MIDTERM EXAM ANSWER OUTLINE

YOUR NAME: ____

EE/Econ 458 — **L. Tesfatsion**
Midterm Exam (90 Points Total) — **Date: Thursday, March 5, 2009**

MIDTERM EXAM INSTRUCTIONS:

- **Please fill in your complete name in the indicated space above. BE SURE TO WRITE CLEARLY.**

- This midterm exam consists of 30 multiple choice questions worth 30 points in total, followed by three problems worth 60 points in total.

- Use this midterm exam packet for all your exam answers, and turn in this entire exam packet at the end of the exam.

- Answers for multiple choice questions should be clearly circled, and answers for problems should be given in legible well-organized form in the spaces provided beneath the problems.

- Several extra blank pages are included at the end of the packet for any needed scratch work.

- Read questions carefully before answering.

- When explanations are called for, justify your assertions carefully. When graphical depictions are used, indicate carefully what the axes variables represent and what is being plotted.

- Watch your time carefully. The exam is 90 minutes long and worth 90 points. Each point corresponds to approximately 1 minute of time.

MIDTERM EXAM: ANSWER OUTLINE

Q1. In electric power systems, ENERGY is defined to be _____ and its standard unit of measurement is _____.

A. the degree of control over grid operations; the operational budget ($)

B. the flow (instantaneous consumption) of power at a point in time; watts (W)

C. the heat content of fuels; British thermal units (Btu)

D D. the integration (accumulation) of electric power over some time interval; watt-hours (Wh)

Q2. In the U.S., the two most commonly used fuels for the generation of electric power are _____.

A A. natural gas and coal

B. coal and nuclear energy (NOTE: ALTHOUGH A IS THE "CORRECT" ANSWER, B WILL ALSO BE ACCEPTED AS AN ANSWER BECAUSE IN FACT THE PERCENTAGES IN THE ON-LINE NOTES FOR NATURAL GAS (20%) AND NUCLEAR ENERGY (19.4%) ARE VERY CLOSE)

C. oil and coal

D. coal and hydroelectric

Q3. The choice of fuels for electric power generation involves a trade-off between different types of costs. For example, typically the production of electric power by means of nuclear and solar energy sources involves _____ whereas the opposite is true for natural gas and oil.

A A. high fixed costs but low variable costs

B. low fixed costs but high variable costs

C. high marginal costs but low average costs

D. low short-run total costs but high long-run total costs

Q4. Key concerns driving the search for alternative fuels for the generation of electric power in the U.S. include _____

A. CO_2 emissions from coal contributing to a rising CO_2 concentration in the atmosphere.

B. eventual depletion of major fossil fuels (oil, gas, coal).

C. a persistent drop in the worldwide temperature.

D. all of the above.

E E. only A and B above.

Q5. Considered together, the U.S. Eastern Interconnection, Western Interconnection, and Texas Interconnection constitute the _____.

- **A.** seven wholesale power markets MISO, ISO-NE, NYISO, PJM, CAISO, SPP, and ERCOT.
- **B B.** U.S. wholesale (high voltage) electric power transmission grid.
- **C.** distribution and retail network servicing commercial, industrial, and household users of electric power.
- **D.** inter-connected collection of companies who buy and sell electric power in the U.S.

Q6. Major U.S. legislation that has strongly affected the electric power industry includes _____

- **A.** the creation in 1920 of the Federal Power Commission, forerunner of the U.S. Federal Energy Regulatory Commission.
- **B.** the 1935 Rural Electrification Act, which brought electricity to rural areas.
- **C.** the 1978 Public Utilities Regulatory Policies Act (PURPA), which began the process of opening up generation markets to competition.
- **D.** the 1992 Energy Policy Act, which forced transmission-owning utilities to provide transmission services to other utilities on a reasonable and nondiscriminatory basis.
- **E E.** all of the above.

Q7. The HEAT RATE for a power plant is _____

- **A.** the amount of heat required to raise the temperature of one pound of liquid water from 60^o F to 61^o F at a constant pressure of one atmosphere.
- **B.** the conversion rate between boiler temperature and steam generation.
- **C.** the maximum amount of heat the plant is allowed to disperse into the atmosphere per hour.
- **D D.** the conversion rate attained by the plant between the Btu content of a fuel and the resulting net output of electric energy (kWh).

Q8. The CAPACITY FACTOR of a power plant is _____

- **A.** the maximum capacity at which this plant can operate.
- **B.** the size of this plant relative to other plants.
- **C C.** the utilization rate of the plant (i.e., the ratio of the actual output of the plant to the output it would have obtained had it had operated at full capacity)
- **D.** the size of this plant relative to the largest plant in the market.

Q9. RESTRUCTURING of wholesale power markets refers to ______.

- **A.** reductions in variable costs of operation through more efficient fuel usage.
- **B.** a change from government to private ownership of utilities.
- **C.** the elimination of regulatory constraints.
- **D D.** changes in market structure and rules of market operation.

Q10. Many commentators agree that key problems hampering the restructuring of wholesale power markets into more "competitive" markets in which prices are determined by demand and supply conditions include ______.

- **A.** it is not feasible for vertically integrated utilities to profitably give up control of their generation assets.
- **B B.** the transmission grid must remain highly regulated to ensure open access
- **C.** it is not possible to price electric power because it cannot be economically stored
- **D.** the most efficient form of organization is the investor-owned utility.
- **E.** both B and C

Q11. According to Huntowski et al., inherent flaws in the REGULATED U.S. electric power industry prior to restructuring include ______

- **A.** a lack of clear price signals regarding true benefits and costs.
- **B.** risks undertaken by retail consumers of electric power were improperly allocated to utilities.
- **C.** risks undertaken by utilities were improperly allocated to retail consumers of electric power.
- **D.** only A and B.
- **E E.** only A and C.

Q12. According to Huntowski et al., a key factor making it difficult to properly evaluate the performance of restructured wholesale power markets since 2000 is ______

- **A A.** fuel costs for the production of electric power have been rising and have exhibited more volatility.
- **B.** there has been a huge diversion of resources to the study of global climate change.
- **C.** the parallel widespread movement to restructure retail markets.
- **D.** the failure of restructuring efforts in the telecommunications and trucking industries.
- **E.** all of the above.

Q13. An AUCTION is _____

A A. a centralized facility permitting buyers and sellers to trade by submitting demand bids and supply offers.

B. a decentralized facility handled by dealers who post bids to buy and offers to sell.

C. a decentralized facility facilitating bilateral trades.

D. a centralized facility in which a regulatory agency sets prices to achieve market efficiency.

Q14. A buyer's TRUE (INVERSE) DEMAND SCHEDULE _____.

A. gives the minimum purchase price that the buyer would be willing to pay for each successive quantity unit he purchases.

B B. gives the maximum purchase price that the buyer would be willing to pay for each successive quantity unit he purchases.

C. gives the maximum quantity that the buyer would be willing to supply for each successive unit price.

D. gives the minimum quantity that the buyer would be willing to purchase for each successive unit price.

Q15. The NET BUYER SURPLUS of a buyer who purchases a quantity amount q is _____

A. the difference between the minimum amount the buyer is willing to pay for q and the actual amount the buyer pays for q.

B B. the difference between the maximum amount the buyer is willing to pay for q and the actual amount the buyer pays for q.

C. the difference between the maximum amount the buyer is willing to pay for q and the minimum amount the seller is willing to accept as payment for q.

D. the difference between the buyer's minimum acceptable payment for q and the seller's maximum acceptable payment for q.

Q16. The PRICE ELASTICITY OF DEMAND for a commodity is _____.

A. the degree to which people change their consumption of the commodity when the price of all other commodities increases.

B. the rate at which demand for the commodity changes over time in response to changes in the general price level.

C C. the percentage change in quantity demanded per the percentage change in its unit price.

D. the percentage change in the price of the commodity per the percentage change in the quantity demanded.

Q17. MARKET EFFICIENCY means that _____

A. the market operator buys low and sells high.

B. the profits of market sellers are maximized.

C. dealer inventories are effectively managed so that, on average, they are zero.

D D. market traders are extracting their maximum possible total net surplus.

E. the payments of market buyers are minimized.

Q18. A COMPETITIVE MARKET CLEARING (CMC) POINT _____.

A. is a quantity and unit price combination at which sellers achieve their maximum possible extraction of net seller surplus and buyers achieve their maximum possible extraction of net buyer surplus.

B. is a quantity and unit price combination that results in the maximum possible extraction of total net surplus in a market.

C. is an intersection point of the true total supply and demand schedules (with vertical segments included).

D. all of the above.

E E. only B and C.

Q19. In economic theory, the main advantage of COMPETITIVE MARKET PRICING (i.e., pricing at a competitive market clearing level) over other forms of pricing is thought to be that it ensures _____

A. the lowest possible price for buyers.

B B. market efficiency.

C. sellers cover their fixed costs.

D. seller market power.

Q20. A seller in a market is said to have MARKET POWER if _____.

A. the seller is a producer of electric power.

B. the seller colludes with other sellers in an attempt to fix prices.

C C. the seller is able to secure additional profits for himself by moving the market price away from the competitive market clearing price level.

D. the seller has a dominant pricing strategy.

Q21. Consider a GenCo that uses different amounts of fuel z_1 and labor z_2 to produce electric power y in the short run with a fixed amount $\bar{k}$ of capital equipment. Then, by definition, the GenCo's SHORT RUN PRODUCTION FUNCTION is a relationship giving _____

A. the minimum amount of inputs z_1 and z_2 needed to produce each successively higher amount of power output y, conditional on $\bar{k}$.

B. the maximum amount of inputs z_1 and z_2 needed to produce each successively higher amount of power output y, conditional on $\bar{k}$.

C C. the maximum possible power output y for each different combination of fuel and labor inputs z_1 and z_2, conditional on $\bar{k}$.

D. the actual power output y attained for each different combination of fuel and labor inputs z_1 and z_2, conditional on $\bar{k}$.

Q22. Consider a GenCo that uses different amounts of coal c to produce electric power y in the short run with a fixed amount $\bar{k}$ of capital equipment. Then, by definition, the GenCo's INPUT-OUTPUT FUNCTION FOR COAL is a relationship that gives _____

A. the minimum amount of coal c and capital equipment k required to produce each successively higher amount of power y.

B. the actual amount of coal c used to produce each successively higher amount of power y, conditional on $\bar{k}$.

C C. the minimum amount of coal c required to produce each successively higher amount of power y, conditional on $\bar{k}$.

D. the actual power output y attained for each different coal amount c, conditional on $\bar{k}$.

Q23. The total variable cost of a firm is _____

A. the rate of change of the firm's total cost of production.

B. the rate of change of the firm's marginal cost of production.

C. the portion of the firm's total cost of production that varies with its level of fuel usage.

D D. the portion of the firm's total cost of production that varies with its level of production.

Q24. For a competitive (price-taking) producer of widgets who wishes to maximize his profits in the short run, the basic rule of thumb is _____

A. keep producing widgets until the short-run marginal cost of producing the next widget falls below the market price.

B. keep producing widgets until the short-run average cost of producing the next widget exceeds the market price.

C C. keep producing widgets until the short-run marginal cost of producing the next widget exceeds the market price.

D. keep producing widgets until the short-run total cost of production exceeds the market price.

Q25. The BERTRAND DUOPOLY MODEL assumes that two firms _____ whereas the COURNOT DUOPOLY MODEL assumes that two firms _____.

A. efficiently set price equal to marginal cost; monopolistically set marginal revenue equal to marginal cost.

B. maximize their joint profits through quantity collusion; maximize their joint profits through price collusion.

C **C.** engage in price competition; engage in quantity competition.

D. engage in quantity competition; engage in price competition.

Q26. A FINANCIAL ASSET is _____

A **A.** a claim against physical assets.

B. an amount of money.

C. any asset owned by a financial firm.

D. an asset that is not durable, in contrast to physical assets.

Q27. A SPOT TRADE is _____

A. a trade agreement between a buyer and a seller that is reached at a particular location.

B. a personalized trade between two traders that is not standardized with respect to quality or quantity.

C **C.** an immediate trade, with both delivery and payment taking place at the same time.

D. a trade that determines a particular time and place for delivery of a physical commodity.

Q28. A HEDGER _____

A. lends assets to borrowers in an attempt to secure a financial return.

B. trades commodities in an attempt to take profitable advantage of price differences.

C. takes risks in the hopes of securing a larger rate of return.

D **D.** engages in financial asset trades to protect wealth.

Q29. PRICE RISK is _____

A. the price that insurance companies charge to their customers for bearing their risk of loss.

B B. the possibility of having to buy at a price higher than expected or sell at a price lower than expected.

C. the possibility of having to buy at a price lower than expected or sell at a price higher than expected.

D. the risk that prices will increase over time (i.e., inflation will occur).

Q30. A key DISTINCTION between a forward contract and a futures contract is that _____

A. a futures contract sets a pre-determined price for future delivery, whereas a forward contract does not.

B. a futures contract involves future delivery, whereas a forward contract involves immediate delivery.

C C. a futures contract is a standardized contract for a commodity whereas a forward contract is a personalized contract between two parties for something of value.

D. a futures contract permits delivery anytime prior to the expiry date whereas a forward contract does not.

PROBLEM 1: (35 Points Total). Suppose a firm's short-run total cost function for the production of gizmos y during a particular month m is given by the following expression:

$$SRTC(y) \; = \; 10y^2 \; + \; 200y \; + \; 100,000 \quad , \tag{1}$$

where $\$100,000$ is the firm's payment obligation for rental of its business facilities during month m. In the following parts, "short run" will consistently be used to refer to the firm's production activities during month m.

Part A (4 Points): What is the firm's short-run total variable cost function and the firm's fixed costs? Explain carefully.

Part B (4 Points):

- (a) Define, in words, what is meant by the firm's *short-run profits* for any nonnegative output level y.
- (b) Provide an expression giving the firm's short-run profits as a function of its output level y, conditional on a given market price π.

Part C (18 Points): Suppose the market price for gizmos in month m is $\pi = \$2400$.

- (a) Determine the entire range of positive output levels y over which it would be profitable for this firm to produce *in the short run*.
- (b) Show, in particular, that the positive output level $y^* = 110$ maximizes short-run profits for the firm, given the market price $\pi = \$2400$, and provide a graphical depiction of your findings.

Part D (9 Points): Now suppose the firm is considering its most profitable production level *in the longer run*, say for the next month $m+1$. Suppose the firm has only two options at the beginning of month $m+1$: (a) renew its agreement for the monthly rental of its business facilities; or (b) shut down all business operations.

- (a) Explain carefully why it is still optimal for the firm to continue producing at the level $y^* = 110$ as long as its monthly rental payment stays at \$100,000.
- (b) Also, explain briefly why it could become optimal for the firm to shut down (set $y = 0$) if its monthly rental payment becomes sufficiently large.

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Answer Outline for Part A (4 Points):

By definition, the firm's *short-run total variable cost* is the portion of the firm's short-run total cost that varies with its production level, and the firm's *fixed cost* is the portion of its short-run total cost that does not vary with its production level. Consequently, it follows from (1) that the firm's short-run total variable cost, as a function of the firm's output level y, is given by

$$SRTVC(y) \; = \; 10y^2 \; + \; 200y \tag{2}$$

and the firm's fixed cost (the same regardless of the output level y) is given by

$$SRTC(0) \; = \; 100,000 \tag{3}$$

Answer Outline for Part B (4 Points):

The short-run profits of this firm for any output level y are its total revenues from sale of y minus its short-run total cost of producing y. Given a market price π, the analytical expression for these short-run profits is thus

$$\text{SRProfits}(y) \ = \ \pi y - SRTC(y) \ \ , \tag{4}$$

where $SRTC(y)$ is as given in (1).

Answer Outline for Part C (18 Points):

By definition, the *short-run marginal cost function* for the firm is given by the following linear relation with a positive slope of 20 and an ordinate of 200:

$$SRMC(y) \ = \ \frac{\partial SRTVC(y)}{\partial y} \ = \ 20y \ + \ 200 \tag{5}$$

The firm's short-run marginal cost at the production level $y = 0$, SRMC(0) = 200, is strictly less than the market price 2400. Consequently, it is profitable for the firm to produce at least some positive amount of y, because the corresponding total revenues ($\$2400 \times y$) over this range for y will be greater than short-run total variable cost (the integral of the SRMC curve over this range for y), implying positive net revenues that can be used to pay down at least part of the firm's obligatory fixed-cost rental payment of $100,000.

Indeed, the firm should keep on producing y in the short run (i.e., during month m) until its short-run marginal cost just equals the market price $2400. This gives one equation in the one unknown y, as follows:

$$20y + 200 \ = \ 2400 \tag{6}$$

The profit-maximizing output level is then the solution y^* to equation (6), $y^* = 110$, and the range of positive output over which it is profitable for the firm to produce in the short run (in the sense that each successive produced unit results in a positive net revenue) is $0 < y \leq y^* = 110$.

Note that production at $y^* = 110$ does *not* necessarily guarantee that the firm's *short-run profits* (4) are positive at the output level $y^* = 110$, i.e., that the firm's resulting revenues cover its short-run total costs given by the sum of its *fixed* cost and its total *variable* cost. Rather, by construction, production at $y^* = 110$ in the short run only guarantees that the firm in the short run attains the highest possible net revenues, given by its total revenues minus its total *variable* cost. Since the firm can ensure for itself zero net revenues in the short run by not producing anything, it will only produce a positive output in the short run if its resulting net revenues are nonnegative.

However, no matter what the firm chooses to produce in the short run, it must still pay its fixed cost because, in the problem at hand, these fixed costs are assumed to be obligatory (non-avoidable). Non-avoidable fixed costs are typically referred to as *sunk* costs in the economics literature.

Some students misinterpreted what I was trying to ask for here because of the ambiguity of my use of the word "profitable". I meant "profitable" in the incremental sense that each additional unit of y brings in additional net revenue (market price > SRMC), which led to my above answer that the "profitable" range is $0 < y \leq y* = 110$. But some students simply found the range of all y for which [Revenues(y) - SRTVC(y)] remained positive, i.e., the range $0 < y < 220$.

It is true that production at any point in the range $0 < y < 220$ would be preferred by the firm to producing nothing at all. Consequently, either answer will be accepted for Part C(a).

For whichever way Part C:(a)-(b) was answered, **a graphical illustration of findings should be given**. Such a graph could take several forms. For example, one could depict the positively sloped linear $SRMC(y)$ function satisfying $SRMC(0) = 200$ at $y = 0$ and rising to intersect the market price line π=2400 at $y* = 110$. The area underneath this $SRMC(y)$ curve through the intersection point $y* = 110$ then represents $SRVC(110)$, and the area between the market price line π=2400 and the $SRMC(y)$ curve through the intersection point $y* = 110$ represents the net revenues earned at $y* = 110$. Alternatively, one could depict the (strictly concave) net revenue function for the firm,

$$NR(y) = 2400y - SRTVC(y) \quad . \tag{7}$$

The function $NR(y)$ equals 0 at y=0, rises to a maximum level of 121,000 at y*=110, falls back down to zero at y=220, and then turning increasingly negative for larger y. The strict concavity of $NR(y)$ follows from the strict negativity of its second derivative for all y: $NR''(y) = -20 < 0$.

IMPORTANT REMARK:

Given the troubles that students had with this question, the on-line EE/Econ 458 lecture notes for Kirschen/Strbac Chapter 2 (Sections 2.3 and 2.5) have been substantially revised to explain more carefully the interpretation, derivation, and relationships among total, average, and marginal cost curves in any given planned production period T. In particular, it is explained how the more traditional decomposition of total costs into "variable" and "fixed" costs is now being replaced in current microeconomic texts by a decomposition of total costs into "avoidable" and "sunk (non-avoidable)" costs.

Answer Outline for Part D (9 Points):

In the longer run (i.e., for month $m + 1$) the firm can reduce its *total cost* to zero simply by shutting down (i.e., setting $y = 0$ and not renewing its rental agreement). Thus the firm will shut down (produce 0) in the longer run unless it can produce at a positive output level where it earns nonnegative *longer-run profits*, here given by total revenues minus total cost in month $m + 1$.

More precisely, the firm earns nonnegative profits at an output level y in the longer run if and only if its revenues are at least as great as its total cost, including the rental payment $R = \$100{,}000$, i.e., if and only if the following condition holds:

$$2400y \ \geq \ TC(y) \ = \ 10y^2 \ + \ 200y + R \tag{8}$$

If y is positive, condition (8) can equivalently be expressed as the requirement that market price be greater or equal to average cost,

$$2400 \ \geq \ TC(y)/y \ = \ AC(y) \ = \ 10y \ + \ 200 + R/y \tag{9}$$

Straightforward calculation shows that the range of positive output levels y satisfying the inequality (9) contains the output level $y^* = 110$, because AC(110) = 2209. Consequently, it turns out that producing at $y^* = 110$ is better than shutting down in the longer run.

Since, by construction, y^* maximizes profits over all positive production levels by setting marginal cost equal to market price, it follows that y^* is the profit-maximizing output level for the firm in the longer run (month $m + 1$) as well as in the short run (month m).

Clearly, however, a sufficiently large level for the rental payment R in (9) would render all positive production levels unprofitable in the longer run.

TECHNICAL REMARK: It can be shown that the range of positive y values satisfying the quadratic inequality (8) is approximately given by

$$155.826 \;\geq\; y \;\geq\; 64.174 \tag{10}$$

For any positive y outside this range, the firm will prefer to shut down. Why? Because the firm earns negative profits outside of the range (10) whereas it can earn zero profits by shutting down since the rental payment is now an "avoidable cost".

PROBLEM 2: (13 Points Total).

Suppose the rules of an electricity market stipulate that all participants must trade energy exclusively through an ISO-managed spot market conducted as a uniform-price double auction. However, two participants in this spot market, GenCo A and LSE B, have signed a two-way contract for difference (CFD) for the delivery by GenCo A of 100MW to LSE B on a continual hourly basis at a strike price of \$10/MWh.

Part A (5 Points): Define, in words, what is meant by a *two-way contract for difference*.

Part B (4 Points): Determine the *flow of electric power and money* between GenCo A and LSE B in hour H if the spot market price for hour H is \$20/MWh.

Part C (4 Points): Determine the *flow of electric power and money* between GenCo A and LSE B in hour H if the spot market price for hour H is \$5/MWh.

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Answer Outline for Part A (5 Points):

As covered by Kirschen/Strbac in Section 2.4.5, as well as in class lectures, a two-way CFD is a type of contract arranged by a buyer and seller who are obliged to trade solely through some centralized market but who wish to trade a desired commodity in amount q^* at a desired per-unit price π^*.

To ensure this desired price, the buyer and seller agree on a combined call option and put option for the commodity amount q^* at a common strike price π^*. If the strike price π^* is higher than the centralized market price at which the buyer and seller are obliged to purchase and sell the desired amount q^*, the buyer agrees to pay the seller the difference between these two prices times the amount q^*. Conversely, if the strike price π^* is less than the centralized market price at which the buyer and seller are obliged to purchase and sell q^*, then the seller agrees to pay the buyer the difference between these two prices times the amount q^*.

Effectively, then, the buyer and seller have guaranteed they can trade their desired commodity amount q^* at their desired price p^*.

Answer Outline for Part B (4 Points):

Under the CFD, the arrangement is for LSE B to pay \$1000/h = [\$10/MWh]×100MW to GenCo A for the hourly delivery of 100MW.

However, the spot market price (\$20/MWh) is higher than the strike price (\$10/MWh) for hour H. Consequently, in hour H, GenCo A produces 100MW and sells this power in the spot market for a total payment of \$2000/h = [\$20/MWh] × 100MW, while LSE B pays \$2,000/h to purchase 100MW from the spot market. Consequently, LSE B pays \$1000 more and the seller receives \$1000 more in payment from the spot market in hour H than arranged under the CFD.

GenCo A then "makes whole" LSE B for hour H by paying LSE B an amount \$1000 = [\$20 - \$10]× 100.

Answer Outline for Part C (4 Points):

As before, under the CFD the arrangement is for LSE B to pay \$1000/h = [\$10/MWh]×100MW to GenCo A for the hourly delivery of 100MW.

However, the spot market price (\$5/MWh) is lower than the strike price (\$10/MWh) for hour H. Consequently, in hour H, GenCo A produces 100MW and sells this power in the spot market for a total payment of \$500/h = [\$5/MWh] $\times$ 100MW, while LSE B pays \$500/h to purchase 100MW from the spot market. Consequently, LSE B pays \$500 less and the seller receives \$500 less in payment from the spot market in hour H than arranged under the CFD.

LSE B then "makes whole" GenCo A for hour H by paying GenCo A an amount \$500 = [\$10 - \$5]$\times$ 100.

PROBLEM 3: (12 Points Total). Consider an economic dispatch problem for a 5-bus transmission grid with three GenCos and a given real-power total demand (load) of $P_D = 975$MW. Suppose the short-run variable cost functions for the three GenCos, in \$/hr, are as follows:

$$VC_1(P_{G1}) \quad = \quad 0.004P_{G1}^2 \; + \; 5.3P_{G1} \;\; ; \qquad (11)$$

$$(12)$$

$$VC_2(P_{G2}) \quad = \quad 0.006P_{G2}^2 \; + \; 5.5P_{G2} \;\; ; \qquad (13)$$

$$(14)$$

$$VC_3(P_{G3}) \quad = \quad 0.009P_{G3}^2 \; + \; 5.8P_{G3} \;\; , \qquad (15)$$

where P_{Gi} denotes the real-power dispatch level of GenCo i. Suppose, also, that there are no GenCo capacity constraints, no congested transmission lines, and no line losses.

Part A (2 Points): Carefully express the objective function and constraint(s) for this economic dispatch problem.

Part B (2 Points): What is the Lagrangian function for this economic dispatch problem.

Part C (4 Points): Use the Lagrangean function from Part B to express the first-order necessary conditions (FONC) for a dispatch $(P_{G1}^*, P_{G2}^*, P_{G3}^*)$ to solve this economic dispatch problem. Write out these FONC explicitly as a set of four equations in four unknowns.

Part D (2 Points): The dispatch solution implied by these FONC is approximately given by

$$P_{G1}^* \quad = \quad 483.0MW \;\; ; \qquad (16)$$

$$(17)$$

$$P_{G2}^* \quad = \quad 305.4MW \;\; ; \qquad (18)$$

$$(19)$$

$$P_{G3}^* \quad = \quad 186.6MW \;\; . \qquad (20)$$

Use these dispatch solution values to obtain an explit equation for λ^*, the Lagrange multiplier (shadow price) solution value that gives the change in the optimized objective function with respect to a change in the total demand. (**Important Note:** What I am seeking here in Part D is a single equation of the form λ^* = numerical expression, where the numerical expression involves products and additions of numbers. You do not have to do out the multiplications and additions of these numbers!)

Part E (2 Points): The solution value λ^* in Part D is the "locational marginal price" at each bus of the transmission grid. Can you explain why?

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Answer Outline for Part A (2 Points):

Minimize

$$\sum_{i=1}^{3}[VC_i(P_{Gi})] \tag{21}$$

with respect to P_{G1}, P_{G2}, and P_{G3} subject to the **Real-Power Balance Constraint:**

$$\sum_{i=1}^{3}[P_{Gi}] \;=\; P_D \tag{22}$$

Answer Outline for Part B (2 Points):

$$L(P_{G1}, P_{G2}, P_{G3}, \lambda, P_D) \;=\; \sum_{i=1}^{3}[VC_i(P_{Gi})] \;+\; \lambda\left(P_D - \sum_{i=1}^{3}[P_{Gi}]\right) \tag{23}$$

Answer Outline for Part C (4 Points):

$$0 \;=\; \frac{\partial L(P_{G1}, P_{G2}, P_{G3}, \lambda, P_D)}{\partial P_{Gi}} \;=\; \frac{dVC_i(P_{Gi})}{dP_{Gi}} - \lambda\;, \quad i = 1, 2, 3 \;\; ; \tag{24}$$

$$0 \;=\; \frac{\partial L(P_{G1}, P_{G2}, P_{G3}, \lambda, P_D)}{\partial \lambda} \;=\; \left(P_D - \sum_{i=1}^{3}[P_{Gi}]\right) \tag{25}$$

Answer Outline for Part D (2 Points):

Any one of the relations (24) can be used to obtain the solution value for λ once the solution value for the corresponding real-power dispatch level is plugged into it. For example, using the first relation:

$$0 \;=\; (0.008P_{G1} + 5.3 \;-\; \lambda) \;=\; ([0.008][483] + 5.3 \;-\; \lambda) \quad . \tag{26}$$

Although not asked for, the approximate solution to equation (26) is $\lambda^* = 9.164$.

Answer Outline for Part E (2 Points):

By definition, a *locational marginal price* at any transmission grid bus is the least cost of servicing one additional MW of demand at this bus.

The objective for the economic dispatch problem is the minimization of total variable cost. The one constraint for the economic dispatch problem is a balance constraint requiring the sum of the real-power dispatch levels to equal given total demand, where given total demand is the "constraint constant" for this balance constraint.

Hence, the claim follows from the ability to express λ^*, the shadow price for the balance constraint, as the change in minimized total variable cost with respect to a change in total demand, as shown in the on-line notes on "Optimization Basics for Electric Power Markets."