Econ 302

Math Review

Function: relationship among two or more variables expressed in mathematical language

Y = F (X, Z, W)   Y = F (X)

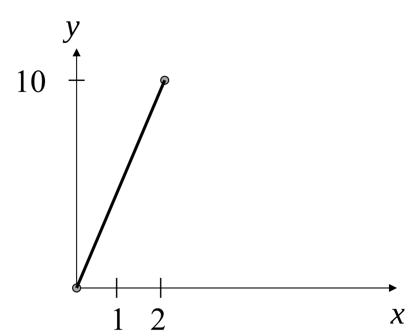
Y = 5 X

How to plot this?

(0,0)

X = 2   Y = 10

Are (2,5) (3,4) (1,3) (5,25) on this line?



Linear functions: straight lines

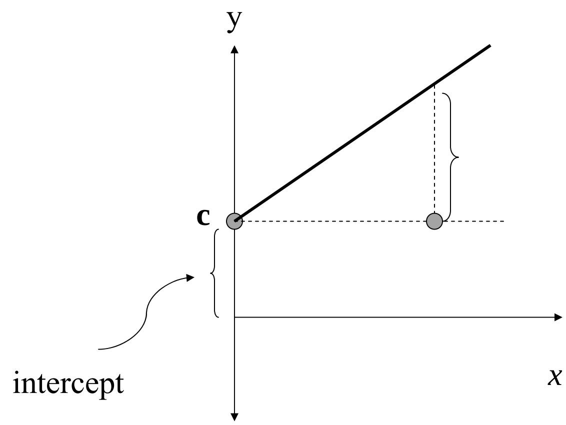
General form of a linear function:

$y = f(x) = \mathbf{m}\, x + \mathbf{c}$

$x = 0 \quad y = \mathbf{c}$

m is slope

By how much does *y* go up when *x* goes up by 1 unit?

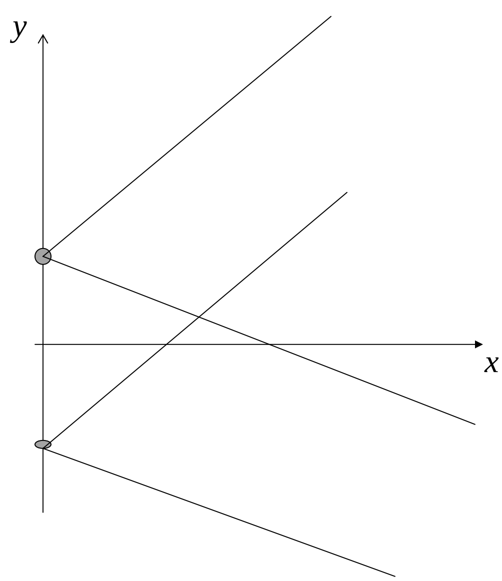


Positive and negative sloped lines

Tiredness vs exercise

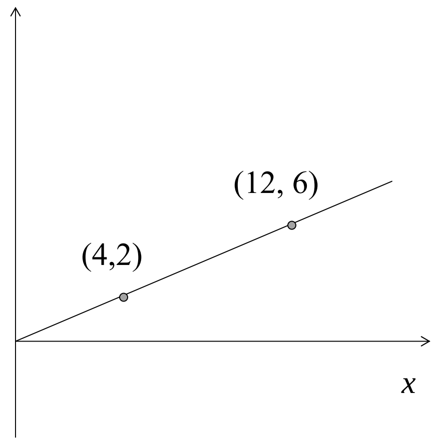
Interest rates vs time

Room temp on really cold day in Iowa vs # of hours the central heat runs



y = happiness

x = hours spent playing video games



Slope = 2/4 = 0.5

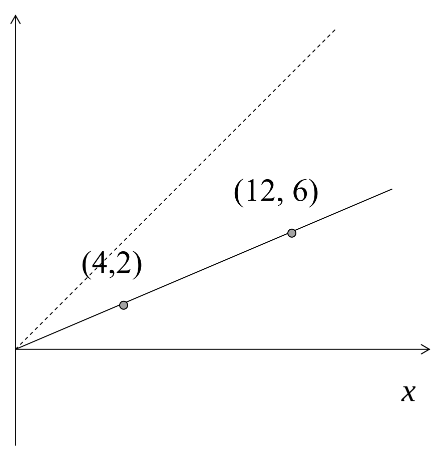
Increase in hours spent playing video games increases happiness but *less* than proportionately

Equation depicting this relationship: $y = 0.5\,x$

What if the relationship was proportional?

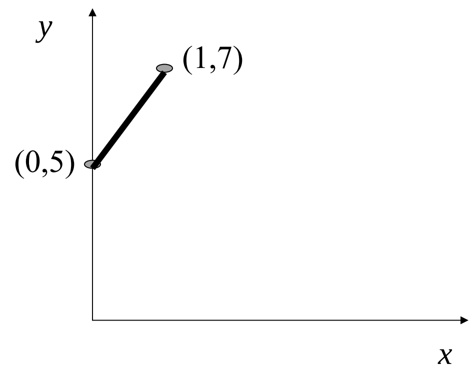
$y = x$

More than proportional?



Example

$y = 2\,x + 5$



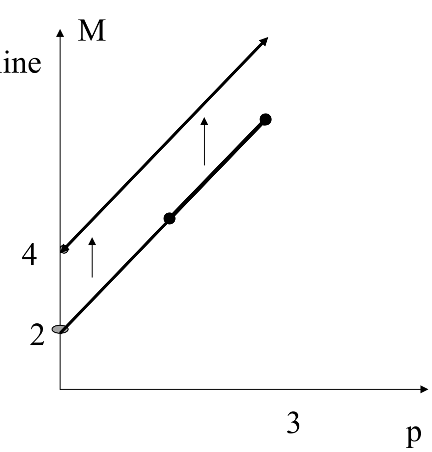
Compare to $y = \mathbf{m}\,x + \mathbf{c}$

Slope = 2

Intercept = 5

Two points is enough: ($x = 0$, $y = 5$) ($x = 1$, $y = 7$)

Shift versus movement along a line



M = 2 p + 2 **I**

M, p : variables

Start with **I** = 1

(p = 0, M = 2), (p = 3, M = 8)

Movement along a line

When p goes down from 3 to 2, M goes down from 8 to 6

When **I** goes up from 1 to 2

(p = 0, M = 4), (p = 3, M = 10) shift

Trickier Example

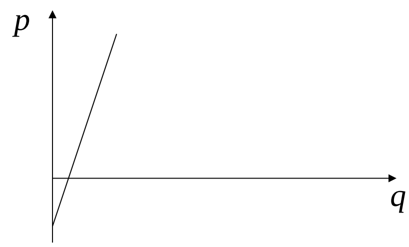
$p = -5 + 100q$

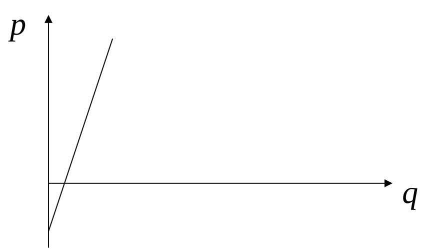
$p = 100q - 5$

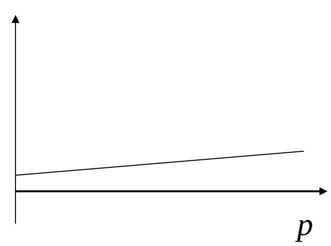
Slope = 100

Compare to $y = \mathbf{m}\, x + \mathbf{c}$

Intercept = -5







What if we wanted to draw *p* on the horizontal axis?

$p = -5 + 100q$

Rewrite as $100q = p + 5$

Or $q = \frac{p+5}{100}$

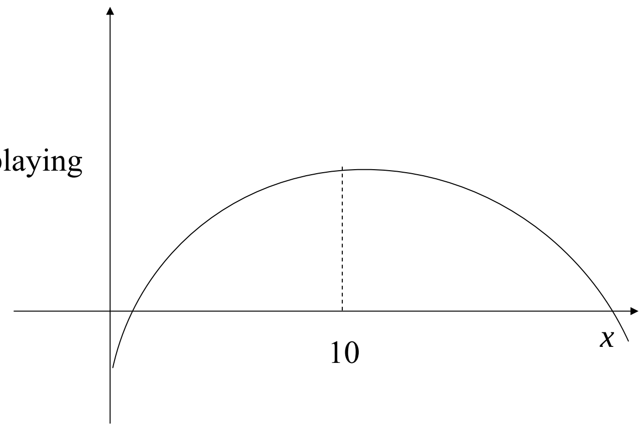
$q = p/100 + 5/100$

Slope = 1/100

Intercept = 5/100

y = happiness

x = hours spent playing video games



Non-linear functions

Pleasure from backrubs vs. time

$y = x^2$

$y = 5x^{0.5}$

$y = 2 - x^2$

$y = 3 + x^{25}$

Always a power different from 1; compare to $y = 5x + 12$

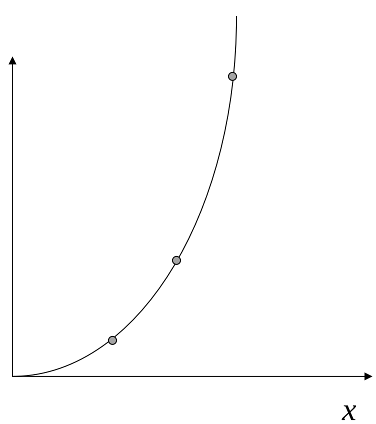
$y = x^2$

$x = 0, y = 0$

$x = 1, y = 1$

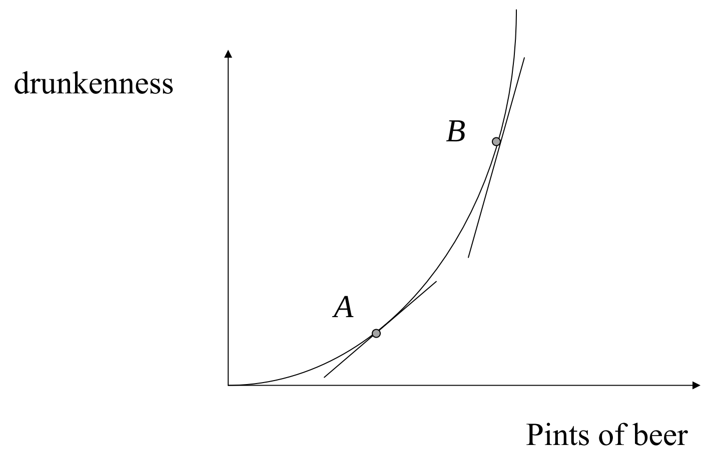
$x = 2, y = 4$

$x = 3, y = 9$

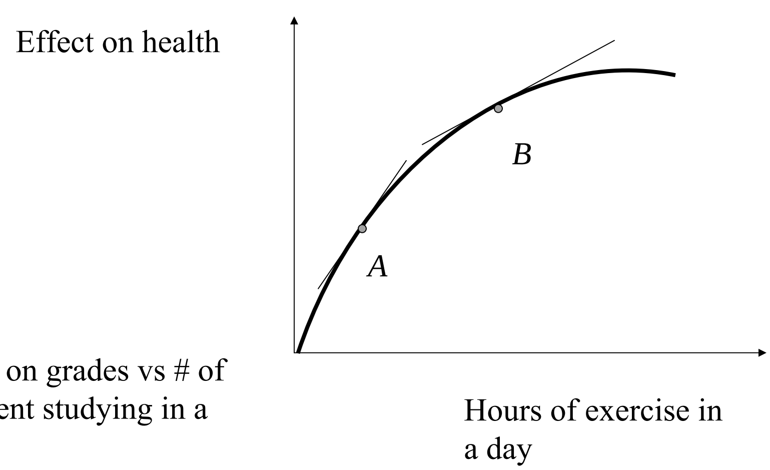


Slope keeps changing

Two points no longer enough



# of days with in-laws and my irritability



Effect on grades vs # of hrs spent studying in a night

Your height vs your age

$y = \mathbf{a} + \mathbf{b}\, x^{\alpha}$ **a**, **b** are constants

α is a exponent

Formula for slope (derivative)

$$\frac{dy}{dx} = b\alpha\, x^{\alpha - 1}$$

$y = x^2$

$y = \mathbf{a} + \mathbf{b}\, x^{\alpha}$

a = 0 **b** = 1

$\alpha = 2$

$$\frac{dy}{dx} = 2x$$

x = 0, y = 0

x = 1, y = 1

x = 2, y = 4

x = 3, y = 9

Solving equations in 1 unknown variable:

y = 2x -5

2x = y + 5

x = (y + 5)/ 2

$y = 25 – 3\, x^2$

$3x^2 = 25 - y$

$x^2 = (25 – y)/ 3$

$$x = \sqrt{\left(\frac{25 - y}{3}\right)}$$

Solving two equations in two unknowns

$3y + 5x = 15$ $\qquad$ $2y + 3x = 10$

$5x = 15 - 3y$

$x = (15 - 3y)/5$

$x = 0$

$2y + 3\,(15 - 3y)/5 = 10$

$$2y + \frac{3}{5}(15 - 3y) = 10$$

$$2y + \frac{45}{5} - \frac{9y}{5} = 10$$

$$2y - \frac{9y}{5} = 10 - 9$$

$$\frac{10y - 9y}{5} = 1$$

$$y = 5$$

Exponents and powers

$2 \times 2 = 2^2$

$a \times a = a^2$

$$\frac{1}{2} = 2^{-1}$$

$$\frac{a}{b} = ab^{-1}$$

$$\frac{2x}{3y^2} = \frac{2}{3}\left(xy^{-2}\right)$$

$$\sqrt{2} = 2^{0.5} = 2^{\frac{1}{2}}$$

$$\sqrt{\frac{wy^2}{2q}} = \left(\frac{wy^2}{2q}\right)^{0.5} = \left(0.5wy^2q^{-1}\right)^{0.5}$$

$(x^2)^{0.5} = (x)^{2\times 0.5}$

$= x$

$(a^b)^c = (a)^{b\times c}$

Solving non-linear equations in 1 variable

$3x^2 = 5$ ⟶ $x^2 = 5/3$ ⟶ (Raise both to power 1/2) $x = \sqrt{(5/3)}$

$4q^2 = 5k$ ⟶ $q^2 = (5/4)\,k$ ⟶ $q = \sqrt{(\,(5/4)\,k)}$

$3k^{0.3} = 5$ ⟶ $k^{0.3} = (5/3)$ ⟶

Raise both to power 1/0.3

$k = (5/3) \wedge (1/0.3)$

Example:

$$2 \times k^{\alpha} = 3k$$

$$\frac{k^{\alpha}}{k} = \frac{3}{2}$$

$$k^{\alpha - 1} = \frac{3}{2}$$

$$(k)^{\frac{\alpha - 1}{\alpha - 1}} = \left(\frac{3}{2}\right)^{\frac{1}{\alpha - 1}}$$

$$k = \left(\frac{3}{2}\right)^{\frac{1}{\alpha - 1}}$$