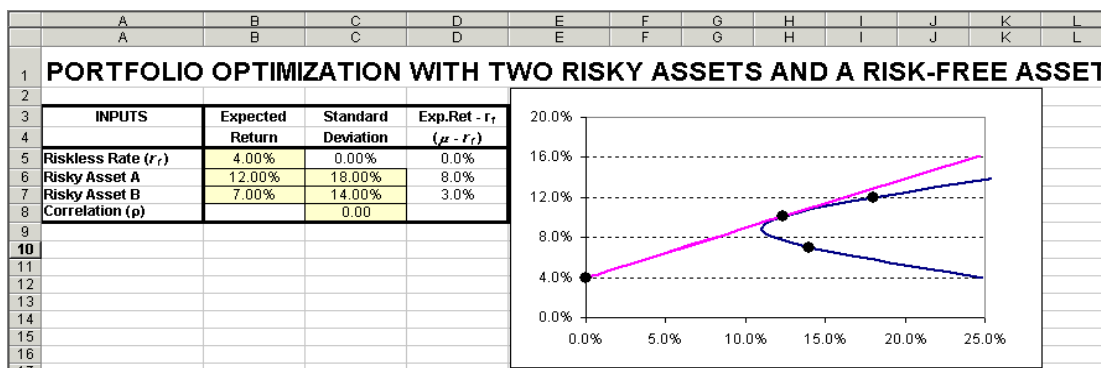


**ECON 653 - HOMEWORK 2**  
(Due: September 22, 2009)

1. Portfolio Analysis: Portfolio problems are typically solved using C++, Matlab, or specialized software. However, given that the main focus of this section of the homework is to help visualize some of the standard results in portfolio analysis, you are asked to use Excel to solve it. Please submit your responses to the portfolio problems as spreadsheets attached to e-mails.  
Note: The instructions below are edited from Holden, C.W., "Spreadsheet Modeling," available from website <http://www.spreadsheetmodeling.com/>.
- 1.1. Graph the efficient trade-off line and the risky asset trade-off curve, assuming that there is a risk-free asset with a net rate of return  $r_f = 4\%$ , and two risky assets A and B with mean net rates of return  $\mu_A = 12\%$  and  $\mu_B = 7\%$ , standard deviations  $\sigma_A = 18\%$  and  $\sigma_B = 14\%$ , and correlation coefficient  $\rho = 0$ . To do this, proceed as follows:
  - a. **Inputs:** Enter the inputs described above into the ranges **B5:B7** and **C6:C8**.



- b. **Expected Return – Riskless Rate:** Calculate the Expected Return minus the Riskless Rate by entering **=B5-\$B\$5** in cell **D5** and copying that cell to the range **D6:D7**.
- c. **Proportion in Risky Asset A:** To graph the Risky Asset Trade-off Curve, we need to evaluate a wide range of values (–60.0% to 140%) for the Proportion in Risky Asset A. Enter **–60.0%** in cell **B23**, **–50.0%** in cell **B24**, and highlight the range **B23:B24**. Then hover the cursor over the lower right corner and it turns to a “fill handle” (which looks like a “+” sign). Drag the fill handle down to **B43**.
- d. **Standard Deviation:** The horizontal axis of our graph is the portfolio’s standard deviation, which is calculated by the formula  $\sigma = [w^2 \sigma_A^2 + (1 - w)^2 \sigma_B^2 + 2 w (1 - w) \rho \sigma_A \sigma_B]^{0.5}$ . Enter **=SQRT(B23^2\*\$C\$6^2+(1-B23)^2\*\$C\$7^2+2\*B23\*(1-B23)\*\$C\$8\*\$C\$6\*\$C\$7)** in cell **C23** and copy the cell to the range **C24:C43**.
- e. **Expected Return:** The formula for a portfolio’s expected return is  $\mu = w \mu_A + (1 - w) \mu_B$ . Enter **=B23\*\$B\$6+(1-B23)\*\$B\$7** in cell **D23** and copy the cell to the range **D24:D43**.
- f. **Optimal Combination of Risky Assets:** Using the notation  $E_A \equiv \mu_A - r_f$  and  $E_B \equiv \mu_B - r_f$ , the formula for the optimal proportion in asset A is  $w = (E_A \sigma_B^2 - E_B \rho \sigma_A \sigma_B) / [E_A \sigma_B^2 + E_B \sigma_A^2 - (E_A + E_B) \rho \sigma_A \sigma_B]$ . In cell **B44**, enter **=(D6\*C7^2-D7\*C8\*C6\*C7)/(D6\*C7^2+D7\*C6^2-(D6+D7)\*C8\*C6\*C7)**
- g. Calculate the corresponding Mean and Standard Deviation by copying the range **C43:D43** to the range **C44:D44**. We want to create a separate column for the Efficient Trade-Off Line, so move the formula in cell **D44** to cell **E44**. To do this, select cell **D44**, click on **Edit Cut**, select cell **E44**, and click on **Edit Paste**.
- h. **Efficient Trade-Off Line:** The Efficient Trade-Off Line is a combination of the Riskless Asset and the Risky Asset Optimal Combination. It can be calculated as follows:
  - Enter **0.0%** in cell **B45**, **100.0%** in cell **B46**, and **200.0%** in cell **B47**. Since the Riskless Asset has a standard deviation of zero, the standard deviation formula simplifies to  $\sigma = w \sigma_T$ , where

$\sigma_T$  is the standard deviation of the Optimal Combination of Risky Assets (or Tangent Portfolio). Enter  $=B45*C\$44$  in cell **C45** and copy the cell to the range **C46:C47**.

- The Expected Return formula for the portfolio including the riskless asset is  $\mu_P = w_P \mu_T + (1 - w_P) r_f$ , where  $\mu_T$  is the expected return of the Tangent Portfolio and  $w_P$  is the proportion invested in the Tangent Portfolio. Enter  $=E\$44*B45+\$B\$5*(1-B45)$  in cell **E45** and copy to the range **E46:E47**.

	A	B	C	D	E
18	<b>OUTPUTS</b>			<b>Risk-Return</b>	<b>Efficient</b>
19		<b>Proportion in</b>		<b>Trade-off</b>	<b>Trade-off</b>
20		<b>Risky Asset A</b>	<b>(x-axis)</b>	<b>Curve</b>	<b>Line</b>
21		<b>or Optimal</b>	<b>Standard</b>	<b>Expected</b>	<b>Expected</b>
22		<b>Combination</b>	<b>Deviation</b>	<b>Return</b>	<b>Return</b>
23	Trade-off Curve	-60.0%	24.9%	4.0%	
24	Trade-off Curve	-50.0%	22.8%	4.5%	
25	Trade-off Curve	-40.0%	20.9%	5.0%	
26	Trade-off Curve	-30.0%	19.0%	5.5%	
27	Trade-off Curve	-20.0%	17.2%	6.0%	
28	Trade-off Curve	-10.0%	15.5%	6.5%	
29	Trade-off Curve	0.0%	14.0%	7.0%	
30	Trade-off Curve	10.0%	12.7%	7.5%	
31	Trade-off Curve	20.0%	11.8%	8.0%	
32	Trade-off Curve	30.0%	11.2%	8.5%	
33	Trade-off Curve	40.0%	11.1%	9.0%	
34	Trade-off Curve	50.0%	11.4%	9.5%	
35	Trade-off Curve	60.0%	12.2%	10.0%	
36	Trade-off Curve	70.0%	13.3%	10.5%	
37	Trade-off Curve	80.0%	14.7%	11.0%	
38	Trade-off Curve	90.0%	16.3%	11.5%	
39	Trade-off Curve	100.0%	18.0%	12.0%	
40	Trade-off Curve	110.0%	19.8%	12.5%	
41	Trade-off Curve	120.0%	21.8%	13.0%	
42	Trade-off Curve	130.0%	23.8%	13.5%	
43	Trade-off Curve	140.0%	25.8%	14.0%	
44	<b>Optimal Combination</b>	61.7%	12.3%		10.1%
45	<b>Eff Trade-Off Line</b>	0.0%	0.0%		4.0%
46	<b>Eff Trade-Off Line</b>	100.0%	12.3%		10.1%
47	<b>Eff Trade-Off Line</b>	200.0%	24.7%		16.2%

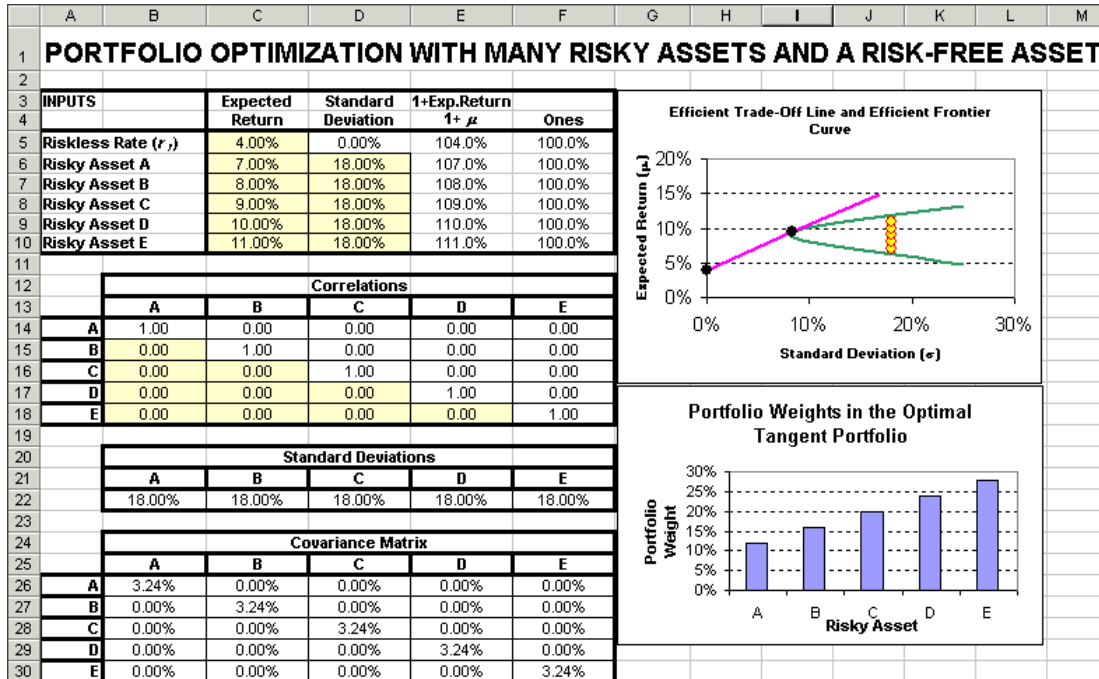
- Create the Graph:** Highlight the range **C23:E47** and then choose **Insert Chart** from the main menu. Select a **Scatter** chart type and make other selections to complete the Chart Wizard. Place the graph in the range **E2:K16**.
- (Optional) Formatting the Graph:** Here are some tips to make the chart look attractive:
  - Click on one of the Chart curves, then click on **Format Selection** in the **Current Selection** tab of the **Format** group. In the Format Data Series dialog box under the **Marker Options** tab, select **None** for the Marker and click on **Close**. Repeat for the other curve.
  - Highlight individual points, such as the Riskless Asset, Tangent Portfolio, and Risky Assets A and B, by clicking on a chart curve, then click a second time on an individual point (the four-way arrows symbol appears), then click on **Format Selection** in the **Current Selection** tab of the **Format** group. In the Format Data Series dialog box select a market Style, Foreground Color, Background Color, and increase the size to **8 pts** and click on **Close**.
  - Click on the x-axis, then click on **Format Selection** in the **Current Selection** tab of the **Format** group. In the Format Axis dialog box under the **Axis Options** tab, enter **0.25** for the Maximum and click on **Close**.

1.2. Use the graph constructed in Problem 1.1 to analyze the effect on (a) the efficient trade-off line, (b) the risky asset trade-off curve, and (c) the optimal proportion invested in asset A in the tangent portfolio in the following situations:

- The expected return of an individual risky asset is *reduced*.
- The standard deviation of an individual risky asset is *increased*.
- The correlation between the two risky assets is *increased*.
- The risk-free rate is reduced.

1.3. Graph the efficient trade-off line and the risky asset trade-off curve, assuming that there is a risk-free asset with a net rate of return  $r_f = 4\%$ , and five risky assets with mean net return vector  $\underline{\mu} = [7\% \ 8\% \ 9\% \ 10\% \ 11\%]$ , standard deviation vector  $\underline{\sigma} = [18\% \ 18\% \ 18\% \ 18\% \ 18\%]$ , and correlation matrix  $\underline{\rho} = [1 \ 0 \ 0 \ 0 \ 0, 0 \ 1 \ 0 \ 0 \ 0, 0 \ 0 \ 1 \ 0 \ 0, 0 \ 0 \ 0 \ 1 \ 0, 0 \ 0 \ 0 \ 0 \ 1]$ . To do this, proceed as follows:

- a. **Inputs:** Enter the expected return inputs into the range **C5:C10**, the standard deviation inputs into the range **D6:D10**, the correlation inputs in the triangular range from **B15 to B18 to E18**, and 0.0% in the cell **D5**.






- b. **One plus the Expected Return and 100%:** It will be useful to have a column based on one plus the expected return and another column with just the number 100%. Enter **=1+C5** in cell **E5** and copy the cell to the range **E6:E10**. Enter **100%** in cell **F6** and copy the cell to the range **F7:F10**
- c. **Fill Out the Correlation Matrix:** The correlation matrix from **B14:F18** has a simple structure. All of the elements on the diagonal represent the correlation of an asset return with itself. For example, **B14** is the correlation of the Asset A return with the Asset A return, which is one. **C15** is the correlation of Asset B with B, and so on. Enter **100.0%** into the diagonal cells from **B14 to F18**. The off-diagonal cells in the *upper* triangular range from **C14 to F14 to F17** are the “mirror image” of the *lower* triangular range from **B15 to B18 to E18**. In other words, the correlation of Asset B with Asset A in C14 is equal to the correlation of Asset A with Asset B in B15. Enter **=B15** in cell **C14**, **=B16** in **D14**, **=B17** in **E14**, etc. Each cell of the *upper* triangular range from **C14 to F14 to F17** should be set equal to its mirror image cell in the *lower* triangular range from **B15 to B18 to E18**
- d. **Transposed Standard Deviations:** In addition to the standard deviation input range which runs vertically from top to bottom, it will be useful to have a range of standard deviations that runs horizontally from left-to-right. This can be done easily by using one of Excel's **Matrix** commands to transpose a range. Highlight the range **B22:F22**. Then, type **=TRANSPOSE(D6:D10)**, hold down the **Shift** and **Control** buttons simultaneously, and while continuing to hold them down, press **Enter**. The resulting formula should have braces around it **{=TRANSPOSE(D6:D10)}**
- e. **Covariance Matrix:** The Covariance Matrix in the range **B26:F30** has a simple structure. All of the elements on the diagonal represent the covariance of an asset return with itself, which equals the variance. For example, **B26** is the covariance of the Asset A return with the Asset A return, which equals the variance of Asset A. **C27** is the variance of Asset B, and so on. The off-diagonal cells are covariances. For example, **C26** is the covariance of the Asset A return with the Asset B

return and is calculated with the formula for the Covariance( $A, B$ ) = Correlation( $A, B$ )  $\times$  (Std. Dev.  $A$ )  $\times$  (Std. Dev.  $B$ ). Enter **=C14\*C\$22\*\$D6** in cell **C26**. Be very careful to enter the \$ absolute references exactly right. Then copy **C26** to the range **B26:F30**

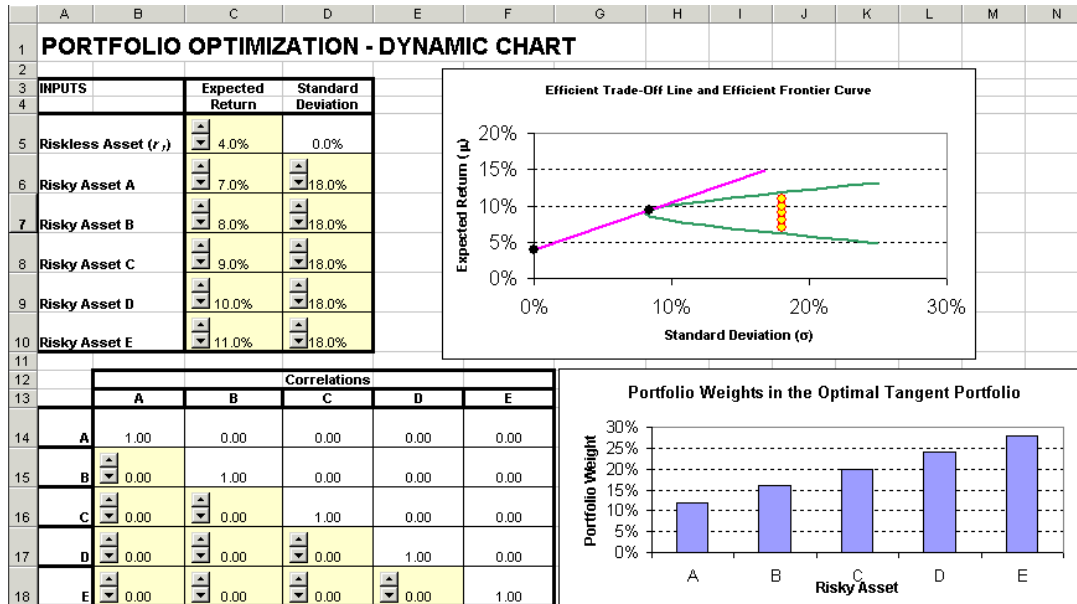
- f. **Hyperbola Coefficients:** In a Mean-Standard Deviation graph, the Efficient Frontier is a hyperbola. The exact location of the hyperbola is uniquely determined by three coefficients  $\alpha$ ,  $\theta$ , and  $\phi$ . They are easy to implement using Excel's **matrix** functions. In each case, you type the formula and then, hold down the **Shift** and **Control** buttons simultaneously, and while continuing to hold them down, press **Enter**
- For  $\alpha$ : **=MMULT(MMULT(TRANPOSE(F6:F10),MINVERSE(B26:F30)),F6:F10)** in cell **C37**
  - For  $\theta$ : **=MMULT(MMULT(TRANPOSE(F6:F10),MINVERSE(B26:F30)),E6:E10)** in cell **C38**
  - For  $\phi$ : **=MMULT(MMULT(TRANPOSE(E6:E10),MINVERSE(B26:F30)),E6:E10)** in cell **C39**

	A	B	C	D	E	F	G	H	I	J
36	<b>OUTPUTS</b>									
37		$\alpha$	154.321							
38		$\theta$	168.210							
39		$\phi$	183.380							
40		$\delta$	4.763							
41		$\gamma$	0.130							
42										
43			<b>Index</b>	<b>Standard Deviation</b>	<b>Expected Return</b>	<b>Efficient Frontier Curve</b>	<b>Efficient Trade-Off Line</b>	<b>Individual Asset Expected Return</b>	<b>Optimal Combination of Risky Assets (Tangent Portfolio)</b>	
44	<b>Risky Asset A</b>			18.00%				7.00%	<b>A</b>	12.00%
45	<b>Risky Asset B</b>			18.00%				8.00%	<b>B</b>	16.00%
46	<b>Risky Asset C</b>			18.00%				9.00%	<b>C</b>	20.00%
47	<b>Risky Asset D</b>			18.00%				10.00%	<b>D</b>	24.00%
48	<b>Risky Asset E</b>			18.00%				11.00%	<b>E</b>	28.00%
49	<b>Trade-off Curve</b>		0	25.00%	4.84%					
50	<b>Trade-off Curve</b>		1	22.77%	5.26%					
51	<b>Trade-off Curve</b>		2	20.57%	5.67%					
52	<b>Trade-off Curve</b>		3	18.42%	6.09%					
53	<b>Trade-off Curve</b>		4	16.32%	6.51%					
54	<b>Trade-off Curve</b>		5	14.31%	6.92%					
55	<b>Trade-off Curve</b>		6	12.43%	7.34%					
56	<b>Trade-off Curve</b>		7	10.73%	7.75%					
57	<b>Trade-off Curve</b>		8	9.34%	8.17%					
58	<b>Trade-off Curve</b>		9	8.39%	8.58%					
59	<b>Trade-off Curve</b>		10	8.05%	9.00%					
60	<b>Trade-off Curve</b>		11	8.39%	9.42%					
61	<b>Trade-off Curve</b>		12	9.34%	9.83%					
62	<b>Trade-off Curve</b>		13	10.73%	10.25%					
63	<b>Trade-off Curve</b>		14	12.43%	10.66%					
64	<b>Trade-off Curve</b>		15	14.31%	11.08%					
65	<b>Trade-off Curve</b>		16	16.32%	11.49%					
66	<b>Trade-off Curve</b>		17	18.42%	11.91%					
67	<b>Trade-off Curve</b>		18	20.57%	12.33%					
68	<b>Trade-off Curve</b>		19	22.77%	12.74%					
69	<b>Trade-off Curve</b>		20	25.00%	13.16%					
70	<b>Optimal Combination</b>			8.4%			9.4%			
71	<b>Eff Trade-Off Line</b>		0.0%		0.0%		4.0%			
72	<b>Eff Trade-Off Line</b>		100.0%		8.4%		9.4%			
73	<b>Eff Trade-Off Line</b>		200.0%		16.7%		14.8%			

- g. **Miscellaneous:** It will simplify matters to create range names for various cells. Put the cursor in cell **C37**, click the **Name** box at the left end of the formula bar  enter the name "ALPHA" and press **RETURN**. Repeat this procedure to give cell **C38** the name "THETA", give cell **C39** the name "PHI". Give cell **C40** the name "DELTA", give cell **C41** the name "GAMMA", and give cell **E5** the name "R." (Excel does not accept plain "R"). Enter **=ALPHA\*PHI-THETA^2** in cell **C40** and enter **=1/(THETA-ALPHA\*R.)** in cell **C41**. Some restrictions do apply on the range of permissible input values. The variable Delta must always be positive or else the calculations will blow-up or produce nonsense results. Simply avoid entering large negative correlations for multiple assets and this problem will be taken care of.
- h. **Individual Risky Assets:** To add the individual risky assets to the graph, reference their individual standard deviations and expected returns. Enter **=D6** in cell **D44** and copy down to the range **D45:D48**. Enter **=C6** in cell **G44** and copy down to the range **G45:G48**
- i. **Expected Return:** Using the three Hyperbola coefficients, we can solve for the expected return on the upper and lower branches of the hyperbola which correspond to a standard deviation of **25%** and then fill in intermediate values in order to generate the Efficient Frontier graph.

- For the upper branch, enter  $= (2 * \text{THETA} - (4 * \text{THETA}^2 - 4 * \text{ALPHA} * (\text{PHI} - (0.25^2) * \text{DELTA}))^{(0.5)}) / (2 * \text{ALPHA}) - 1$  in cell E49
  - For the lower branch, enter  $= (2 * \text{THETA} + (4 * \text{THETA}^2 - 4 * \text{ALPHA} * (\text{PHI} - (0.25^2) * \text{DELTA}))^{(0.5)}) / (2 * \text{ALPHA}) - 1$  in cell E69
  - Fill out in index for expected return by entering **0** in cell C49, entering **1** in cell C50, selecting the range C49:C50, and dragging the fill handle (in the lower right corner) down the range C51:C69
  - Fill in the intermediate values by entering  $= \$E\$49 + (\$E\$69 - \$E\$49) * (C50/20)$  in cell E50 and copying the cell down the range E51:E68
- j. **Standard Deviation:** Again using the three Hyperbola coefficients, we can solve for the Efficient Frontier standard deviation which corresponds to any particular value of expected return. Enter  $= ((\text{ALPHA} * (1 + E49)^2 - (2 * \text{THETA} * (1 + E49)) + \text{PHI}) / (\text{ALPHA} * \text{PHI} - (\text{THETA}^2)))^{(1/2)}$  in cell D49 and copy it down the range D50:D69
- k. **Tangent Portfolio:** The Optimal Combination of Risky Assets (or Tangent Portfolio) can be calculated using Excel's **matrix** functions. In each case, you type the formula and then, hold down the **Shift** and **Control** buttons simultaneously, and while continuing to hold them down, press **Enter**
- For the portfolio weights: Select the range I44:I48, then type  $= \text{GAMMA} * \text{MMULT}(\text{MINVERSE}(\text{B26:F30}), (\text{E6:E10} - \text{R} * \text{F6:F10}))$
  - For expected returns: Enter  $= \text{MMULT}(\text{TRANSPOSE}(\text{I44:I48}), \text{E6:E10}) - 1$  in cell F70
  - For standard deviations: Enter  $= \text{SQRT}(\text{MMULT}(\text{MMULT}(\text{TRANSPOSE}(\text{I44:I48}), \text{B26:F30}), \text{I44:I48}))$  in cell D70
- l. **Efficient Trade-Off Line:** The Efficient Trade-Off Line is a combination of the Riskless Asset and the Risky Asset Optimal Combination. It can be calculated as follows
- Enter **0.0%** in cell C71, **100.0%** in cell C72, and **200.0%** in cell C73
  - As before, the standard deviation formula simplifies to  $\sigma = w \sigma_T$ . Enter  $= C71 * \$D\$70$  in cell D71 and copy the cell to the range D72:D73
  - The Expected Return formula is  $\mu_P = w_P \mu_T + (1 - w_P) r_f$ . Enter  $= \$F\$70 * C71 + \$C\$5 * (1 - C71)$  in cell F71 and copy to the range F72:F73
- m. **Create and Locate the Graphs:** Highlight the range D44:G73 and then choose **Insert Chart** from the main menu. Select a **Scatter** chart type and make other selections to complete the Chart Wizard. Place the graph in the range G3:L16. Highlight the range H44:I48 and then choose **Insert Chart** from the main menu. Select a **Column** chart type and make other selections to complete the Chart Wizard. Place the graph in the range G17:L28. Optionally, one can format the graph as discussed in the problem 1.1.
- 1.4. Use the spreadsheet constructed in Problem 1.3 to perform an analysis analogous to Problem 1.2.
- 1.5. To expedite the analysis requested in Problem 1.4, construct a "dynamic" mean-variance portfolio graph. To this end, proceed as follows:
- a. **Start with the Spreadsheet Constructed in Problem 1.3, Add Rows, and Rearrange a Bit:** Open the spreadsheet that you created in Problem 1.3 and immediately save it under a new name using the **Microsoft Office Button**  **Save As** command. Move both of the graphs out of the way. Select each graph and drag them below row 22 and to the right of column H. Select the range E3:F10 and drag the range (hover the cursor over the lower highlighted line, click on the left mouse button, and hold it down while you move it) to cell H23.
- b. **Increase Row Height for the Spinners:** Select the range A5:A10. Then click on **Cells Format Row Height** from the **Home** menu. Enter a height of **30** and click on **OK**. Select the range A14:A18. Then click on **Cells Format Row Height** from the **Home** menu. Enter a height of **30** and click on **OK**.
- c. **Display the Developer Tab if it is Not Available:** Click the **Microsoft Office Button** , and then click **Excel Options**. In the **Popular** category, under **Top Options for Working with Excel**, select the **Show Developer tab in the Ribbon** check box, and then click **OK**.


- d. **Create the Spinners:** Look for the up-arrow/down-arrow button on the **Developer Controls Insert** toolbar (which will display the word “**Spin Button (Form Control)**” if you hover the cursor over it) and click on it. Then draw the box for a spinner covering the *left half* of cell **C5**. Then a spinner appears in the *left half* of cell **C5**. Right click on the spinner (press the right mouse button while the cursor is above the spinner) and a small menu pops up. Click on **Copy**. Then select the cell **C6** and click on **Paste**. This creates an identical spinner in the cell **C6**. Repeat the process four times more in cells **C7:C10**. Repeat the process five times more in cells **D6:D10**. Click on the spinner button on the **Developer Controls Insert** toolbar and draw the box for a spinner covering the *left half* of cell **B15**. Click on **Copy**. Then select the cell **B16** and click on **Paste**. Repeat the process eight times more in cells **B17, B18, C16, C17, C18, D17, D18, and E18**.



- e. **Create the Cell Links:** Right click on the first spinner in cell **C5** and a small menu pops up. Click on **Format Control** and a dialog box pops up. Click on the **Control** tab, then enter the cell link **K5** in the **Cell link** edit box and click on **OK**. Repeat this procedure for the spinners in the range **C6:C10** (linking to cells in the range **K6:K10**) and in the range **D6:D10** (linking to cells in the range **L6:L10**). Right click on the spinner in the cell **B15** and a small menu pops up. Click on **Format Control** and a dialog box pops up. Click on the **Control** tab, then enter cell **K15** as the cell link in the **Cell link** edit box and click on **OK**. Repeat this procedure for the spinners in the range **B16:B18** and in the triangle **C16 to C18 to E18** (linking each spinner to the cells in the range **K16:K18** and in the triangle **L16 to L18 to N18**, respectively). Test your spinners by clicking on the up-arrows and down-arrows of the spinners to see how they change the values in the linked cells.
- f. **Create Scaled Inputs:** The values in the linked cells are always integers, but they can be scaled appropriately to the problem at hand. In cell **C5**, enter **=K5/500** and copy this cell to the range **C6:C10**. In cell **D6**, enter **=L6/500** and copy this cell to the range **D7:D10**. In cell **B15**, enter **=K15/10-1** and copy the cell to the range **B16:B18** and to the triangle **C16 to C18 to E18**.
- g. **Relocate Graphs:** Move both of the graphs back into place. Move the Efficient Frontier graph into the range **G11:K22**. Move the Portfolio Weights graph into the range **I5:K10**.
- 1.6. Assume that the portfolio optimization is subject to constraints (e.g., no short sales, or upper limits on the holdings of individual assets). Graph the efficient trade-off line, assuming that there is a risk-free asset with a net rate of return  $r_f = 4\%$ , and five risky assets with mean return vector  $\underline{\mu} = [7.6\% \ 8\% \ 1\% \ 0.6\% \ 16\%]$ , standard deviation vector  $\underline{\sigma} = [12\% \ 16\% \ 18\% \ 18\% \ 20\%]$ , and correlation matrix  $\underline{\rho} = [1 \ 0.2 \ -0.1 \ 0.1 \ 0, \ 0.2 \ 1 \ 0.4 \ 0.1 \ 0, \ -0.1 \ 0.4 \ 1 \ 0.2 \ 0.2, \ 0.1 \ 0.1 \ 0.2 \ 1 \ 0.3, \ 0 \ 0 \ 0.2 \ 0.3 \ 1]$ . Analyze how such

constraints affect the location of the efficient set in mean-variance space. To this end, construct the spreadsheet as indicated in the following instructions.

- a. **Start with the Spreadsheet Constructed in Problem 1.3:** Open the spreadsheet that you created in Problem 1.3 and immediately save it under a new name using the **Microsoft Office**

**Button**  **Save As** command. In the range **A80:M94** type the titles shown in the figure below.

	A	B	C	D	E	F	G	H	I	J	K	L	M
80	<b>Restricted Frontier</b>												
81		<b>Restricted</b>	<b>Restricted</b>										
82		<b>Frontier</b>	<b>Frontier</b>			<b>Restricted</b>	<b>Restricted</b>						
83		<b>Curve</b>	<b>Curve</b>			<b>Curve</b>	<b>Optimal</b>						
84		<b>Standard</b>	<b>Expected</b>							<b>Restricted Optimal</b>			<b>Expected</b>
85		<b>Deviation</b>	<b>Return</b>			<b>Variance</b>	<b>Riskfree Asset</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>E</b>	<b>Restriction</b>
86	<b>Eff. Trade Off Line</b>	20.00%	16.00%			0.0400	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	25.00%
87													
88	<b>Eff. Trade Off Line</b>	0.00%	4.00%			0.0000	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%	4.00%
89	<b>Eff. Trade Off Line</b>	2.86%	6.00%			0.0008	73.72%	8.90%	5.07%	0.00%	0.00%	12.31%	6.00%
90	<b>Eff. Trade Off Line</b>	5.73%	8.00%			0.0033	47.44%	17.80%	10.15%	0.00%	0.00%	24.61%	8.00%
91	<b>Eff. Trade Off Line</b>	8.59%	10.00%			0.0074	21.16%	26.70%	15.22%	0.00%	0.00%	36.92%	10.00%
92	<b>Eff. Trade Off Line</b>	11.49%	12.00%			0.0132	0.00%	30.35%	18.13%	0.00%	0.00%	51.52%	12.00%
93	<b>Eff. Trade Off Line</b>	15.35%	14.00%			0.0236	0.00%	12.29%	12.09%	0.00%	0.00%	75.61%	14.00%
94	<b>Eff. Trade Off Line</b>	20.00%	16.00%			0.0400	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	16.00%

**Solver Parameters**


Set Target Cell:

Equal To:  Max  Min  Value of:

By Changing Cells:

Subject to the Constraints:

- 
- 
- 
- 
- 
-

- b. **Calculate Portfolio Variance:** In cell **F86** type **=MMULT(MMULT(H86:L86,\$B\$26:\$F\$30),TRANSPOSE(H86:L86))**. Since this is intended to be a **matrix** function, after typing you must hold down the **Shift** and **Control** buttons simultaneously, and while continuing to hold them down, press **Enter**.
- c. **Restricted Optimal Proportion of Risk-Free Asset:** In cell **G86** type **=1-SUM(H86:L86)**.
- d. **Standard Deviation and Expected Return of Optimal Restricted Portfolio:** In cells **C86** and **D86** type **=SQRT(F86)** and **=MMULT(H86:L86,\$C\$6:\$C\$10)+C5\*G86**, respectively.
- e. **Insert Solver Add In if it is Not Available:** Click the **Microsoft Office Button** , and then click **Excel Options**. Click **Add-Ins**, and then in the **Manage** box, select **Excel Add-ins** and click **Go**. In the **Add-Ins available** box, select the **Solver Add-in** check box, and then click **OK**. (Note: If **Solver Add-in** is not listed in the **Add-Ins available** box, click **Browse** to locate the add-in. If you get prompted that the Solver Add-in is not currently installed on your computer, click **Yes** to install it.) Once you insert the **Solver Add In**, the **Solver** command is available in the **Analysis** group on the **Data** tab.
- f. **Restricted Portfolio Optimization:** In cell **M86** type the desired expected return of the restricted portfolio (e.g., 16%). Then select **Analysis Solver** from the **Data** tab. A menu like the one shown in the figure above will appear on the screen. Type the values shown in the figure (Note: Due to space limitations, the figure above does not show the additional constraint **\$L\$86 >= 0**. Click **Solve**, and then **Keep Solver Solution OK**. Select range **C86:M86**, click on **Edit Copy**, select cell **C94**, click on **Edit Paste Special**, and click **Values OK**. Repeat the procedure for desired restricted portfolio returns of 4%, 6%, 8%, 10%, 12%, 14%, and 25% to obtain a table like the one shown in the figure above.
- g. **Add the Restricted Frontier to the Mean-Variance Graph:** Select the mean-standard deviation chart and select **Chart Source Data** from the main menu. Click on **Series Add** and select the range **C86:C94** as **X Values** axis and **D86:D94** as **Y Values**. Edit the points so that they show as a curve.

- 1.7. Based on your response to Problem 1.6, are convex combinations of the restricted frontier portfolios necessarily on the restricted portfolio frontier?
2. Solve Exercise 6 on p. 55 of Pennacchi.
3. (Exercise 6 on p. 171 of Luenberger) Suppose there are  $n$  assets which are uncorrelated. (They might be  $n$  different “wild cat” oil prospects.) You may invest in any one, or in any combination of them. The mean rate of return  $\bar{r}$  is the same for each asset, but the variances are different. The return on asset  $i$  has a variance of  $\sigma_i^2$  for  $i = 1, 2, \dots, n$ .
  - a. Show the situation in an  $\bar{r}$ - $\sigma$  diagram.
  - b. Find the minimum-variance point. Express your result in terms of  $\bar{\sigma}^2 = \left( \sum_{i=1}^n \frac{1}{\sigma_i^2} \right)^{-1}$ .

**References:**

Luenberger, D. G. *Investment Science*. New York: Oxford University Press, 1998.  
Pennacchi, G. *Theory of Asset Pricing*. Boston: Pearson, 2007.