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3D Rybczynski Theorem with More Goods than Factors

Consider the smallest uneven HO case with more goods than factors (3 × 2 case).

$$\begin{bmatrix} a_{K1} & a_{K2} & a_{K3} \\ a_{L1} & a_{L2} & a_{L3} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} K \\ L \end{bmatrix}. \tag{1}$$

● Many different solutions are consistent with the resource constraint, and hence the output vector is not unique. The trade pattern cannot be predicted.

● A typical approach is laid out in Feenstra (Chapter 3, page 66). When adapted to the 3 × 2 case, the problem is to choose K_i, L_i to maximize the GDP function,

$$G(p,K,L) = p_1 y_1(K_1,L_1) + p_2 y_2(K_2,L_2) + p_3 y_3(K_3,L_3)$$

$$K_1 + K_2 + K_3 \leq K,$$

$$L_1 + L_2 + L_3 \leq L,$$

The Lagrangian function is written as

$$\mathcal{L} = \sum_i p_i F(K_i, L_i) + r\left[K - \sum_i K_i\right] + w\left[L - \sum_i L_i\right]. \tag{2}$$

FOCs for optimal choice of inputs (K_i, L_i) are:

$$p_i F_{Li} - w = 0,$$
$$p_i F_{Ki} - r = 0, \tag{3}$$

● From (3), $MRTS_i = w/r,$ for all three industries. That is, each industry chooses input combinations only along the expansion paths, $(k_1, k_2, k_3).$ Moreover, recall that each production function $F(K_i, L_i)$ exhibits CRS.

● As pointed out earlier, the solution to (3) is not unique! Equation (3) only yields marginal conditions, which under CRS amounts to zero profit conditions (P = MC = AC), and does not yield unique employment levels. $\Rightarrow$ The Reciprocity relations should be derived by another method. One cannot derive the reciprocity relations $\left(\partial w_j / \partial p_i = \partial y_i / \partial V_j\right)$ from this problem, contrary to Leamer (1984) or Feenstra (2003).

● If prices are arbitrarily chosen, the above problem should be converted to one to find a single solution in terms of outputs that satisfies all constraints with equality or inequality (not maximizing GDP). To take into account inequality constraints, add slack variables for each inequality, z and v.

$$\mathcal{L} = p_1 y_1 + p_2 y_2 + p_3 y_3 + w\left[L - a_{L1} y_1 - a_{L2} y_2 - a_{L3} y_3 - z^2\right]$$
$$+ r[K - a_{K1} y_1 - a_{K2} y_2 - a_{K3} y_3 - v^2]. \quad (4)$$

(For simplicity, we will not explicitly include nonnegative constraints on the three goods, $y_i > 0$).

FOCs are:

$$\begin{aligned} &p_1 - (a_{L1} w + a_{k1} r) \leq 0, \quad (= 0, \quad \text{for } y_1 > 0) \\ &p_2 - (a_{L2} w + a_{k2} r) \leq 0, \quad (= 0, \ \text{for } y_2 > 0) \\ &p_3 - (a_{L3} w + a_{k3} r) \leq 0, (= 0, \quad \text{for } y_3 > 0) \end{aligned} \quad (5)$$

In addition, with respect to the slack variables,

$$-2wz = 0, \quad -2rv = 0. \quad (6)$$

● This implies that if w > 0, then z = 0 (i.e., L constraint must be binding). Likewise, if r > 0, then v = 0 (K constraint must be binding.)

● It further implies that (i) if L constraint is not binding ($z \neq 0$), then w = 0 (its shadow price is zero), and that (ii) if K constraint is not binding ($v \neq 0$), then the shadow price of K is zero (r = 0).

● Assume that two of these conditions are satisfied with equality, p_1 and p_2. These two output prices completely determine two factor prices (w,r).

$$p_1 - (a_{L1}w + a_{k1}r) = 0, \text{ for } y_1 > 0$$
$$p_2 - (a_{L2}w + a_{k2}r) = 0, \text{ for } y_2 > 0$$
$$p_3 - (a_{L3}w + a_{k3}r) < 0, \; y_3 = 0.$$

● For good 3, if $p_3 - (a_{L3}w + a_{k3}r) < 0$, then $\partial G/\partial y_3 < 0$ for $y_3 > 0$. Since a nonnegative output should be chosen, optimal output $y_3 = 0$.

Leamer (1984): Assume goods outnumber factors (n > m). If n output prices are arbitrarily chosen, only m outputs are produced.

[“If A, then B” is true, its contrapositive “Not B, then Not A” is also true.]

Choi (2003, Handbook of International Trade, Volume 1.)
● Contrapositive of Leamer’s Theorem: If more than m outputs are produced, then output prices are interdependent.

● In the real world, the number of outputs produces lies between n and m, but much closer to n.

● In the 3 × 2 case, the two output prices completely determine factor prices, which in turn determine the price of good 3. ⇒ The output vector is indeterminate.

● In the n × m case where n is much greater than m, if P = AC for all industries (i.e., prices are interdependent), the output vector is indetermiate. So is the trade vector.

● Choi (2003): In the n × 2 world, output indeterminacy implies that exports of a capital abundant need not be capital intensive.

● Jones-Deardorff's attempt to resolve output intedeterminacy. Rank industries in terms of capital intensities.

References

Leamer, Edward E., *Sources of International Comparative Advantage*, Cambridge, Mass: MIT Press, 1984.

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