

1. A family with wealth W solves the problem

$$\max_{c,b} u = c^{0.5}b^{0.5}$$

s.t

$$c + b = W$$

The solution to this problem (see lecture notes) is

$$b^* = 0.5W$$

For this problem, $\varepsilon = \alpha^\alpha (1 - \alpha)^{1-\alpha} = 0.5$. So, utility is given by $0.5W$.

2. The Larsen child starts off life with an inheritance of 2 units of corn. This means that if she were to consider getting an education, she would have to borrow to finance it. Her lifetime utility from this choice will be

$$U_s^b(x) = 0.5 [(x - h)(1 + i) + w_s] = 0.5 [(2 - 4.8)(1 + 0.1) + 9] = 2.96$$

If she considers not going to school and working at the Open Flame, her lifetime utility will be

$$U_n(x) = \varepsilon [(x + w_n)(1 + r) + w_n] = 0.5 [(2 + 2)(1 + 0.02) + 2] = 3.04$$

Since $U_s^b(x) < U_n(x)$ holds, then she cannot afford to go to Gilbert High. No education for her!

3. By a very similar logic as above, you can check that the Johansen child (with inheritance 4.1) will choose education, because for her, $U_s^b(x) > U_n(x)$ holds.
4. When will *all* families prefer to let their kids remain uneducated? When returns to not getting an education exceed the maximum return to getting one¹, i.e.,

$$[(x + w_n)(1 + r) + w_n] > [(x - h)(1 + r) + w_s]$$

or,

$$(2 + r)w_n > w_s - h(1 + r)$$

here, $r = 0.02$, $w_n = 2$, $w_s = 9$. For these numbers, if $h = 5$ (in fact any number bigger than 4.9) no family will send their children to Gilbert High.

¹recall that the utility to getting an education is highest when a person does not have to borrow to get an education, i.e., all those with inheritance $x > h$.

5. From part (3), it follows that Johansen's child will go to school. Her lifetime wealth will be

$$W = (x - h)(1 + i) + w_s = (4.1 - 4.8)(1 + 0.1) + 9 = 8.246.$$

The Johansens' grandchild will then start off life with an inheritance of $0.5 * 8.23 = 4.115$. This means that if she were to consider getting an education, she would have to borrow to finance it. Her lifetime utility from this choice will be

$$U_s^b(x) = \varepsilon [(x - h)(1 + i) + w_s] = 0.5 [(4.115 - 4.8)(1 + 0.1) + 9] = 4.123$$

If she considers not going to school and working at the Open Flame, her lifetime utility will be

$$U_n(x) = \varepsilon [(x + w_n)(1 + r) + w_n] = 0.5 [(4.115 + 2)(1 + 0.02) + 2] = 4.1187$$

Since $U_s^b(x) > U_n(x)$ holds, she can afford to go to Gilbert High. But, using the same logic, you can check that her daughter her start off her life with an inheritance of $\frac{8.246}{2} = 4.123$, and will be able to afford school, and so will her kid.....

6. The Hansen child starts off life with an inheritance of 3.9 units of corn. This means that if she were to consider getting an education, she would have to borrow to finance it. Her lifetime utility from this choice will be

$$U_s^b(x) = 1.412 [(x - h)(1 + i) + w_s] = 0.5 [(3.9 - 4.8)(1 + 0.1) + 9] = 4.005$$

If she considers not going to school and working at the Open Flame, her lifetime utility will be

$$U_n(x) = \varepsilon [(x + w_n)(1 + r) + w_n] = 0.5 [(3.9 + 2)(1 + 0.02) + 2] = 4.009$$

Since $U_s^b(x) < U_n(x)$ holds, then she cannot afford to go to Gilbert High. No education for her! But check for yourself that her child goes to school.