

Money and the Overlapping Generations Model

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What is money?

- barter economies
- the lack of double coincidence of wants problem
- money avoids this problem of *lack* of double coincidence of wants
- money refers to any asset that is widely used and accepted as a form of payment
- characteristics of money

Types of money

- fiat vs. commodity money
- historically, money has been metallic (gold or silver)
- last 200 years, paper money has slowly become the most important form of money
- Why? Gresham's Law

Functions of money

■ Functions of Money

- 1 medium of exchange
- 2 unit of account
- 3 store of value; assets like stocks, bonds are all stores of value; higher rates than money.

OG model – demographic structure

- overlapping-generations model: a simple model of money where there is a lack of double coincidence of wants problem in the *time* dimension
- $t = 1, 2, \dots, \infty$; basic demographic structure: two-period lived agents; generation t , G_t is born at t and lives till the end of $t + 1$; overlaps with G_{t+1} who are born at the start of $t + 1$ and live till the end of period $t + 2$; know for sure when they will die; all identical carbon copies
- initial old (Adam & Eve)
- population growth: in each period t , N_t young people are born who then live for a period alongside N_{t-1} old people, where
$$N_t = (1 + n)N_{t-1}.$$

Preliminaries

- endowments: “manna” from heaven, strictly positive, private ownership economy; each young agent gets $w_1 > 0$ and *nothing* when old
- utility $U(c_{1t}^t, c_{2,t+1}^t)$; consumption smoothing motive built in
- assume good is **perishable** and no savings instruments
- where can the old get income?
- maybe trade with others? no trade is possible with fellow old people; no trade is possible with young.

Autarky

- Autarky: no trade equilibrium. People eat their endowment when young and starve when old; trade cannot take place; old have nothing to offer plus (current old will not be around when today's young become old tomorrow);
- lack of double coincidence of wants
- if utility function is given by

$$U(c_1, c_2) = \log c_1 + \log c_2$$

then what is utility under autarky?

- What can a social planner do? He can take all the goods in the economy and allocate it to the old and young, hoping to make everyone's life better off.

Planning problem

- Total supply of the good in the economy at date t :
- Total demand for the good at t is:
- Economy-wide resource constraint:

$$c_1 + \frac{c_2}{1+n} = w_1$$

- In a steady state, planner's problem:

$$\max U(c_1, c_2) = \log c_1 + \log c_2$$

subject to resource constraint.

Planning problem

- Picture of constraint.
- “biological interest rate” $1 + n$
- solution to planner’s problem:

$$c_1^{SP} = \frac{w_1}{2} > 0, \quad c_2^{SP} = \frac{w_1(1+n)}{2} > 0$$

- can social planner “do better” than autarky?

$$\log c_1^{SP} + \log c_2^{SP} > -\infty$$

Intuition

- What is the planner doing? endowment pattern is $(w_1, 0)$; he takes $1/2$ of young-age endowment of the initial young and gives it to the initial old. This is repeated every period till ∞ .
- Initial old is better off than before. Why? Is every generation now better off than autarky?
- Gamow $1, 2, 3, \dots, \infty$ hotel-bed problem.
- Do we need a social planner to get us this good outcome? can the market deliver this nice outcome?

Social contrivance of money

Paul Samuelson's "social contrivance of money": suppose the initial old invents an object they call money; not valued for its own sake but only if other individuals believe that it has value. Then, by passing this piece of chocolate wrapper down as money, the initial old can convince the young to give up $1/2$ to them in exchange for this chocolate wrapper. If the young believe that the young tomorrow will do the same, then they will gladly give up $1/2$ to the old and will in turn get $1/2$ from tomorrow's young. Then money will have value.

Money

- we need to assume that the economy goes on forever.
- intergenerational trust – social contract
- Fiat money: *intrinsically worthless*
- fiat money opens up trading possibilities: young can sell part of their endowment for money and next period buy some endowment from the future young with that money

Prices

- define prices: it costs 20 cents to buy an apple. Then $p_t = \$\frac{1}{5}$ is the price of an apple. Then, the *value* of 1\$ is five apples.
- $v_t = \frac{1}{p_t}$ is the value of \$1. If $v_t > 0$, then money has value. With one good, p_t is the price level in the economy.
- we begin by assuming that the initial old together have $\$M$.
- how much money will a young agent hold? back to budget constraints.

Agent's budget constraints

- Suppose he “buys” m_t \$. In exchange, he has to give up — amount of goods. Then,

$$c_1 + v_t m_t = w_1$$

- second period of his life, he gets no endowment; so all he has is the \$ bills. How much endowment (how many units of the good) can he buy with \$ m_t ? The value of his money holdings from when he was young is?

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$$c_2 = v_{t+1} m_t$$

Agent's budget constraints

- rewrite:

$$c_1 + \frac{m_t}{p_t} = w_1; \quad c_2 = \frac{m_t}{p_{t+1}}$$

$$m_t = \frac{w_1 - c_1}{v_t}$$

then,

$$c_2 = v_{t+1} \left(\frac{w_1 - c_1}{v_t} \right) = w_1 \left(\frac{v_{t+1}}{v_t} \right) - c_1 \left(\frac{v_{t+1}}{v_t} \right)$$

or,

$$c_1 + \left(\frac{v_t}{v_{t+1}} \right) c_2 = w_1$$

- picture; optimal combination (c_1^m, c_2^m)

Interpretation

- $\left(\frac{v_{t+1}}{v_t}\right)$ is the gross real return on fiat money because it expresses how many goods can be obtained in $t + 1$ if one unit of the good is sold for money at t .
- 1 unit of good sold at t earns \$ p_t . Next period, \$1 buys v_{t+1} goods, so \$ p_t buys $p_t v_{t+1}$ goods. Since $v_t = \frac{1}{p_t}$, $p_t v_{t+1} = v_{t+1}/v_t$. So 1 good invested in \$ today earns v_{t+1}/v_t goods tomorrow.
- $\left(\frac{v_{t+1}}{v_t}\right) = \frac{p_t}{p_{t+1}}$; connection with inflation
- how to compute $\left(\frac{v_{t+1}}{v_t}\right)$?
- Money market must clear. Money demand has to equal money supply.

Money demand = Money Supply

- money demand per young person is $\$m_t$. But,
 $c_1 + v_t m_t = w_1 \Rightarrow m_t = \frac{w_1 - c_1}{v_t}$. There are N_t young people. So total demand is $N_t \left(\frac{w_1 - c_1}{v_t} \right)$ and this should equal total number of dollar bills “out there” which is $\$M \Rightarrow$

$$N_t \left(\frac{w_1 - c_1}{v_t} \right) = M$$

- $$v_t = \frac{N_t (w_1 - c_1)}{M}; \quad v_{t+1} = \frac{N_{t+1} (w_1 - c_1)}{M}$$

then,

$$\frac{v_{t+1}}{v_t} = (1 + n) \quad (1)$$

Real money demand

- now compute money demand and consumptions:

$$\max U = \log c_1 + \log c_2$$

subject to

$$c_1 + v_t m_t = w_1$$

$$c_2 = v_{t+1} m_t$$

- $$\max_{m_t} \log (w_1 - v_t m_t) + \log (v_{t+1} m_t)$$

- real money demand

$$v_t m_t = \frac{w_1}{2}$$

Consumptions

- what will consumptions in the two periods be? recall

$$c_1^m = w_1 - v_t m_t = \frac{w_1}{2}$$

$$c_2^m = v_{t+1} m_t = v_{t+1} \frac{w_1}{2} \frac{1}{v_t} = \frac{w_1}{2} \frac{v_{t+1}}{v_t} = \frac{w_1}{2} (1 + n)$$

using equation (1).

Money vs. autarky vs. planner

- Do people get higher utility with money than with autarky?
- Are people happier with what the social planner gave them over what they could do on their own with money?

$$c_1^{SP} = \frac{w_1}{2} > 0$$

$$c_2^{SP} = \frac{w_1(1+n)}{2} > 0$$

$$c_1^m = \frac{w_1}{2}$$

$$c_2^m = v_{t+1}m_t = v_{t+1}\frac{w_1}{2}\frac{1}{v_t} = \frac{w_1}{2}\frac{v_{t+1}}{v_t} = \frac{w_1}{2}(1+n)$$

money can replicate what the social planner can do!