

### Problem Set No. 6

1. Consider the following three-player game:

|          |          |          |
|----------|----------|----------|
|          | <i>L</i> | <i>R</i> |
| <i>T</i> | 0, 0, 3  | 0, 0, 0  |
| <i>B</i> | 1, 0, 0  | 0, 0, 0  |
|          | <i>A</i> |          |

|          |          |          |
|----------|----------|----------|
|          | <i>L</i> | <i>R</i> |
| <i>T</i> | 2, 2, 2  | 0, 0, 0  |
| <i>B</i> | 0, 0, 0  | 2, 2, 2  |
|          | <i>B</i> |          |

|          |          |          |
|----------|----------|----------|
|          | <i>L</i> | <i>R</i> |
| <i>T</i> | 0, 0, 0  | 0, 0, 0  |
| <i>B</i> | 0, 1, 0  | 0, 0, 3  |
|          | <i>C</i> |          |

- (a) Show that the pure strategy equilibrium payoffs are  $(1, 0, 0)$ ,  $(0, 1, 0)$ , and  $(0, 0, 0)$ .
- (b) Show that there is a correlated equilibrium in which player 3 chooses  $B$  and players 1 and 2 play  $(T, L)$  and  $(B, R)$  with equal probabilities.
2. An all-pay auction is similar to a first price sealed bid auction in that the highest bidder gets the object. The only difference is that in the all-pay auction all players pay their bids, even if they do not win.
- (a) Define formally the Bayesian game that represents an all-pay auction, where the players' valuations are identically and independently distributed.
- (b) Find a symmetric equilibrium of the all-pay auction.
- (c) Calculate the equilibrium expected payment of a bidder whose valuation is  $v$ .
- (d) Calculate the ex-ante expected payment of a bidder.
3. Calculate the symmetric equilibrium of the first price auction when the bidders' valuations are independent random draws from the distribution  $F : [0, 1] \rightarrow [0, 1]$  given by  $F(v) = v$ . What happens to the equilibrium bid when the number of bidders increases.
4. Calculate the symmetric equilibrium of the first price auction when the bidders' valuations are independent random draws from the distribution  $F : [0, \infty] \rightarrow [0, 1]$  given by  $F(v) = 1 - e^{-2v}$ , and the number of bidders is two.