## COMPETITION

## 1. Assumptions of Perfect Competition

1.: All firms produce homogeneous goods and consumers view them as identical.
2.: All buyers and sellers have perfect information regarding the price and quality of the product.
3.: Buyers and sellers cannot influence the price at which the product can be purchased or sold. In other words buyers and sellers are price takers.
4.: There are no transaction costs of participating in the market.
5.: Each firm bears the full cost of its production process. That is, there are no externalities that the firm imposes on others. An example of an externality would be pollution.
6.: Firms can enter and exit the industry quickly at any time without having to incur special expenses. Thus firms have freedom of entry and exit.
7.: There is perfect divisibility of output.

## 2. Behavior of a Single Competitive Firm

2.1. The profit maximization decision: Choosing the optimal output level. The profit maximization decision is often divided into two steps. In the first step the firm determines the minimum cost way to produce any given output level given fixed input prices. In the second step the firm determines the optimal level of output by maximizing profit given the cost of producing any output level from the first step. The cost function for a firm is derived by determining the least costly way to produce each output level. The cost function is defined for all possible output price vectors and all positive input price vectors $w=\left(w_{1}, w_{2}, \ldots\right.$, $\left.\mathrm{w}_{n}\right)$. An output vector, y , is producible if y belongs to the effective domain of $\mathrm{V}(\mathrm{y})$, i.e,

$$
\operatorname{Dom} V=\left\{y \in R_{+}^{m}: V(y) \neq \emptyset\right\}
$$

where $\mathrm{V}(\mathrm{y})$ is the input requirement set for the output vector $y$. The cost function does not exist it there is no technical way to produce the output in question. The cost function is defined by

$$
\begin{equation*}
C(y, w)=\min _{x}\{w x: x \in V(y)\}, \quad y \in \operatorname{Dom} V, w>0 \tag{1}
\end{equation*}
$$

The single output firm solves the following problem to obtain its cost function

$$
\begin{equation*}
C(w, y)=\min _{x}\left\{\Sigma_{j=1}^{n} w_{j} x_{j}: f(x) \geq y\right\} \tag{2}
\end{equation*}
$$

The firm will choose the levels of the various inputs $x_{1}, x_{2}, \ldots x_{n}$ in a way that minimizes the cost of producing a target level of output $y$. The firm then maximizes profits by choosing the level of output. Specifically it has the following maximization problem

$$
\begin{equation*}
\pi(p, w)=\max _{y}[p y-C(y, w)] \tag{3}
\end{equation*}
$$

where $\pi(\mathrm{p}, \mathrm{w})$ is the maximum profits attainable when output sells for the price $p$, and the vector of input prices is $w$. Here, $y$ is output and $C(y, w)$ is the function giving the minimum cost of producing output y with input prices w . In a competitive market it is assumed that the firm cannot affect the output price $p$ or the input prices $w$. It is profitable for the firm to expand output as long as the increase in revenue

[^0](py) is larger than the increase in cost, $C(y, w)$. The increase in revenue from the sale of one more unit of output is p , while the cost of producing one more unit of output is given by marginal cost $\frac{d C\left(y, w_{1}, w_{2}, \ldots, w_{n}\right)}{d y}$. At the optimum, the costs will be such that $p=\frac{d C\left(y, w_{1}, w_{2}, \ldots, w_{n}\right)}{d y}$. This result can be seen formally by differentiating the expression for profit (equation 2)with respect to $y$. We obtain
\[

$$
\begin{align*}
\frac{d \pi}{d y} & =p-\frac{d C(y, w)}{d y}=0  \tag{4}\\
\Rightarrow p & =M C
\end{align*}
$$
\]

where MC is the cost of producing the last unit of output or marginal cost. The optimal output level is a function of the price of the output and the prices of the various inputs $w$. We usually denote this function as $y(p, w)$.
2.2. Long-term versus short-term decisions. If some of the firm's inputs are fixed and cannot be varied in the current decision period, then we define the cost function based on this fact. If the variable inputs are denoted $x$ and the fixed ones $z$, then the cost function is given by

$$
\begin{align*}
C(y, w, z) & =w_{z} z+\min _{x}\left[\sum_{j=1}^{n} w_{j} x_{j}\right] \text { such that }(x, z) \in V(y)  \tag{5}\\
& =w_{z} z+\min _{x}\left[w_{1} x_{1}+w_{2} x_{2}+\ldots+w_{n} x_{n}\right] \text { such that } y=f(x, z)
\end{align*}
$$

where the vector of fixed and variable inputs ( $\mathrm{x}, \mathrm{z}$ ) must be able to produce the output level y . The technology can be represented by either the input requirement set $V(y)$ or the production function $y=f(x$, $z)$. Here, $\mathrm{w}_{z} z$ is the fixed cost, and the remainder is the variable cost. In this case the profit maximization problem is given by

$$
\begin{equation*}
\pi(p, w, z)=\max _{y}[p y-C(y, w, z)] \tag{6}
\end{equation*}
$$

where the optimum output is now a function of $z$, i.e. $y=y(p, w, z)$. Graphically we can see this solution in figure 1. The firm will produce at the output level $y_{0}$ when the price is $p_{0}$. At a higher output level marginal cost would exceed price. Profits are the box bounded by $p_{0}, \mathrm{~b}$, and $\mathrm{p}_{A C}$ where $\mathrm{p}_{A C}$ is the average cost of production at output $y_{0}$. If the price rises above $p_{0}$ the firm's profits will rise while if it falls below $\mathrm{p}_{c}$, profits will be negative at output level $\mathrm{y}_{0}$. At any price above average total cost for a given output level, the firm will have positive profits. At prices less than average total cost the firm may still cover variable costs if the price is above the average variable cost at a given output level. If the price is below average variable cost at a given output level then the firm loses money by producing at that output level, even in the short-run.

### 2.3. The shutdown decision.

a.: The firm produces only if doing so is more profitable than not producing which means that it produces only if the revenues (py) from production exceed avoidable costs. Avoidable costs are those costs that are not incurred if the firm ceases production. They include variable costs and the subset of fixed costs that are not sunk. Revenue earned in excess of avoidable costs is called quasirent. Quasi-rents are thus revenues in excess of the amount necessary to keep the firm operating in the short run. If all fixed costs are sunk, then avoidable costs are equivalent to variable costs.
b.: If all fixed costs are sunk then the minimum point on the average variable cost curve represents the price level below which the firm will not produce in the short run. Consider the diagram above and a price that lies everywhere below the average total cost curve but above the average variable

cost curve at some output levels. By producing where price equals marginal cost at any of these output levels the firm will have revenue (py) that exceeds average variable costs at that level and thus will remain in business since it can use the revenues in excess of the average variable cost as a payment toward covering the fixed (and in this case sunk) costs. If the price available to the firm lies anywhere below the average variable cost curve ( $\mathrm{p}_{\mathrm{s}}$ in the diagram), the firm will be better off by shutting down and simply eating its fixed costs, since production will lead to no revenue above the avoidable costs. Thus the shutdown decision depends only on avoidable cost. Sunk costs are not relevant to this decision.
c.: At the minimum point on any average cost curve, average cost will be equal to marginal cost. We can find the level of $y$ at which average cost has its minimum by setting average cost equal to marginal cost and then solving for $y$.
2.4. The supply curve for a competitive firm. The supply curve of the firm is equivalent to the marginal cost curve above the minimum point of average variable cost. At prices less than this cost the firm will supply zero because the value of output is less than variable (avoidable) cost. The supply curve is made up of two parts, a portion where it is zero because prices are less than variable (avoidable) costs, and a portion where it is equivalent to the marginal cost curve for the firm. In figure 2 , the supply curve is as in the diagram below, where the darker portion of the curve represents the short-run supply function. In the long run, if prices are not greater than minimum average total cost, the firm will not replace the machinery, equipment, fixed labor etc. that leads to the fixed costs associated with production and will shut down since it cannot cover these costs at price $\mathrm{p}_{s}$. The less fixed costs the firm has, the closer the average variable and average total cost curves will be, and the closer the short and long-run shut-down decisions will be as well.
2.5. Profit maximization example with always upward sloping variable costs. Consider a firm with cost function as follows

Figure 2. Short Run Supply Curve


$$
\begin{equation*}
C\left(y, w_{1}, w_{2}, \ldots, w_{n}\right)=400+16 y+y^{2} \tag{7}
\end{equation*}
$$

Fixed cost is equal to 400 while variable cost is

$$
\begin{equation*}
V C\left(y, w_{1}, w_{2}, \ldots, w_{n}\right)=16 y+y^{2} \tag{8}
\end{equation*}
$$

Marginal cost (MC)is the derivative of the variable or total cost function. For this problem we obtain

$$
\begin{equation*}
M C\left(y, w_{1}, w_{2}, \ldots, w_{n}\right)=16+2 y \tag{9}
\end{equation*}
$$

We can find average variable cost (AVC) and average (AC) cost by dividing costs by the output level y as follows

$$
\begin{align*}
A C\left(y, w_{1}, w_{2}, \cdots, w_{n}\right) & =\frac{400+16 y+y^{2}}{y} \\
& =\frac{400}{y}+16+y  \tag{10}\\
A V C\left(y, w_{1}, w_{2}, \cdots, w_{n}\right) & =\frac{16 y+y^{2}}{y} \\
& =16+y
\end{align*}
$$

The minimum point of the average variable cost curve is found by setting AVC = MC and solving for y . This gives

$$
\begin{align*}
& \operatorname{AVC}\left(y, w_{1}, w_{2}, \cdots, w_{n}\right)=16+y=16+2 y=M C\left(y, w_{1}, w_{2}, \cdots, w_{n}\right) \\
& \Rightarrow 16+y=16+2 y \\
& \Rightarrow y=2 y  \tag{11}\\
& \Rightarrow y=0 \\
& \Rightarrow A V C_{\min }=16
\end{align*}
$$

So in the short run the firm will not produce if prices are less than $\$ 16$. The minimum point of the average cost curve is found by setting $\mathrm{AC}=\mathrm{MC}$ and solving for y . This gives

$$
\begin{align*}
A C\left(y, w_{1}, w_{2}, \cdots, w_{n}\right)=\frac{400}{y}+16+y & =16+2 y=M C\left(y, w_{1}, w_{2}, \cdots, w_{n}\right) \\
\Rightarrow \frac{400}{y}+16+y & =16+2 y \\
\Rightarrow \frac{400}{y}+y & =2 y \\
\Rightarrow \frac{400}{y}-y & =0  \tag{12}\\
\Rightarrow 400-y^{2} & =0, y \neq 0 \\
\Rightarrow 400 & =y^{2} \\
\Rightarrow y & =20 \\
\Rightarrow A C_{\min } & =56
\end{align*}
$$

So in the long run the firm will not produce if prices are less than $\$ 56$. The portion of the supply function where the firm produces is given by setting price equal to marginal cost and solving for $y$. For this problem this gives

$$
\begin{align*}
p & =16+2 y=M C\left(y, w_{1}, w_{2}, \cdots, w_{n}\right) \\
\Rightarrow p & =16+2 y \\
\Rightarrow 2 y & =p-16  \tag{13}\\
\Rightarrow y(p) & =\frac{1}{2} p-8
\end{align*}
$$

In the short run this is relevant if the price is greater than $\$ 16$. In the long run this is relevant if the price is greater than $\$ 56$. So we can write the short run supply function for the firm as follows

$$
y= \begin{cases}\frac{1}{2} p-8, & p \geq 16  \tag{14}\\ 0, & p<16\end{cases}
$$

The long run supply function is given by

$$
y= \begin{cases}\frac{1}{2} p-8, & p \geq 56  \tag{15}\\ 0, & p<56\end{cases}
$$

Now let the price of output for this firm be $\$ 80$. The firm will maximize profits by setting price equal to marginal cost and solving for y . This gives

$$
\begin{align*}
p=80 & =16+2 y=M C\left(y, w_{1}, w_{2}, \cdots, w_{n}\right) \\
\Rightarrow 80 & =16+2 y \\
\Rightarrow 2 y & =64 \\
\Rightarrow y^{*} & =32  \tag{16}\\
\Rightarrow A V C\left(y^{*}\right) & =48 \\
\Rightarrow A C\left(y^{*}\right) & =60.5
\end{align*}
$$

We could get the same answer by plugging $\$ 80$ into the supply equation as in

$$
\begin{align*}
y(p) & =\frac{1}{2} p-8 \\
& =\frac{1}{2}(80)-8  \tag{17}\\
& =40-8=32
\end{align*}
$$

Price is higher than average variable and average total cost and the firm makes a profit. Profit is given by

$$
\begin{align*}
\pi & =p y-C\left(y, w_{1}, w_{2}, \ldots, w_{n}\right) \\
& =80 y-\left(400+16 y+y^{2}\right) \\
& =(80)(32)-\left(400+(16)(32)+32^{2}\right)  \tag{18}\\
& =2560-400-512-1024 \\
& =624
\end{align*}
$$

In figure 3, we can see that at price of $\$ 80$, the firm will produce 32 units of output and more than cover its total costs.

Now consider a drop in prices to $\$ 36$. If we set price equal to marginal cost we obtain

$$
\begin{align*}
p=36 & =16+2 y=M C\left(y, w_{1}, w_{2}, \ldots, w_{n}\right) \\
\Rightarrow 36 & =16+2 y \\
\Rightarrow 2 y & =20 \\
\Rightarrow y^{*} & =10  \tag{19}\\
\Rightarrow A V C\left(y^{*}\right) & =26 \\
\Rightarrow A C\left(y^{*}\right) & =66
\end{align*}
$$

The firm will cover its variable costs but will not cover its fixed costs. So the firm will shut down in the long run at prices of $\$ 36$.

Now assume that $\$ 225$ of the fixed cost is avoidable if the firm shuts down. The avoidable cost function (VDC) is then given by

$$
\begin{equation*}
V D C\left(y, w_{1}, w_{2}, \cdots, w_{n}\right)=225+16+y^{2} \tag{20}
\end{equation*}
$$

Average avoidable cost (AVDC) is given by

Figure 3. Profit Maximization Example with Upward Sloping Average Variable Costs


$$
\begin{align*}
\operatorname{AVDC}\left(y, w_{1}, w_{2}, \ldots, w_{n}\right) & =\frac{225+16 y+y^{2}}{y}  \tag{21}\\
& =\frac{225}{y}+16+y
\end{align*}
$$

The minimum of average avoidable cost $\left(\mathrm{AVDC}_{m} i n\right)$ is obtained by setting $\mathrm{AVDC}=\mathrm{MC}$ and solving for y

$$
\begin{align*}
A V D C\left(y, w_{1}, w_{2}, \ldots, w_{n}\right)=\frac{225}{y}+16+y & =16+2 y=M C\left(y, w_{1}, w_{2}, \ldots, w_{n}\right) \\
\Rightarrow \frac{225}{y}+16+y & =16+2 y \\
\Rightarrow \frac{225}{y}+y & =2 y \\
\Rightarrow \frac{225}{y}-y & =0  \tag{22}\\
\Rightarrow 225-y^{2} & =0, y \neq 0 \\
\Rightarrow 225 & =y^{2} 0 \\
\Rightarrow y & =15 \\
\Rightarrow A V D C_{\min } & =46
\end{align*}
$$

Now the firm will shut down in the short run if prices fall below $\$ 46$ rather than $\$ 16$ because the firm can get rid of some of the fixed costs if it shuts down. So the short run avoidable) cost function is given by

$$
y= \begin{cases}\frac{1}{2} p-8, & p \geq 46  \tag{23}\\ 0, & p<46\end{cases}
$$

We represent this graphically in the figure 4 which shows the various cost curves and prices equal to $\$ 80$ and \$36.


At a price of $\$ 80$, the firm should produce 32 units of output. The price is higher than average variable or average total cost. At a price of $\$ 36$, the output where price is equal to marginal cost is $y=10$. While this price does not cover average total cost, it is higher than the average variable cost of $\$ 26$ in figure 3 . When avoidable fixed costs are $\$ 225$, the $\$ 80$ price covers all costs as before. But the price of $\$ 36$ (and the $\mathrm{P}=\mathrm{MC}$ output of 10 units) does not generate enough revenue to cover avoidable costs. So the firm will shut down in the short run and in the long run at these prices.
2.6. Profit maximization example with $U$-shaped variable costs. Consider a firm with cost function as follows

$$
\begin{equation*}
C\left(y, w_{1}, w_{2}, \ldots, w_{n}\right)=600+200 y-20 y^{2}+y^{3} \tag{24}
\end{equation*}
$$

Fixed cost is equal to 600 while variable cost is

$$
\begin{equation*}
V C\left(y, w_{1}, w_{2}, \ldots, w_{n}\right)=200 y-20 y^{2}+y^{3} \tag{25}
\end{equation*}
$$

Marginal cost (MC)is the derivative of the variable or total cost function. For this problem we obtain

$$
\begin{equation*}
M C\left(y, w_{1}, w_{2}, \ldots, w_{n}\right)=200-40 y+3 y^{2} \tag{26}
\end{equation*}
$$

We can find average variable cost (AVC) and average (AC) cost by dividing costs by the output level y as follows

$$
\begin{align*}
A C\left(y, w_{1}, w_{2}, \cdots, w_{n}\right) & =\frac{600+200 y-20 y^{2}+y^{3}}{y} \\
& =\frac{600}{y}+200-20 y+y^{2}  \tag{27}\\
A V C\left(\left(y, w_{1}, w_{2}, \ldots, w_{n}\right)\right. & =\frac{200 y-20 y^{2}+y^{3}}{y} \\
& =200-20 y+y^{2}
\end{align*}
$$

The minimum point of the average variable cost curve is found by setting AVC $=\mathrm{MC}$ and solving for y . This gives

$$
\begin{align*}
A V C\left(y, w_{1}, w_{2}, \ldots, w_{n}\right)=200-20 y+y^{2} & =200-40 y+3 y^{2}=M C\left(y, w_{1}, w_{2}, \ldots, w_{n}\right) \\
\Rightarrow 20 y-2 y^{2} & =0 \\
\Rightarrow 10 y-y^{2} & =0  \tag{28}\\
\Rightarrow y(10-y) & =0 \\
\Rightarrow y & =0, \text { or } y=10
\end{align*}
$$

There are 2 roots in this case. At $\mathrm{y}=0, \mathrm{AVC}=200$ and is a maximum. At $\mathrm{y}=10, \mathrm{AVC}=100$ and is a minimum. So $y=10$ is the appropriate root. So in the short run the firm will not produce if prices are less than $\$ 100$. Finding the minimum of the average cost curve in this case involves solving a cubic equation. Solving it numerically we obtain $y=12.062$ and cost $=153.995 \approx 154$. By examining table 1 , it is also clear that AC reaches a minimum at around 12 units of output with an average cost of approximately $\$ 154$. The portion of the supply function where the firm produces is given by setting price equal to marginal cost and solving for y . For this problem this gives

$$
\begin{align*}
& p=200-40 y+3 y^{2}=M C\left(y, w_{1}, w_{2}, \cdots, w_{n}\right) \\
& \Rightarrow p=200-40 y+3 y^{2}  \tag{29}\\
& \Rightarrow 3 y^{2}-40 y+(200-p)=0
\end{align*}
$$

This is a general quadratic equation. We solve using the quadratic formula and pick the larger root.

$$
\begin{align*}
& 3 y^{2}-40 y+(200-p)=0 \\
& \Rightarrow y=\frac{40+\sqrt{1600-(4)(3)(200-p)}}{6} \\
& \Rightarrow y=\frac{40+\sqrt{1600-2400+12 p}}{6}  \tag{30}\\
& \Rightarrow y=\frac{40+\sqrt{12 p-800}}{6}
\end{align*}
$$

This is relevant in the short run as long as the price is greater than $\$ 100$. In the long run this is relevant if the price is greater than approximately $\$ 154$. So we can write the short run supply function for the firm as follows

$$
y= \begin{cases}\frac{40+\sqrt{12 p-800}}{6}, & p \geq 100  \tag{31}\\ 0, & p<100\end{cases}
$$

The long run supply function is given by

$$
y= \begin{cases}\frac{40+\sqrt{12 p-800}}{6}, & p \geq 154  \tag{32}\\ 0, & p<154\end{cases}
$$

Table 1. Cost, Revenue, and Profit Data

| $\mathrm{c}\left(\mathrm{y}, \mathrm{w}_{1}, \mathrm{w}_{2}, \ldots, \mathrm{w}_{n}\right)=600+200 \mathrm{y}-20 \mathrm{y}^{2}+\mathrm{y}^{3}, \mathrm{y}=\$ 275$ |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| y | FC | VC | C | AVC | ATC | MC | P | Revenue | Profit |
| 0 | 600 | 0 | 600 |  |  |  | 275 | 0 | -600 |
| 1 | 600 | 181 | 781 | 181 | 781 | 163 | 275 | 275 | -506 |
| 2 | 600 | 328 | 928 | 164 | 464 | 132 | 275 | 550 | -378 |
| 3 | 600 | 447 | 1047 | 149 | 349 | 107 | 275 | 825 | -222 |
| 4 | 600 | 544 | 1144 | 136 | 286 | 88 | 275 | 1100 | -44 |
| 5 | 600 | 625 | 1225 | 125 | 245 | 75 | 275 | 1375 | 150 |
| 6 | 600 | 696 | 1296 | 116 | 216 | 68 | 275 | 1650 | 354 |
| 7 | 600 | 763 | 1363 | 109 | 194.71 | 67 | 275 | 1925 | 562 |
| 8 | 600 | 832 | 1432 | 104 | 179 | 72 | 275 | 2200 | 768 |
| 9 | 600 | 909 | 1509 | 101 | 167.67 | 83 | 275 | 2475 | 966 |
| 10 | 600 | 1000 | 1600 | 100 | 160 | 100 | 275 | 2750 | 1150 |
| 11 | 600 | 1111 | 1711 | 101 | 155.55 | 123 | 275 | 3025 | 1314 |
| 12 | 600 | 1248 | 1848 | 104 | 154 | 152 | 275 | 3300 | 1452 |
| 13 | 600 | 1417 | 2017 | 109 | 155.15 | 187 | 275 | 3575 | 1558 |
| 14 | 600 | 1624 | 2224 | 116 | 158.86 | 228 | 275 | 3850 | 1626 |
| 15 | 600 | 1875 | 2475 | 125 | 165 | 275 | 275 | 4125 | 1650 |
| 16 | 600 | 2176 | 2776 | 136 | 173.50 | 328 | 275 | 4400 | 1624 |
| 17 | 600 | 2533 | 3133 | 149 | 184.29 | 387 | 275 | 4675 | 1542 |
| 18 | 600 | 2952 | 3552 | 164 | 197.33 | 452 | 275 | 4950 | 1398 |
| 19 | 600 | 3439 | 4039 | 181 | 212.58 | 523 | 275 | 5225 | 1186 |
| 20 | 600 | 4000 | 4600 | 200 | 230 | 600 | 275 | 5500 | 900 |

Now let the price of output for this firm be $\$ 275$ The firm will maximize profits by setting price equal to marginal cost and solving for $y$. This gives

$$
\begin{align*}
p=275 & =200-40 y+3 y^{2}=M C\left(y, w_{1}, w_{2}, \cdots, w_{n}\right) \\
\Rightarrow 3 y^{2}-40 y-75 & =0 \\
\Rightarrow(3 y+5)(y-15) & =0  \tag{33}\\
\Rightarrow y & =15, \text { or } y=-\frac{3}{5}
\end{align*}
$$

The appropriate root is $\mathrm{y}=15$. This gives an average total cost of $\$ 165$ and an average variable cost of $\$ 125$. We could obtain the same answer by plugging $\$ 275$ into the supply equation as follows

$$
\begin{align*}
y(p) & =\frac{40+\sqrt{12 p-800}}{6} \\
& =\frac{40+\sqrt{(12)(275)-800}}{6} \\
& =\frac{40+\sqrt{3300-800}}{6}  \tag{34}\\
& =\frac{40+\sqrt{2500}}{6} \\
& =\frac{40+50}{6} \\
& =\frac{90}{6}=15
\end{align*}
$$

We can see the solution graphically in figure 5 .

Figure 5. Profit Maximization Example with U-Shaped Average Variable Costs, Price $=275$


Price is higher than average variable and average total cost and the firm makes a profit. Profit is given by

$$
\begin{aligned}
\pi & =p y-C\left(y, w_{1}, w_{2}, \ldots, w_{n}\right) \\
& =275 y-\left(600+200 y-20 y^{2}+y^{3}\right) \\
& =(275)(15)-(600+(200)(15)-(20)(225)+3375) \\
& =4125-2475 \\
& =1650
\end{aligned}
$$

Now consider a drop in prices to $\$ 123$. If we set price equal to marginal cost we obtain

$$
\begin{aligned}
p=123 & =200-40 y+3 y^{2}=M C\left(y, w_{1}, w_{2}, \cdots, w_{n}\right) \\
\Rightarrow 3 y^{2}-40 y+77 & =0 \\
\Rightarrow(3 y-7)(y-11) & =0 \\
\Rightarrow y & =11, \text { or } y=\frac{3}{7}
\end{aligned}
$$

The appropriate root is $y=11$. With an output of 11 units average cost is $\$ 155.55$ and average variable cost is $\$ 101$. The firm covers variable costs but not total costs and will shut down in the long run. We can see this in figure 6 where price is equal to $p=\$ 123$.

Figure 6. Profit Maximization Example with U-Shaped Average Variable Costs, Price $=123$


Price is equal to marginal cost at an output level of $y=11$. This price is higher than average variable cost but less then average total cost in the diagram.

## 3. The Competitive Industry and Market Equilibrium

3.1. The short-run industry supply curve. The industry supply curve, which we denote $Q$, is the horizontal summation of the individual firm supply curves $y(p, w, z)$ accounting for the fact that $y(p, w, z)$ will be zero at some price levels. Thus

$$
\begin{align*}
Q & =\sum_{\ell=1}^{L} y_{\ell}(p, w, z)  \tag{36}\\
& =y_{1}(p, w, z)+y_{2}(p, w, z)+\ldots+y_{L}(p, w, z)
\end{align*}
$$

If there are L identical firms this will be given by

$$
\begin{equation*}
Q=L * y(p, w, z) \tag{37}
\end{equation*}
$$

If the firms are identical, the supply of an individual firm is represented graphically figure 7 . There will be no output below price $\mathrm{p}_{A V C}$.

Figure 7. Short Run Industry Supply


If we horizontally sum the supply curves for 20 identical firms we obtain the industry supply curve in figure 8 . Output will be $L$ times an individual firm's supply at prices above $p_{A V C}$. If the price in figure 8 is given by $p_{0}$, industry supply will be $L y_{0}=Q$.
3.2. Short-run market equilibrium. Now consider adding the demand side to the market. Consider then a linear market demand curve given by $\mathrm{D}(\mathrm{p})$ in figure 9 . The market equilibrium will occur where $\mathrm{D}(\mathrm{p})$ intersects the industry supply curve. This occurs at price $p_{0}$ in figure 9 . At the price level $p_{0}$, the market is at equilibrium. With this price, individual firms earn a profit as is clear from figure 10. Thus there will be an incentive for other firms to enter the industry.

Figure 8. Short Run Industry Supply


Figure 9. Short Run Industry Equilibrium

3.3. Long-run market equilibrium. If a firm is not in the industry and the price is above its minimum long-run average total cost, then this firm has an incentive to enter. If entrance occurs, the industry supply

Figure 10. Profits at Short Run Industry Equilibrium

curve will shift out and price will fall in as in figure 11. The long-run equilibrium is determined by the intersection of the demand curve and long-run industry supply curve. In long run equilibrium, the number of firms in the industry, $\mathrm{L}_{l e}$, is such that the industry is in both short- and long-run equilibrium. In this equilibrium, the demand curve intersects both the long-run and the short-run supply curves, corresponding to the equilibrium number of firms, at the price $\mathrm{p}_{l e}$ which is equal to the minimum of long average cost, $\left(\mathrm{p}_{A C}\right.$ in figure 11), and equilibrium output $\mathrm{Q}_{l e}$ This output is given by

$$
Q_{l e}=L_{l e} y_{l e}
$$

In the long run when firms can enter or exit freely and there are no sunk costs, if all potential firms are identical, the supply curve for the industry will be horizontal at the minimum of the long run average cost curve.
3.4. An example industry equilibrium with two heterogeneous firms. Consider a market equilibrium model where there are only two price taking firms. Assume that they behave competitively (are price takers) even though they could behave in a non-competitive manner. Assume that there is a linear market inverse demand curve given by

$$
\begin{equation*}
p=75-Q \tag{38}
\end{equation*}
$$

The cost functions for the two (only) firms in the industry are given by

$$
\begin{align*}
& \operatorname{cost}\left(y_{1}\right)=100+5 y_{1}+y_{1}^{2} \\
& \operatorname{cost}\left(y_{2}\right)=200+10 y_{2}+0.5 y_{2}^{2} \tag{39}
\end{align*}
$$

The marginal cost equations are

Figure 11. Long Run Industry Equilibrium


Average costs for each firm are given by

$$
\begin{align*}
A C\left(y_{1}\right) & =\frac{100+5 y_{1}+y_{1}^{2}}{y_{1}} \\
& =\frac{100}{y_{1}}+5+y_{1} \\
A V C\left(y_{1}\right) & =5+y_{1} \\
A C\left(y_{2}\right) & =\frac{200+10 y_{2}+0.5 y_{2}^{2}}{y_{2}}  \tag{41}\\
& =\frac{200}{y_{2}}+10+0.5 y_{2} \\
A V C\left(y_{2}\right) & =10+0.5 y_{2}
\end{align*}
$$

The long run shut down price for each firm is determined by setting average cost equal to marginal cost and solving for y . First for firm 1.

$$
\begin{align*}
A C\left(y_{1}\right)=\frac{100}{y_{1}}+5+y_{1} & =5+2 y_{1}=M C\left(y_{1}\right) \\
\Rightarrow \frac{100}{y_{1}}+y_{1} & =2 y_{1} \\
\Rightarrow \frac{100}{y_{1}}-y_{1} & =0  \tag{42}\\
\Rightarrow 100-y_{1}^{2} & =0 \\
\Rightarrow y_{1}^{2} & =100 \\
\Rightarrow y_{1} & =10 \\
\Rightarrow A C_{\min }\left(y_{1}\right) & =A C(10)=25
\end{align*}
$$

Now for the firm 2.

$$
\begin{align*}
A C\left(y_{2}\right)=\frac{200}{y_{2}}+10+0.5 y_{2} & =10+y_{2}=M C\left(y_{2}\right) \\
\Rightarrow \frac{200}{y_{2}}+0.5 y_{2} & =y_{2} \\
\Rightarrow \frac{200}{y_{2}}-0.5 y_{2} & =0 \\
\Rightarrow 200-0.5 y_{2}^{2} & =0  \tag{43}\\
\Rightarrow 0.5 y_{2}^{2} & =200 \\
\Rightarrow y_{2}^{2} & =400 \\
\Rightarrow y_{2} & =20 \\
\Rightarrow A C_{\min }\left(y_{2}\right) & =A C(20)=30
\end{align*}
$$

So firm 1 will not produce at prices less than $\$ 25$ and firm 2 will not produce at prices less than $\$ 30$ in the long run. The short run shut down price for each firm is determined by setting average variable cost equal to marginal cost and solving for y. First for firm 1.

$$
\begin{align*}
A V C\left(y_{1}\right)=5+y_{1} & =5+2 y_{1}=M C\left(y_{1}\right) \\
y_{1} & =2 y_{1} \\
\Rightarrow y_{1} & =0  \tag{44}\\
\Rightarrow A V C_{\min }\left(y_{1}\right) & =A V C(0)=5
\end{align*}
$$

Now for the firm 2.

$$
\begin{align*}
A C\left(y_{2}\right)=10+0.5 y_{2} & =10+y_{2}=M C\left(y_{2}\right) \\
\Rightarrow 0.5 y_{2} & =y_{2} \\
\Rightarrow y_{2} & =0  \tag{45}\\
\Rightarrow A V C_{\min }\left(y_{2}\right) & =A V C(0)=10
\end{align*}
$$

So firm 1 will not produce at prices less than $\$ 5$ and firm 2 will not produce at prices less than $\$ 10$ in the short run. In equilibrium the total supplied by both firms will equal the market demand

$$
\begin{equation*}
Q=y_{1}+y_{2} \tag{46}
\end{equation*}
$$

We can rewrite the inverse demand function as

$$
\begin{align*}
p & =75-Q \\
\Rightarrow p & =75-y_{1}-y_{2}  \tag{47}\\
\Rightarrow y_{1}+y_{2} & =75-p
\end{align*}
$$

Each firm will maximize profits by setting marginal cost equal to price and then solving for $\mathrm{y}_{i}$ to determine its supply function

$$
\begin{align*}
M C\left(y_{1}\right) & =5+2 y_{1}=p \\
\Rightarrow 2 y_{1} & =p-5 \\
\Rightarrow y_{1} & =\frac{1}{2} p-\frac{5}{2}  \tag{48}\\
M C\left(y_{2}\right) & =10+y_{2}=p \\
\Rightarrow y_{2} & =p-10
\end{align*}
$$

Total supply $(\mathrm{Q})$ is given by adding $\mathrm{y}_{1}$ and $\mathrm{y}_{2}$.

$$
\begin{align*}
y_{1}+y_{2} & =Q=\left(\frac{1}{2} p-\frac{5}{2}\right)+(p-10) \\
& =\frac{3}{2} p-\frac{25}{2}  \tag{49}\\
& =\frac{3}{2} p-12.5
\end{align*}
$$

This will be relevant in the short run if price is higher than $\$ 10$ and in the long run if price is higher than $\$ 30$. The short run supply when price is between $\$ 5$ and $\$ 10$ will be the short run supply of firm 1 or

$$
\begin{equation*}
y=y_{1}=\frac{1}{2} p-\frac{5}{2} \tag{50}
\end{equation*}
$$

Short run supply is then written

$$
y= \begin{cases}\frac{3}{2} p-\frac{25}{2}, & p \geq 10  \tag{51}\\ \frac{1}{2} p-\frac{5}{2}, & 5 \leq p \leq 10 \\ 0, & p<5\end{cases}
$$

Long run supply is

$$
y= \begin{cases}\frac{3}{2} p-\frac{25}{2}, & p \geq 30  \tag{52}\\ \frac{1}{2} p-\frac{5}{2}, & 25 \leq p \leq 30 \\ 0, & p<25\end{cases}
$$

We can then set supply equal to demand and solve for price as follows

$$
\begin{align*}
Q_{S}=y_{1}+y_{2} & =\frac{3}{2} p-\frac{25}{2}=75-p=Q_{D} \\
\Rightarrow \frac{5}{2} p & =\frac{175}{2}  \tag{53}\\
\Rightarrow 5 p & =175 \\
\Rightarrow p & =35
\end{align*}
$$

Because the market equilibrium price is greater than either firms shutdown price, this equilibrium is sustainable. The quantities supplied by each firm are found by substituting the price in the individual supply equations

$$
\begin{align*}
y_{1} & =\frac{1}{2} p-\frac{5}{2} \\
& =\left(\frac{1}{2}\right)(35)-\frac{5}{2} \\
& =\frac{30}{2}=15  \tag{54}\\
y_{2} & =p-10 \\
& =35-10=25
\end{align*}
$$

This gives a market supply of 40. Profit for firm 1 and firm 2 is obtained by substituting $p$ and $y_{i}$ in the expressions for profit.

$$
\begin{align*}
\pi\left(y_{1}\right) & =(35)(15)-100-(5)(15)-15^{2} \\
& =525-100-75-225 \\
& =125 \\
\pi\left(y_{2}\right) & =(35)(25)-200-(10)(25)-(0.5) 25^{2}  \tag{55}\\
& =875-200-250-(0.5)(625) \\
& =425-312.5 \\
& =112.5
\end{align*}
$$

This is a short run equilibrium in the sense that other firms will want to enter the industry.
3.5. An example industry equilibrium with forty identical firms. Consider a market equilibrium model where there are forty identical price taking firms. Assume that there is a market demand curve given by

$$
\begin{equation*}
Q^{D}=2600-50 p \tag{56}
\end{equation*}
$$

Denote the output of a representative firm by $q$ and aggregate output by $\mathrm{Q}^{S}=40 \mathrm{q}$. The long-run cost function for each firm in the industry is given by

$$
\begin{equation*}
\operatorname{cost}(q)=50+2 q+0.1 q^{2} \tag{57}
\end{equation*}
$$

The marginal cost equation is

$$
\begin{equation*}
M C(q)=0.2 q+2 \tag{58}
\end{equation*}
$$

Average costs for each firm are given by

$$
\begin{align*}
A C(q) & =\frac{50+2 q+0.1 q^{2}}{q} \\
& =\frac{50}{q}+2+0.1 q  \tag{59}\\
A V C(q) & =2+0.1 q
\end{align*}
$$

The long run shut down price for each firm is determined by setting average cost equal to marginal cost and solving for q .

$$
\begin{align*}
& A C(q)=\frac{50}{q}+2+0.1 q=2+0.2 q=M C(q) \\
& \Rightarrow \frac{50}{q}=0.1 q \\
& \Rightarrow 50=0.1 q^{2}  \tag{60}\\
& \Rightarrow 500=q^{2} \\
& \Rightarrow q=22.3606=10 \sqrt{5} \\
& \Rightarrow A C_{\min }(q)=A C(10 \sqrt{5})=6.4721
\end{align*}
$$

So a firm will not produce at prices less than $\$ 6.47$ in the long run. The short run shut down price for each firm is determined by setting average variable cost equal to marginal cost and solving for $y$.

$$
\begin{align*}
A V C(q)=2+0.1 q & =2+0.2 q=M C(q) \\
0.1 q & =0.2 q \\
\Rightarrow q & =0  \tag{61}\\
\Rightarrow A V C_{\min }(q) & =A V C(0)=2
\end{align*}
$$

So a firm will not produce at prices less than $\$ 2$ in the short run. In equilibrium the total supplied by all firms will equal the market demand

$$
\begin{equation*}
Q^{S}=40 q \tag{62}
\end{equation*}
$$

Each firm will maximize profits by setting marginal cost equal to price and then solving for $q$ to determine its supply function.

$$
\begin{align*}
M C & =0.2 q+2=p \\
\Rightarrow 0.2 q & =p-2  \tag{63}\\
\Rightarrow q & =5 p-10
\end{align*}
$$

Total supply $\left(\mathrm{Q}^{S}\right)$ is

$$
\begin{align*}
Q^{S} & =40 q \\
& =(40)(5 p-10)  \tag{64}\\
& =200 p-400
\end{align*}
$$

This will be relevant in the short run if price is higher than $\$ 2$ and in the long run if price is higher than $\$ 6.4721$. Short run supply is then written

$$
Q^{S}= \begin{cases}200 p-400, & p \geq 2  \tag{65}\\ 0, & p<2\end{cases}
$$

Long run supply is

$$
Q^{S}= \begin{cases}200 p-400, & p \geq 6.4721  \tag{66}\\ 0, & p<6.4721\end{cases}
$$

We can then set supply equal to demand and solve for price as follows

$$
\begin{align*}
Q^{S}=200 p-400 & =2600-50 p=Q^{D} \\
\Rightarrow 250 p & =3000 \\
\Rightarrow p & =12  \tag{67}\\
\Rightarrow Q^{D}=Q^{S} & =2600-(50)(12)=2000
\end{align*}
$$

Because the market equilibrium price is greater than the shutdown price, this equilibrium is sustainable. Individual firm quantities are found by substituting the price in firm level supply equation

$$
\begin{align*}
q & =5 p=10 \\
& =(5)(12)-10  \tag{68}\\
& =50
\end{align*}
$$

This gives a market supply of 2000 [(50) (40)]. Profit for each firm is obtained by substituting p and q in the expression for profit.

$$
\begin{align*}
\text { Revenue } & =R=p q=(12)(50)=600 \\
\operatorname{cost}(q) & =50+0.1 q^{2}+2 q \\
& =50+0.1(2500)+2(50)  \tag{69}\\
& =50+250+100=400 \\
\pi=R & -\operatorname{cost}(q)=200
\end{align*}
$$

This is a short run equilibrium in the sense that other firms will want to enter the industry.
3.6. An example industry equilibrium with three dissimilar firms. Consider a market with three potential firms and an industry demand given by

$$
\begin{equation*}
Q^{D}=36-p \tag{70}
\end{equation*}
$$

We can describe the three firms by their cost functions.
Firm 1 The long run cost function is given by

$$
\begin{equation*}
\operatorname{cost}\left(y_{1}\right)=36+10 y_{1}+0.25 y_{1}^{2} \tag{71}
\end{equation*}
$$

Of the fixed cost of $\$ 36$, $\$ 20$ is sunk (at least in the short run), and $\$ 16$ is avoidable. In the long run, all costs are avoidable.

Firm 2 The long run cost function is given by

$$
\begin{equation*}
\operatorname{cost}\left(y_{2}\right)=16+7 y_{2}+y_{2}^{2} \tag{72}
\end{equation*}
$$

Of the fixed cost of $\$ 16, \$ 7$ is sunk (at least in the short run), and $\$ 9$ is avoidable. In the long run, all costs are avoidable.

Firm 3 The long run cost function is given by

$$
\begin{equation*}
\operatorname{cost}\left(y_{3}\right)=8+8 y_{3}+0.5 y_{3}^{2} \tag{73}
\end{equation*}
$$

Of the fixed cost of $\$ 8, \$ 6$ is sunk (at least in the short run), and $\$ 2$ is avoidable. In the long run, all costs are avoidable.

We will first analyze the three firms seperately.
3.6.1. Supply for firm 1. For firm 1, average cost is given by

$$
\begin{equation*}
A C\left(y_{1}\right)=\frac{36+10 y_{1}+0.25 y_{1}^{2}}{y 1} \tag{74}
\end{equation*}
$$

For firm 1, marginal cost is given by

$$
\begin{equation*}
M C\left(y_{1}\right)=10+0.5 y_{1} \tag{75}
\end{equation*}
$$

We first find the level of output at which average cost is minimized by setting it equal to marginal cost.

$$
\begin{align*}
A C_{1} & =\frac{36+10 y_{1}+0.25 y_{1}^{2}}{y_{1}}=10+0.5 y_{1}=M C_{1} \\
& \Rightarrow 36+10 y_{1}+0.25 y_{1}^{2}=10 y_{1}+0.5 y_{1}^{2} \\
& \Rightarrow 36=0.25 y_{1}^{2}  \tag{76}\\
& \Rightarrow 144=y_{1}^{2} \\
& \Rightarrow 12=y_{1}
\end{align*}
$$

The minimum level of average cost is

$$
\begin{align*}
M C_{1}(12) & =10+0.5(12) \\
& =16 \tag{77}
\end{align*}
$$

In the long run, price must be at least $\$ 16$ for the firm to continue operating. To determine the minimum price for the firm to operate in the short run we compute average avoidable cost.

$$
\begin{equation*}
\text { Avoidable cost }\left(y_{1}\right)=16+10 y_{1}+0.25 y_{1}^{2} \tag{78}
\end{equation*}
$$

and

$$
\begin{equation*}
A V D C\left(y_{1}\right)=\frac{16+10 y_{1}+0.25 y_{1}^{2}}{y_{1}} \tag{79}
\end{equation*}
$$

Average avoidable cost is minimized where it is equal to marginal cost.

$$
\begin{align*}
\frac{16+10 y_{1}+0.25 y_{1}^{2}}{y_{1}} & =10+0.5 y_{1} \\
\Rightarrow 16+10 y_{1}+0.25 y_{1}^{2} & =10 y_{1}+0.5 y_{1}^{2} \\
\Rightarrow 16 & =0.25 y_{1}^{2}  \tag{80}\\
\Rightarrow 64 & =y_{1}^{2} \\
\Rightarrow 8 & =y_{1}
\end{align*}
$$

The minimum level of average avoidable cost is

$$
\begin{align*}
M C_{1}(8) & =10+(0.5)(8)  \tag{81}\\
& =14
\end{align*}
$$

To obtain the supply function for the firm we set price equal to marginal cost.

$$
\begin{align*}
M C_{1} & =10+0.5 y_{1}=p \\
\Rightarrow 0.5 y_{1} & =p-10  \tag{82}\\
\Rightarrow y_{1} & =2 p-20
\end{align*}
$$

This function will be the short run supply function above the minimum of average aviodable cost. This function will be the long run supply function above the minimum of average cost.

The long run supply function is

$$
y_{1}^{L R}= \begin{cases}0, & p<16  \tag{83}\\ 2 p-20 p, & p \geq 16\end{cases}
$$

The short run supply function is

$$
y_{1}^{S R}= \begin{cases}0, & p<14  \tag{84}\\ 2 p-20 p, & p \geq 14\end{cases}
$$

3.6.2. Supply for firm 2. For firm 2, average cost is given by

$$
\begin{equation*}
A C\left(y_{2}\right)=\frac{16+7 y_{2}+y_{2}^{2}}{y 2} \tag{85}
\end{equation*}
$$

For firm 2, marginal cost is given by

$$
\begin{equation*}
M C\left(y_{2}\right)=7+2 y_{2} \tag{86}
\end{equation*}
$$

We first find the level of output at which average cost is minimized by setting it equal to marginal cost.

$$
\begin{align*}
A C_{2} & =\frac{16+7 y_{2}+y_{2}^{2}}{y_{2}}=7+2 y_{2}=M C_{2} \\
& \Rightarrow 16+7 y_{2}+y_{2}^{2}=7 y_{2}+2 y_{2}^{2}  \tag{87}\\
& \Rightarrow 16=y_{2}^{2} \\
& \Rightarrow 4=y_{2}
\end{align*}
$$

The minimum level of average cost is

$$
\begin{align*}
M C_{2}(4) & =7+2(4)  \tag{88}\\
& =15
\end{align*}
$$

In the long run, price must be at least $\$ 15$ for the firm to continue operating. To determine the minimum price for the firm to operate in the short run we compute average avoidable cost.

$$
\begin{equation*}
\text { Avoidable cost }\left(y_{2}\right)=9+7 y_{2}+y_{2}^{2} \tag{89}
\end{equation*}
$$

and

$$
\begin{equation*}
A V D C\left(y_{2}\right)=\frac{9+7 y_{1}+y_{2}^{2}}{y_{2}} \tag{90}
\end{equation*}
$$

Average avoidable cost is minimized where it is equal to marginal cost.

$$
\begin{aligned}
\frac{9+7 y_{2}+y_{2}^{2}}{y_{2}} & =7+2 y_{2} \\
\Rightarrow 9+7 y_{2}+y_{2}^{2} & =7 y_{2}+2 y_{2}^{2} \\
\Rightarrow 9 & =y_{2}^{2} \\
\Rightarrow 3 & =y_{2}
\end{aligned}
$$

The minimum level of average avoidable cost is

$$
\begin{align*}
M C_{2}(3) & =7+(2)(3) \\
& =13 \tag{92}
\end{align*}
$$

To obtain the supply function for the firm we set price equal to marginal cost.

$$
\begin{align*}
M C_{2} & =7+2 y_{2}=p \\
\Rightarrow 2 y_{2} & =p-70  \tag{93}\\
\Rightarrow y_{2} & =\frac{1}{2} p-\frac{7}{2}
\end{align*}
$$

This function will be the short run supply function above the minimum of average aviodable cost. This function will be the long run supply function above the minimum of average cost.

The long run supply function is

$$
y_{2}^{L R}= \begin{cases}0, & p<15  \tag{94}\\ \frac{1}{2} p-\frac{7}{2}, & p \geq 15\end{cases}
$$

The short run supply function is

$$
y_{2}^{S R}= \begin{cases}0, & p<13  \tag{95}\\ \frac{1}{2} p-\frac{7}{2}, & p \geq 13\end{cases}
$$

3.6.3. Supply for firm 3. For firm 3, average cost is given by

$$
\begin{equation*}
A C\left(y_{3}\right)=\frac{8+8 y_{3}+0.5 y_{3}^{2}}{y_{3}} \tag{96}
\end{equation*}
$$

For firm 3, marginal cost is given by

$$
\begin{equation*}
M C\left(y_{3}\right)=8+y_{3} \tag{97}
\end{equation*}
$$

We first find the level of output at which average cost is minimized by setting it equal to marginal cost.

$$
\begin{align*}
A C_{1} & =\frac{8+8 y_{3}+0.5 y_{3}^{2}}{y_{3}}=8+0.5 y_{3}=M C_{1} \\
& \Rightarrow 8+8 y_{3}+0.5 y_{3}^{2}=8 y_{3}+0.5 y_{3}^{2} \\
& \Rightarrow 8=0.5 y_{3}^{2}  \tag{98}\\
& \Rightarrow 16=y_{3}^{2} \\
& \Rightarrow 4=y_{3}
\end{align*}
$$

The minimum level of average cost is

$$
\begin{align*}
M C_{3}(4) & =8+(4) \\
& =12 \tag{99}
\end{align*}
$$

In the long run, price must be at least $\$ 12$ for the firm to continue operating. To determine the minimum price for the firm to operate in the short run we compute average avoidable cost.

$$
\begin{equation*}
\text { Avoidable } \operatorname{cost}\left(y_{3}\right)=2+8 y_{3}+0.5 y_{3}^{2} \tag{100}
\end{equation*}
$$

and

$$
\begin{equation*}
A V D C\left(y_{3}\right)=\frac{2+8 y_{3}+0.5 y_{3}^{2}}{y_{3}} \tag{101}
\end{equation*}
$$

Average avoidable cost is minimized where it is equal to marginal cost.

$$
\begin{align*}
\frac{2+8 y_{3}+0.5 y_{3}^{2}}{y_{3}} & =8+y_{3} \\
\Rightarrow 2+8 y_{3}+0.5 y_{3}^{2} & =8 y_{3}+y_{3}^{2} \\
\Rightarrow 2 & =0.5 y_{3}^{2}  \tag{102}\\
\Rightarrow 4 & =y_{3}^{2} \\
\Rightarrow 2 & =y_{3}
\end{align*}
$$

The minimum level of average avoidable cost is

$$
\begin{align*}
M C_{2}(2) & =8+(2)  \tag{103}\\
& =10
\end{align*}
$$

To obtain the supply function for the firm we set price equal to marginal cost.

$$
\begin{align*}
& M C_{3}=8+y_{3}=p \\
& \Rightarrow y_{3}=p-8 \tag{104}
\end{align*}
$$

This function will be the short run supply function above the minimum of average aviodable cost. This function will be the long run supply function above the minimum of average cost.

The long run supply function is

$$
y_{3}^{L R}= \begin{cases}0, & p<12  \tag{105}\\ p-8, & p \geq 12\end{cases}
$$

The short run supply function is

$$
y_{3}^{S R}= \begin{cases}0, & p<10  \tag{106}\\ p-8, & p \geq 10\end{cases}
$$

3.6.4. Market equilibrium with firm 1 and firm 2 in the market. Now consider a market containing the first two firms. Suppose the technology for firm 3 has been invented yet. Assume that firms 1 and 2 behave competitively (are price takers) even though they could behave in a non-competitive manner. In equilibrium the total supplied by both firms will equal the market demand

$$
\begin{align*}
Q^{S} & =y_{1}+y_{2}  \tag{107}\\
Q^{S}=y_{1}+y_{2} & =2 p-20+\frac{1}{2} p-\frac{7}{2} \\
& =\frac{5}{2} p-\frac{47}{2}  \tag{108}\\
& =2.5 p-23.5
\end{align*}
$$

The long run market supply equation will have 3 parts, one for when there is zero output, one for when only firm 2 produces and one for when both firms produce. It is as follows

$$
Q_{12}^{L R}= \begin{cases}0, & p<15  \tag{109}\\ \frac{1}{2} p-\frac{7}{2}, & 15 \leq p \leq 16 \\ \frac{5}{2} p-\frac{47}{2} & p \geq 16\end{cases}
$$

The short run market supply equation will also have 3 parts, one for when there is zero output, one for when only firm 2 produces and one for when both firms produce. It is as follows

$$
Q_{12}^{S R}= \begin{cases}0, & p<13  \tag{110}\\ \frac{1}{2} p-\frac{7}{2}, & 13 \leq p \leq 14 \\ \frac{5}{2} p-\frac{47}{2} & p \geq 14\end{cases}
$$

We find the market equilibrium price by setting supply equal to demand.

$$
\begin{align*}
Q^{D}=36-p & =2.5 p-23.5=Q_{12}^{S} \\
\Rightarrow 59.5 & =3.5 p  \tag{111}\\
\Rightarrow 17 & =p
\end{align*}
$$

This price is higher than the minimum of short run and long run average costs. To find the equilibrium quantity supplied for each firm we substitute price in the supply equations.

$$
\begin{align*}
& y_{1}=2(17)-20=34-20=14 \\
& y_{2}=\frac{1}{2}(17)-3.5=8.5-3.5=5 \tag{112}
\end{align*}
$$

Aggregate quantity of 19 is consistent with a price of $\$ 17$.
Profits are given by

$$
\begin{align*}
\pi_{1} & =(17)(14)-\left[36+(10)(14)+(0.25)\left(14^{2}\right)\right] \\
& =238-[36+140+49] \\
& =238-225 \\
& =13 \\
\pi_{2} & =(17)(5)-\left[16+(7)(5)+\left(5^{2}\right)\right]  \tag{113}\\
& =85-[16+35+25] \\
& =85-76 \\
& =9 \\
& \begin{aligned}
\pi_{2} & =(17)(5)-\left[16+(7)(5)+\left(5^{2}\right)\right] \\
& =85-[16+35+25] \\
& =85-76 \\
& =9
\end{aligned}
\end{align*}
$$

3.6.5. Short-run market equilibrium with firm 1, firm 2, and firm 3 in the market. If all firms supply then the total quantity supplied is

$$
\begin{align*}
Q_{123}^{S}=y_{1}+y_{2}+y_{3} & =2 p-20+\frac{1}{2} p-\frac{7}{2}+p-8 \\
& =\frac{7}{2} p-\frac{63}{2}  \tag{115}\\
& =3.5 p-31.5
\end{align*}
$$

If firm 1 and firm 3 are in the market, supply is given by

$$
\begin{align*}
Q_{13}^{S}=y_{1}+y_{3} & =2 p-20+p-8 \\
& =3 p-28 \tag{116}
\end{align*}
$$

If firm 2 and firm 3 are in the market, supply is given by

$$
\begin{align*}
Q_{23}^{S}=y_{2}+y_{3} & =\frac{1}{2} p-\frac{7}{2}+p-8 \\
& =\frac{3}{2} p-\frac{23}{2}  \tag{117}\\
& =1.5 p-11.5
\end{align*}
$$

The long-run market supply function will have four parts.

$$
Q_{123}^{L R}= \begin{cases}0, & p<12  \tag{118}\\ p-8, & 12 \leq p \leq 15 \\ \frac{3}{2} p-\frac{23}{2}, & 15 \leq p \leq 16 \\ \frac{7}{2} p-\frac{63}{2}, & p \geq 16\end{cases}
$$

The short-run market supply function is given by

$$
Q_{123}^{S R}= \begin{cases}0, & p<10  \tag{119}\\ p-8, & 10 \leq p \leq 13 \\ \frac{3}{2} p-\frac{23}{2}, & 13 \leq p \leq 14 \\ \frac{7}{2} p-\frac{63}{2}, & p \geq 14\end{cases}
$$

We find the short-run market equilibrium price if all firms participate in the market by setting supply equal to demand.

$$
\begin{align*}
Q_{123}^{S} & =3.5 p-31.5=36-p=Q^{D} \\
& \Rightarrow 4.5 p=67.5  \tag{120}\\
& \Rightarrow p=15
\end{align*}
$$

This price is higher than the minimum of short-run costs for all firms. The equilibrium quantity supplied for each firm is given by

$$
\begin{align*}
y_{1} & =2(15)-20=30-20=10 \\
y_{2} & =(1 / 2)(15)-7 / 2=\frac{15}{2}-\frac{7}{2}=\frac{8}{2}=4  \tag{121}\\
y_{3} & =15-8 \\
& =7
\end{align*}
$$

Short-run profits are are follows

$$
\begin{align*}
\pi_{1} & =(15)(10)-\left[16+(10)(10)+0.25\left(10^{2}\right)\right] \\
& =150-[16+100+25] \\
& =150-141 \\
& =9 \\
\pi_{2} & =(15)(4)-\left[9+(7)(4)+4^{2}\right] \\
& =60-[9+28+16]  \tag{122}\\
& =60-53=7 \\
\pi_{3} & =(15)(7)-\left[2+(8)(7)+(0.5)\left(7^{2}\right)\right] \\
& =105-[2+56+24.5] \\
& =105-[82.5] \\
& =22.5
\end{align*}
$$

The three firms will stay in the industry in the short run. Now consider long run profits

$$
\begin{align*}
\pi_{1} & =(15)(10)-\left[36+(10)(10)+0.25\left(10^{2}\right)\right] \\
& =150-[36+100+25] \\
& =150-161 \\
& =-11 \\
\pi_{2} & =(15)(4)-\left[16+(7)(4)+4^{2}\right] \\
& =60-[16+28+16]  \tag{123}\\
& =60-60=0 \\
\pi_{3} & =(15)(7)-\left[8+(8)(7)+(0.5)\left(7^{2}\right)\right] \\
& =105-[8+56+24.5] \\
& =105-[88.5] \\
& =16.5
\end{align*}
$$

Firm 1 will want to exit the market in the long run. When firm 1 exits, the industry supply will be given by

$$
Q_{23}^{L R}= \begin{cases}0, & p<12  \tag{124}\\ p-8, & 12 \leq p \leq 15 \\ \frac{3}{2} p-\frac{23}{2}, & p \geq 15\end{cases}
$$

Setting supply equal to demand we obtain

$$
\begin{align*}
Q^{S} & =3 / 2 p-\frac{23}{2}=36-p=Q^{D} \\
& \Rightarrow \frac{5}{2} p=\frac{95}{2}  \tag{125}\\
& \Rightarrow 5 p=95 \\
& \Rightarrow p=19
\end{align*}
$$

The amount supplied for each firm is given by

$$
\begin{align*}
& y_{2}=1 / 2(19)-7 / 2=\frac{19}{2}-\frac{7}{2}=\frac{12}{2}=6  \tag{126}\\
& y_{3}=19-8=11
\end{align*}
$$

Profits are given by

$$
\begin{align*}
\pi_{2} & =(19)(6)-\left[16+(7)(6)+6^{2}\right] \\
& =114-[16+42+36] \\
& =114-94 \\
& =20 \\
\pi_{3} & =(19)(11)-\left[8+(8)(11)+(0.5)\left(11^{2}\right)\right]  \tag{127}\\
& =209-[8+88+60.5] \\
& =209+[156.5] \\
& =52.5
\end{align*}
$$

Both firms make profits, but if firm 1 were to enter, supply would increase and price fall such that firm 1 would exit again. Firm 1 has large fixed costs and so can not stay in the market, though it has relatively low marginal costs as is clear in table 2.
3.7. Rising input prices and the industry supply curve. For the individual competitive firm, it is reasonable to assume that input prices will not rise as the firm expands its output and its input use. For an industry that is a significant user of a specific input, this is not a good assumption. As industry demand for the input rises, so will the price of the input. Thus the costs of the individual firms will rise, causing their marginal costs to rise and their supply curves to shift back. This will cause an leftward shift in the industry supply function. Thus the long-run industry supply function may not be horizontal as claimed above.
3.8. Different cost structures and the industry supply curve. In some situations there may be only a few firms in an industry that can produce at a particular low cost level due to special resource endowments, skills, previous luck, etc. For output to rise higher cost firms must enter the market or the firm in question must produce at a point beyond the minimum of its long run average cost curve. Thus the long run supply curve may not be horizontal but may rise after the output of these low cost firms is exhausted. Thus the owners of the low cost firms will earn quasi-rents in the long run if demand is sufficiently high. Over time though, these quasi-rents will be bid into the opportunity cost of the fixed assets and these quasi-rents will accrue only to the asset owner and not to the firm.

Table 2. Marignal Cost for Three Firms

| y | Firm 1 <br> MC | Firm 2 <br> MC | Firm 3 <br> MC |
| :--- | :--- | :--- | :--- |
| 0 | 10.0 | 7 | 8 |
| 1 | 10.5 | 9 | 9 |
| 2 | 11.0 | 11 | 10 |
| 3 | 11.5 | 13 | 11 |
| 4 | 12.0 | 15 | 12 |
| 5 | 12.5 | 17 | 13 |
| 6 | 13.0 | 19 | 14 |
| 7 | 13.5 | 21 | 15 |
| 8 | 14.0 | 23 | 16 |
| 9 | 14.5 | 25 | 17 |
| 10 | 15.0 | 27 | 18 |
| 11 | 15.5 | 29 | 19 |
| 12 | 16.0 | 31 | 20 |
| 13 | 16.5 | 33 | 21 |
| 14 | 17.0 | 35 | 22 |
| 15 | 17.5 | 37 | 23 |
| 16 | 18.0 | 39 | 24 |
| 17 | 18.5 | 41 | 25 |
| 18 | 19.0 | 43 | 26 |
| 19 | 19.5 | 45 | 27 |
| 20 | 20.0 | 47 | 28 |

## 4. Entry and Exit and Perfect Competition

4.1. Freedom of Entry and Exit is a Key to Perfect Competition. If efficient firms are not able to enter the market, firms in the industry may be able to set the price above marginal cost. Thus, any welfare advantages of the market break down. If there is not freedom of entry and exit, the assumption of price taking may not hold.
4.2. Restrictions on Entry. If there are restrictions on entry in to an industry due to government regulation, licensing or other means there may be a reduction in societal welfare.
4.3. Contestability. Even if the number of firms in an industry is small, there may still be competition such that price is no higher than long-run average cost. Contestable markets are those in which the market price is independent of the number of firms currently serving the market because the mere possibility of entry suffices to discipline the actions of incumbent suppliers. If firms not in the market can enter with few sunk costs, they may be able to make a quick profit and then leave as the incumbent firms lower prices. The threat of this serves to police the incumbent firms. An example might be the airline industry where the threat that other carriers might fly a given route serves to discipline incumbents. Another example might be meat packing, where a dominant firm may not be able to pay lower procurement prices for fear of bringing other buyers into its market areas.

### 4.4. Types of Entry Barriers.

4.4.1. Absolute cost advantage. Such advantages are usually due to unique abilities or resources. Examples might be KFC's secret recipe, Michael Jordan's athletic ability, DeBeer's ownership of certain diamond mines, the location of Sam's burro rides into the Grand Canyon, etc.
4.4.2. Economies of size. Any particular firm may be able to produce with the same cost structure, but industry demand may support only one firm. The incumbent firm (first one there) will usually remain the only one, since others cannot enter without lowering the price. Natural monopolies such as local electric service are a good example of this.
4.4.3. Product differentiation. If a firm can differentiate its product sufficiently that no other firm will be able to enter at the market price because consumers will regard the new product as inferior. An example might be Pioneer Seed Corn.

### 4.4.4. Regulatory barriers.

a. patents
b. copyrights
c. franchises
d. licenses
4.5. Exit Barriers. The absence of exit barriers also promotes competition since their absence allows firms not in the industry to enter quickly, make a profit and exit before the incumbent firm can lower the price and cause them to have zero or negative profits, as they are unable to recover sunk costs. For example, if a particular industry has specialized equipment that will not have much value elsewhere, a firm not in the industry will not enter if it seems likely that its profits will be short-lived.

## 5. Problems with Externalities and Competition

5.1. Externalities and Social Welfare. Perfect competition leads to maximum social welfare because the cost of producing the product is equal to its full social value. In other words, the producer must incur the full social cost of production in order to supply the product. If the producer is able to produce the product for less than its cost to society then price will fall below the level that reflects actual social cost, and consumption will rise above the optimal level.
5.2. Example of Water Pollution. Consider, for example, a large hog feeder who must dispose of animal waste as a by-product of his operation. If he can just dump this into the local river with no penalties, he will probably do so in order to minimize costs. The marginal cost of production will then reflect this lower cost, the price of hogs will fall, the quantity demanded will rise and output will be higher than what may be socially optimal, given that some consumers may have to drink water drawn from this river. Thus, allowing competition to work in this market may not lead to the optimal welfare level. On the other hand, if a local community owns the rights to release waste into the river, it may choose to restrict or eliminate such releases to the extent that their social damage is higher than the benefits in terms of lower cost pork.
5.3. Grazing Example. Assume that all sheep farmers can use a particular range since it is publicly owned land and there are no user fees. Each producer will consider only its individual profit. In this case, there will be overgrazing since the sheep herders do not take into account the decline in foliage as the number of sheep rises. While everyone would probably be better off with restrictions on use, many producers dislike restrictions on their gazing rights and so the overuse continues. This is an example of the commons problem. Other examples are fisheries, slash and burn agriculture, etc.

## 6. Efficiency and Equity (FAirness)

Economists generally favor markets that are competitive because they maximize social welfare for a given initial allocation of resources and wealth. Perfectly competitive markets are not necessarily fair in the sense that they lead to "equitable" outcomes. An outcome where one person got almost all the wealth and others got little may be no less efficient than one with a more equitable distribution. All competition will do is ensure that given initial wealth, no one can be made better off without making at least one person worse off. But economic theory also says that any income distribution can be supported by a competitive equilibrium plus a system of income transfers. Thus if one is not happy with an income distribution the answer is not to abandon competition but to make income transfers to obtain equity objectives and then let the market determine what is produced, by whom and who consumes it. For example it may be better to simply transfer income to poor farmers via a direct subsidy rather than providing price supports that cause production beyond the social optimum.


[^0]:    Date: December 16, 2004.

