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Concepts from the Theory of the Firm:

Kirschen/Strbac Chapter 2 (Parts 2.3 & 2.5)

Important Acknowledgement:

These notes on Kirschen/Strbac (Chapter Sections 2.3 and 2.5, 2004) are based on slides prepared by Daniel Kirschen (U Manchester) with substantial edits by Leigh Tesfatsion (Iowa State U).

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Short-Run Cost: Accountant's Perspective

- Some production costs are variable and others are fixed:

  Variable cost:
  - Labor, fuel, transport…

  Fixed cost (amortised):
  - Equipment, land, overhead

- Some production costs are "sunk" and some are avoidable

  Sunk (Not Avoidable)
  - Loan obligations, special assets with no resale value,…

  Avoidable
  - GenCo "start-up" costs (if GenCo self commits), labor, fuel, transport, …

Two Different Taxonomies for Total Cost

- Let T denote ***any*** given production planning period.
- Consider a firm at the beginning of period T making its production plans for period T.
- The period-T total cost of production for this firm can be decomposed in ***two different ways***:

- Total Cost = Fixed Cost + Variable Cost
- Total Cost = Sunk Cost + Avoidable Cost

Two Different Taxonomies for Total Cost …Continued

- ***Fixed Cost:*** Does NOT vary with changes in the firm's output level y over the range $y \geq 0$
- ***Variable Cost:*** VARIES with changes in the firm's output level y
- ***Sunk Cost:*** Payment obligations that the firm has irrevocably committed to, hence CANNOT BE RECOVERED.
- ***Avoidable Cost:*** Payments that CAN BE RECOVERED by the firm because the firm has not committed to them, or because they involve costs for purchased assets that can be recovered through resale, etc.

EXAMPLE: A Farmer's Costs for 2009

- A corn farmer takes out a loan on 1/1/09 to finance a used tractor worth \$138,000. As down payment for this loan, he gives the bank \$28,000. He is then also obliged to make a \$1,000 payment on this loan at the end of every month for the next 10 years (i.e., Jan 2009 - Dec 2019).
- The farmer's labor and other operating expenses incurred if he produces corn in amount y during 2009 are given by $h(y) = ay + by^2$.
- If the farmer goes out of business during 2009, the bank keeps the down payment, takes ownership of the tractor, and retires the loan (i.e., the farmer is not obliged to make any additional loan payments).
- ***Total Production Cost for 2009 As Of 1/2/09: \$38,000 + h(y)***

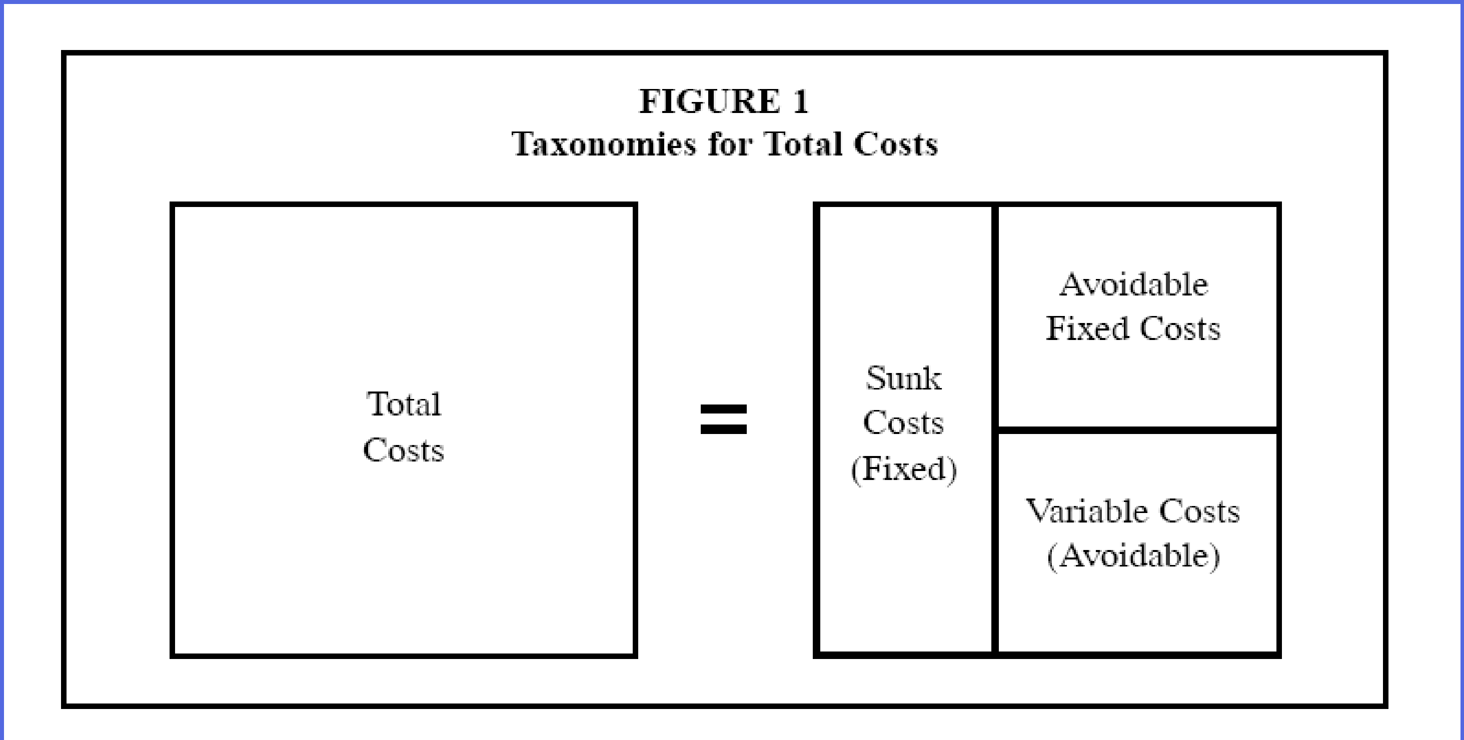
Fixed Cost: \$40,000

Variable Cost: h(y)

Sunk Cost: \$28,000

Avoidable Cost: [\$12,000 + h(y)] = [Avoidable Fixed Cost + Variable Cost]

Total cost decomposition for a planner at the beginning of a particular planning period T:



Source: X. H. Wang & B. Z. Yang, “Fixed and Sunk Costs Revisited,” *Journal of Economic Education*, Spring 2001, pp. 178-185

Total Cost Decomposition … Continued

We may classify total costs by asking the following two questions sequentially:

Question 1: Is this cost now irrevocably committed over the relevant time period?

Yes → It is a sunk cost. (It is now irrelevant whether the firm intended the costs to be fixed or variable at the time they were irrevocably committed.)

No

↓

It is an avoidable cost. Then, we ask,

Question 2: Does this cost vary with the output level to be produced over that time period?

No → It is an (avoidable) fixed cost.

Yes → It is a variable cost (avoidable).

Source: X. H. Wang & B. Z. Yang, "Fixed and Sunk Costs Revisited," *Journal of Economic Education*, Spring 2001, pp. 178-185

Two Different Taxonomies for Total Cost…Continued

- In any given time period T:

  Total Cost = Fixed Cost + Variable Cost

  Total Cost = Sunk Cost + Avoidable Cost

- ***Second*** way of decomposing cost is easier to understand & use.

  Example: **How to classify a "start up cost" incurred by a GenCo when it first starts up a generation unit in some period T?** The start-up cost varies with production level y from y=0 to y > 0 but not thereafter. This is an example of ***Kirschen's "quasi-fixed cost"***.

  Easy answer under 2): It's a sunk cost in period T if the GenCo is *obliged* to start up the generation unit in period T. Otherwise it's an avoidable cost in period T.

Costing of Durable-Benefit Expenditures Requires Care

- How should producers cost out expenditures whose benefits last over successive time periods?
  - Purchase of physical assets with long-term durability such as equipment, buildings, and land
  - Start-up costs (once started up, a generation unit can be run over successive hours without incurring additional start-up costs)
- Prior to such expenditures, there must be SOME production planning period T over which full recovery of all avoidable cost is anticipated, else the cost should not be incurred.
- Once incurred, such expenditures become either sunk (e.g., start-up costs) or avoidable fixed costs (e.g., resale value of purchased equipment).
- When deciding whether or not to produce in the presence of avoidable fixed costs, there must be SOME planning period T over which full recovery of all avoidable costs is anticipated, including all avoidable fixed costs, else the firm should shut down and act to reduce its avoidable costs to zero.
- **BOTTOM LINE:** When durable-benefit expenditures are involved, planning periods must be suitably long to permit proper consideration of cost recovery.

Determining Optimal Production for a Competitive (Price-Taking) Firm at the Beginning of a "Suitably Long" Planning Period T:

- **DEFINE:**

  Net_Revenue = [Revenue – Avoidable Cost]

- **Step 1:**

  Find production level $y^* \geq 0$ that maximizes net revenue for a given output price π and input prices $w_1, w_2, \ldots$

- **Step 2:**

  If Net_Revenue at y^* is positive, produce y^* ;

  Otherwise shut down (set $y = 0$) and take action to reduce all avoidable cost to zero so that net revenue = 0 .

Modern Take on Traditional Economic Distinction:
Long Run versus Short Run

- Some factors of production can be adjusted faster than others.
  - *Example:* Fertilizer vs. planting more trees

- ***"Long Run":*** All costs can be avoided
- ***"Short Run":*** At least some costs are sunk
- **Additional assumption for K/S Chapter 2:** For simplification and consistency with notation of Kirschen/Strbac, we will assume for now that **all fixed costs are sunk costs**! This implies

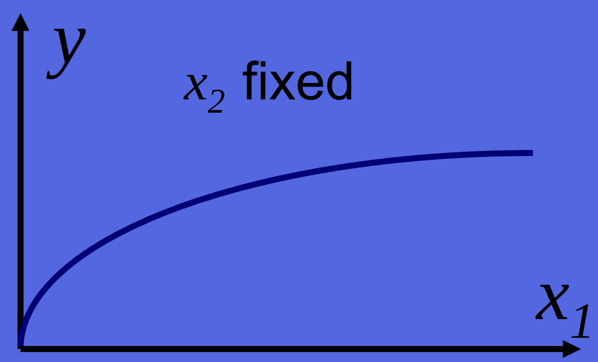
  (a) **Fixed cost = Sunk cost (i.e., there are NO avoidable fixed costs);**

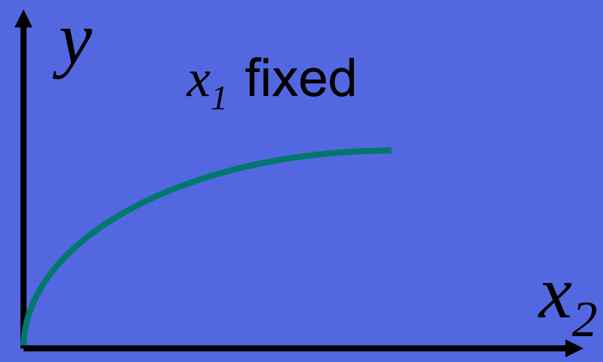
  (b) **Variable cost = Avoidable cost** .

Production function: A function giving maximum possible output for each amount of inputs

$$y = f\left(x_1, x_2\right)$$

- y: output
- x_1 , x_2: inputs (or "factors of production")





*****Law of diminishing marginal product*****

Input-Output Function:

$$y = f\left(x_1, \overline{x_2}\right) \qquad \overline{x_2} \text{ fixed}$$

The inverse of a production function for some input x_1 (all else equal) is the ***input-output function for x_1***

$$x_1 = g(y) \ \text{ for } x_2 = \overline{x_2}$$

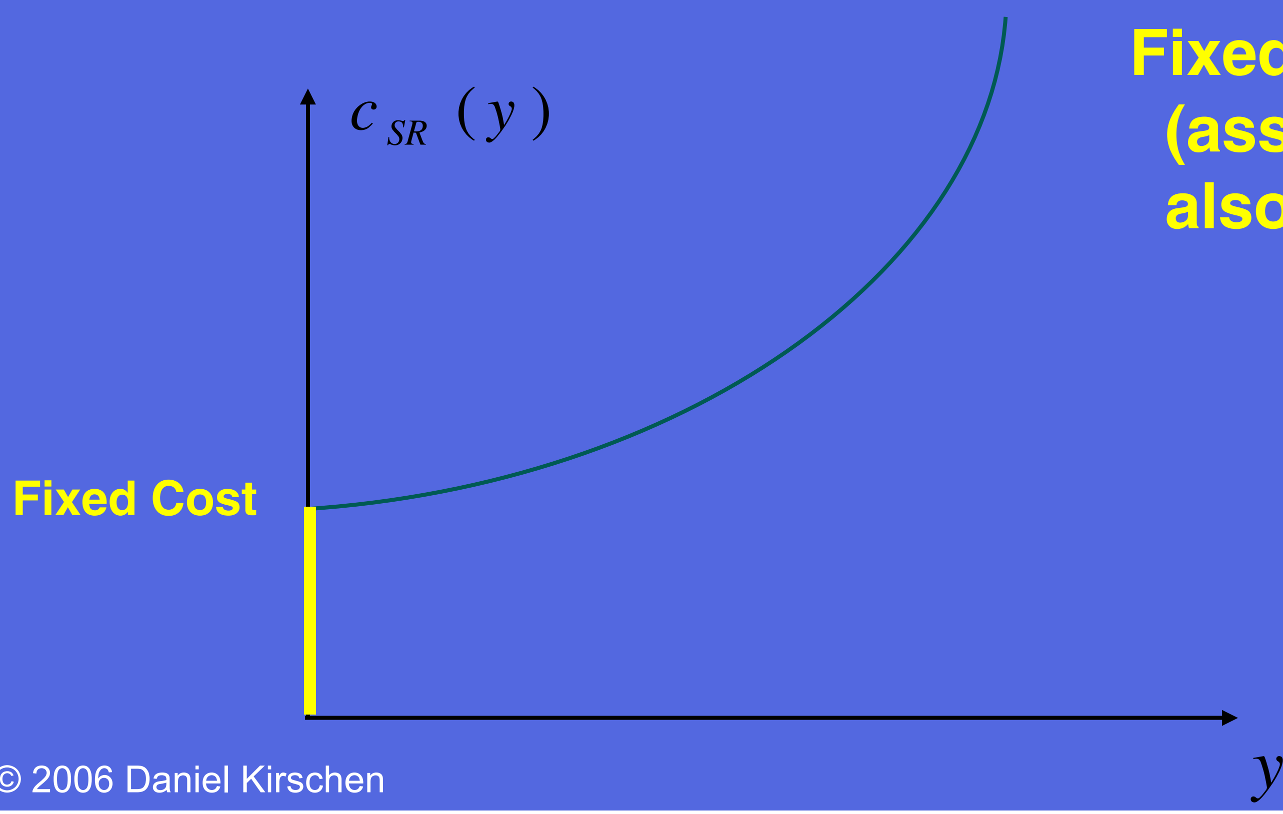
Example: Minimum amount of fuel x_1 required to produce successively higher amounts of electric power y, given a particular generation plant x_2

Short-Run Cost Function: $C_{SR}(y)$

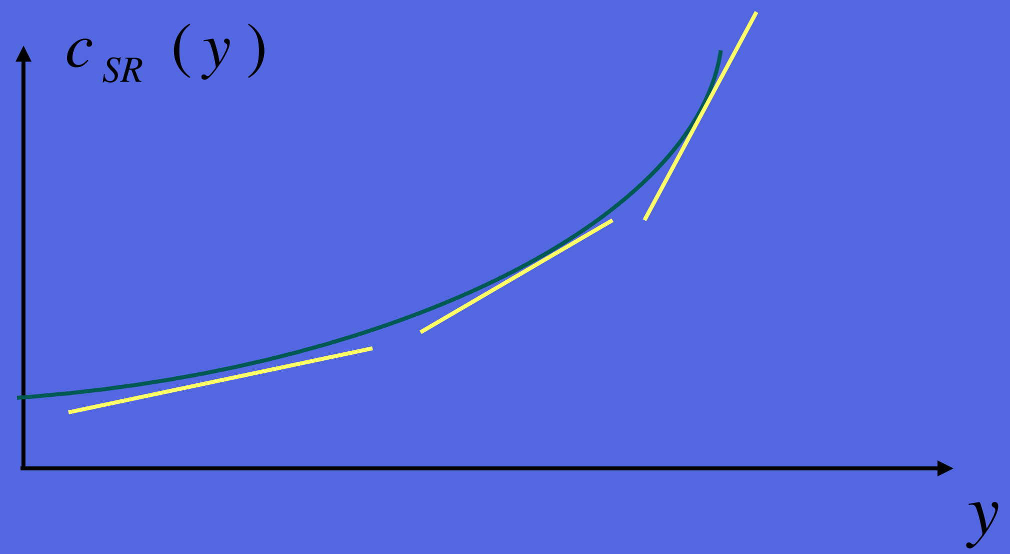
Given fixed input x_2, (unit) prices w_1, w_2 for inputs x_1, x_2, and $x_1 = g(y)$ = input-output function for variable input x_1, define:

$$c_{SR}(y) = w_1 \cdot x_1 + w_2 \cdot \overline{x_2} = w_1 \cdot g(y) + \underline{w_2 \cdot \overline{x_2}}$$

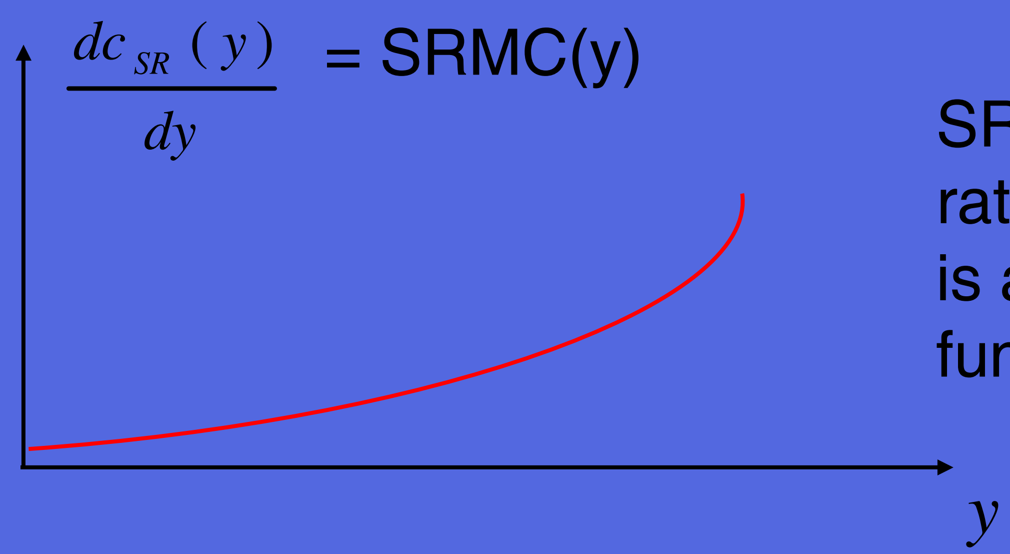
Fixed Cost FC (assumed to also be sunk)



Short-Run Marginal Cost: SRMC(y)



$C_{SR}(y)$ is convex due to the law of diminishing marginal product

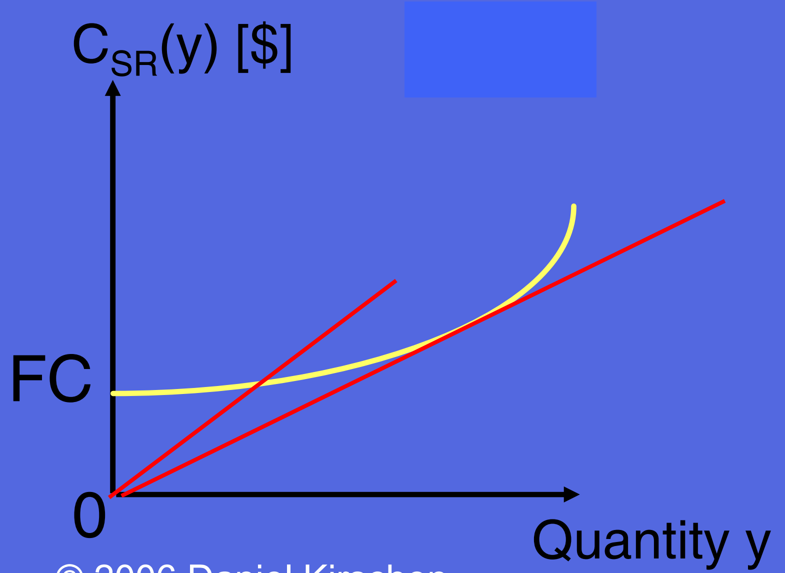


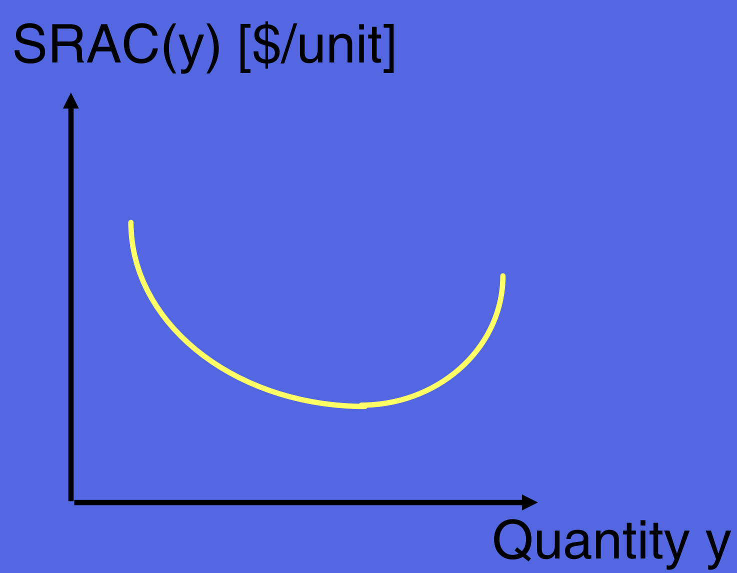
SRMC(y) = $dC_{SR}(y)/dy$ = rate of change of $C_{SR}(y)$ is a ***non-decreasing*** function

SR Average Cost SRAC(y), SR Average Variable Cost SRAVC(y), and Average Fixed Cost AFC

$$C_{SR}(y) = c_v(y) + c_f$$ = Variable Cost + Fixed Cost (with $C_v(0) = 0$)

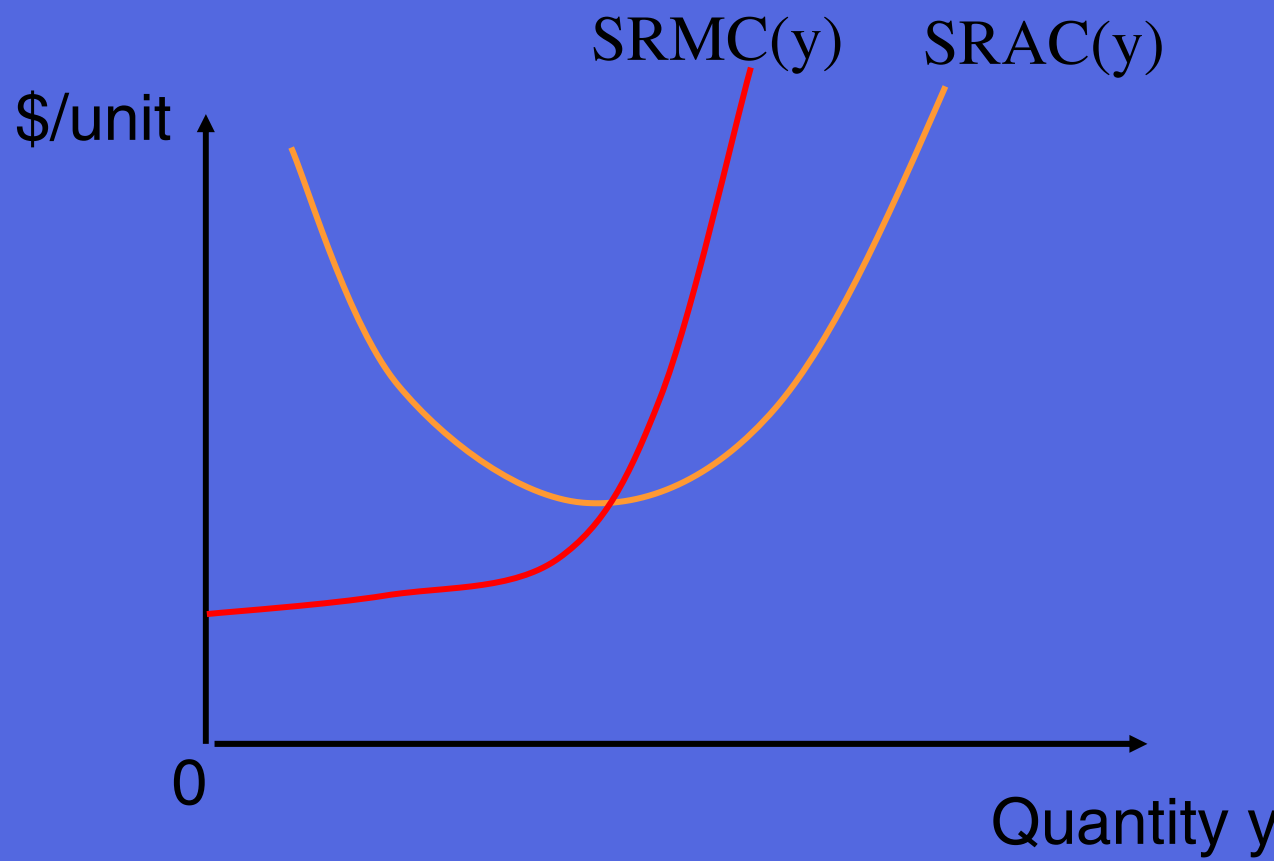
$$SRAC(y) = \frac{C_{SR}(y)}{y} = \frac{c_v(y)}{y} + \frac{c_f}{y} = SRAVC(y) + AFC$$



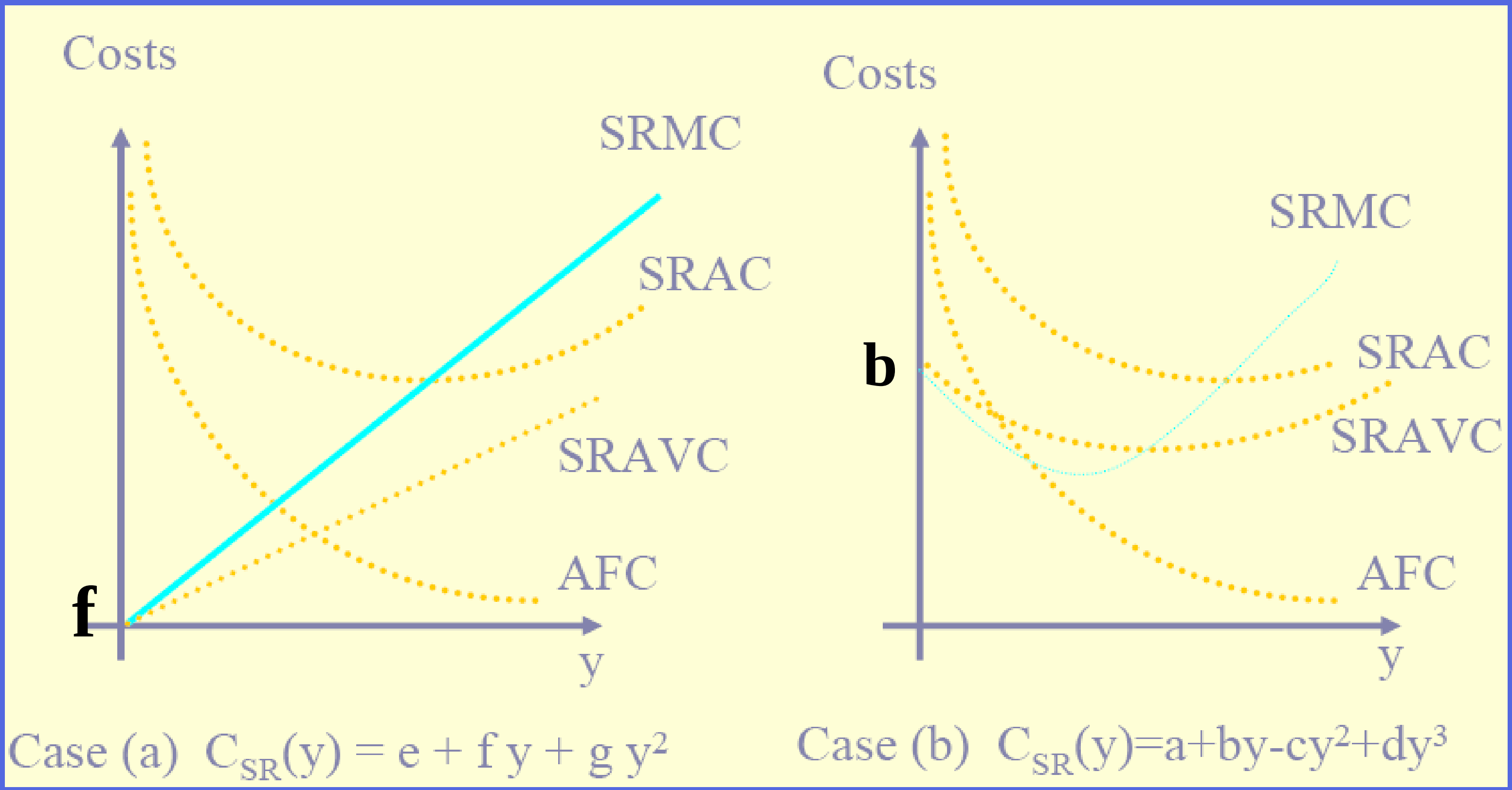


Relation between SR Marginal Cost and SR Average Cost

Note: $dSRAC(y)/dy = [\,SRMC(y) - SRAC(y)\,] / y$



A Fuller Depiction of SR Cost Relationships: Two Examples



Optimal period-T production for a competitive (price- taking) firm with short-run cost function $C_{SR}(y) = C_v(y) + C_f$ when all of its fixed cost C_f is sunk

- **Define:** For given output price π and input prices w_1, w_2,…

Profit (y) = [Revenue (y) – SR_Cost (y)]

= [Revenue – $C_{SR}(y)$]

Net_Revenue (y) = [Revenue (y) – Avoidable_Cost (y)]

= [πy - $C_v(y)$]

★ **Since "sunk costs are sunk," the firm's proper focus should be on NET REVENUE, not on profit!**

Special K/S Chapter 2 assumption made here that ALL fixed costs are sunk, so there are NO avoidable fixed costs!

Optimal production…Continued

- For given output price π and given input prices w_1, w_2,…

Net_Revenue (y) = [Revenue (y) – Avoidable_Cost (y)]

= **[** $\pi y - C_v(y)$]

Recall $C_v(0) = 0$!

- **Step 1:**

  Find output level y* that maximizes Net_Revenue (y) over all y ≥ 0, where y = 0 corresponds to "shut down"

- **Step 2:**

  If y* > 0 , produce y* with Net_Revenue (y*) ≥ 0 .

  If y* = 0 , shut down and attain Net_Revenue (0) = 0

Additional Details About Step 1:

- CAUTION: The following is a necessary condition for optimal y* **ONLY** if y* > 0:

$$\max_{y} \left\{ \pi \cdot y - c_V(y) \right\}$$

$$\Downarrow$$

$$\frac{d\left\{ \pi \cdot y - c_V(y) \right\}}{dy} = 0$$

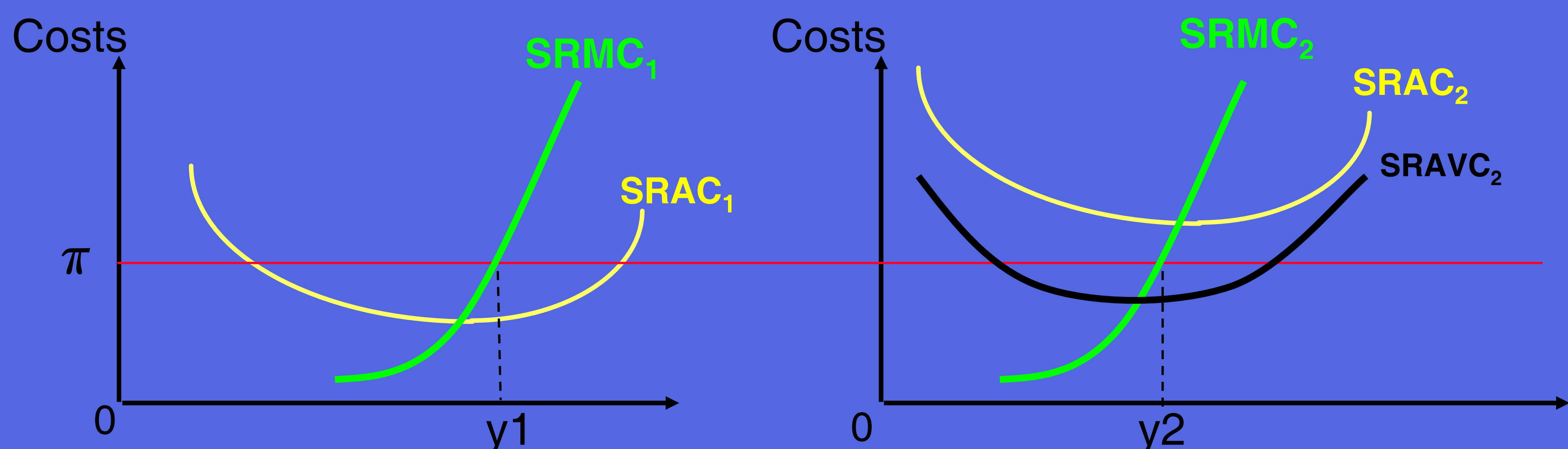
$$\Downarrow$$

$$\pi = \frac{dc_V(y)}{dy} = \text{SRMC}(y)$$

Equality here only holds if changes in y do not affect market prices ⟺ Competitive (small) producer who has no power to affect market prices through changes in its output

Additional details about Step 1…Continued

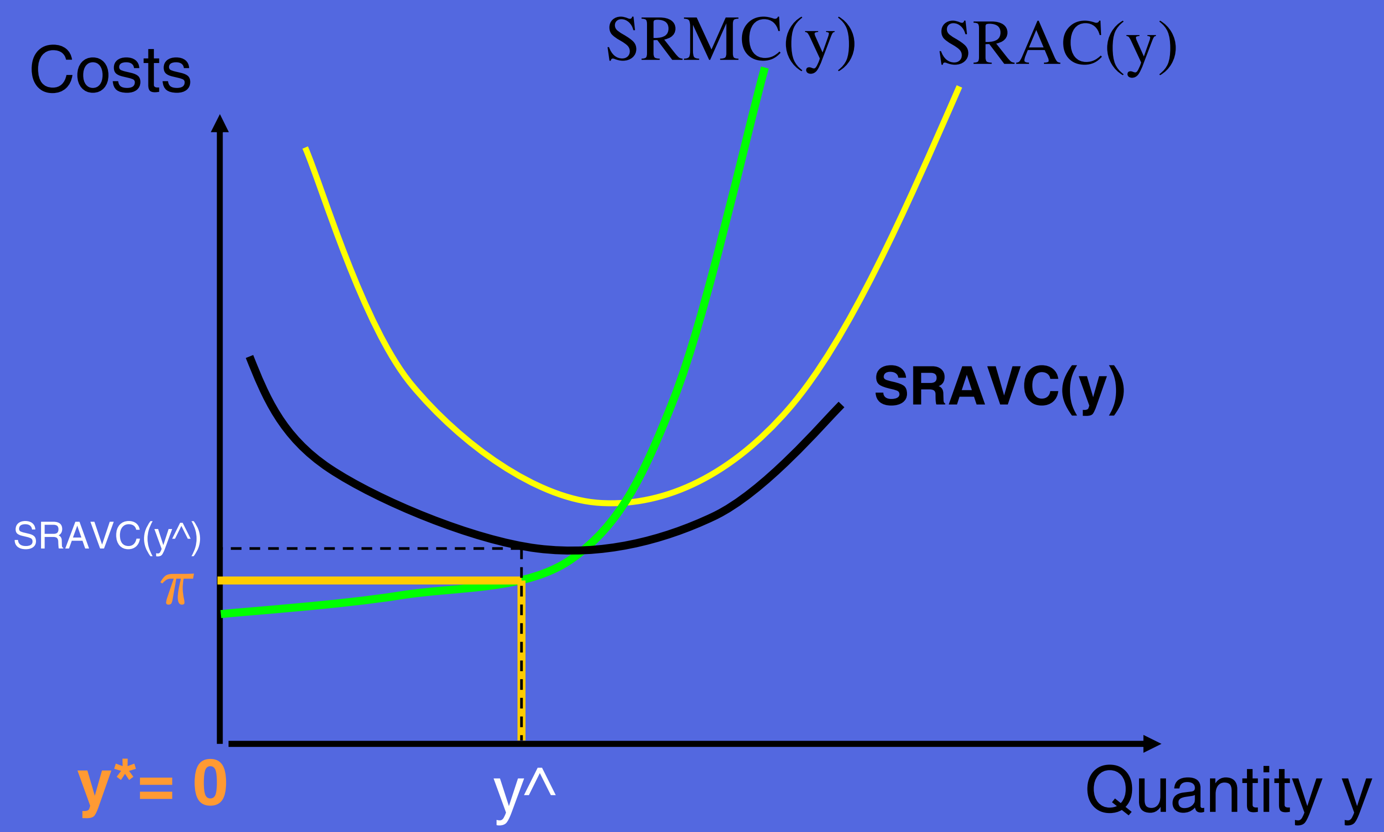
- SRMC = $dC_{SR}(y)/dy = dC_v(y)/dy$ = SR cost of producing 1 more unit
- If SRMC < π , the next unit costs ***less*** than it returns.
- If SRMC > π , the next unit costs ***more*** than it returns.
- Find y^ where SRMC(y^) just equals π (cutting from below)
- If Net_Revenue(y^) > 0, produce y*=y^. Otherwise, set y* = 0 (shut down).



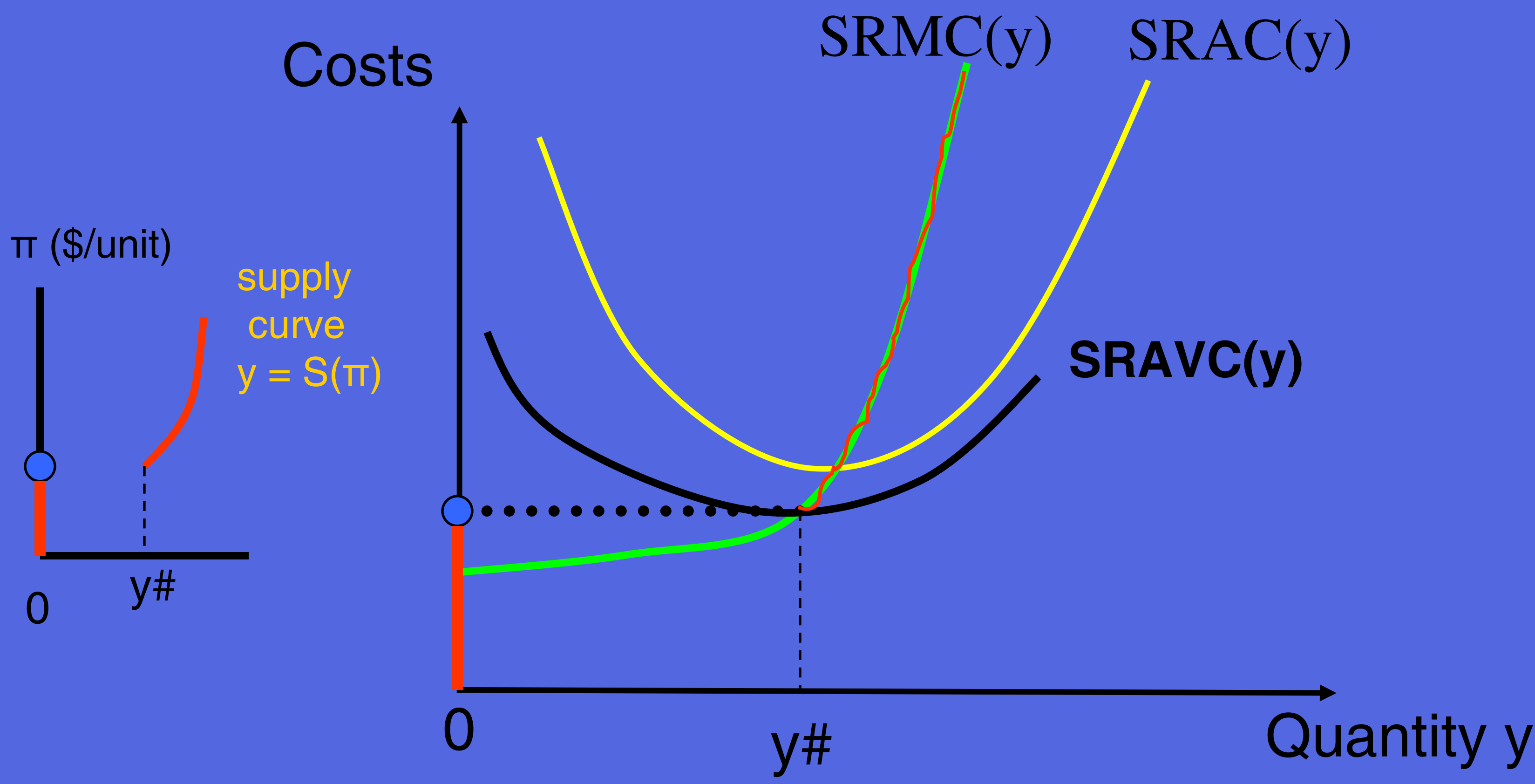
Avoidable and sunk cost are both fully covered at optimal output level y*=y1 but only avoidable cost is fully covered at optimal output level y*=y2

In figure, below, the output level y^ where SRMC = π results in Net_Revenue (y^) = [πy^ - C_v(y^)] < 0, meaning avoidable cost is not covered. Firm should select y* = 0 (shut down).

Note: **[π] y^ = Revenue (y^) and [SRAVC(y^)] y^ = C_v(y^)**

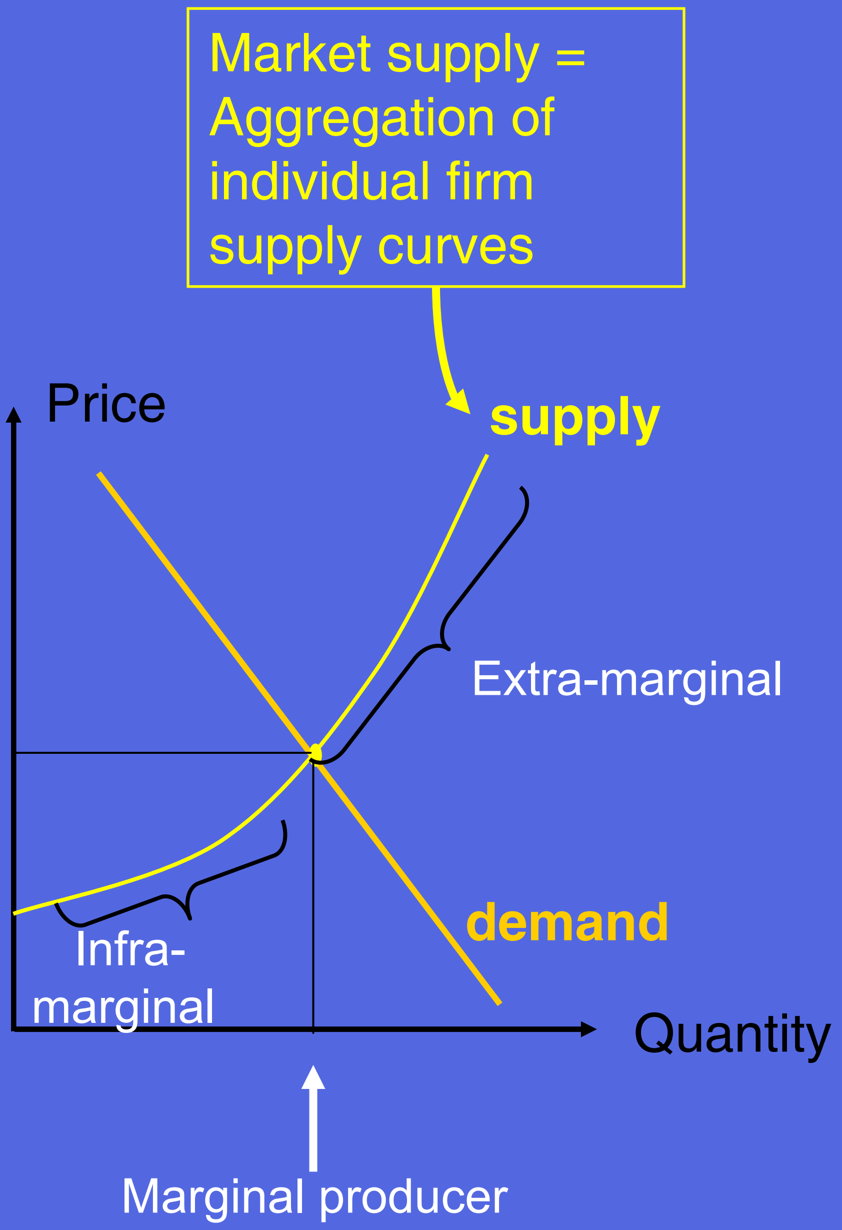


When all fixed costs are sunk, the short-run supply curve for a competitive firm coincides with the portion of its SRMC curve lying above its SRAVC curve.



Perfect Competition

- Perfect competition
  - The volume handled by each market participant is small compared to the overall market volume
  - No market participant can influence the market price by its own unilateral actions
  - All market participants act as price takers



Costs: Economist's Perspective

- ***Opportunity cost:***
  - What is the **next best** use of the money that is currently being spent to produce something ?
  - Not taking advantage of an opportunity to invest this money in a more profitable use represents an "opportunity cost"
- Examples:
  - Growing apples or growing kiwis?
  - Using money to grow apples or instead depositing this money in a bank where it would earn interest?
- "Avoidable costs" can include income lost by not investing in next best opportunity (i.e., opportunity cost)
- Selling "at cost" (break even point) does not necessarily mean zero net dollar earnings. It means producer is just indifferent between current use and next best use.

Imperfect Competition

- One or more "strategic player" firms can influence the market price through their actions
- Strategic player firms
  - Can influence the market price through their actions (price SETTERS rather than price takers)
  - Perceive and actively exploit potential demand for their output
  - Do not have "supply curves" in previously developed sense
  - Typically have a large market share of capacity/output/revenues
- A "competitive fringe" could still exist
  - Participants with a small market share
  - Take the market price as given
- ***Examples:*** Monopoly (single seller) models, Cournot and Bertrand multi-seller models of competition

Single-Price Monopoly Models

- A firm is a ***monopoly*** if it is the only supplier of a product y for which there is no close substitute.
- A monopoly that is limited to charging the same price π for each unit of its product y is called a ***single-price monopoly.***
- This is the case treated in Kirschen/Strbac, Chapter 2, Section 2.5.3.

Single-Price Monopoly…Continued

- A single-price monopolist perceives it faces a downward sloping inverse demand curve $\pi(y) = D^{-1}(y)$.
- The monopolist exploits this knowledge by setting its price equal to $\pi(y) = D^{-1}(y)$ = (maximum willingness of buyers to pay for y), given it produces y.
- The monopolist sets its price with no worry this price can be undercut by *existing* rivals.
- However, *new* market entrants could be a concern (i.e., the market could be “contestable”).

Marginal Revenue for a Single-Price Monopolist

- ***Total Revenue Function:*** TR(y) = π(y)•y
- ***Marginal Revenue Function:***

$$
\begin{aligned}
MR(y) &= \frac{dTR(y)}{dy} \\
&= \pi(y) + \frac{d\pi(y)}{dy} \cdot y \\
&= \pi(y) + \frac{d\pi(y)}{dy} \cdot \frac{y}{\pi(y)} \cdot \pi(y) \\
&= \pi(y) \left[1 - \frac{1}{\mid \epsilon(y) \mid} \right]
\end{aligned}
$$

- **Note:** MR(y) < π(y) unless demand is perfectly elastic (i.e., $\varepsilon(y) = -\infty$)

Profix Maximization for a Single-Price Monopolist

- **Basic Observation:**

  A small increase in output by a monopolist will ***increase*** its profit if MR > MC and ***decrease*** its profit if MR < MC.

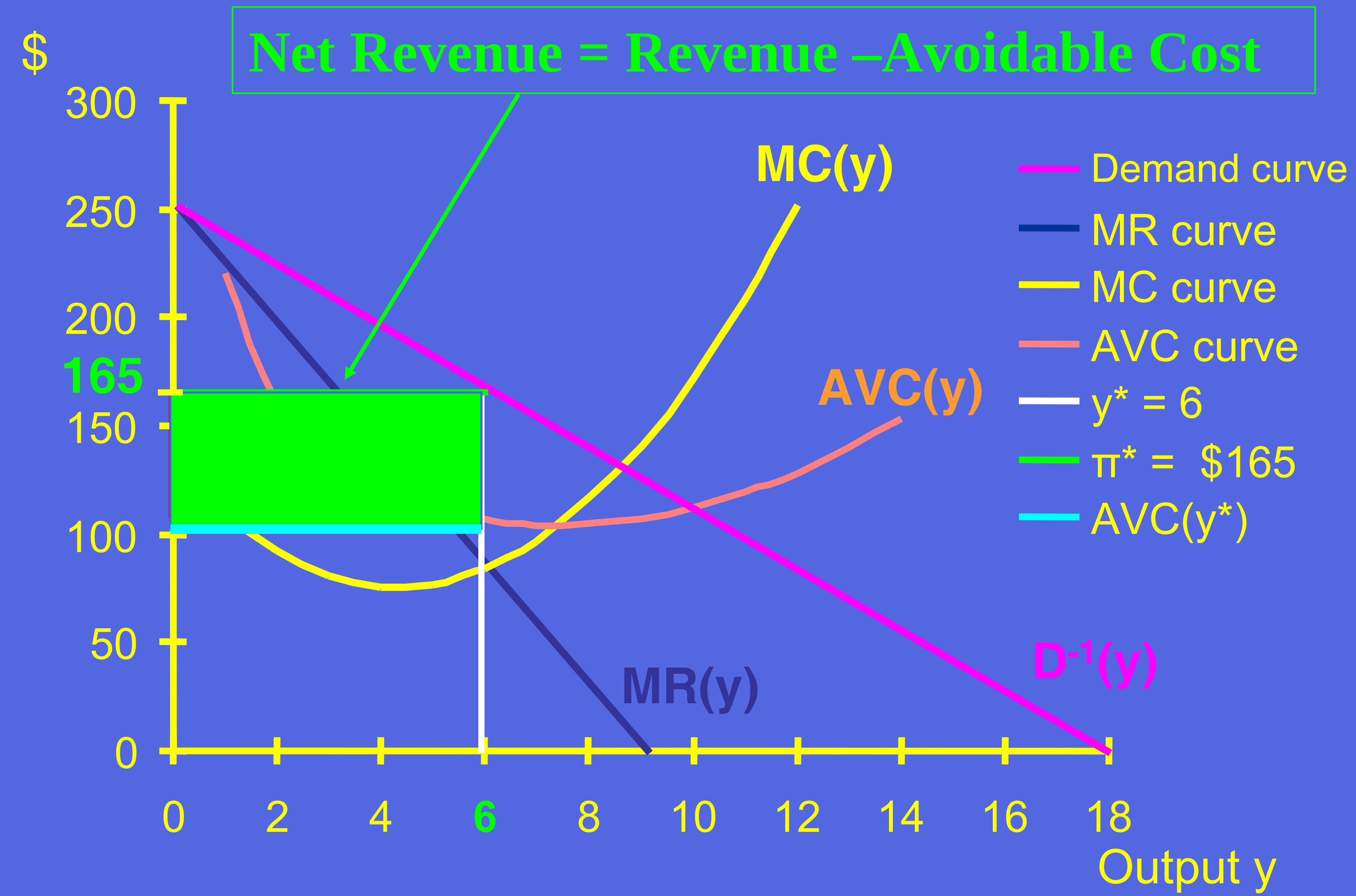
- ***Single-Price Monopolist's Profit Maximization Rule:***

  - Increase output if MR > MC ;
  - Decrease output if MR < MC ;
  - Choose output y* where MR = MC and set price equal to the consumer's maximum willingness to pay at y*, i.e., set

$$\pi^* = \pi(y^*) = D^{-1}(y^*) .$$

  - If revenue $\pi^* y^*$ covers avoidable cost at y*, produce y*. Otherwise, shut down.

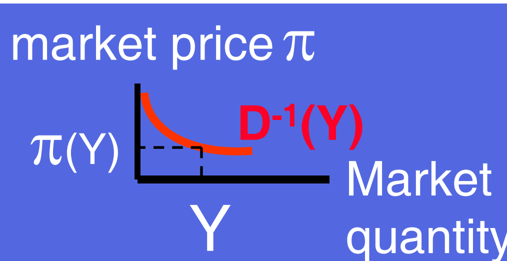
Single-Price Monopolist's Profit Maximization:



Natural Monopoly

- A production process is said to exhibit ***economies of scale*** if average costs of production decrease with increases in production.
- A monopoly is said to be a ***natural monopoly*** if it exhibits such extensive economies of scale that the one monopolist firm can produce at a lower cost per unit (i.e., at lower average cost) than can two or more firms.
- ***Key Issue for Power Industry:*** What aspects of electric power production (if any) are properly considered to be "natural monopolies" and hence potential candidates for regulation?

Cournot Duopoly (Two-Firm) Model of Quantity Competition



Let $Y = y_1 + y_2$, and let $\pi(Y)$ = market price given Y.

Problem for firm 1: $$\max_{y_1} \pi(y_1 + y_2^e)\,y_1 - c(y_1)$$

$$\Rightarrow \quad y_1 = f_1(y_2^e)$$ *(y^e_2 = expected y_2)*

Similar problem for firm 2:

$$\Rightarrow \quad y_2 = f_2(y_1^e)$$ *(y^e_1 = expected y_1)*

Cournot Nash Equilibrium:
$$y_1^* = f_1(y_2^*)$$
$$y_2^* = f_2(y_1^*)$$

Neither firm has any incentive to deviate from the equilibrium

Cournot Oligopoly (Multi-Firm) Model

Total industry output: $Y = y_1 + \cdots + y_n$, $s_i = y_i / Y$

Firm i:

$$\max_{y_i} \{ y_i \cdot \pi(Y) - c(y_i) \}$$

$$\frac{d}{dy_i} \{ y_i \cdot \pi(Y) - c(y_i) \} = 0$$

$$\pi(Y) + y_i \frac{d\pi(Y)}{dy_i} = \frac{dc(y_i)}{dy_i}$$

$$\pi(Y) \left\{ 1 + \frac{y_i}{Y} \frac{Y}{dy_i} \frac{d\pi(Y)}{\pi(Y)} \right\} = \frac{dc(y_i)}{dy_i}$$

$$\pi(Y) \left\{ 1 - \frac{s_i}{|\varepsilon(Y)|} \right\} = \frac{dc(y_i)}{dy_i}$$

This equals 1 for perfect competition

This equals 1 – | 1/ε(Y) | for a monopolist (i.e., when n = 1)

Cournot Oligopoly Model … Continued

$$\pi(Y)\left\{1-\frac{s_i}{\left|\varepsilon(Y)\right|}\right\}=\frac{dc(y_i)}{dy_i} = MC(y_i)$$

$$\underbrace{\left\{1-\frac{s_i}{\left|\varepsilon(Y)\right|}\right\}}_{< 1}$$

- Cournot firm i operates at a point where its marginal cost $MC(y_i)$ is ***less*** than the market price $\pi(Y)$
- Ability of firm i to profitably deviate from "competitive" $\pi(Y) = MC(y_i)$ point is a function of:
  - Market share $s_i = y_i / Y$
  - Inverse of price elasticity of demand $\varepsilon^{-1}(Y) = -\frac{d\pi(Y)}{dy_i}\frac{Y}{\pi(Y)} < 0$

Cournot Oligopoly Model…Continued

- We can now complete this model in the same way we did for the duopoly case (N=2)
- Consider the following equation:

$$\pi(Y)\left\{1-\frac{s_i}{|\varepsilon(Y)|}\right\}=\frac{dc(y_i)}{dy_i}$$

- Let Y be replaced by [$y_i + h(y^e_{-i})$] = Sum of y_i plus expected outputs for all firms but firm i
- Then optimal y_i choice for each firm i can be expressed as a function $y_i^* = f(y^e_{-i})$ of the expected output of other firms.
- Cournot Nash equilibrium obtained at $y^* = (y^*_1,\ldots,y^*_N)$ if the expectations of each firm i are fulfilled at y^*.

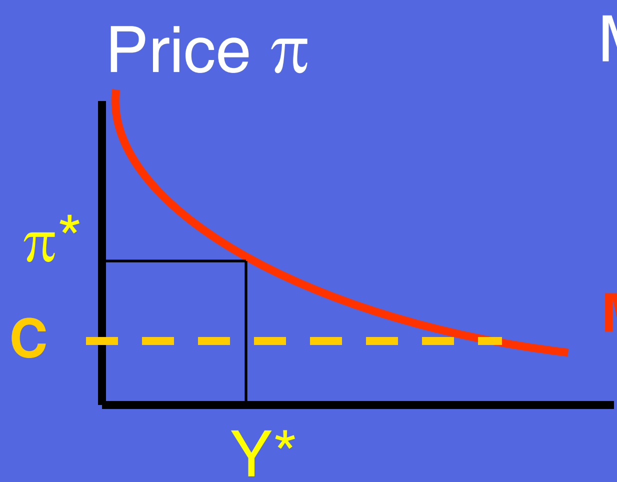
Bertrand Duopoly (Two-Firm) Model of Price Competition

- Two identical firms with constant marginal cost of production, c, are competing for demand
- Each firm offers a price with the promise to supply all quantity demanded from it at that price

IF a firm was a **monopolist**, it would solve:

$$\max_{\pi} \left[\pi D(\pi) - cD(\pi) \right] \Rightarrow \pi^* = c\left[\varepsilon^*/(1+\varepsilon^*)\right]$$

(with $\varepsilon^* = \varepsilon(\pi^*) < -1$)



Bertrand Duopoly … Continued

- However, the firm that sets the lowest price captures the entire market, so firms keep undercutting each other's price offers
- But neither firm will bid below its marginal cost of production, c, because it would sell at a loss
- Thus the only equilibrium is when each firm sells at the same price, equal to the marginal cost of production c
- Price=MC: Equivalent to competitive equilibrium!
- Note that the division of Y between the firms is indeterminate when $\pi = c$