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3E Rybczynski Theorem with More Factors than Goods

● The number of goods is smaller than the number of factors, $n < m$. This case was examined by Jones and Scheinkaman (JPE, 1977).

● The smallest uneven case with more factors than goods is the 2×3 case (two goods and three factors).

$$\begin{bmatrix} a_{K1} & a_{K2} \\ a_{L1} & a_{L2} \\ a_{N1} & a_{N2} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \leq \begin{bmatrix} K \\ L \\ N \end{bmatrix}. \quad (1)$$

● The weak inequality simply means that each factor is either (a) fully employed or (b) not fully employed. In this case, there are two unknown outputs but there are three (equality or inequality) constraints.

● It is necessary to have two independent equations in order to determine two unknown variables uniquely.

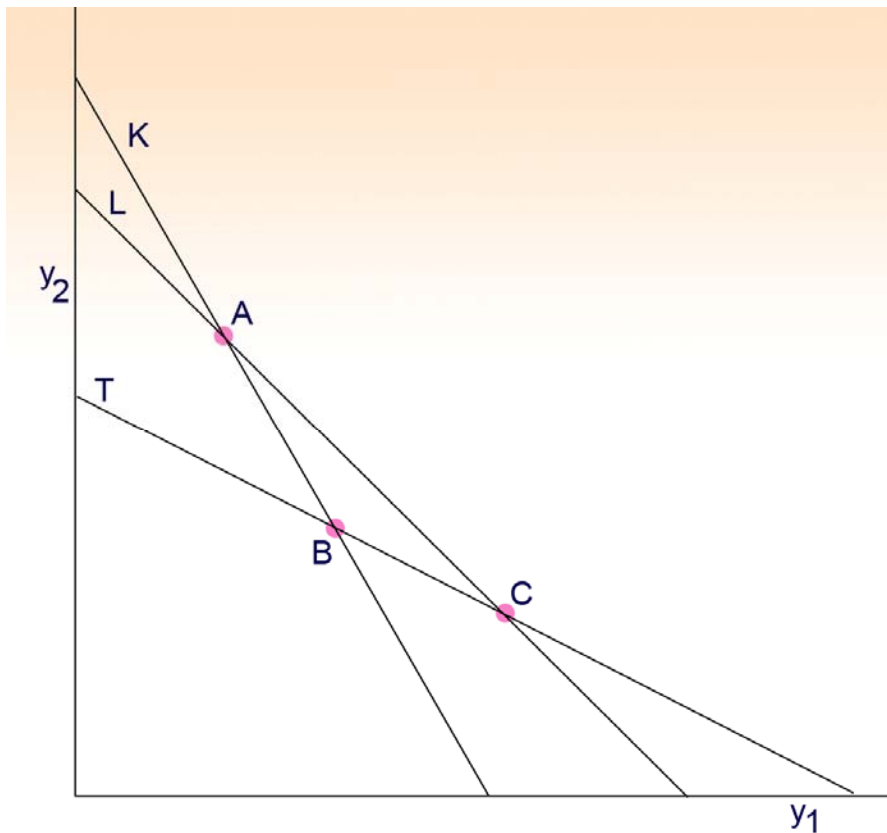
⇒ One of the constraints is redundant, i.e., **that resource will not be fully employed**. This result also has an implication on the three factor prices.

Let (r, w, s) denote a given factor price vector. Then a_{ij} 's in (1) are constant.

● The first equation in (1) defines a locus of outputs (y_1, y_2) that satisfy the capital constraint. Differentiating the first equation gives $a_{K1}dy_1 + a_{K2}dy_2 = 0$. The slope of this locus is $dy_2 / dy_1 = -a_{K1} / a_{K2} < 0$.

● The labor constraint have different a slope and an intercept, $dy_2 / dy_1 = -a_{L1} / a_{L2} < 0$.

● The land constaint: $dy_2 / dy_1 = -a_{T1} / a_{T2} < 0$.



● Feasible area: In Figure 1, the three resource constraints are labeled, K, L and T. The feasible area for each constraint lies on or below the line.

● Point A does not satisfy T constraint, whereas Point C does not satisfy K constraint. Neither will, therefore, be a solution that satisfies all three constraints with equality.

● In contrast, point B satisfies all three constraints with equality or inequality. Upon closer inspection, however, point B satisfies L constraint with strict inequality. This implies that the associated Lagrange multiplier, or the shadow price of L is zero, meaning that the constraint is not binding (although the market price $w > 0$).

● This means that the initial set of factor prices (r, w, s) are not compatible with the full employment condition, and one shadow price must be zero. At the given wage w , there will be unemployment of this factor, as shown by point B. This scenario is based on a given shadow price vector (w, r, s) .

● When three factor prices are arbitrarily chosen, only two constraints can be satisfied with equality, and the third constraint is simply redundant (and there is unemployment).

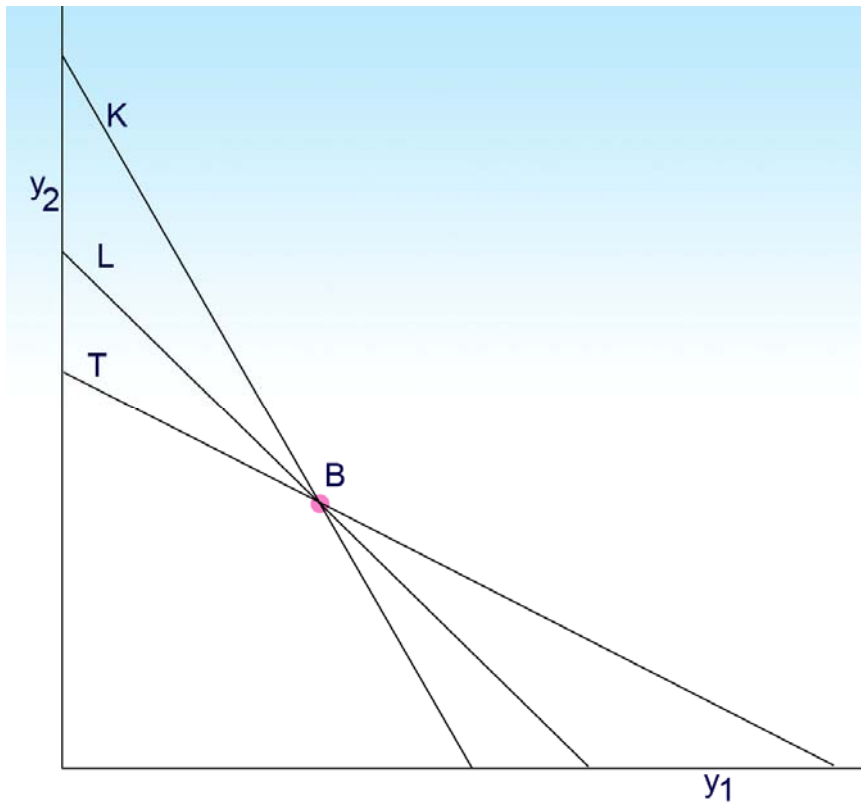
Rybczynski Effects in the Uneven Case (When factor prices w, r, s are arbitrarily chosen) : Two of the three equations completely determine a unique output vector.

● If both K and L are binding constraints, a change in K or L will have the usual Rybczynski effects in the 2×2 or $n \times n$ case.

- An increase in T , which is not a binding constraint, will have no effect on the outputs of the two sectors, because the resource is not fully employed.
- This result can be extended into the $m \times n$ case, where $m > n > 3$.
- All the binding constraints will have the Rybczynski effects, i.e., an increase in the endowment of a factor whose constraint is binding will have output effects (it will increase some output and decrease some other output, as shown in the $n \times n$ even case).
- An increase in the endowment of any factor whose constraint is not binding has no output effect.
- If all three constraints were binding at the outset, an increase in the endowment of any factor will make that constraint nonbinding. \Rightarrow no Rybczynski effect immediately.

Nonuniqueness of Factor Prices

- The above discussion of 2×3 case is based on the assumption that an arbitrary factor price vector (w,r,s) is chosen.
- Suppose now that factor prices are interdependent.
- Assume two constraints, K and L, are binding, and w and r are fixed. Then the price of an unemployed resource T declines. Since T is not fully employed, s declines, which encourages each industry to use more of T input. $\Rightarrow a_{Ti} \uparrow$. This shifts the land constraint inward.
- As s declines, each industry needs less of other inputs. $\Rightarrow a_{Ki} \downarrow, a_{Li} \downarrow$, which shifts the K and L constraints outward. This process continues until all constraints meet in one spot.
- For all three factors to be fully employed, all three resource constraints must meet one another at the same point as shown in Figure 2.



● It is important to note that three factor prices cannot be uniquely determined from a pair of output prices.

● Zero profit conditions are given by

$$a_{K1}r + a_{L1}w + a_{T1}s = p_1,$$

$$a_{K2}r + a_{L2}w + a_{T2}s = p_2,$$

(2)

● For a given pair of output prices, this system of two equations with three unknown factor prices cannot be uniquely determined.

● Two factor prices can be solved for in terms of the other factor price and output prices. For example, if T constraint is binding and its price $s > 0$. Then for any prices (p_1, p_2, s) , one can uniquely solve for the other factor prices (w, r) from the following zero profit conditions:

$$\begin{aligned} a_{K1}r + a_{L1}w &= p_1 - a_{T1}s, \\ a_{K2}r + a_{L2}w &= p_2 - a_{T2}s, \end{aligned} \tag{3}$$

● Thus, there are infinitely many solutions, depending the chosen value of s .

● When a constraint, say T, is not binding, its shadow price is zero. Hence, its price must fall, encouraging industries to use more of T input. For this constraint to be binding, its price must adjust downward. But there is no unique way to adjust the three factor prices to make all three constraints binding, as can be shown in (3).

● In the real world, we do not observe zero factor prices.
⇒ In a world of two goods and three factors, the factor prices cannot be chosen independently. They are dependent on each other. The factor prices move together if all factors are to be fully employed.

● Jones and Scheinman (1977) claim that both the Rybczynski and Stolper-Samuelson effects can be integrated in the following equations:

$$\begin{bmatrix} S & A \\ A' & 0 \end{bmatrix} \begin{bmatrix} dw \\ dy \end{bmatrix} = \begin{bmatrix} dV \\ dp \end{bmatrix}, \quad (4)$$

where S is a symmetric and negative semidefinite matrix, dV is the $(m \times 1)$ vector of changes in factor endowments, and dp is the $(n \times 1)$ vector of changes in output prices. Thus, the above $(m + n) \times (m + n)$ B matrix is invertible. Then Jones-Scheinman's approach implies that (dw, dy) vector can be uniquely determined from the (dV, dp) vector.

● However, we have already demonstrated that if factor price vector is not unique. Hence, factor prices in two countries are not likely to be equalized, except by chance.

● Moreover, if factor prices are arbitrarily chosen in each country, there will be unemployed resources or nonbinding resource constraints.

● If factor prices are interdependent so as to guarantee full employment of all resources, such factor price vector in one country may not be the same as that in another country.

● Also, when factor prices are interdependent to guarantee full employment of all resources, then the Rybczynski effects are zero (for any factor).