

Problem Set No. 5

1. It is common knowledge that Gino and Polly always tell the truth. The state space is $\Omega = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. The agents' initial information partitions are given by:

$$\Pi_G = \{\{1, 2, 3\}, \{4, 5, 6\}, \{7, 8, 9\}\}$$

and

$$\Pi_P = \{\{1, 2, 3, 4\}, \{5, 6, 7, 8\}, \{9\}\}.$$

The agents alternate in announcing how many elements their current partition cell contains, starting with Gino.

- (a) Why does Gino begin announcing “three” in all states of the world?
 - (b) How does Gino’s announcement change Polly’s partition?
 - (c) Now Polly makes an announcement. Let $f_P^1 : \Omega \rightarrow \mathbb{N}$ be the function that assigns to each state $\omega \in \Omega$ Polly’s announcement. That is, for each $\omega \in \Omega$, $f_P^1(\omega)$ is the number of elements of Polly’s current partition cell, $\Pi_P(\omega)$, at the time of his announcement. Write down $f_P^1(\omega)$ for each $\omega \in \Omega$.
 - (d) What is Gino’s partition after he learns Polly’s announcement?
 - (e) Continue updating the agents’ partitions as announcements are made, until there are no further changes. What are the limit partitions?
2. As in the previous question, it is common knowledge that Gino and Polly always tell the truth. The state space is $\Omega = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. The agents’ initial information partitions are given by:

$$\Pi_G = \{\{1, 2, 3\}, \{4, 5, 6\}, \{7, 8, 9\}\}$$

and

$$\Pi_P = \{\{1, 2, 3, 4\}, \{5, 6, 7, 8\}, \{9\}\}.$$

The agents’ share a common prior probability π on Ω , according to which, all states are equally likely. Consider the event $F = \{3, 4\}$. Instead of announcing how many elements their current partition cell contains, they announce their current conditional probability of the event F .

- (a) For each agent and for each state, what is his posterior probability of the event F , conditional he knows at that state? That is, what is $\pi(F|\Pi_i(\omega))$ for $\omega \in \Omega$ and $i = G, P$?
- (b) What is the event “Gino announces 1/3”? What is the event “Gino announces 0”?
- (c) What is Polly’s partition after Gino’s initial announcement? Explain why the event “Polly announces 1/2” is $\{1, 2, 3, 4\}$, and the event he announces 0 is $\{5, 6, 7, 8, 9\}$.

- (d) what is Gino's new partition after Polly's announcement? What does Gino announce now at each possible state?
- (e) What is Polly's new partition? What does he announce at each one the the states?
- (f) Explain why each agent's posterior probability is common knowledge, whatever the true state of the world.
- (g) Using the initial partitions explain why no player's posterior probability for F is common knowledge in any state.
- (h) What will the sequence of announcements be when the true state of the world is $\omega = 2$?