

## 1B Ricardian Model with a Continuum of Goods

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- DFS assume a continuum of goods. Goods are indexed in an interval  $[0,1]$ . This interval is a real line with uncountably infinite number of goods. (This is logically inconsistent with a finite number of consumers consuming a finite number of goods.)
- Countable numbers: one-to-one correspondence to the natural numbers,  $N = \{1, 2, 3, \dots\}$ . For example, the set of even numbers has the same number of elements as  $N$ . So it is countably infinite.
- There is no one-to-one correspondence between  $N$  and the interval  $[0,1]$ . In the set of natural numbers, there is a successor to  $n$ , which is  $(n+1)$ . There is no successor for any number in  $[0,1]$ .
- Assume  $n$  goods. Rank the foreign-domestic labor requirement ratios,

$$\frac{a_{L1}^*}{a_{L1}} > \frac{a_{L2}^*}{a_{L2}} > \dots > \frac{a_{Ln}^*}{a_{Ln}} .$$

$\Rightarrow$  HC has a CA in good 1, and the FC in good  $n$ .

- Under balanced trade, one country exports at least one good and imports at least one other good.
- Whether a country imports or exports any intermediate good  $k$  ( $1 < k < n$ ) depends on the world prices, which obviously incorporate the demand and supply conditions of both countries. Accordingly, economic size matters.)

### Continuum of Goods

Assume there are infinitely many goods indexed by a variable  $z$  over the interval  $[0,1]$ . Labor requirements are written as  $a(z)$  and  $a^*(z)$ . The foreign-domestic labor requirement function is written as

$$A_L(z) = \frac{a_L^*(z)}{a_L(z)}, \quad A_L' < 0.$$

- I would call this Labor Comparative Advantage function (They do not).  $A_L(z)$  decreases as  $z \uparrow$ .

(In a world of many factors, one can derive similar functions for other factors. For example, let  $a_K(z)$  and  $a_K^*(z)$  denote the capital requirement functions. One can also have a ranking of the relative capital requirement functions,

$$A_K(z) = \frac{a_K^*(z)}{a_K(z)}, \quad A_K' < 0.$$

● No reason why labor and capital requirement functions,  $A_L(z)$  and  $A_K(z)$ , to have the same rankings. Also, there are other factors, such as natural resources, and one can use another ranking based on land or natural resource requirements,

$$A_T(z) = \frac{a_T^*(z)}{a_T(z)}, \quad A_T' < 0.$$

## Wage Ratio

Wage ratio is needed to find the cutoff or demarcation between CA and CD (comparative disadvantage) industries.

Foreign price of  $z$ :  $p^*(z) = w^*a_L^*(z)$ .

Domestic price of  $z$ :  $p(z) = wa_L(z)$ .

HC has a CA in  $z$ , if  $wa_L < w^*a_L^*$ , or

$$\frac{w}{w^*} < \frac{a_L^*(z)}{a_L(z)} \equiv A_L(z). \quad (1)$$

● Let  $Z$  be the value of  $z$ , which satisfies (1) with equality.

Then for all  $z < Z$ , the HC has a CA in  $z$ , and exports that commodity.

If both sides are multiplied by the foreign-domestic wage ratio, we get

$$1 < \frac{a_L^*(z)w^*}{a_L(z)w} = \frac{p_1^*}{p_1}.$$

● DFS did not because they were not interested in extending the model to include many factors. Even if they had, their method cannot be directly applied to recover factor prices from output prices. (This discussion is deferred until we face it again in the HO model.)

● We need to determine the wage ratio,  $w/w^*$ .

Let  $b_i = p_i x_i / I =$  budget share of good  $i$ .

● Assume balance of trade. (If trade is not balanced, this approach needs to be modified)

● Let  $b(z)$  denote the budget share of product  $z$ ,

$$b(z) = \frac{p(z)x(z)}{I}.$$

● Identical and homothetic tastes  $\Rightarrow b(z) = b^*(z)$ .  
(Homothetic tastes: to be discussed later)

● The sum of the budget shares is unity.

$$\int_0^1 b(z) dz = 1.$$

● The fraction of income spent on HC's goods is

$$\int_0^Z b(z)dz = B(Z), \quad B'(Z) = b(Z).$$

- The fraction of income spent on FC's goods is

$$B^*(Z) = \int_Z^1 b(z)dz = 1 - B(Z).$$

- HC's income is

$$wL = B(Z)(wL + w^*L^*)$$

Or

$$\frac{wL}{B} = \frac{w^*L^*}{1-B}.$$

Divide both sides by  $w^*$ . We get relative wage,

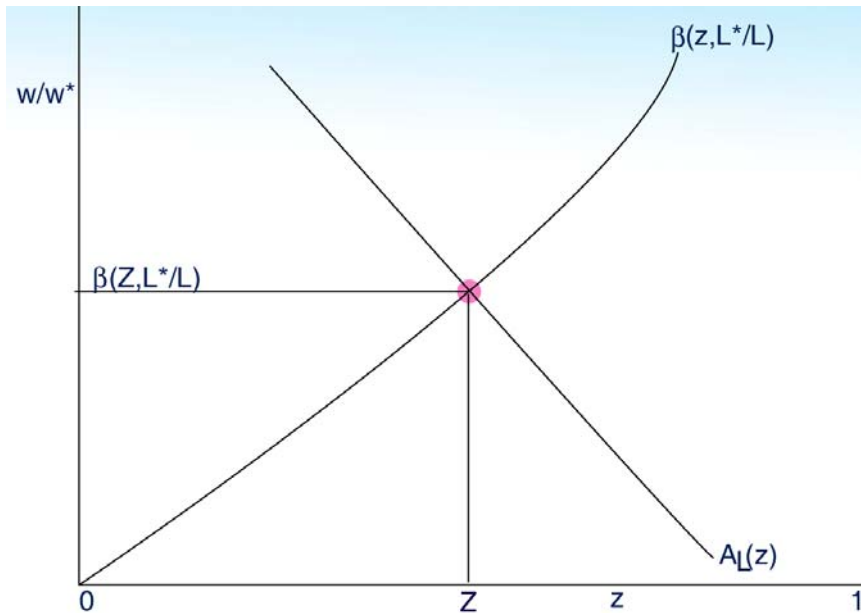
$$\frac{w}{w^*} = \frac{B(Z)}{B^*(Z)} \frac{L^*}{L} = \beta\left(Z, \frac{L^*}{L}\right).$$

- Relative Wage

Note that  $\beta(0, L^*/L) = 0$ , and  $\partial\beta / \partial Z > 0$ .

$\Rightarrow \beta(Z, L^*/L)$  is positively sloped.

The intersection of the  $\beta$  curve and  $A(z)$  curve determines the equilibrium  $w/w^*$  ratio.



- An increase in  $L$  shifts  $\beta$  curve downward, and HC exports more.
- An increase in  $L$  endowment affects the wage ratio. Not so in the HO model.
- Remark: RDS works only for one factor model. This is a limitation of the model (because it claims to be a generalization of Ricardian model).
- RDS cannot be used to extend the Ricardian to include many factors, such as  $K$  and  $T$ . Comparing labor comparative advantage is not good enough.
- For capital requirements, one can also rank the capital CA,

$$\frac{a_{K1}^*}{a_{K1}} > \frac{a_{K2}^*}{a_{K2}} > \dots > \frac{a_{Kn}^*}{a_{Kn}}. \quad (2)$$

- K CA ranking and L CA ranking are different.
- (Choi) Compare foreign-domestic price ratios under balanced free trade condition,

$$\frac{p_1^*}{p_1} > \frac{p_2^*}{p_2} > \dots > \frac{p_k^*}{p_k} > 1 > \dots > \frac{p_n^*}{p_n}. \quad (3)$$

- These prices (in lowercase) are prices under balanced free trade scenario. These are not autarky prices.
- Need to prove that exporting the first k goods and importing the rest balances the budget or trade.
- Let  $P_1^A, P_1^B$  denote the autarky prices of countries A and B, so as to distinguish them from their prices under the balanced trade scenario.
- Comparing autarky prices of a good, one cannot predict whether the country with a lower autarky price will export it, although it is very likely.

- If  $P_1^A < P_1^B$ , there is no guarantee that country A will necessarily export good 1. It does not guarantee that under balanced trade, the price of good 1 in country A is lower than that in country B ( $p_1^A < p_1^B$ ).
- We cannot compare only the domestic-foreign K or L requirements, etc. Go back to the other model of Choi (a previous note) that compares price ratios, which works when there are many factors.