Let $G = \langle N, (A_i)_{i\in N}, (u_i)_{i\in N}\rangle$ be a strategic game.
Let the following be exogenously given:

- A finite probability space (Ω, π)
- For each player $i \in N$ a partition $\mathcal{P}_i$ of Ω
- For each player $i \in N$ a function $\sigma_i : \Omega \to A_i$ which is measurable with respect to $\mathcal{P}_i$.

Definition 1 Player $i \in N$ is *Bayes rational* at $\omega \in \Omega$ if his expected payoff at ω, $E(u_i(\sigma)|\mathcal{P}_i)(\omega)$ is at least as large as the amount $E(u_i(\sigma_{-i}, a_i)|\mathcal{P}_i)(\omega)$ that he would have got had he chosen action $a_i \in A_i$ instead of $\sigma_i(\omega)$.

Theorem 1 If each player is rational at each state of the world, then the distribution of the action n-tuple is a correlated equilibrium distribution.

Proof : We need to show a correlated equilibrium whose induced probability distribution over the action profiles coincides with the distribution of σ. We claim that the correlated strategy we are looking for is the function $\sigma : \Omega \to A$ itself. Let $\tau_i : \Omega \to A_i$ be a function that is measurable with respect to $\mathcal{P}_i$. Since i is Bayes rational at ω

$$E(u_i(\sigma)|\mathcal{P}_i)(\omega) \geq E(u_i(\sigma_{-i}, a_i)|\mathcal{P}_i)(\omega) \qquad \forall a_i \in A_i.$$

That is,

$$\sum_{\omega'\in\mathcal{P}_i(\omega)} \frac{\pi(\omega')}{\pi(\mathcal{P}_i(\omega))} u_i(\sigma_{-i}(\omega'), \sigma_i(\omega')) \geq \sum_{\omega'\in\mathcal{P}_i(\omega)} \frac{\pi(\omega')}{\pi(\mathcal{P}_i(\omega))} u_i(\sigma_{-i}(\omega'), a_i) \qquad \forall a_i \in A_i.$$

In particular, for $a_i = \tau(\omega) = \tau(\omega')$ for all $\omega' \in \mathcal{P}_i(\omega)$,

$$\sum_{\omega'\in\mathcal{P}_i(\omega)} \frac{\pi(\omega')}{\pi(\mathcal{P}_i(\omega))} u_i(\sigma_{-i}(\omega'), \sigma_i(\omega')) \geq \sum_{\omega'\in\mathcal{P}_i(\omega)} \frac{\pi(\omega')}{\pi(\mathcal{P}_i(\omega))} u_i(\sigma_{-i}(\omega'), \tau(\omega')).$$

Multiplying both sides by $\pi(\omega)$ and adding over all ω we get

$$\sum_{\omega\in\Omega} \pi(\omega) u_i(\sigma_{-i}(\omega), \sigma_i(\omega)) \geq \sum_{\omega\in\Omega} \pi(\omega) u_i(\sigma_{-i}(\omega), \tau(\omega)).$$

$\square$