

Problem Set No. 1

1. Reuben and Simon may divide 2 indivisible candies between themselves if they agree on how they should be divided.
 - a) What is the set Z of possible physical outcomes? Draw it.
 - b) Taking into account that they may decide to use some random device to allocate the two candies, what is the set $\text{Lott}(Z)$ of lotteries over Z ?
 - c) Reuben's utility function is $u_R(x; y) = x^{\frac{1}{4}}$ and Simon's utility function is $u_S(x; y) = y^{\frac{3}{4}}$, where x represents the number of candies Reuben gets and y represents the number of candies Simon gets. Namely, Reuben cares only for the candies he gets and Simon cares only for the candies he gets. For each of the feasible physical outcomes, calculate the corresponding pair of utilities. Draw.
 - d) Calculate the set of all the expected utility pairs, associated with the lotteries in $\text{Lott}(Z)$. What is the disagreement point?
 - e) Repeat c) and d) for the case where Reuben's utility function is

$$v_R = 2u_R + 1 = (2x^{\frac{1}{4}} + 1)$$

and Simon's utility function is

$$v_S = \frac{1}{2}u_S + 2 = \left(\frac{1}{2}x^{\frac{3}{4}} + 2\right).$$

- f) For which representation of Reuben's and Simon's preferences respectively, the disagreement point is $(0; 0)$ and the utility pair that corresponds to equal division is $(2; 2)$?
- g) For which representation of their preferences the disagreement point is $(1; 1)$ and the utility pair that corresponds to equal division is $(3; 6)$?

2. Consider the bargaining problem $\langle S, d \rangle$ where $d = (0, 0)$ and

$$S = \left\{ (s_1, s_2) : \frac{s_1}{2} + s_2 \leq 1 \text{ and } s_1 + \frac{s_2}{2} \leq 1; s_1 \geq 0, s_2 \geq 0 \right\}.$$

- i) Draw the bargaining problem (the set of feasible utility pairs and the disagreement point).
 - ii) Show that $\langle S, d \rangle$ is a symmetric bargaining problem.
 - iii) Calculate the recommendation of the Nash bargaining solution to this bargaining problem. (Hint: use the fact that the Nash bargaining solution satisfies PAR and SYM).
 - iv) What is the recommendation given by the Kalai-Smorodinsky bargaining solution?
3. Consider this new bargaining problem $\langle S', d \rangle$ where $d = (0, 0)$ and

$$S' = \left\{ (s_1, s_2) : \frac{3}{5}s_1 + \frac{s_2}{5} \leq 1; s_1 \geq 0, s_2 \geq 0 \right\}.$$

- i) Draw the bargaining problem $\langle S', d \rangle$ and show that $S \subseteq S'$.
 - ii) Calculate the recommendation of the Nash bargaining solution to this bargaining problem.
 - iii) What is the recommendation given by the Kalai-Smorodinsky bargaining solution?
4. John and Mary may divide a dollar between them if they agree on how it should be divided (this time the dollar is perfectly divisible). How will they divide the dollar if they use the Nash bargaining solution in the case when John's utility for $\$x$ is $u_1(x) = x^{1/3}$ and Mary's is $u_2(x) = x^{2/3}$? Whose share of the dollar would increase if the utility functions were changed to $v_1(x) = x^{1/2}$ and $v_2(x) = x^{1/2}$? What can you conclude about the effect that risk aversion has on the bargaining outcome?