Problem Set No. 2

1. Dominoes can be placed on a $m \times n$ board so as to cover two squares exactly. Two players alternate in doing this. The first to be unable to place a domino is the loser.

   a) Draw the game tree for the case $m = 2$ and $n = 3$.

   b) Find the value of the 2 by 3 version of the game and determine a winning strategy for one of the players.

   c) Who has a winning strategy when m and n are even?

2. Two players use the following procedure to divide a cake. Player 1 divides the cake into two pieces, and then player 2 chooses one of the pieces, player 1 obtaining the remaining piece. The cake is continuously divisible, and each player cares only about the size of the piece of cake he obtains. How is the cake divided in a subgame perfect equilibrium?

3. Two people engage in the following procedure to divide \$10. Person 1 offers person 2 some amount (not restricted to be an integral number of cents) $0 \leq x \leq 10$. If 2 accepts this offer, then 1 receives $10 - x$. If 2 rejects the offer then neither person receives anything. Assume that each person cares only about the amount of money he receives, and prefers to receive as much as possible.

   (a) Find the values of x for which there is a Nash equilibrium in which person 1 offers x.

   (b) Find the subgame perfect equilibrium of the game.

4. Suppose that a parent and child play the following game, first analyzed by G. Becker in the *Journal of Political Economy* 82: 1063–93, (1974). First the child takes an action $a > 0$ and as a consequence the child gets a monetary income of $I_c = a(500 - a)$ and the parent gets a monetary income of $I_p = a(1000 - a)$. Second, after observing the child's action (and consequently both incomes), the parent chooses a bequest b —which can be positive or negative— to leave to the child. The child's preferences are perfectly egoistic: they depend only on the money he gets,and are represented by the utility function $U_c = (I_c + b)^{1/2}$. The parent, on the other hand, is concerned about his child's well-being: his preferences depend both on the money he gets and on the child's utility, and are represented by the utility function $U_p = (I_p - b)^{1/2} + U_c$.

   (a) What is the action that maximizes the child's income I_c?

   (b) What is the action that maximizes the family's aggregate income $I_c + I_p$?

   (c) Prove the "rotten kid" theorem: in the backward induction (subgame perfect equilibrium) outcome, the child chooses the action that maximizes the family's aggregate income, even though only the parent's payoff exhibits altruism.