# Economics 376
## Optional Problem Set

**Instructions:**

- If you complete this problem set, it will be worth 10% of grade. The weight on your exams (but not on the term paper, if you do it) will be reduced proportionally.
- This problem set is due on April 21, 2008, in class.
- Your answers must be your own work: collaboration is not allowed.

1. Recall that with two ethnic groups, the Index of Dissimilarity is the minimum proportion of either group that must change neighborhoods in order for each neighborhood to have the same ethnic distribution as the city as a whole.

   (a) Compute the Index of Dissimilarity for the following city in two ways: once by moving blacks and once by moving whites. Confirm that both ways give the same answer. Show your work.

   | | City X | | |
   |---|---|---|---|
   | | Nbhd A | Nbhd B | Nbhd C |
   | Blacks | 10 | 20 | 90 |
   | Whites | 900 | 200 | 100 |

   (b) Now suppose that the number of blacks in each neighborhood is multiplied by 10 while the number of whites is unchanged:

   | | City X | | |
   |---|---|---|---|
   | | Nbhd A | Nbhd B | Nbhd C |
   | Blacks | 100 | 200 | 900 |
   | Whites | 900 | 200 | 100 |

   What is the Index of Dissimilarity now? Explain the intuition.

   (c) Suppose now that the size of neighborhood B quadruples while the other neighborhoods remain unchanged:

| | City X | | |
|---|---|---|---|
| | Nbhd A | Nbhd B | Nbhd C |
| Blacks | 100 | 800 | 900 |
| Whites | 900 | 800 | 100 |

What is the Index of Dissimilarity now? Explain the intuition

2. Segregation by Income. In class we defined segregation by income as follows. Let there be $H$ households, $h = 1, ..., H$. Let the income of household $h$ be $x_h$. Let $\overline{x}_{\text{City}}$ be the average household income in the city, and let $\overline{x}_{h\text{'s nbhd}}$ be the average income in household $h$'s neighborhood. Define the following types of variances:

$$
\begin{aligned}
V_{\text{Total}} &= \frac{1}{H}\sum_{h=1}^{H}(x_h - \overline{x}_{\text{City}})^2 \text{ (the total variance of household incomes in the city)} \\
V_{\text{Between}} &= \frac{1}{H}\sum_{h=1}^{H}(\overline{x}_{h\text{'s nbhd}} - \overline{x}_{\text{City}})^2 \text{ (the between-neighborhood variance of incomes)} \\
V_{\text{Within}} &= \frac{1}{H}\sum_{h=1}^{H}(x_h - \overline{x}_{h\text{'s nbhd}})^2 \text{ (the within-neighborhood variance of incomes)}
\end{aligned}
$$

We defined the income segregation index to be $S = V_{\text{Between}}/V_{\text{Total}}$. Since total variance equals between-neighborhood variance plus within-neighborhood variance ($V_{\text{Total}} = V_{\text{Between}} + V_{\text{Within}}$), this index equals the proportion of income variance that is due to variance between neighborhoods.

(a) Consider the following city: in neighborhood $A$, there are three households, with incomes 30, 50, and 100; in neighborhood $B$, there are three households, with incomes 20, 100, and 150. For this city, compute $V_{\text{Total}}$, $V_{\text{Between}}$, $V_{\text{Within}}$, and the segregation index $S$.

(b) Now suppose that incomes in neighborhood $B$ all double while incomes in neighborhood $A$ remain the same. That is, in $A$ there are now three households with incomes 30, 50, and 100, while in $B$ there are three households with incomes 40, 200, and 300. Once again, compute $V_{\text{Total}}$, $V_{\text{Between}}$, $V_{\text{Within}}$, and the segregation index $S$. What happens to income segregation in the city? Explain the intuition.

(c) Finally, suppose that the income of each household in the city doubles. That is, in $A$ there are now three households with incomes 60, 100, and 200, while in $B$ there are three households with incomes 40, 200, and 300. Once again, compute $V_{\text{Total}}$, $V_{\text{Between}}$, $V_{\text{Within}}$, and the segregation index $S$. What happens to income segregation in the city? Explain, intuitively, why this result differs from (b).

3. A jurisdiction has five voters, numbered 0 through 4. Each voter $n = 0, 1, 2, 3, 4$ wants to spend exactly $400n + 100$ dollars per pupil on the schools in the community. (For instance, voter 0 wants to spend $400 * 0 + 100 = \$100$ per pupil; voter 1 wants to spend $400 * 1 + 100 = \$500$ per pupil; voter 2 wants to spend $400 * 2 + 100 = \$900$ per pupil; and so on.)

   (a) According to the median voter theorem, how much will the community decide to spend per pupil?

   (b) If the assumptions of the Tiebout model held, so that voters could "vote with their feet," how would your answer change?

4. Consider a single jurisdiction with seven households. Four of them each have a marginal benefit from park acreage equal to $900 - 3A$, where $A$ is park acreage in the jurisdiction. The marginal benefit of each of the other three households equals $600 - 2A$. The price of an acre of parks is constant at \$1200.

   (a) What acreage is socially optimal?

   (b) Now suppose the park costs are divided equally among the 7 households and the number of acres is determined by the median voter rule. What acreage will be chosen?

5. The year is 1900 and would-be movie moguls are trying to decide where to locate the movie production industry. One possibility is Los Angeles, where sunny weather permits you to film outside all year around. Actors are willing to work for less if there are more potential employers, so the wage for actors in Los Angeles will be $W = 10 - 0.01N$, where $N$ is the number of movie companies there. Each movie company produces a single movie and its total cost is just the wage, $W$. Movies are sold to distribution companies whose demand curve for movies is $P = 40 - 0.11N$ (since $N$, the number of companies in L.A., also equals the number of movies). A movie company's profit equals $P - W$.

   (a) Suppose there are 100 movie companies in L.A. ($N = 100$). What is the wage $W$ of an actor?

   (b) Suppose movie companies enter Los Angeles until their profits are zero. What are $N$ and $W$ now?

6. Consider a model in which agents live evenly spaced along an endless (or at least very long) street. The number of agents per mile is 500. If an agent consumes $C$ units of food, her utility is $U(C) = C$. There are two ways to produce food: cottage production and large-scale manufacturing. With cottage production, an agent makes a unit of food at home at zero cost. With large scale manufacturing, food is produced in a single factory located somewhere on the street. (You can

assume the factory is very far from either end of the street, so that from the factory's point of view, the street extends endlessly in both directions.) If $N$ agents work at the factory, the factory can produce $4N$ units of food. However, travelling to the factory is costly. For a worker located $x$ miles from the factory, the total cost of travelling back and forth to work at the factory is $x$. Normalize the price of factory food to 1. Then if a worker located $x$ miles from the factory chooses to work there at the wage $W$, her consumption is just her wage less her commuting costs: $C = W - x$. If the worker works at home, her consumption is $C = 1$.

(a) Suppose the factory offers the wage $W$ per worker. What is the distance $x^*$ at which an agent is indifferent between working at the factory and producing at home? Given this, how many agents are willing to work at the factory if the wage is $W$?

(b) Use your answer to (a) to compute the factory's profit-maximizing wage. You might use the fact that the value of $W$ that maximizes the formula $aW - bW^2$ is $W = a/2b$.

(c) Now compute the workers' bid-rent as a function of their distance from the factory, assuming that rent is zero at a distance greater than $x^*$.