

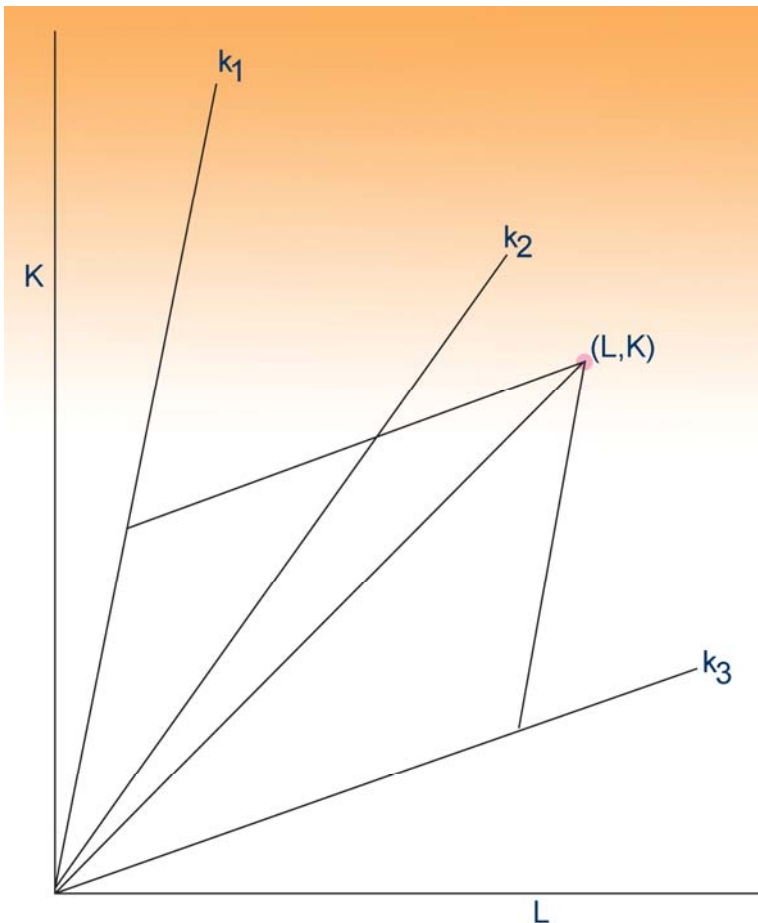
3F. Broken Chain

Jones' Version of the Chain Proposition

In the world with n goods and two factors ($n > 2$), if factor prices are equalized and $k_1 > k_2 > \dots > k_n$, then there exists r such that the capital abundant country exports the output of every industry i , $i \leq r$, for which $k_1 > k_2 > \dots > k_r$, and imports the rest.

● Bhagwati's Proof that there is no chain

If only those industries produced positive outputs, not all resources will be fully employed. The country exports at least one good which is labor intensive.



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Abstract

In the many good world with two factors of production, the so-called Chain Proposition has been widely accepted as a valid proposition. If factor prices differ between countries, a capital-(labor-) abundant country exports the most capital-(labor-) intensive goods, while the product with the middle capital-intensity could be produced in either country. We show that this proposition is false, and provide a counter example.

What is wrong with the Chain Proposition?

1. Introduction

More than five decades ago, Ronald Jones (1956) proposed the so-called Chain Proposition. Jones (1956, p. 6) wrote:

“Ordering the commodities with respect to the capital-labor ratios employed in production is to rank them in order of comparative advantage. Demand conditions merely determine the dividing line between exports and imports; it is not possible to break the chain of comparative advantage by exporting, say, the third and fifth commodities and importing the fourth when they are ranked by factor intensity”

Jagdish Bhagwati (1972) convincingly demonstrated that the chain proposition cannot hold where factor price equalization occurs. However, he also conjectured (p. 1052) that the chain “proposition, although correct for the case where factor prices are *not* equalized, is untenable as literally stated. When factor-price equalization is realized, a nont unimportant case, a variety of crisscrossings are possible.”

Subsequently, Deardorff (1979) worked on Bhagwati’s conjecture and provided a graphical proof that the chain proposition holds where factor prices are not equalized. Deardorff’s result is widely cited (e.g., Bhagwati, Panagariya and Srinivasan, 1998), Feenstra, 2004 and Romalis, 2004). The chain proposition also was deemed desirable to resolve production indeterminacy in the many-commodity world.

The purpose of this paper is to refute Bhagwati's conjecture and Deardorff's graphical proof.

In Section 2, we review Deardorff's graphical argument and point out an error. In Section 3, we provide a numerical counterexample.

2. Review of Deardorff's Proof

In most empirical studies on trade patterns commodities outnumber factors. When the number of goods, n , exceeds the number of factors, m , it is well known that the output vector is indeterminate, in which case the trade vector cannot be predicted either. The chain proposition may have been motivated to resolve this indeterminacy. For instance, Bhagwati (1972, p. 1054) argues that when transport costs are introduced on every commodity, factor prices will not be equalized, and that "the trade pattern cannot register a crisscrossing of the chain; each exportable must thus have a higher K/L ratio than each importable, in the K-abundant country (with identical homothetic tastes across countries)."

Now, let us review Deardorff's graphical proof of the Chain Proposition.

Assume:

1. No trade impediments so that output prices are equal
2. Production of each good requires capital (K) and labor (L).
3. Supply of each input is fixed in each country.
4. Production functions are identical between countries, concave and (linearly) homogenous.
5. Perfect competition prevails in both countries.
6. Goods are ranked in terms of capital intensity, $k_1 > \dots > k_n$, where $k_i \equiv K_i / L_i$.

No factor intensities are reversed.

7. Unequal factor prices.

The first six assumptions imply that factor prices will be equalized. Since Deardorff claimed that the chain proposition holds for unequal factor prices, we assume one of the first six is relaxed, but continue to assume that all resources are fully employed. This is true, since both factor prices are different but positive. If one factor were not fully employed, its price would be zero.

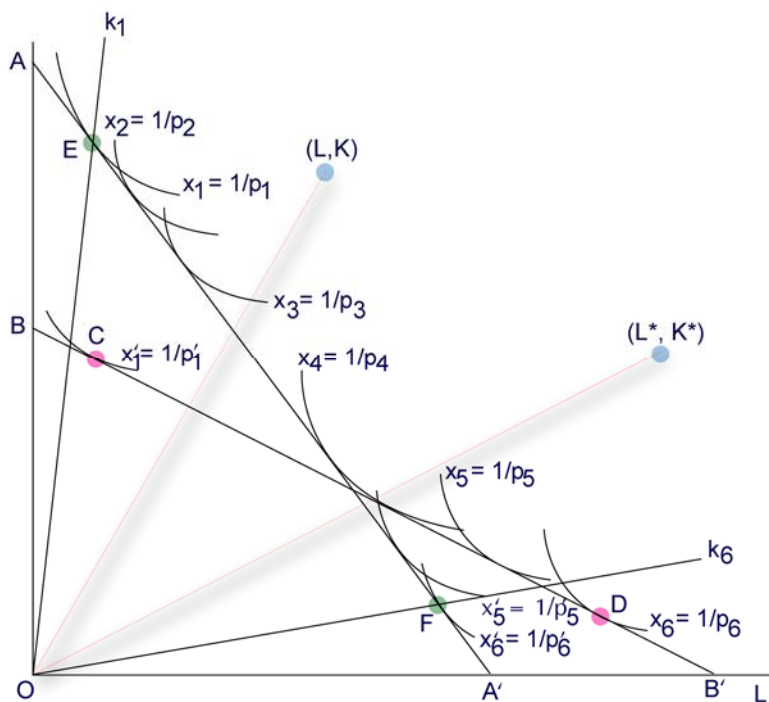


Figure 1 reproduces Deardorff's Figure 1 (p. 200). The endowment points (L, K) and (L^*, K^*) of the two countries and a couple new unit value isoquants are added.

Unit value isoquant for good 4 is tangent to both isocost curves, AA' and BB' , which would cost one dollar in each country. In country A, all unit value isoquants, $X_1 = 1/p_1$, $X_2 = 1/p_2$, $X_3 = 1/p_3$, $X_4 = 1/p_4$, and $X_5' = 1/p_5'$ are tangent to the common unit isocost curve, AA' .

Even though the unit value isoquant $X_5' = 1/p_5'$ lies below $X_5 = 1/p_5$, the cost of inputs along Country A's domestic isocost curve AA' is unity. Similarly, the cost of factors along Country B's isocost curve BB is unity.

Thus, Country A can produce any of these five goods, and good 6, because its unit value isoquant, $X_6' = 1/p_6'$ (not included in Deardorff's original Figure 1) is also tangent to AA' . If there are only six industries, then Country A's cone of diversification is spanned by EOF. As long as Country A's endowment point (L, K) belongs to this cone, the country can produce some combination of these goods. Country B's cone of diversification is denoted by the cone COD (not drawn). Due to a lower wage-rent ratio, each industry has a lower capital-labor ratio in Country B than in Country A.

Note in Figure 1 that capital intensities of industries 1, 2 and 3 are greater than Country A's capital-labor ratio, while those of industries 4, 5 and 6 are less. Country A cannot just produce positive outputs of only industries 1, 2 and 3, because in that case no positive output combinations of these three industries will match the country's endowment points. Country A must produce positive output of at least one industry whose capital intensity $k_i = K_i / L_i$ is greater than K/L , and a positive output of at least one industry whose capital intensity is less than K/L .

For instance, if Country A produces some output of the most capital-intensive good x_1 , then it must also produce a positive output of another industry whose capital-intensity is less than K/L . In this case, it could be industry 4, 5 or 6. While industry 6 is most labor-intensive, it is permissible for Country to operate on the expansion path k_6 , because at point F, unit value isoquant $X'_6 = 1/p'_6$ is tangent to the common unit iso-cost curve AA' . Thus, even the most labor-intensive industry can survive in the capital-abundant country.

Deardorff (1972, p. 201) wrote:

“It is now immediately evident from fig. 1 that the pattern of trade must agree with the ranking of the goods by factor intensity. The most capital intensive good (1, 2 and 3 in fig. 1) can only be produced in the high wage country, A, and must therefore be exported by A, while the most labor intensive (5 and 6) must be produced and exported by B. Good 4, in this case, may be produced in both countries and may be exported by either. It therefore constitutes the division of the chain of comparative advantage.”

We now know this chain breaks down. In other words, in the $n \times 2$ world, it is possible for the capital abundant HC to produce some output of the least capital-intensive product. Similarly, it is also possible for the labor abundant country to produce some amount of the most capital-intensive product. For instance, in Figure 1, it is possible for Country B to produce goods 2, 4 and 6, and Country A to produce goods 1, 3, and 5, contrary to the presumed chain proposition.

Thus, we present the broken chain proposition:

Proposition: Assume that (i) two countries produce n goods, $n > 2$, using two inputs, K and L , and (ii) the home country is capital-abundant ($k \equiv K/L > k^* \equiv K^*/L^*$), and (iii) all resources are fully employed. Let $k_i \equiv K_i/L_i$ denote the capital-intensity. Then its production vector is indeterminate, but

- (i) the K -abundant country cannot produce outputs of only the goods for which $k_i > k$, and
- (ii) if the K -abundant country produces the output of good i for which $k_i > K/L$, then it must also produce the output of another good j for which $k_j < K/L$.

Note that this proposition holds, whether factor prices are equalized or not. It is tempting to define that industry i is capital-intensive if $k_i > K/L$. However, an industry's capital intensity may lie between the capital-labor ratios of the two countries. For instance, if $K/L > k_i > K^*/L^*$, then industry i is labor-intensive from the home country's viewpoint, but it is capital-intensive in the eyes of the foreign country.

3. Numerical Example

We construct a numerical example, which illustrates the possibility that the capital abundant country can export the most labor intensive product. Each of the two countries, A and B , produces two different products and export them to the other country.

- (1) Assume output prices are equal, but factor prices are not. This must be due to different technologies or factor intensity reversal, so that the two countries have overlapping but not identical cones of diversification.
- (2) Assume the home country is capital abundant $\Rightarrow K/L = 4 > K^*/L^* = 3$.
- (3) Two industries must have capital intensities greater than K/L , and two other industries must have intensities less than K^*/L^* . There must be at least four industries. Of course, capital intensities depend on the wage-rent ratio, w/r . Assume that no FIR occurs. Thus, $k_1 > k_2 > K/L > K^*/L^* > k_3 > k_4$. for both (w,r) and (w^*,r^*) .

(Assume $6 > 5.5 > 4 > 3 > 2.5 > 1$ for (w,r) and $5.5 > 5 > 4 > 3 > 2 > 1.5$ for (w^*,r^*) .) $k_1 = 6 > 5.5 = k_2$ at (w,r) and $k_1 = 5.5 > 5 = k_2$ at (w^*,r^*) . A similar relationship occurs between k_3 and k_4 , at (w,r) and (w^*,r^*) .

We note

- (1) The output vector is indeterminate.
- (2) Certain pairs of goods cannot be produced. Specifically, in the (4×2) world, the capital abundant (Home) country cannot produce only the two most capital intensive goods. A positive linear combination of these goods will yield a capital-labor ratio greater than the capital-labor endowment ratio.

For instance, if the HC produces the most capital intensive good (good 1), then it cannot produce good 2. This is because, $k_1 > k_2$ if and only if

$$\frac{K_1}{L_1} > \frac{K_1 + K_2}{L_1 + L_2} > \frac{K_2}{L_2}.$$

Thus, if $k_1 > k_2 > K/L$, then the capital labor ratio in production exceeds that in endowment, $(K_1 + K_2)/(L_1 + L_2) > K/L$. For simplicity, assume that $a_{Li} = 1$, for all industries, so that $a_{Ki} = k_i$.

If the HC chooses to produce a good whose capital intensity is greater than K/L , it must also produce another output whose capital intensity is less than K/L ratio.

Assume the HC produces goods, 1 and 4. Then

$$\begin{bmatrix} 6 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_4 \end{bmatrix} = \begin{bmatrix} K \\ L \end{bmatrix}.$$

Then

$$y_1 = \frac{K - L}{5} = L > 0, y_4 = \frac{6L - K}{5} = \frac{2L}{5} > 0.$$

In this case, the HC exports both goods (1 and 4) and imports goods 2 and 3. That is, the capital abundant HC can import the most labor intensive good, 4.

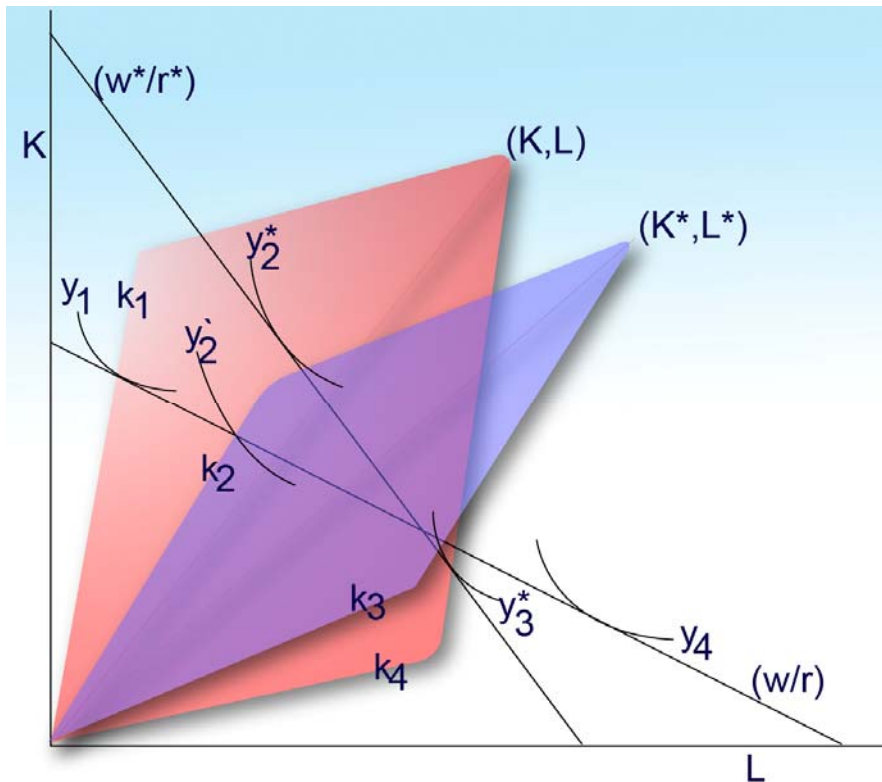
Next, suppose the FC produces good 3 with $a_{k2} = 2$, which is less than its capital-labor ratio, 3. Thus, it is not possible for the FC to produce two goods, 3 and 4, both of which have capital intensities less than 3. Suppose the FC produces goods 2 and 3. Then

$$\begin{bmatrix} 5 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} K^* \\ L^* \end{bmatrix}.$$

Then

$$y_2^* = \frac{K^* - 2L^*}{3} = \frac{L^*}{3}, y_3^* = \frac{5L^* - K^*}{4} = \frac{L^*}{2}.$$

The FC produces and exports goods 2 and 3, and imports goods 1 and 4. That is, even though trade is balanced, the labor abundant FC can import the most labor-intensive good, 4.



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