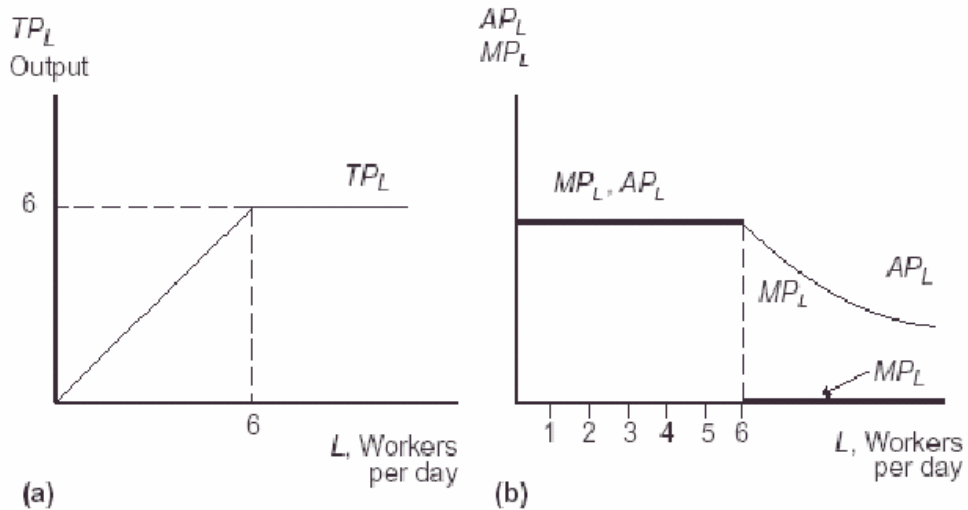


**Intermediate Microeconomics 301**  
**Problem Set # 3 - Solution**  
**Due Wednesday July 13, 2005**

1. Each extra worker produces an extra unit of output up to six workers. After six, no additional output is produced. Draw the total product of labor, average product of labor and marginal product of labor curves.

After the sixth unit, marginal product of labor falls to zero. Total product remains at 6 units, and average product of labor falls after the sixth unit.



2. True or false, explain your answer. “Marginal products in the Cobb-Douglas function cannot be negative.”

In the Cobb-Douglas production function  $Q = AK^\alpha L^\beta$ , as long as the output elasticities are positive, the marginal products cannot be negative. For example, increases in labor will always have the marginal effect  $\beta Q/L$ .

3. A firm’s cost curve is  $C = F + 10q - bq^2 + q^3$ , where  $b > 0$ . Answer the followings  
a) For what values of  $b$  are cost, average cost, and average variable cost positive (From now on, assume that all these measures of cost are positive at every output level)

When  $b < F/q^2 + 10/q + q$ , all cost values are positive.

b) What is the shape of the AC curve? At what output level is the AC minimized?

The average cost curve is U-shaped. AC is minimized at  $dAC/dq = -F/q^2 - b + 2q = 0$ .

c) At what output levels does the MC curve cross the AC and the AVC curves?

MC crosses AC when the functions are equal.  $MC = AC$  where  $10 - 2bq + 3q^2 = F/q + 10 - bq + q^2$ .  $MC = AVC$  where  $10 - 2bq + 3q^2 = 10 - bq + q^2$ .

d) Use calculus to show that the MC curves must cross the AVC at its minimum point.

AVC is minimized where  $dAVC/dq = 0$ .

$$dAVC/dq = -b + 2q = 0$$

$$\text{or } b = 2q$$

$$MC = AVC \text{ where } 10 - 2bq + 3q^2 = 10 - bq + q^2.$$

Then, by substituting  $2q$  for  $b$  on both sides yields

$$10 - q^2 = 10 - q^2$$

4. If input prices are  $w = 4$ , and  $r = 1$  and  $q = 4K^{0.5}L^{0.5}$ , what is the least cost input combination required to produce 40 units of output? Suppose instead that capital was fixed at 16 units. What would be the implications for labor usage and total cost?

Minimizing cost requires that  $MRTS = w/r$ . Since  $MRTS = MPL/MPK$ , set the ratio of marginal products equal to the ratio of input prices, then substitute into the output constraint.

$$K/L = 4/1$$

$$40 = 4(4L)^{0.5}L^{0.5} = 8L$$

$$L^* = 5$$

$$K^* = 20$$

$$C = 4(5) + 1(20) = 40$$

If capital is fixed at 16 units, least cost production is not possible. Instead, labor must be increased to 6.25 units. Total cost increases from \$40 to \$41.

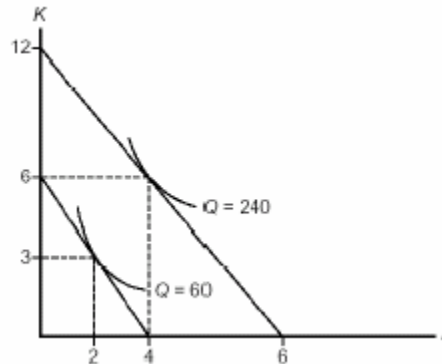
5. If input prices are  $w = 3$  and  $r = 2$ , and  $q = 10KL$ , what is the least cost input combination required to produce 60 units of output? How would input usage change if output is increased to 240 units? Sketch the solutions on a graph.

The solution is

$$L^* = 2.$$

$$K^* = 3.$$

If output is increased but input prices remain the same with a Cobb-Douglas function, the input ratio does not change.  $L^* = 4$ ,  $K^* = 6$ . See the figure:



6. Two firms currently produce the goods  $q_1$  and  $q_2$  separately. Their cost functions are  $C(q_1) = 25 + q_1$  and  $C(q_2) = 35 + 2q_2$ . By merging, they can produce the two goods jointly with costs described by the function  $C(q_1, q_2) = 45 + q_1 + q_2$ . Are there scope economies in this case that would justify the merger?

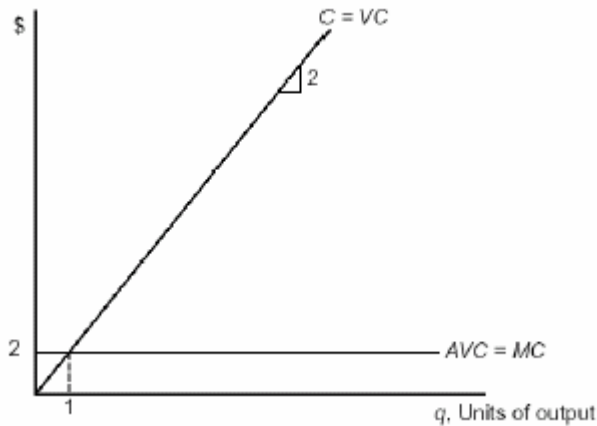
Using the equation for scope economies given in Section 7.5 of the chapter, scope economies exist if  $SC > 0$ . In this case, scope economies do exist as the following expression is greater than zero for all values of both outputs.

$$SC = [25 + q_1 + 35 + 2q_2 - (45 + q_1 + q_2)] / (45 + q_1 + q_2)$$

7. The only variable input a janitorial services firm uses to clean offices is workers who are paid a wage,  $w$ , of \$8 an hour. Each worker can clean four offices in an hour. Use math to determine the variable cost, the average variable cost, and the marginal cost of cleaning one more office. Draw a diagram like Figure 7.1 to show the variable cost, average variable cost, and marginal cost curves.

Let  $q$  equal the number of offices cleaned per hour. Then,  $C = 2q$  (15 minutes of labor is

required per office). Variable cost is also  $2q$  because there are no fixed costs. Average variable cost and marginal cost are  $\$2$ . See figure below.



8. A firm has two plants that produce identical output. The cost functions are  $C_1 = 10q - 4q^2 + q^3$  and  $C_2 = 10q - 2q^2 + q^3$ .

a) At what output levels does the average cost curve of each plant reach its minimum?

You must set  $dAC/dq = 0$  for each firm. The minimum point of  $AC_1$  is at  $q = 2$ . At plant 2, the minimum point is at  $q = 1$ .

b) If the firm wants to produce 4 units of output, how much should it produce in each plant?

The firm chooses  $q_1 + q_2 = 4$  such as to minimize its cost functions, i.e.,

$$\begin{cases} \text{Min}_{q_1, q_2} \{TC_1(q_1) + TC_2(q_2)\} \\ \text{s.t. } q_1 + q_2 = 4 \end{cases} \Rightarrow \text{Min}_{q_1} \{TC_1(q_1) + TC_2(4 - q_1)\} \\ \Rightarrow \text{Min}_{q_1} \{q_1(-8 + 2q_1) + 4(18 - 6q_1 + q_1^2)\}$$

(the last equation comes from substitution  $q_2 = 4 - q_1$  into the cost function  $C_2$ . The math is quite tedious but it works out just fine.)

**You will not be taken points off if you do unit by unit analysis. In that case, the answer is  $q_1 = 3$  and  $q_2 = 1$**

The FOC gives  $q_1 = \frac{8}{3}$

and thus  $q_2 = 4 - q_1 = \frac{4}{3}$

9. If a competitive firm's cost function is  $C(q) = 100 + 10q - q^2 + 1/3 q^3$ , what is the firm's marginal cost function? What is the firm's profit-maximizing condition?

Marginal cost is computed by taking the derivative  $dC/dq$ . Profits are maximized by setting  $MC = MR = p$ . For the function given,  $MC = 10 - 2q + q^2$ . Thus profits are maximized when  $p = 10 - 2q + q^2$ .

10. Each firm in a competitive market has a cost function of  $C = 16 + q^2$ . The market demand function is  $Q = 24 - p$ . Determine the equilibrium price, quantity per firm, market quantity, and number of firms.

In the long run price equals marginal cost, and profits are zero. Thus, given that industry output  $Q = nq$ , the following will be true in long-run equilibrium,  $p = 24 - nq$ . Therefore,

$$\begin{aligned} 24 - nq &= 2q \\ (24 - nq)q &= 16 + q^2. \end{aligned}$$

Solving these equations for  $q$ ,  $n$ ,  $Q$ , and  $p$  yields

$$\begin{aligned} q &= 4 \\ n &= 4 \\ Q &= 16 \\ p &= 8. \end{aligned}$$

11. Suppose a perfectly competitive firm has the short-run cost function  $C(q) = 125 + q^2$ . Use the derivative formula or marginal cost to determine the firm's output level and profit at prices of \$30 and \$20. At what price does the firm reach the shut-down point?

The marginal cost equation is  $MC = 2q$ . When  $p = \$30$ ,  $q^* = 15$ ,  $\pi = \$100$ . When  $p = \$20$ ,  $q^* = 10$ ,  $\pi = -\$25$ . The firm should keep operating since  $TR > TVC$ . The firm should shut down when  $AVC > MC$  (normally, the minimum point of  $AVC$ ). In this case, however, variable cost is linear with slope of 1, and  $MC$  is linear with slope of 2, making all positive output levels above the shutdown point.