ANSWER OUTLINE: FINAL EXAM - VERSION A

YOUR NAME: ____

EE/Econ 458 **L. Tesfatsion**
Final Exam (120 Points Total) **Date: Thursday, May 7, 2009**

FINAL EXAM INSTRUCTIONS:

- **Please fill in your complete name in the indicated space above. BE SURE TO WRITE CLEARLY.**

- This final exam consists of Part I (20 multiple choice questions worth 20 points in total), followed by Part II (three problems worth 100 points in total).

- Use this final exam packet for all your exam answers, and turn in this entire exam packet at the end of the exam.

- Answers for multiple choice questions should be clearly circled, and answers for problems should be given in legible well-organized form in the spaces provided beneath the problems.

- Several extra blank pages are included at the end of the packet for any needed scratch work.

- Read questions carefully before answering.

- When explanations are called for, justify your assertions carefully. When graphical depictions are used, indicate carefully what the axes variables represent and what is being plotted.

- Watch your time carefully. The exam is 120 minutes long and worth 120 points. Each point corresponds to approximately 1 minute of time.

VERSION A
PART I: TWENTY MULTIPLE CHOICE QUESTIONS

Q1. In electric power systems, ELECTRIC ENERGY is defined to be _____ and its standard unit of measurement is _____.

A. the heat content of fuels; British thermal units (Btu)

B. the flow of electrons at a point in time; watts (W)

C. the degree of control over grid operations; the operational budget ($)

D **D.** the integration (accumulation) of electric power over some time interval; watt-hours (Wh)

Q2. A WHOLESALE electric power market refers to the _____ whereas a RETAIL electric power market refers to the _____.

A. production and end-use consumption of electric power; wheeling of electric power through retail intermediaries.

B **B.** generation and high-voltage transmission of electric power; distribution of lower-voltage electric power to final customers.

C. generation of electric power; transmission of electric power

D. purchase and sale of electric power through centralized facilities; purchase and sale of electric power through bilateral trades.

Q3. RESTRUCTURING of wholesale power markets refers to _____.

A. reductions in production costs through more efficient fuel usage.

B. a change from government to private ownership of utilities.

C. the elimination of regulatory constraints.

D **D.** changes in market structure and rules of market operation.

Q4. At present, characteristics of electric energy important for the operation of restructured wholesale power markets include _____.

A. electric energy cannot be economically stored in large quantities

B. load (demand) for electric power tends to be insensitive to price

C. injections and withdrawals of electric power on the transmission grid have to be kept in balance

D D. all of the above

E. only A and B above

Q5. In the U.S., approximately _____ of generation capacity is now operating under some form of restructured wholesale power market.

A. 30%

B B. 50%

C. 80%

D. 100%

Q6. In the lower 48 states of the U.S., the high-voltage transmission grid is divided into _____ major components.

A. two

B B. three

C. four

D. five

E. seven

Q7. By definition, the SUNK COSTS of a firm during a particular time period are the _____

A. expenditures of the firm on capital equipment.

B. amount of borrowing the firm has undertaken.

C C. payment obligations the firm has irrevocably committed to and hence cannot recover.

D. costs of the firm that do not vary with its level of production.

Q8. The PRICE ELASTICITY OF DEMAND for a commodity is _____.

A. the degree to which people change their consumption of the commodity when the price of all other commodities increases.

B. the rate at which demand for the commodity changes over time in response to changes in the general price level.

C C. the percentage change in quantity demanded per the percentage change in its unit price.

D. the percentage change in the price of the commodity per the percentage change in the quantity demanded.

Q9. A key characteristic of PERFECTLY COMPETITIVE markets is that _____

A. every seller perceives it is in direct competition with all other sellers.

B B. all buyers and sellers take the market price as given when determining their quantity decisions.

C. buyers and sellers perceive they can influence the market price through their quantity decisions.

D. each seller attempts to undercut the sale prices offered by other sellers.

Q10. A profit-seeking firm participating in an imperfectly competitive market is said to have a DOMINANT STRATEGY for selecting its price and quantity decisions if _____.

A A. this strategy is most profitable for the firm regardless of the price and quantity decisions of other market participants.

B. this strategy is the best the firm can do, given the current price and quantity decisions of all other market participants.

C. this strategy ensures minimal production costs for the firm, regardless of the price and quantity decisions of all other market participants.

D. this strategy minimizes the firm's total costs.

Q11. A wholesale power power market is said to be EFFICIENT if _____.

A A. the total net surplus extracted by the buyers and sellers is as large as possible.

B. no individual buyer or seller has an incentive to deviate from his current bid or offer given the current bids and offers of all other buyers and sellers.

C. the total quantity demanded equals the total quantity supplied.

D. the number of buyers and sellers is so large that no trader is able to affect the price.

Q12. A seller in a market is said to have MARKET POWER if _____.

A. the seller is a producer of electric power.

B. the seller colludes with other sellers in an attempt to fix prices.

C C. the seller is able to secure additional profits for himself by moving the market price away from the competitive market clearing price level.

D. the seller has a dominant pricing strategy.

Q13. By definition, a DOUBLE AUCTION is _____

- **A.** a decentralized facility facilitating bilateral (double) trades.
- **B.** a decentralized facility handled by dealers who engage in double posting of both bids to buy and offers to sell.
- **C C.** a centralized facility permitting buyers and sellers to trade by submitting demand bids and supply offers, respectively.
- **D.** a centralized facility in which a regulatory agency sets prices to achieve double goals (market efficiency and market reliability).

Q14. By definition, a FINANCIAL ASSET is _____

- **A.** any type of paper money.
- **B B.** a claim against physical assets.
- **C.** any type of asset owned by a financial institution.
- **D.** anything of value available for purchase and sale in standardized form.
- **E.** a contract that promises the issuer a definite stream of payments.

Q15. By definition, a FUTURES CONTRACT is _____

- **A A.** a standardized contract between parties for the future delivery of some commodity at a pre-determined price.
- **B.** a bet that a specified price will move in a specified direction at a particular future time.
- **C.** a personalized contract between parties for the future delivery of something of value at a pre-determined price.
- **D.** a promise to deliver a specified amount of futures at a specified time and at a pre-determined price.
- **E.** a right (not an obligation) to buy a specified quantity at a specified price on a specified future date.

Q16. As defined by Kirschen/Strbac, RISK is the _____.

A. probability that a party to a financial contract will make a promised physical delivery.

B. likelihood that a disaster will occur.

C. probability that a party to a financial contract will meet a payment obligation.

D D. possibility that an actual outcome will deviate from an expected outcome.

Q17. Examples of SPOT TRADES include _____

A. you purchase a loaf of bread from HyVees.

B. you purchase a share of Alliant Energy Corporation for $24.00 on the New York Stock Exchange.

C. a generation company sells electric energy today for delivery during the noon hour one week from today.

D. all of the above.

E E. only A and B above.

Q18. By definition, a LOCATIONAL MARGINAL PRICE at a particular transmission grid bus A at a particular time T is _____

A. the price for electric power at bus A at time T as determined by a regulated rate.

B. the price of electric power at bus A at time T as determined by the condition that total load equals total generation.

C. the marginal cost of the least expensive GenCo operating at bus A at time T.

D D. the least cost to the system of servicing an additional MW of load at bus A at time T.

Q19. A generation company (GenCo) and load-serving entity (LSE) located on an ISO-managed transmission grid who wish to enter into a one-year bilateral contract with each other for the purchase/sale of electric power at a fixed price of \$80/MWh might need to consider a CONTRACT FOR DIFFERENCE if _____

A. there is a chance that fuel prices will vary during the life of the contract.

B **B.** injections and withdrawals of electric power on the transmission grid are subject to locational marginal pricing (LMP).

C. there is a chance that labor costs will vary during the life of the contract.

D. there is a chance the GenCo or LSE will default on the contract.

Q20. By definition, a GenCo is said to be MARGINAL if _____

A. the GenCo is operating at a point where its marginal cost is strictly below the price at its bus.

B. the GenCo is operating at a point where its marginal cost is strictly above the price at its bus.

C **C.** the GenCo is operating at a point strictly between its minimum and maximum operating capacities.

D. the GenCo is operating at a point where total load equals total generation.

VERSION A
PART II: THREE PROBLEMS

PROBLEM 1: (32 Points Total; 4 Points Each Part A through H).

Consider a Generation Company (GenCo) G at the beginning of some hour H that is trying to decide its optimal level P∗ for the production of active power P (MW) during hour H. This question first asks you to sort out and categorize the different types of costs for GenCo G. It then asks you to set out the criterion that GenCo G should use to determine P∗ assuming that GenCo G wishes to make as much money as possible during hour H.

Suppose GenCo G owns a single gas-fired generating unit, U. Unit U is shut down at the beginning of hour H, and the start-up cost for unit U in hour H is 10 (\$/h). The cost of gas during hour H is 4.00 (\$/MBtu) and the labor cost associated with maintaining unit U in operation during hour H at any non-negative production level P is estimated to be 40 P (\$/h).

During the previous year GenCo G borrowed funds to finance the purchase of specialized equipment E to help with the operation of unit U. The pro-rated hourly interest payment that GenCo G is obliged to make to service this loan is 3.00 (\$/h), and the specialized equipment E has no resale value. However, GenCo G estimates that the value that it could get from E in its next best alternative use in its own plant during hour H is 1.00 (\$/h).

Based on the current production facilities of GenCo G, the heat rate H(P) for unit U is given by

$$\text{H}(P) = 15 + 0.1P \quad \text{(MBtu/MWh)} \tag{1}$$

where P (MW) denotes GenCo G's production of active power. Finally, the market price of active power P (MW) in hour H is 180 (\$/MWh).

Derive a quantitative expression for each of the following seven terms, being sure to justify your assertions with care:

A. Sunk cost (\$/h) for GenCo G for hour H.

Answer Outline for A:

By definition, the *sunk cost* for GenCo G for hour H is the portion of its fixed production cost that it *cannot* recover by shutting down production and taking suitable alternative actions (e.g., sale or alternative use of assets). The portion of GenCo G's fixed cost that it *can* recover by shutting down production and taking suitable alternative actions is called *avoidable fixed cost*.

GenCo G is obliged to make a payment of \$3.00 in hour H for the equipment E regardless whether it produces or not in hour H, so this portion of its production cost is fixed. If GenCo G chooses not to produce, it can earn \$1.00 from a best alternative use of E that can be used to cover a portion of its fixed cost. By definition, then, GenCo G's sunk cost (i.e., unavoidable fixed cost) is \$2.00 and GenCo G's avoidable fixed cost is \$1.00.

Remark: As will be clarified below in Parts G and H, GenCo G should go ahead and produce power in hour H only if its revenues from production – net of all variable costs (i.e., fuel and labor costs) – are at least \$1.00. Otherwise it earns more by not producing and by instead using E in its next best alternative use. The correct criterion for production is thus arrived at by lumping together all avoidable cost, both variable cost and avoidable fixed cost, and requiring revenues to cover all of these avoidable costs in order for production to proceed.

B. Avoidable cost (\$/h) for GenCo G during hour H as a function of P.

Answer Outline for B: By definition, *avoidable cost* during a planning period T is the production cost that a firm can avoid by shutting down production and taking suitable alternative actions (e.g., sale or alternative use of assets). For the case at hand, for planning hour H, there are four types of avoidable cost:

1. avoidable fixed cost (\$/h) equal to the 1.00 (\$/h) that is recoverable by alternative usage of the equipment E during hour H;
2. start-up cost Start(P) (\$/h) for unit U, a variable cost given by 10 (\$/h) for $P > 0$ and 0 (\$/h) for P=0. (Note – for simplicity, multi-period unit commitment issues are clearly being ignored here!);
3. fuel cost (\$/h) for the operation of unit U at various power levels P, a variable cost;
4. labor cost (\$/h) for the operation of unit U at various power levels P, a variable cost, which for the problem at hand is given as 40P (\$/h).

The calculation of FuelCost(P) ($/h) as a function of the power level P is found by first multiplying the heat rate H(P) (MBtu/MWh) in (1) by the per-MBtu cost of gas in hour H, 4.00 ($/MBtu), and then multiplying the resulting per-MWh fuel cost by the power level P (MW), as follows:

$$\text{FuelCost}(P) \;=\; H(P) \times [4] \times P \;=\; 60P \;+\; 0.4P^2 \;\; (\$/h) \qquad (2)$$

Consequently, the desired function AvoidCost(P) for GenCo G in hour H is given by

$$\text{AvoidCost}(P) = 1.00 + \text{Start}(P) + 100P + 0.4P^2 \;\; (\$/h) \qquad (3)$$

for $P \geq 0$.

C. Total cost ($/h) for GenCo G during hour H as a function of P.

Answer Outline for C: By definition, *total cost* for GenCo G during hour H as a function of P is given by sunk cost plus avoidable cost, as follows:

$$\begin{aligned} \text{TotCost}(P) &= \text{SunkCost} + \text{AvoidCost}(P) \\ &= 2.00 + [1.00 + \text{Start}(P) + 100P + 0.4P^2] \\ &= 3.00 + \text{Start}(P) + 100P + 0.4P^2 \;\; (\$/h) \qquad (4) \end{aligned}$$

for $P \geq 0$.

D. Marginal cost of production ($/MWh) for GenCo G during hour H as a function of P.

Answer Outline for D: By definition, the *marginal cost of production* MC(P) for GenCo G during hour H as a function of P is given by the change in TotCost(P) — or equivalently, by the change in AvoidCost(P) — with respect to a change in P. For $P > 0$, MC(P) is therefore given as follows:

$$\text{MC}(P) \;=\; \frac{\text{dTotCost}(P)}{\text{d}P} \;=\; \frac{\text{dAvoidCost}(P)}{\text{d}P} \;=\; 100{+}0.8P \;\; (\$/MWh) \qquad (5)$$

Note that neither TotCost(P) nor AvoidCost(P) is differentiable at P=0 due to the discrete jump in start-up cost.

E. Profit ($/h) for GenCo G during hour H as a function of P.

Answer Outline for E: By definition, *profit* for GenCo G during hour H as a function of P is given by revenue from sale of P minus TotCost(P). Using the fact that the market price of P in hour H is 180 ($/MWh), it follows that

$$\begin{aligned} \text{Profit}(P) &= 180P - \text{TotCost}(P) && (6) \\ &= 180P - [3.00 + \text{Start}(P) + 100P + 0.4P^2] && (7) \\ &= 80P - 0.4P^2 - \text{Start}(P) - 3.00 \quad (\$/h) && (8) \end{aligned}$$

for P $\geq$ 0.

F. Net revenue ($/h) for GenCo G during hour H as a function of P.

Answer Outline for F: By definition, *net revenue* for GenCo G during hour H as a function of P is given by revenue from sale of P minus AvoidCost(P). Using the fact that the market price of P in hour H is 180 ($/MWh), it follows that

$$\begin{aligned} \text{NetRev}(P) &= 180P - \text{AvoidCost}(P) && (9) \\ &= 180P - [1.00 + \text{Start}(p) + 100P + 0.4P^2] && (10) \\ &= 80P - 0.4P^2 - \text{Start}(P) - 1.00 \quad (\$/h) && (11) \end{aligned}$$

for P $\geq$ 0.

G. The criterion that GenCo G should use during hour H to decide its optimal production level P$*$ assuming the objective of GenCo G is to make as much money as possible in hour H (ignoring future time periods).

Answer Outline for G: By construction, unless there exists a positive production level P for which NetRev(P) is positive, the best thing that GenCo G can do in hour H is shut down production (set P=0) and earn $1.00 from the use of the equipment E in its next best alternative. The desired criterion is therefore as follows: Determine the production level P^m that maximizes NetRev(P). If NetRev(P^m) > 0 and $P^m > 0$, set $P* = P^m$; otherwise, set $P* = 0$.

H. The optimal production level P$*$ for GenCo G in hour H. (Remark – If you find yourself doing any complicated calculations to get the answer for Part H, you have made some kind of calculation error.)

Answer Outline for H: Taking the derivative of the (strictly concave) function NetRev(P) with respect to P over P $>$ 0, and setting this derivative to zero, one obtains the equation $80 = .8\text{P}$. The P value that strictly maximizes NetRev(P) over $\text{P} > 0$ is thus given by P^m=100 (MW). Since $\text{NetRev}(100) = 3989 > 0$, it follows that $\text{P}* = 100$ (MW).

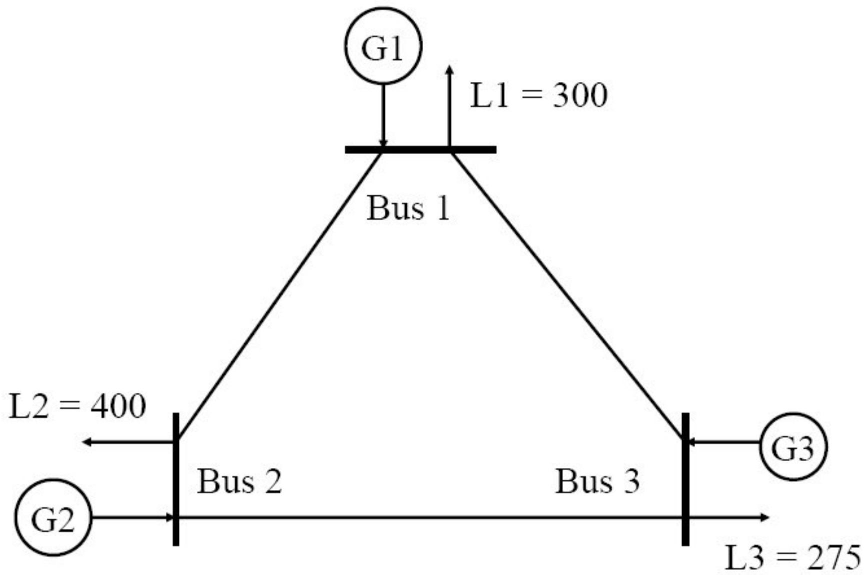

Figure 1: A Three-Bus System with No Branch Capacity Constraints or Losses

PROBLEM 2: (33 Points Total). Consider the 3-bus power system depicted in Figure 1 with three GenCos {G1, G2, G3}, three loads {L1, L2, L3}, and no branch capacity constraints or losses. Suppose the avoidable cost functions for the three GenCos are as follows:

$$VC_1(P_{G1}) \quad = \quad 0.004P_{G1}^2 \ + \ 5.3P_{G1} \ \ (\$/h) \ ; \tag{12}$$

$$VC_2(P_{G2}) \quad = \quad 0.006P_{G2}^2 \ + \ 5.5P_{G2} \ \ (\$/h) \ ; \tag{13}$$

$$VC_3(P_{G3}) \quad = \quad 0.009P_{G3}^2 \ + \ 5.8P_{G3} \ \ (\$/h) \ , \tag{14}$$

where P_{Gi} denotes the active power dispatch level of GenCo i.

Part A (15 Points): First suppose that there are no GenCo operating capacity constraints.

1 Carefully express in quantitative form the objective function and constraint(s) for the economic dispatch problem for this 3-bus system.

2 Carefully write down in quantitative form the Lagrangian function for this economic dispatch problem.

3 Use this Lagrangean function to express the first-order necessary conditions (FONC) for a solution to this economic dispatch problem. Write out these FONC in explicit quantitative form as a set of four equations in four unknowns.

4 Explain carefully how the solution to these FONC determines a *common* locational marginal price (LMP) for the 3-bus system in Figure 1, the same LMP at each bus.

5 Explain, intuitively, why the marginal costs of G1, G2, and G3 must be the *same* at the optimal solution for this economic dispatch problem. Illustrate your arguments with a simple graphical depiction.

Answer Outline for Problem 2-Part A:

Note that the total load in Figure 1 is given by 975 = L1+L2+L3 Consequently, in answer to [1], the objective function and constraints for this economic dispatch problem are as follows: Minimize total avoidable cost

$$[VC_1(P_{G1}) + VC_2(P_{G2}) + VC_3(P_{G3})] \tag{15}$$

with respect to the choice of the dispatch levels P_{G1}, P_{G2}, and P_{G3} subject to the system-wide balance constraint

$$P_{G1} + P_{G2} + P_{G3} \;=\; 975 \tag{16}$$

Let $\mathbf{P} = (P_{G1}, P_{G2}, P_{G3})$ and let $TVC(\mathbf{P}) = [VC_1(P_{G1}) + VC_2(P_{G2}) + VC_3(P_{G3})]$. In answer to [2], the Lagrangean function for this economic dispatch problem then takes the form

$$L(\boldsymbol{P}, \lambda) \;=\; [VC(\boldsymbol{P}) \tag{17}$$

$$+\lambda[975 - P_{G1} - P_{G2} - P_{G3}] \tag{18}$$

In answer to [3], the FONC for this economic dispatch problem take the form

$$0 = \frac{dL(\boldsymbol{P},\lambda)}{dP_{G1}} = MC_1(P_{G1}) - \lambda; \tag{19}$$

$$0 = \frac{dL(\boldsymbol{P},\lambda)}{dP_{G2}} = MC_2(P_{G2}) - \lambda; \tag{20}$$

$$0 = \frac{dL(\boldsymbol{P},\lambda)}{dP_{G3}} = MC_3(P_{G3}) - \lambda; \tag{21}$$

$$0 = \frac{dL(\boldsymbol{P},\lambda)}{d\lambda} = [975 - P_{G1} - P_{G2} - P_{G2}]. \tag{22}$$

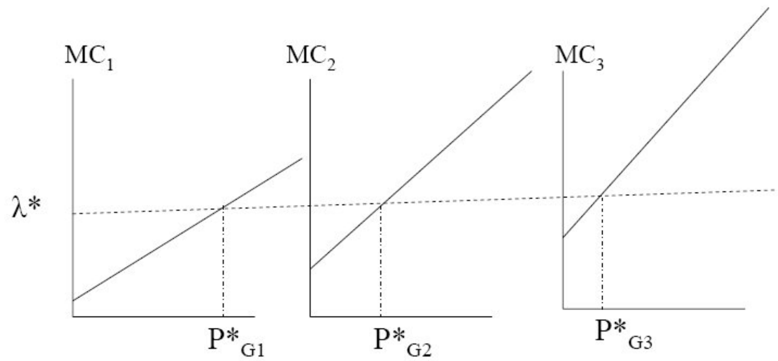

Figure 2: Graphical Depiction of Economic Dispatch Solution for Part A with No Branch Capacity Constraints and No GenCo Operating Capacity Limits

In answer to [4], by definition a *Locational Marginal Price (LMP)* at a bus is the least system cost of servicing one additional MW of load at this bus. By construction (cf. class notes on "Optimization Basics"), the solution for the Lagrange multiplier λ for the system-wide balance constraint (16) gives the change in minimized total avoidable cost with respect to a change in load anywhere on the grid, hence in particular at any bus. It follows that the FONC determine a common LMP λ across the grid.

In answer to [5], the intuitive reason that the marginal cost of each GenCo must be the same at the economic dispatch solution point is as follows: If some GenCo has a higher MC than another GenCo at this point, then total avoidable cost could be lowered by reducing the dispatch level of the GenCo with the higher MC and increasing the dispatch of the GenCo with the lower MC. This can always be done for the economic dispatch problem in Part A because, by assumption, there are no branch capacity constraints and no GenCo operating capacity limits. Graphically speaking, one is finding the particular common level λ for MC across all three GenCos that is also consistent with the system-wide balance constraint (16); see Figure 2.

Part B (18 Points): Now suppose, instead, that GenCo G1 has the following operating capacity constraints that limit its feasible power generation levels P_{G1}:

$$10 \;\leq\; P_{G1} \;\leq\; 450 \;\; (MW)\;. \tag{23}$$

i Carefully express in quantitative form the objective function and constraint(s) for the economic dispatch problem for this modified 3-bus system.

ii Carefully write down in quantitative form the Lagrangian function for this economic dispatch problem.

iii Use this Lagrangean function to express in analytical form the first-order necessary conditions (FONC) for a solution to this economic dispatch problem.

iv Explain carefully how the solution to these FONC still determines a common locational marginal price (LMP) for the modified 3-bus system, the same LMP at each bus.

v The approximate solution to the economic dispatch problem in Part A of Problem 2 is as follows:

$$P_1 = 482.9MW; P_2 = 305.3MW; P_3 = 186.5MW; \lambda = 9.2\$/MWh\ .$$

Explain how you might use this information to go about determining a solution for the economic dispatch problem in Part B.

vi For the economic dispatch problem for Part B, what will be the relationship among the marginal costs of the three GenCos at the optimal dispatch point? Explain, using a simple graphical depiction for illustration.

Answer Outline for Problem 2-Part B:

As in Part A of Problem 2, let $\mathbf{P} = (\mathrm{P}_{G1}, \mathrm{P}_{G2}, \mathrm{P}_{G3})$ and let $\mathrm{TVC}(\mathbf{P}) = [\mathrm{VC}_1(\mathrm{P}_{G1}) + \mathrm{VC}_2(\mathrm{P}_{G2}) + \mathrm{VC}_3(\mathrm{P}_{G3})]$. In answer to [i], the objective function and constraints for the Part B economic dispatch problem are as follows: Minimize total avoidable cost

$$[TVC(\boldsymbol{P})] \tag{24}$$

with respect to the choice of the vector $\mathbf{P}$ of dispatch levels subject to the system-wide balance constraint:

$$P_{G1} + P_{G2} + P_{G3} \ = \ 975 \tag{25}$$

and the operating capacity limits for GenCo G1:

$$10 \ \leq \ P_{G1} \ \leq \ 450 \ \ (MW)\ . \tag{26}$$

In answer to [ii], the Lagrangean function for the economic dispatch problem for Part B then takes the form

$$\begin{aligned} L(\boldsymbol{P}, \lambda, \mu_1, \mu_2) \quad = \quad & [TVC(\boldsymbol{P})] \\ & +\lambda[975 - P_{G1} - P_{G2} - P_{G3}] \\ & +\mu_1[P_{G1} - 450] \\ & +\mu_2[10 - P_{G1}] \quad . \end{aligned} \tag{27}$$

In answer to [iii], the FONC for this economic dispatch problem take the following form:

$$0 = \frac{\mathrm{dL}(\boldsymbol{P},\lambda,\mu_1,\mu_2)}{\mathrm{d}P_{G1}} = MC_1(P_{G1}) - \lambda + \mu_1 - \mu_2; \tag{28}$$

$$0 = \frac{\mathrm{dL}(\boldsymbol{P},\lambda,\mu_1,\mu_2)}{\mathrm{d}P_{G2}} = MC_2(P_{G2}) - \lambda; \tag{29}$$

$$0 = \frac{\mathrm{dL}(\boldsymbol{P},\lambda,\mu_1,\mu_2)}{\mathrm{d}P_{G3}} = MC_3(P_{G3}) - \lambda; \tag{30}$$

$$0 = \frac{\mathrm{dL}(\boldsymbol{P},\lambda,\mu_1,\mu_2)}{\mathrm{d}\lambda} = [975 - P_{G1} - P_{G2} - P_{G2}]; \tag{31}$$

$$0 = \mu_1\frac{\mathrm{dL}(\boldsymbol{P},\lambda,\mu_1,\mu_2)}{\mathrm{d}\mu_1} = \mu_1[P_{G1} - 450]; \tag{32}$$

$$0 \geq \frac{\mathrm{dL}(\boldsymbol{P},\lambda,\mu_1,\mu_2)}{\mathrm{d}\mu_1} = [P_{G1} - 450]; \tag{33}$$

$$\mu_1 \geq 0; \tag{34}$$

$$0 = \mu_2\frac{\mathrm{dL}(\boldsymbol{P},\lambda,\mu_1,\mu_2)}{\mathrm{d}\mu_2} = \mu_2[10 - P_{G1}]; \tag{35}$$

$$0 \geq \frac{\mathrm{dL}(\boldsymbol{P},\lambda,\mu_1,\mu_2)}{\mathrm{d}\mu_2} = [10 - P_{G1}]; \tag{36}$$

$$\mu_2 \geq 0. \tag{37}$$

In answer to [iv], by definition a *Locational Marginal Price (LMP)* at a bus is the least system cost of servicing one additional MW of load at this bus. By construction (cf. class notes on "Optimization Basics"), the solution for the Lagrange multiplier λ for the system-wide balance constraint (25) gives the change in minimized total avoidable cost with respect to a change in load anywhere on the grid, hence in particular at any bus. It follows that the FONC determine a common LMP λ across the grid.

In answer to [v], this "solution" shows that the upper operating capacity constraint 450 MW in (26) is binding on P_{G1}. A reasonable next step would therefore be to try for a solution with P_{G1} set at this upper operating capacity limit of 450 MW. Substituting this value for P_{G1} into the FONC for Part B, note that μ_2 is now forced to be 0 by (35). One therefore obtains the following greatly reduced set of FONC:

$$
\begin{aligned}
0 &= \frac{dL(\boldsymbol{P},\lambda,\mu_1,0)}{dP_{G1}} = MC_1(P_{G1}) - \lambda + \mu_1; && (38)\\
0 &= \frac{dL(\boldsymbol{P},\lambda,\mu_1,0)}{dP_{G2}} = MC_2(P_{G2}) - \lambda; && (39)\\
0 &= \frac{dL(\boldsymbol{P},\lambda,\mu_1,0)}{dP_{G3}} = MC_3(P_{G3}) - \lambda; && (40)\\
0 &= \frac{dL(\boldsymbol{P},\lambda,\mu_1,0)}{d\lambda} = [975 - P_{G1} - P_{G2} - P_{G2}]; && (41)\\
\mu_1 &\geq 0. && (42)
\end{aligned}
$$

It is seen from these reduced FONC that G2 and G3 must still have the same marginal cost λ at the optimal dispatch point, but that G1 will now have a (left hand) marginal cost that is strictly lower than λ. The intuitive reason for this is as follows: If either G2 or G3 has a higher MC than the other at the optimal dispatch point, then total avoidable cost could be further lowered by reducing the dispatch level of the GenCo with the higher MC and increasing the dispatch of the GenCo with the lower MC – a contradiction of optimality. This adjustment in dispatch levels for G2 and G3 can be done for the economic dispatch problem in Part B because, by assumption, there are no branch capacity constraints and no GenCo operating capacity limits on G1 and G2.

Consequently, in Part B one is finding the particular common level λ for MC across G2 and G3, with G1 constrained to be generating at the level 450 MW, such that the three dispatch levels are consistent with the system-wide balance constraint (25); see Figure 3. Since P_{G1} is now constrained at 450 MW, lower than the dispatch level of G1 in Part A, it follows that the dispatch levels of G2 and G3 must be correspondingly increased to meet the load, resulting in a higher solution value λ^{**} for λ than in Part A. To see this graphically, compare Figure 3 with Figure 2.

Remark: Using the above FONC for Part B, it can be shown by simple substitution that the optimal dispatch solution for Part B is as follows:

$$P^{**}_{G1} = 450MW; P^{**}_{G2} = 325MW; P^{**}_{G3} = 200MW \quad . \qquad (43)$$

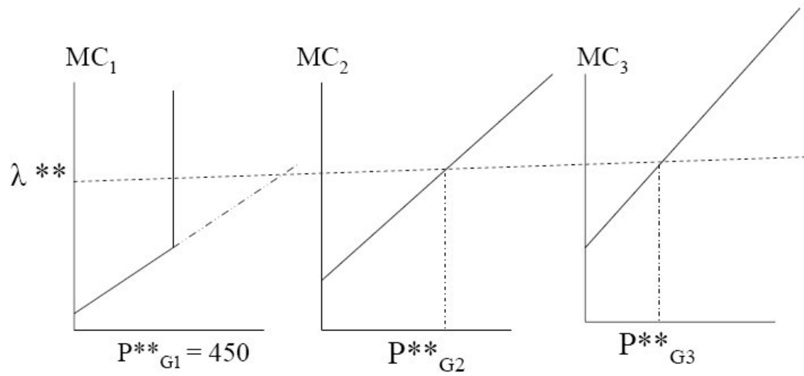

Figure 3: Graphical Depiction of Economic Dispatch Solution for Part B with No Branch Capacity Constraints and with Operating Capacity Limits for G1

It then follows that the Lagrange multiplier solutions are given by

$$MC_1(450) \quad = \; 0.008[450] + 5.3 \; = \; \lambda - \mu_1; \tag{44}$$
$$MC_2(325) \quad = \; 0.012[325] + 5.8 \; = \; \lambda \; . \tag{45}$$

This yields

$$\lambda^{**} = 9.7; \quad \mu_1^{**} = 0.80 \; . \tag{46}$$

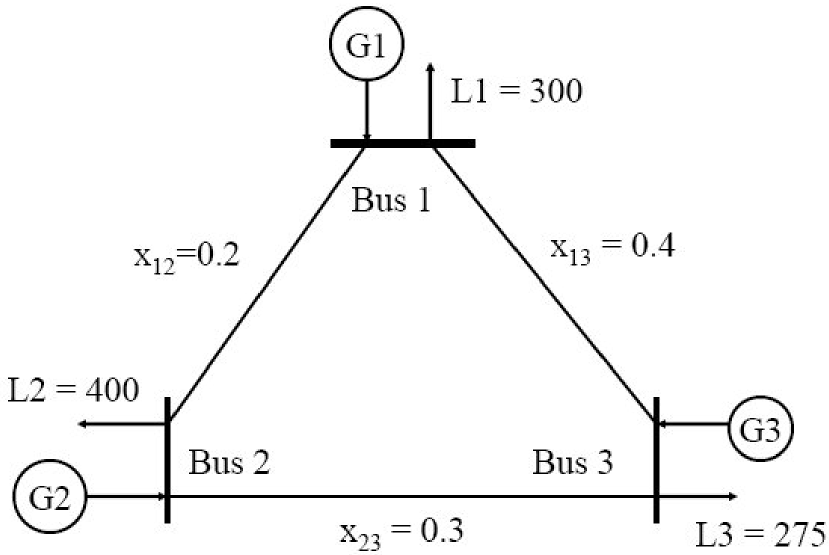

Figure 4: A Three-Bus System with Potential Branch Congestion and No Losses

PROBLEM 3: (35 Points Total). The economic dispatch solution for Problem 2 (Part B) is as follows:

$$P_{G1}^{**} = 450MW; P_{G2}^{**} = 325MW; P_{G3}^{**} = 200MW \quad . \qquad (47)$$

Now consider the 3-bus system in Problem 2 (Part B) with the following two modifications: (a) branch 1-2 has an upper capacity limit of 81.66 MW; and (b) the branch reactances are now explicitly given as depicted in Figure 4.

a (14 Points) Outline the procedure (K/S Chapter 6) by which the reactances in Figure 4 and Kirchhoff's Current Law (KCL) can be used to determine the branch power flows that result under the economic dispatch solution (47) for Problem 2 (Part B). Use graphical depictions to illustrate your assertions.

b (14 Points) It can be shown that the branch power flows that result under the economic dispatch solution (47) for Problem 2 (Part B) are approximately given by F_{12} = 91.66 MW, F_{13} = 58.34 MW, and F_{23} = 16.66 MW . Note that F_{12} violates the upper branch capacity limit of 81.66 MW on branch 1-2 by 10 MW.

Outline the procedure (K/S Chapter 6) for finding a least-cost modification of the economic dispatch solution (47) for Problem 2 (Part B) that removes this overload on branch 1-2. Illustrate your assertions graphically.

c (7 Points) It can be shown that the least-costly economic redispatch that eliminates the overload on branch 1-2 is approximately given by

$$P_{G1} = 437.5MW; \quad P_{G2} = 337.5MW; \quad P_{G3} = 200MW \quad . \qquad (48)$$

Given (48), outline the procedure (K/S Chapter 6) for determining the LMPs at buses 1, 2, and 3. Justify your assertions carefully.

Answer Outline for Problem 3-Part [a]: Let P_i denote the total amount of power injected at bus i. Also, let F_{ij} denote the power flowing on branch i-j, where F_{ij} = - F_{ji}. Then, using the KCL, the economic dispatch solution for Problem 2 (Part B) must satisfy the following three equations:

$$P_1 - L1 \quad = \quad 150 = F_{12} + F_{13} \quad ; \qquad (49)$$

$$P_2 - L2 \quad = \quad -75 = F_{21} + F_{23} \quad ; \qquad (50)$$

$$P_3 - L3 \quad = \quad -75 = F_{31} + F_{32} \quad . \qquad (51)$$

Since the sum of these three equations is 0, this gives two independent equations in the three unknown flows F_{12}, F_{13}, and F_{23}. One additional equation is therefore needed to solve for these flows. As in Exercise 9, this additional equation can be obtained by superposition.

In particular, let the net injection 150 MW at bus 1 be divided into two parts: (1) a net injection of 75 MW withdrawn at bus 2; and (2) a net injection of 75 MW withdrawn at bus 3. See Figures 5, 6, and 7.

From the given reactances, the *Power Transfer Distribution Factors (PTDFs)* for the A injection at bus 1 imply the following flows:

$$F_{12}^A \quad = \quad \frac{[x_{13} + x_{23}]}{[x_{12} + x_{13} + x_{23}]} P_1^A \quad = \quad \frac{0.7}{0.9} 75; \qquad (52)$$

$$F_{13}^A \quad = \quad \frac{[x_{12}]}{[x_{12} + x_{13} + x_{23}]} P_1^A \quad = \quad \frac{0.2}{0.9} 75 \quad . \qquad (53)$$

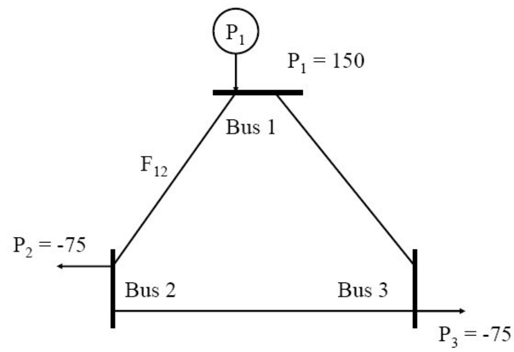

Figure 5: Graphical Depiction of the Use of Superposition to Determine an Additional Flow Equation

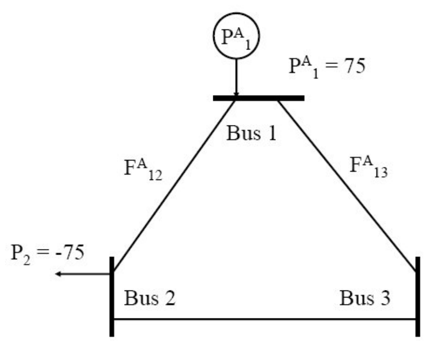

Figure 6: Decomposing P_1 into two Parts: The “A” Flows

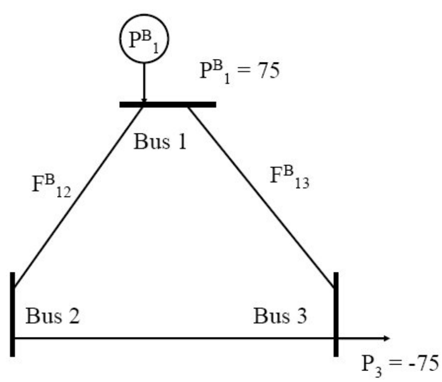

Figure 7: Decomposing P_1 into two Parts: The “B” Flows

Similarly, the PTDFs for the B injection at bus 1 imply the following flows:

$$F_{12}^B = \frac{[x_{13}]}{[x_{12}+x_{13}+x_{23}]}P_1^B = \frac{0.4}{0.9}75; \quad (54)$$

$$F_{13}^B = \frac{[x_{12}+x_{23}]}{[x_{12}+x_{13}+x_{23}]}P_1^B = \frac{0.5}{0.9}75 \ . \quad (55)$$

Thus, by superposition, the flow F_{12} satisfies

$$F_{12} = F_{12}^A + F_{12}^B \quad (56)$$

Equations (50), (51), and (56) then provide three independent equations for solving for the three unknown flows.

Remark: Finishing out the calculations in the above equations (with a calculator), it can be shown that

$$F_{12}^A = 58.33MW \ ; \quad (57)$$

$$F_{12}^B = 33.33MW \ ; \quad (58)$$

$$F_{12} = [F_{12}^A + F_{12}^B] = 91.66 \ . \quad (59)$$

Consequently,

$$P_1 - L1 = 150 = 91.66 + F_{13} \ ; \quad (60)$$

$$P_2 - L2 = -75 = -91.66 + F_{23} \ . \quad (61)$$

$$(62)$$

It follows that the desired flows are approximately given by

$$F_{12} = 91.66; \quad F_{13} = 58.34; \quad F_{23} = 16.66 \ . \quad (63)$$

Answer Outline for Problem 3-Part [b]: To offset the [91.66 - 81.66] = 10 MW overload on branch 1-2, one needs to inject additional power either at bus 2 or at bus 3.

The first step is to determine the effects on the flow of power on branch 2-1 if one additional MW of power is injected at bus 2 and withdrawn at bus 1. Using the given reactances, this effect is given by $PTDF_{21}$, determined as follows:

$$PTDF_{21} = \frac{x_{23}+x_{13}}{x_{12}+x_{13}+x_{23}} = \frac{0.7}{0.9} \approx 0.8 \ . \quad (64)$$

To get a full offset of 10MW, one would then need to inject more power at bus 2 in amount ΔP_2 determined by the following equation:

$$10MW \;=\; \Delta F_{21} \;=\; PTDF_{21}\Delta P_2 \;\approx\; 0.8\Delta P_2 \;\; . \qquad (65)$$

The needed change in power at bus 2 (by G2) is thus approximately given by $10/0.8 = 100/8 \approx +\ 12.5$ MW.

The second step is to determine the effects of injecting one additional MW at bus 3 and withdrawing this 1 MW at bus 1. Using the given reactances, this effect is given by $PTDF_{321}$, determined as follows:

$$PTDF_{321} = \frac{x_{13}}{x_{12} + x_{13} + x_{23}} = \frac{0.4}{0.9} \approx 0.4 \;\; . \qquad (66)$$

To get a full offset of 10MW, one would then need to inject power at bus 3 in amount Δ P_3 determined by the following equation:

$$10MW \;=\; \Delta F_{21} \;=\; PTDF_{321}\Delta P_3 \;\approx\; 0.4\Delta P_3 \;\; . \qquad (67)$$

The needed change in power at bus 3 (by G3) is thus approximately given by $10/0.4 = 100/4 \approx +\ 25$ MW.

The third step is to calculate which of these two feasible redispatches results in the least increase in system costs. The first redispatch involves a [-12.5 MW] redispatch of G1 at bus 1 and a [+12.5 MW] redispatch of G2 at bus 2. The second redispatch involves a [-25 MW] redispatch of G1 at bus 1 and a [+25 MW] redispatch of G3 at bus 3. These two redispatch solutions need to be priced out using the avoidable cost functions of G1, G2, and G3.

Remark: Using a calculator, it can be shown that the two redispatch options result in the following total avoidable costs:

$$\begin{aligned}
TVC_1 &= [VC_1(450 - 12.5) + VC_2(325 + 12.5) + VC_3(200)] && (68)\\
&= [VC_1(437.5) + VC_2(337.5) + VC_3(200) && (69)\\
&\approx [3084.4 + 2539.7 + 1520] && (70)\\
&\approx 7144(\$/h); && (71)\\
TVC_2 &= [VC_1(450 - 25) + VC_2(325) + VC_3(200 + 25)] && (72)\\
&= [VC_1(425) + VC_2(325)_V C_3(225)] && (73)\\
&\approx [2975.0 + 2421.3 + 1760.6] && (74)\\
&\approx 7156.9(\$/h) && (75)
\end{aligned}$$

Consequently, option 1 (redispatch of G1 and G2) is the least costly redispatching option.

Answer Outline for Problem 3-Part [c]:

In general, the LMP at a bus with a marginal GenCo is equal to the marginal cost of this marginal GenCo at its current operating point. If there exists some bus A without a marginal GenCo, the LMP at this bus can be found by considering suitable linear combinations of the LMPs at marginal buses, i.e., at buses that do have marginal GenCos. This involves a consideration of changes in injections of power by the marginal GenCos at the marginal buses that add up to the addition of 1 MW of power at bus A while not increasing the flow of power on any branch currently at a capacity limit. These changes in injection at the marginal buses (positive and negative) are valued using the cost functions of the marginal GenCos at the marginal buses, and this then gives the LMP at bus A.

However, for the specific case at hand, each GenCo G1, G2, and G3 is marginal at the operating point (48). Second, each of the three buses 1, 2, and 3 has a marginal GenCo. It follows that the LMP at each bus 1, 2, and 3 is given by the MC of the (marginal) GenCo located at that bus, evaluated at the operating point (48).

To see this more clearly, consider the LMP at bus 2. The issue is how can one additional MW of load at bus 2 be most cheaply serviced?

G2 is marginal (i.e., not capacity constrained in either the up or down direction), hence it can service an additional MW of power at bus 2 without inducing any overload of branch 1-2. If it were optimal to service an additional MW of load at bus 2 by increased generation of G1 and/or G3 rather than by increased generation of G2, taking into account the capacity limit on branch 1-2, then — since G1, G2, and G3 are all marginal GenCos – the current operating point (48) could not be achieving minimum total avoidable cost. G1 should be backed down and G2 and/or G3 should be called up.

Similar arguments can be given for the LMPs at buses 1 and 3.