Problem Set No. 7

1. Alice's valuation for an object you own is drawn from a uniform distribution on $[0, 1]$. If you make her a take-it-or-leave-it offer to sell the object at price p,

   (a) what is the probability that you will make a sale?

   (b) what is the revenue maximizing take-it-or-leave-it offer p?

2. Suppose now that both Alice's and Bob's valuations for a single object that you own are independently drawn from the same uniform distribution on $[0, 1]$. If you conduct a take-it-or-leave-it auction by posting a price p (if both bidders are willing to buy the object at p, then you select the buyer at random),

   (a) what is the probability that you will make a sale?

   (b) what is the the revenue maximizing take-it-or-leave-it offer p?

   (c) check that a standard auction with *optimal reserve price* yields a higher expected revenue than the optimal take-it-or-leave-it offer p.

3. Consider a second price auction where the bidders valuations take positive *integer* values, and the allowable bids are restricted to integer amounts of money. Show that there is a symmetric equilibrium in which all bidders bid according to $\beta(v) = v - 1$. Show that all bidders prefer this equilibrium to the one at which they bid their true valuations.

4. A single object is sold at an all-pay auction. Assume that there are four potential bidders: Alice, Bob, Carol, and Dan. Their valuations are independently drawn from the uniform distribution on $[0, 1]$. All subsets of bidders have the same probability of showing up at the auction. For example, the probability that Bob and Carol are the only bidders is equal to the probability that Alice, Bob and Dan are the only bidders. A bidder that shows up at the auction does not know how many bidders will participate. Consequently, a strategy is a function that assigns a bid to each possible valuation a participating bidder may have.

   (a) Given that Alice participates in the auction, what is the probability that she assigns to the event that the total number of bidders in the auction is n for $n = 1, 2, 3, 4$?

   (b) Find a symmetric equilibrium of the game.