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Game Theory: Basic Concepts and Terminology Relevant for Power Markets

A **GAME** consists of:

- a collection of decision-makers, called *players*;
- the possible information states of each player at each decision time;
- the collection of possible moves (decisions, actions, plays,...) that each player can choose to make in each of his possible information states;
- a procedure for determining how the move choices of all the players collectively determine the possible outcomes of the game;
- preferences of the individual players over these possible outcomes, typically measured by a *utility* or *payoff* function.

GenCo2

		C	D
GenCo1	**C**	**(40,40)**	**(10,60)**
	D	**(60,10)**	**(20,20)**

Illustrative Modeling of Price-Setting by GenCos Modeled as a "Prisoner's Dilemma Game"

D = Defect (Price Low)

C = Cooperate (Price High),

(P1,P2) = (GenCo1 Profit, GenCo2 Profit)

A **PURE STRATEGY** for a player in a particular game is a complete contingency plan, i.e., a plan describing what move that player should take in each of his possible information states.

A **MIXED STRATEGY** for a player in a particular game is a probability distribution defined over the collection A of the player's possible pure strategy choices. That is, a mixed strategy assigns a nonnegative probability $\mathrm{Prob}(a)$ to each pure strategy a in A, with

$$\sum_{a \in A} \mathrm{Prob}(a) = 1 .$$

EXPOSITIONAL NOTE:

For simplicity, the remainder of these brief notes will develop definitions in terms of pure strategies. Unless otherwise indicated, the term "strategy" will always refer to a *pure* strategy. Extension to mixed strategies is conceptually straightforward.

A One-Stage Simultaneous-Move N-Player Game

- The game is played just once among N players.
- Each of the N players *simultaneously* chooses a strategy based on his current information state, where this information state does *not* include knowledge of the strategy choices of any other player.
- A payoff (reward, return, utility outcome,...) for each player is then determined as a function of the N simultaneously-chosen strategies of the N players.

Note: For ONE-stage games, there is only one decision time. Consequently, a choice of a strategy based on a current information state is the same as the choice of a move based on this current information state.

Extension to a Multi-Stage (i.e., Iterated) Simultaneous-Move N-Player Game

- The game is played among N players over successive iterations $T = 1, 2, \ldots \text{TMax}$.
- In each iteration T, each of the N players *simultaneously* chooses a move conditional on his current information state, where this information state does *not* include the iteration-T move of any other player.
- An iteration-T payoff (reward, return, utility outcome,...) is then determined for each player as a function of the N simultaneously-chosen moves of the N players in iteration T.
- If $T < \text{TMax}$, the next iteration T+1 then commences.
- The information states of the players at the beginning of iteration T+1 are typically updated to include at least some information regarding the outcomes (e.g., moves, payoffs,...) from the previous iteration T.

Note: For ITERATED games there are multiple decision times. Consequently, a choice of a move based on a current information state does not constitute a strategy (complete contingency plan). Rather, a strategy is the choice of a move for the current iteration, given the current information state, together with a designation of what move to choose in each future iteration conditional on every possible future information state.

"PAYOFF MATRIX" FOR A ONE-STAGE SIMULTANEOUS-MOVE 2-PLAYER GAME:

Consider a one-stage simultaneous-move 2-player game in which each player must choose to play one possible strategy in a collection A consisting of M possible strategy choices.

The *Payoff Matrix* for this 2-player game then consists of an $M \times M$ table that gives the payoff received by each of the two players under each of the $M \times M$ possible strategy combinations that the two players can choose.

This definition is easily generalized to the case in which the set of strategies for each player is different by type and/or by number.

GenCo2

GenCo1		C	D
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	D	(60,10)	(20,20)

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NASH EQUILIBRIUM FOR AN N-PLAYER GAME:

Let S_i^* denote the particular strategy choice of a player i in an N-player game, where S_i^* is drawn from a collection A_i of M_i possible strategy choices for player i.

A specific combination $(S_1^*, \ldots, S_N^*)$ of possible strategy choices for an N-player game, one strategy choice S_i^* for each player i, is called a *(Pure Strategy) Nash equilibrium* if no player i perceives any possible way of achieving a higher payoff by switching unilaterally to another possible strategy choice S_i' in A_i.

DOMINANT STRATEGY FOR AN N-PLAYER GAME:

A feasible strategy for a player in an N-player game is said to be a *dominant strategy* for this player if it is this player's *best response* to *any* feasible choice of strategies for the other players.

For example, suppose S_1^* is a dominant strategy for player 1 in an N-player game. This means that, no matter what feasible combination of strategies $(S_2, \ldots, S_N)$ players 2 through N might choose to play, player 1 attains the highest possible (expected) payoff if he chooses to play strategy S_1^*.

QUESTIONS:

(1) Does the GenCo game have a Nash equilibrium?

(2) Does either player in the GenCo game have a dominant strategy?

(3) What is the key distinction between a dominant strategy and a strategy constituting part of a Nash equilibrium?

PARETO EFFICIENCY:

Intuitive Definition:

A possible combination of decisions for a collection of agents is said to be *Pareto efficient* if there does *not* exist another possible combination of decisions under which each agent is at least as well off and some agent is strictly better off.

More Rigorous Definition: N-Player Game Context

For each $i = 1, \ldots, N$, let P_i denote the payoff attained by player i under a possible strategy combination $S = (S_1,\ldots,S_N)$ for the N players. The strategy combination S is said to be *Pareto efficient* if there does *not* exist another possible strategy combination S' under which each player i achieves at least as high a payoff as P_i and some player j achieves a strictly higher payoff than P_j. The payoff outcome $(P_1,\ldots,P_N)$ is then said to be a *Pareto efficient payoff outcome*.

QUESTION:

Does the GenCo game have a Pareto efficient strategy combination?

PARETO DOMINATION:

Intuitive Definition: A possible combination of decisions for a collection of agents is said to be *Pareto dominated* if there *does* exist another possible combination of decisions under which each agent is at least as well off and some agent is strictly better off.

More Rigorous Definition: N-Player Game Context For each $i = 1, \ldots, N$, let P_i denote the payoff attained by player i under a strategy combination $S = (S_1, \ldots, S_N)$ for the N players. The strategy combination S is said to be *Pareto dominated* if there *does* exist another possible strategy combination S' under which each player i achieves at least as high a payoff as P_i and some player j achieves a strictly higher payoff than P_j.

QUESTION:

Does the GenCo game have strategy combinations that are Pareto dominated?

COORDINATION FAILURE:

Intuitive Definition: A combination of decisions for a collection of agents is said to exhibit *coordination failure* if mutual gains, attainable by a collective switch to a different possible combination of decisions, are not realized because no individual agent perceives any feasible way to increase their own gain by a unilateral deviation from their current decision.

More Rigorous Definition: N-Player Game Context A strategy combination $S = (S_1, \ldots, S_N)$ is said to exhibit *coordination failure* if it is a Pareto-dominated Nash equilibrium.

QUESTIONS:

Does the GenCo game have a move combination that exhibits coordination failure?

Might the *iterative* play of this GenCo game help alleviate coordination failure problems?