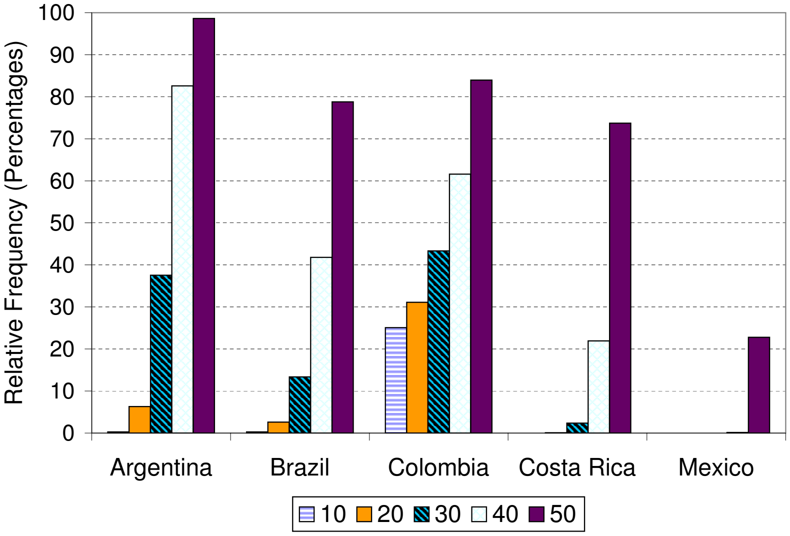
(a) Examined Countries



(b) Reference Countries

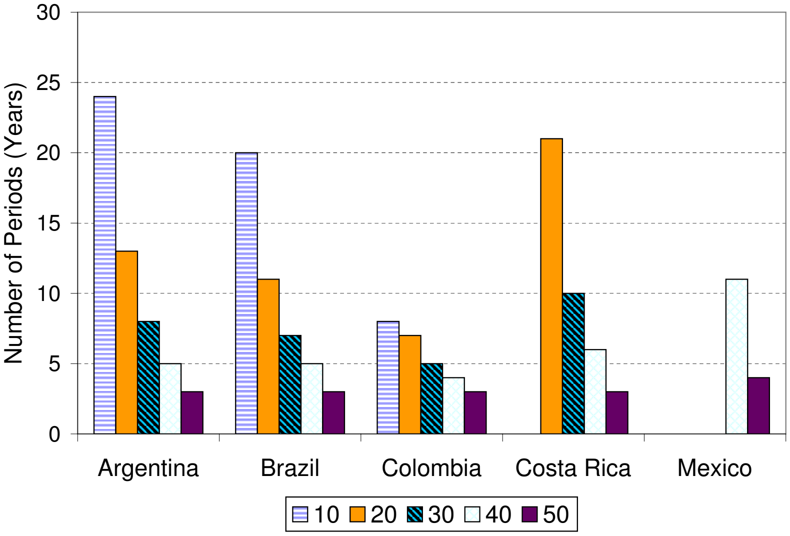
Source: see Appendix.

Figure 2: Relative Frequency of Fiscal Crises for Different Starting Values of the Public Debt-to-GDP Ratio



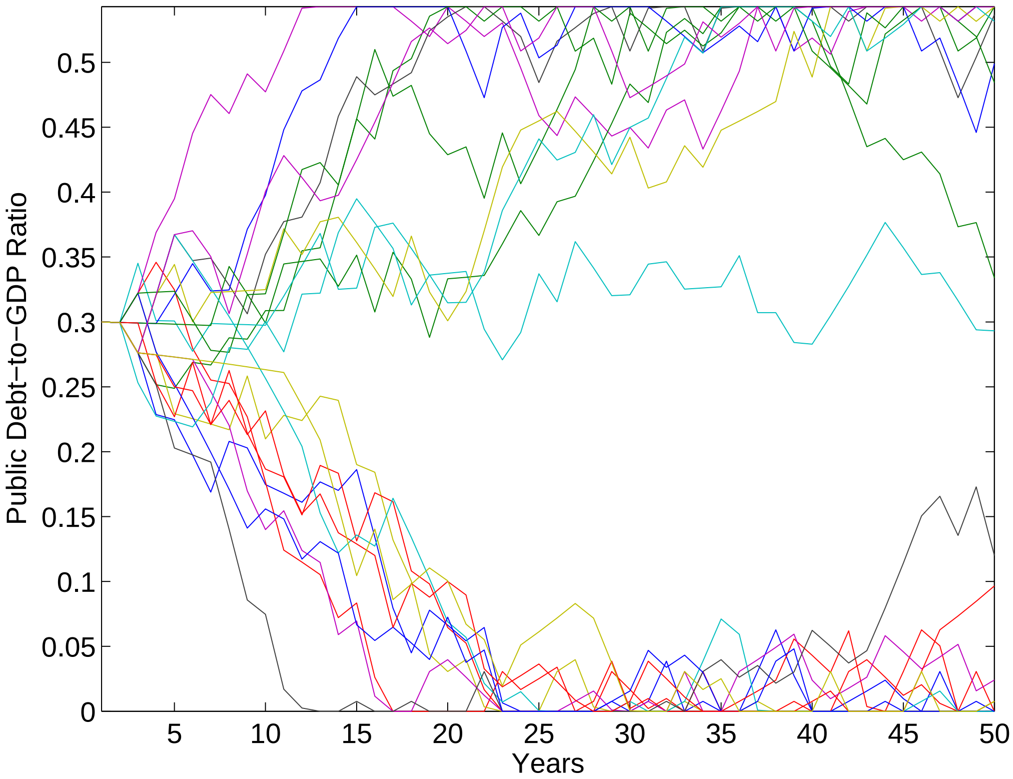
Notes: Results of 10,000 simulations of the basic MO model with country specific fiscal-revenue processes and (fixed) government expenditures. For each country there is a bar for each starting value of the debt ratio. The relative frequencies report the probabilities of hitting a fiscal crisis within the next 50 periods.

Figure 3: Extreme Adverse Scenarios of Fiscal Crises for Different Starting Values of the Public Debt-to-GDP Ratio



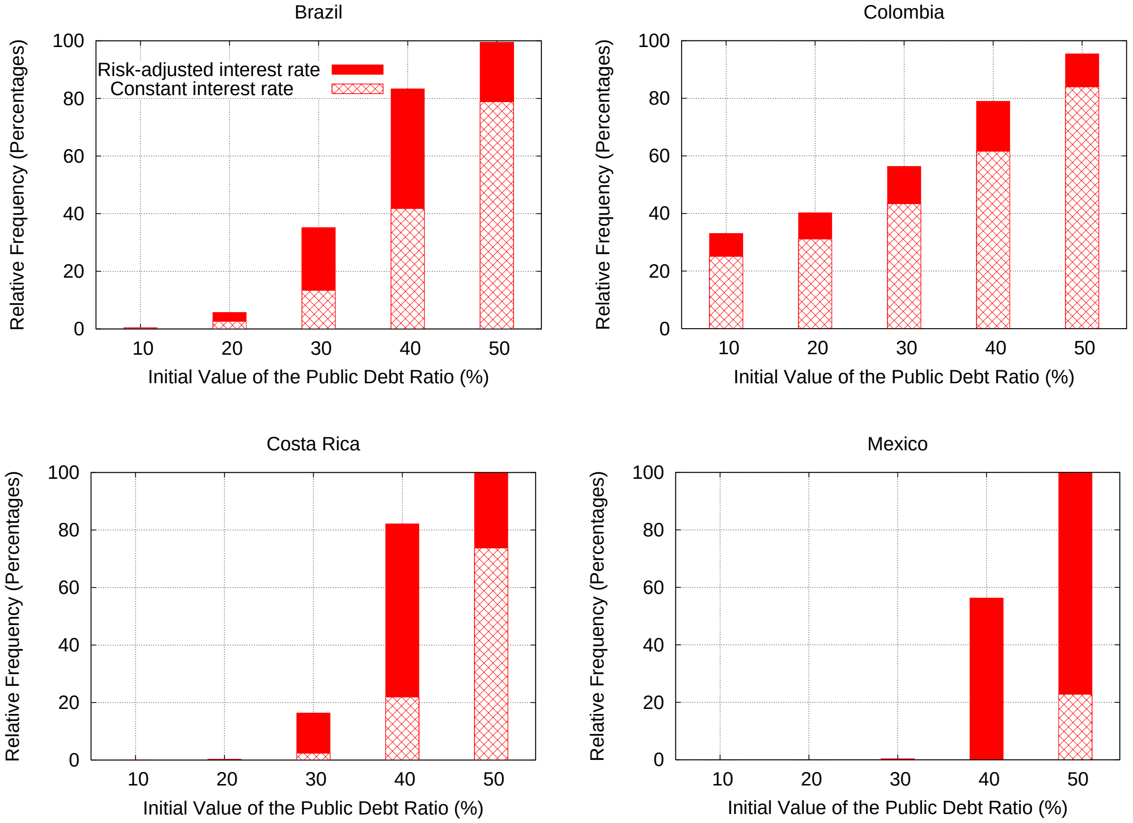
Notes: Results of 10,000 simulations of the basic MO model with country specific fiscal-revenue processes and (fixed) government expenditures. For each country, there is a bar for each starting value of the debt ratio that indicates the minimum number of periods that could take to hit a fiscal crisis in the most adverse scenario.

Figure 4: Simulations of Debt-to-GDP Ratios in Argentina with a Starting Ratio of 30 Percent



Notes: Results of 20 simulations of the basic MO model calibrated to Argentina. The starting debt ratio in all simulations at $t = 1$ is 30 percent. From time $t = 2$ on, random draws of tax revenues drive the dynamics of the debt ratio according to the government's budget constraint and the ad-hoc decision rules of the model.

Figure 5: Extreme Adverse Scenarios of Fiscal Crises with and without Default Risk


Brazil

Colombia

Costa Rica

Mexico

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