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## TWO VERBAL DEFINITIONS OF VOLATILITY

- The relative rate at which the price of a security moves up and down. If the price of a stock moves up and down rapidly over short time periods, it has high volatility. If the price almost never changes, it has low volatility.

  Source: http://www.investorwords.com/5256/volatility.html

- Volatility is the degree of fluctuation in the value of a security, mutual fund, or index. The greater a fund's volatility, the wider the fluctuations between its high and low prices.

  Source: http://www.franklintempleton.co.uk/uk/jspcm/tools/glossary.jsp#v

## QUANTITATIVE VOLATILITY MEASURE: SAMPLE STANDARD DEVIATION

Let $(Z_1, Z_2, \ldots, Z_T)$ be a time-dated sequence of realized values for some empirical datum. For example, $Z_t$ could be a daily "stock return rate" defined as the difference $Z_t = [\ln S_{t+1} - \ln S_t]$ in the natural log of some stock index $S_t$ for day $t = 1, 2, \ldots, T$. Then the *sample mean*, the *sample variance*, and the *sample standard deviation* for this sequence are defined as follows:

$$m \;=\; \text{SampleMean}(Z_1, \ldots, Z_T) \;=\; \frac{\sum_{t=1}^{T} Z_t}{T} \quad . \tag{1}$$

$$\sigma^2 \;=\; \text{SampleVariance}(Z_1, \ldots, Z_T) \;=\; \frac{\sum_{t=1}^{T} [Z_t - m]^2}{T} \quad . \tag{2}$$

$$\sigma \;=\; \text{SampleStandardDeviation}(Z_1, \ldots, Z_T) \;=\; \sqrt{\sigma^2} \quad . \tag{3}$$

**Remarks:** Notice that the first verbal definition of volatility presented above focuses only on frequency of fluctuations, while the second verbal definition of volatility only focuses on amplitude of fluctuations. In contrast, the sample standard deviation $\sigma$ takes into account not only the *frequency* of the deviations of the $Z_t$ values from their mean $m$ over the observation period from 1 to $T$ but also the *amplitude* of these deviations.

## QUESTIONS TO PONDER:

(1) If the sample standard deviation $\sigma$ for the daily return rate $Z_t$=$[\ln S_{t+1} - \ln S_t]$ for each of the three major stock indices $S_t$ (DJIA, S&P 500, and NASDAQ Composite) were separately calculated over some extended time period, say 1985.01 through 2006.08, for which stock index would $\sigma$ be the *smallest* and for which the *largest*, and why?

(2) Many finance economists now argue that measuring "volatility" of a stock index by the sample standard deviation $\sigma$ of its return rate is meaningless, since this return rate has a "thick-tailed" distribution with undefined variance – meaning the average (2) fails to converge to a finite limit as $T$ gets bigger and bigger. What other measures (if any) do these researchers suggest should instead be used to measure the "volatility" of stock returns?