Game Theory Fall 2007

Problem Set No. 4

1. Consider the infinite version of the alternating offer game where player I is the first proposer. Suppose both players have a common discount factor δ=0.9. Suppose you did not know that this game has a unique subgame perfect equilibrium. Could you immediately argue why the split (0,1) is not a subgame perfect equilibrium outcome?

2. In Rubinstein's infinite horizon bargaining game $G(I;\infty)$ that we studied in class, suppose that the players are restricted to proposing either that Player I should get the whole dollar or that Player II should get the whole dollar.

   i) Show that there is a subgame perfect equilibrium in which Player I begins by proposing that he get the whole dollar and Player II agrees.

   ii) Show that there is a subgame perfect equilibrium in which Player I begins by proposing that the whole dollar be given to Player II and Player II agrees.

   iii) Show that there are other subgame perfect equilibria in which agreement does not occur immediately.

3. John and Mary may divide a dollar between them if they agree on how it should be divided (the dollar is perfectly divisible).

   i) How will they divide the dollar if they use the Nash bargaining solution in the case when John's utility for \$$x$ is $u_1(x) = x^{1/4}$ and Mary's is $u_2(x) = x^{3/4}$?

   Assume now that John and Mary play the infinite horizon bargaining game of alternating offers in order to divide the dollar, and that they have a common discount factor δ. (You may assume that $\delta = .9$ if this makes things easier).

   ii) What is the unique subgame perfect equilibrium division of the dollar, if the first one to make a proposal is John?

   iii) What is the unique subgame perfect equilibrium division of the dollar, if the first one to make a proposal is Mary?

   iv) What happens to your answers to ii) and iii) as δ goes to 1? Compare to your answer to your answer to i).