Exercise 4 (Team Exercise)

Leigh Tesfatsion
VII Trento Summer School

Game Theory Analysis of Strategic Market Pricing

Basic References:

** [1] L. Tesfatsion, **"Game Theory: Basic Concepts and Terminology"**, lecture notes posted on-line in Section II of my syllabus, accessible at
http://www.econ.iastate.edu/classes/econ308/tesfatsion/gamedef.pdf

** [2] L. Tesfatsion, **"Notes on Price Discovery with Price-Setting Agents"**, required lecture notes posted on-line in Section II of my syllabus, accessible at
http://www.econ.iastate.edu/classes/econ308/tesfatsion/pricedis.pdf

This exercise asks each exercise team to undertake a game-theoretic analysis of a distinct version of the 2-firm 1-consumer example discussed in detail in L. Tesfatsion, "Notes on Price-Discovery with Price-Setting Agents." As will be clarified below, the eight versions of this example involve eight different settings (treatments) for various key parameter values.

SET-UP OF 2-FIRM 1-CONSUMER MODEL FOR EXERCISE 4:

Consider an economy that consists of:

- two profit-maximizing firms, Firm 1 and Firm 2, producing distinct consumption goods Q_1 and Q_2;
- one budget-constrained consumer who obtains utility (happiness) from the consumption of these two goods.

Each firm is a price-*setter*, meaning the firm sets it own output price. Each firm has only two possible settings for its price: a low price value $L = 5$ and a high price value $H = 10$. Consequently, there are only four possible price strategy combinations (p_1,p_2) for the two firms in your economy: (10,10), (10,5), (5,10), and (5,5).

The profit obtained by Firm n from the sale of good Q_n is given by

$$p_n q_n - c_n q_n \quad , \quad n = 1, 2, \tag{1}$$

where

- p_n = (unit) price of good Q_n;
- q_n = amount sold of good Q_n;
- c_n = positive constant per-unit marginal cost.

The utility obtained by the consumer from consumption of goods Q_1 and Q_2 is measured by a *utility function* given by

$$U(q_1, q_2) \quad = \quad log(q_1 - b_1) + log(q_2 - b_2) \quad , \tag{2}$$

where

- b_1 and b_2 are given nonnegative constants;
- $\log(\cdot)$ denotes the natural (base e) logarithm function.

The income I of the consumer is assumed to be $I = 100$, and the consumer takes prices as given. The *utility maximization problem* faced by the consumer then takes the following form:

> Given goods prices p_1 and p_2, maximize
>
> $$U(q_1, q_2) \tag{3}$$
>
> with respect to the choice of q_1 and q_2 subject to the budget and nonnegativity constraints
>
> $$p_1 q_1 \ + \ p_2 q_2 \quad \leq \quad 100 \quad ; \tag{4}$$
> $$q_1, q_2 \geq 0 \quad . \tag{5}$$

For any given pair of prices p_1 and p_2 that Firm 1 and Firm 2 might post, the following condition is essential for consumer survival since it represents a necessary condition for the consumer to be able to purchase and consume his subsistence needs b_1 and b_2.

Consumer Survival Condition:

$$p_1 b_1 + p_2 b_2 \ \leq \ 100 \tag{6}$$

Given (6), the solution to the consumer's utility maximization problem yields the following *demand functions* for q_1 and q_2:

$$q_1^d \quad = \quad b_1/2 + [100 - b_2 p_2]/2p_1 \quad = \quad D_1(p_1, p_2) \quad ; \tag{7}$$
$$q_2^d \quad = \quad b_2/2 + [100 - b_1 p_1]/2p_2 \quad = \quad D_2(p_1, p_2) \quad , \tag{8}$$

where dependence of these demand functions on the exogenous variables b_1, b_2, and I has been supressed for expositional simplicity.

The profit function of Firm 1 takes the form

$$\pi_1(p_1, p_2) \quad = \quad [p_1 - c_1] D_1(p_1, p_2) \quad , \tag{9}$$

and the profit function of Firm 2 takes the form

$$\pi_2(p_1, p_2) \quad = \quad [p_2 - c_2]D_2(p_1, p_2) \quad . \tag{10}$$

BASIC EXERCISE INSTRUCTIONS:

As noted above, the consumer's income is assumed to be $I = 100$, and each firm is assumed to have only two possible values it can set for its price, a low price value $L = 5$ and a high price value $H = 10$. Each exercise team should assume these three parameter values hold.

However, the values taken on by the *remaining* four parameters $\{b_1, b_2, c_1, c_2\}$ for the above-described model economy are specified *differently* for each of the six exercise teams. These six distinct *treatments* (parameter value settings) are as follows:

TREATMENT FOR EXERCISE TEAM 1:

$$b_1 = 1;\ b_2 = 2;\ c_1 = 1;\ c_2 = 2 \quad . \tag{11}$$

TREATMENT FOR EXERCISE TEAM 2:

$$b_1 = 1;\ b_2 = 2;\ c_1 = 2;\ c_2 = 2 \quad . \tag{12}$$

TREATMENT FOR EXERCISE TEAM 3:

$$b_1 = 1;\ b_2 = 2;\ c_1 = 3;\ c_2 = 2 \quad . \tag{13}$$

TREATMENT FOR EXERCISE TEAM 4:

$$b_1 = 2;\ b_2 = 2;\ c_1 = 1;\ c_2 = 2 \quad . \tag{14}$$

TREATMENT FOR EXERCISE TEAM 5:

$$b_1 = 2;\ b_2 = 2;\ c_1 = 2;\ c_2 = 2 \quad . \tag{15}$$

TREATMENT EXERCISE TEAM 6:

$$b_1 = 2;\ b_2 = 2;\ c_1 = 3;\ c_2 = 2 \quad . \tag{16}$$

TREATMENT EXERCISE TEAM 7:

$$b_1 = 3;\ b_2 = 2;\ c_1 = 1;\ c_2 = 2 \quad . \tag{17}$$

TREATMENT EXERCISE TEAM 8:

$$b_1 = 3;\ b_2 = 2;\ c_1 = 2;\ c_2 = 2 \quad . \tag{18}$$

Important Caution:

Each team should use its own assigned treatment, and ONLY this treatment, to answer the 10 questions below. These 10 questions exactly parallel, step by step, the questions answered in reference [2]; but please be aware that reference [2] answers these 10 questions for a particular treatment of the above model economy that is **distinct** from any of the eight treatments specified above for the eight exercise teams. Consequently, your answers could differ from the specific answers determined in reference [2] to these ten questions.

Consequently, each exercise team should use reference [2] **only as a guideline** to obtain answers to the 10 questions below using its own particular assigned treatment.

EXERCISE QUESTIONS:

Question 1: Using reference [2] as a guide, derive the demand $D_1(p_1, p_2)$ faced by Firm 1 in your economy for each of the four possible price strategy combinations (p_1,p_2). Show your work.

Question 2: Using reference [2] as a guide, derive the demand $D_2(p_1, p_2)$ faced by Firm 2 in your economy for each of the four possible price strategy combinations (p_1,p_2). Show your work.

Question 3: Using reference [2] as a guide, carefully derive and present the 2×2 *profit payoff matrix* faced by the two firms in analogy to the 2×2 profit payoff matrix depicted in Table 1 in reference [2]. If you wish, for easier readability you can normalize these profits to whole numbers by multiplying each profit value by some common appropriate value. (For example, in reference [2] each profit value is multiplied by 48 in order to express these values in whole numbers.) Show your work.

Question 4: Using the 2×2 profit payoff matrix you derived in Question 3, above, determine for *each* of the four possible price strategy combinations whether or not the strategy combination is a Nash equilibrium. Explain and justify your answers carefully, using reference [1] for basic concepts and terminology.

Question 5: Using the 2×2 profit payoff matrix you derived in Question 3, above, determine for each firm whether or not either $H = 10$ or $L = 5$ is a dominant price strategy. Explain and justify your answer carefully, using reference [1] for basic concepts and terminology.

Question 6: Using only the 2×2 profit payoff matrix you derived in Question 3, above – that is, focusing only on producer surplus without considering the surplus of the consumer – determine for *each* of the four possible price strategy combinations whether or not the strategy combination is Pareto efficient. Explain and justify your answer carefully, using reference [1] for basic concepts and terminology.

Question 7: Referring only to the 2×2 profit payoff matrix you derived in Question 3, above – that is, focusing only on producer surplus without considering the surplus of the consumer – determine for *each* of the four possible price strategy combinations whether coordination failure is exhibited at this strategy combination. Explain and justify your answer carefully, using reference [1] for basic concepts and terminology.

Question 8: Using reference [2] as a guide, carefully determine what utility level would be achieved by the *consumer* under each of the four possible price strategy combinations. Present this information in the form of a 2×2 table in analogy to Table 2 in reference [2]. Show your work. *Note:* If you normalized your profit values in Question 3 by multiplying through by some number R, you should normalize your utility levels by multiplying each utility level by R as well.]

Question 9: As in reference [2], define *total net surplus* for your economy to be the sum of attained consumer utility AND profits for the two firms (where for simplicity it is assumed that utility is in dollar terms so that utility and profits can be added together). Carefully derive and present in a 2×2 table (analogous to Table 3 in reference [2]) the total net surplus that would be attained by your economy under each of the four possible price strategy combinations.

Question 10: Referring to the 2×2 total net surplus table you derived in Question 9, determine which of the four possible price strategy combinations achieve(s) the highest social welfare in the sense that total net surplus is maximized. Given that the two firms in your economy are free to choose either price level $H = 10$ or price level $L = 5$, and that they only care about their own profits, is it likely for your economy that social welfare will be maximized? Explain carefully.