Essay I

The Economics of Domestic Cultural Content Protection in Radio Broadcasting

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Abstract

Many countries claim foreign cultural goods are a threat to their national identities and engage in protectionism to limit the competition from foreign cultural goods with various policy interventions in these markets. We analyze the economics of domestic cultural content protection in terrestrial radio broadcasting. Direct regulation of the proportion of domestic music in the total volume of broadcasting is the most widespread policy instrument in terrestrial radio broadcasting. Using the love-of-variety approach, we model a representative consumer deriving utility from broadcasting services net of advertising, and optimally allocating scarce time between consuming the various broadcasting services and leisure. Advertising is a nuisance, costing time and bringing no utility. Terrestrial radio broadcasting is assumed to be a pure public good. However, broadcasting firms are able to make profit in the monopolistic competition environment by bundling advertising with valuable cultural content which cannot be unbundled by the consumer. We impose a discrete domestic content requirement and then investigate the effects of its marginal changes on consumption of domestic broadcasting. We found that marginal changes in the domestic content requirement may reduce consumption of domestic content when the consumer has highly elastic demands and high degree of preference of foreign content to domestic content, and the sensitivity of the price of advertising to size of audience is high. This occurs because the consumer reshuffles his/her consumption bundle towards leisure away from high domestic content stations thereby reducing the overall aggregate consumption of broadcasting, and subsequently, the overall aggregate consumption of domestic programs.

I. Introduction

Globalization brings numerous benefits to trading countries yet it is considered to be the single most serious threat to countries' national identities, especially by policymakers and mercantilist interests[1]. A perceived tradeoff between increasing economic integration and diminishing national identity is at the center of trade and cultural debates[2], as evidenced by the current negotiations on trade in services mandated by the General Agreement on Trade in Services (GATS) of the World Trade Organization (WTO). Many countries use exemptions clauses of GATS[3] in order to cope with "cultural externality" of economic integration by engaging in "cultural protectionism" favoring and implicitly subsidizing domestic producers of cultural goods over their foreign competitors. This is especially true in radio and television broadcasting and the movie industries. Decreasing domestic programming content in the broadcasting industry or increasing dominance of blockbusters (mostly American) in the movie industry are example of the perceived threats.[4] Quantifying culture and its loss from trade in cultural goods is an impossible task which would depend on arbitrary metrics of culture[5]. Nonetheless, it is possible to analyze the effectiveness of some of the instruments used by governments to protect culture. In the case of terrestrial broadcasting policymakers often choose linguistic erosion as an indicator of the cultural loss even though this choice is not devoid of its problems, such as a potential violation of the Most-Favored-Nation (MFN) principle imbedded in the GATS for countries sharing the same language.[6] Economists long theorized that the rationale for domestic content protection is the existence of a market failure that leads to lower than optimal consumption of domestic programming. The most prevalent of those are abuses of market power by providers of entertainment (Farchy (1999), Sapir (1991), and Shao (1995)) and failure of consumers to endogenize the positive externalities generated by higher domestic cultural

1 An example of such concern is 600 experts representing 132 UNESCO member countries, as well as numerous observers, 19 international organizations, and 20 nongovernmental organizations gathering in Paris in September 2004 to discuss "Preliminary Draft Convention on the Protection of the Diversity of Cultural Contents and Artistic Expressions" for the 33rd UNESCO general convention that will take place in the Fall of 2005.

2 See Cowen (2002) for an excellent review of the cultural issues brought up by globalization as well as columns by Bernier (2003-2004).

3 For example, articles XIV(a) of GATS and Annex on Communications to GATS 2(b).

4 An anecdotic example is the position of French Gerard Depardieu, one of the most prominent opponents of domination of the U.S. blockbusters yet who does not mind casting in such or being exported to foreign markets.

5 How does one measure the loss of culture in France from introduction of, say, Big Macs.

6 For example, restrictions on foreign content by Spain that applies to U.S. broadcasting services but does not to Mexican ones could violate the MFN principle.

content (Cwi (1980), Globerman (1983), Sapir (1991), and Shao (1995)). The first rationale applies primarily to the movie industry where producers of domestic programming are marginalized by vertically integrated Hollywood studios. Francois and van Ypersele (2002) show that restrictions on trade in the movie industry, characterized by increasing returns to scale technologies, where individuals have discrete valuations of domestic (cultural genres) and foreign movies, may help resurrect production of valuable cultural genres in both the exporter and importer which subsequently may raise welfare in both countries. The second reason for market failure applies to radio and television broadcasting industries. The latter is often a public good that lacks direct pricing. This paper addresses this second case and looks at domestic cultural content requirement in terrestrial radio broadcasting.

There exist a large body of literature regarding domestic content protection of private goods where governments require that a certain fraction of the final product be of domestic origin. The most prominent papers are Grossman (1981), Mussa (1984), Hollander (1987), Vosden (1987) and Krishna and Itoh (1988). However, the literature has not yet provided a conceptualization of the effects of domestic content protection of public goods. Our paper fills this void. We elucidate the allocative effect of cultural protection policies and in particular, on preserving the cultural identity in the case of terrestrial radio broadcasting as defined by the production and consumption of domestic broadcasting.[7]

The most popular tool used to limit foreign influence in broadcasting is the imposition of a minimum proportion of domestic cultural content. Only a handful of countries, including the United States, have not passed such legislations. The EU has regulation requiring broadcasters in member states to reserve a majority proportion of their transmission time for EU work[8]. Within the EU, The most active proponent of content regulations is France where, for example, at least 40% of all songs should be in French after the infamous “Loi Toubon”.[9] Similarly, Canadian regulation stipulates that each week at least 35% of popular musical selections by commercial stations are Canadian and 65% of the popular

7 There are a few empirical analyzes of the effect of content protection on welfare (Anderson, Swimmer and Suen (1997)), however, they do not provide sufficient theoretical foundations encompassing the various existing content protection initiatives.

8 Council Directive 89/552/EEC of 3 October 1989 adopted by the European Union, Chapter III, Article 4.1.

9 Minister Toubon was nicknamed Mr. Allgood after he imposed his cultural policy (The Economist (1996)).

vocal music selections French-language radio stations broadcast are in French.[10] For television, the Canadian requirement is stricter and requires that 60% of all programming be of a local origin. Even in states that are viewed as culturally conservative, like South Korea, legislators passed laws limiting foreign content.[11] The regulation takes an extreme form on some of the countries of the former Soviet Union. For example, in Kazakhstan, a country in which the Russian language dominates the official language of the state, the Kazakh language, the government requires that half of all programming to be done in Kazakh.[12]

However, despite all of these regulations there is little evidence that DCR are actually successful, as Acheson and Maule [2002] recently noted about the Canadian cultural protection initiatives. The key stylized fact for the analysis of content protection is that government content regulation is an attempt to increase the *absolute consumption* of domestic programming (proxy for the cultural identity) by imposing a *relative* restriction on *production* (broadcasting), and when broadcasted content is a non-excludable (public) good.

For our analysis we adopt the love-of-variety model in which stations compete over consumers in a monopolistic competition environment. We assume that individuals have preferences over various genres, say rock, pop, rap, classical music or their combination for radio broadcasting so that each genre is covered only by one station.[13] Since broadcasting industry is characterized by increasing returns to scale technology we assume that broadcaster face only fixed costs and derive revenue by selling air time to advertisers. Advertising is modeled as a nuisance,- it brings zero utility but costs scarce time. However, broadcasters bundle advertising with real content in fixed proportions “forcing” people to consume advertising whenever they consume broadcasting services. This feature of our model allows us to derive

10 Canadian Broadcasting Act, R.S.C., 1991, c. 11, Article 10.1.

11 Article 71(1) of Broadcasting Act says that “A broadcasting business operator shall program, among the total programs of the relevant channel, domestically produced broadcast programs in excess of a specified ratio … prescribed by Presidential Decree”.

12 The law of the RK of 23 July 1999, #451-1 “About Means of Mass Information”, article 3.2, “The volume of broadcasting on tele- and radio channels… on the language of the state, in terms of time, exceeds that of all other languages combined”.

13 This assumption is made to avoid the non-existence of equilibrium problem in Bertrand games with fixed costs. However, this assumption is not restrictive because a mixture of any two or more genres could constitute a new genre and given that the number of stations is a finite number, it guarantees that no station chooses exactly the same combination of genre as the other station. Alternatively, one may argue that government licenses radio and television licenses thus artificially limiting number of entrants into the market. In this case fixed costs can be normalize at zero, we, however, do not pursue such approach.

the price of consumption of broadcasting in term of time units[14]. We analyze the economics of marginal domestic content requirement (DCR) policy on the aggregate consumption of domestic programming. We find that the effectiveness of domestic content protection policies depends crucially on consumer preferences. The larger is the elasticity of substitution between genres, between consumption of broadcasting and leisure, the larger is the preference of foreign produced music over the domestically produced music, and the larger is the sensitivity of price of advertising to size of audience, the more likely the DCR is to be counterproductive to preserve domestic culture. The implication of this result is that minimum DCR may be an effective policy is some EU countries or Canada but likely to fail in countries where language is the main obstacle for consumption of domestic programming. The latter might be some of the Baltic States and Central Asian where consumers strictly prefer foreign music to domestic music .

II. The Model

We assume that there exist two types of music, (M_1, M_2). For example, type M_1 could be folklore music and type M_2 could refer to popular music. Define $\beta \equiv \frac{M_1}{M_1 + M_2}$ as a share of type 1 music. We refer to β as genre[15]. We assume that $\beta \sim U[0,1]$. We assume that each firm obtains license to operate genre on the exclusive basis. Each individual derives utility from consumption of different genres and leisure. We assume that the utility function of individual j, where $j \sim h(j)$, has quasilinear CES specification of the form

$$U_j \equiv \frac{j}{\lambda} \frac{\sigma - 1}{\sigma} \left(\int_0^1 \left(\gamma(\beta) q_{j,d}(\beta)^{\beta} q_{j,f}(\beta)^{1-\beta} \right)^{\frac{\sigma-1}{\sigma}} d\beta \right)^{\frac{\sigma}{\sigma-1}\lambda} + l_j \qquad (1),$$

where triplet $\left(q_{j,d}(\beta), q_{j,f}(\beta), l_j\right)$ refers to consumption of domestic programs of genre β, foreign programs of genre β and leisure. Parameter $\sigma > 1$ is the elasticity of substitution between genres.

14 The pioneering works in the modeling broadcasting industry in continuous setups are papers by Berry and Waldfogel (1996) and Anderson and Coate (2003).

15 Our model permits more than two types of music as long as there exist a unique mapping between combinations of types of music and genres.

Parameter λ adds concavity to the subutility from broadcasting and allows us to derive individual broadcasting demands (a combination of parameters (σ, λ) gives us the elasticity of substitution between broadcasting and leisure). We assume that $0 < \lambda < \frac{\sigma - 1}{\sigma}$ which guarantees that this utility function satisfies all the regularity conditions,- increasing marginal utility and negative semi-definite Hessian. Further, preferences reflect the ability of consumers to substitute broadcasting away towards leisure.[16] Function $\gamma(\beta)$ is the weight of each genre. We have Cobb-Douglas relationship between consumption of domestic and foreign program which varies from genre to genre. Naturally one would expect that the larger is the proportion of M_1 type of music/program the larger is the intensity of consumption of domestic programs. For simplicity we assume that there exist 1-to-1 correspondence between genres and the intensity of consumption of domestic and foreign programming[17].

A key feature of the above utility function is that individuals derive utility from consumption of only foreign and domestic content, however, because stations bundle advertising with broadcasting, advertising plays the role of nuisance. This specification allows us to price broadcasting and generate revenues for broadcasters.

Consumers maximizes their utility function subject to the constraint that the total time spent on consumption of broadcasting and leisure does not exceed unity. Let us denote as $b(\beta)$ the consumption of broadcasting and $(d(\beta), f(\beta))$ as shares of domestic and foreign music in total volume of broadcasting for genre β. Since market is served by commercial stations we assume that broadcasters sell advertising by including it as a proportion of total broadcasting. Calling consumption of advertising of genre β as $q_a(\beta)$ and its proportion as $a(\beta)$ we have the following identities

16 This utility specification essentially assumes the existence of a representative consumer. It can be generalized into a case where people are indexed and distributed according to some probability density function, as long as demands can be aggregated. The simplest form would be multiplicative index where subutility from broadcasting is multiplied by index.

17 This assumption is not necessary as long as intensity is an increasing function of the proportion of

$q_d(\beta)+q_f(\beta)+q_a(\beta)\equiv b(\beta)$ and $d(\beta)+f(\beta)+a(\beta)\equiv 1$. Then, the utility function can be restated as follows:

$$U_j = \frac{j}{\lambda}\frac{\sigma-1}{\sigma}\left(\int_0^1 \left[b_j(\beta)\gamma(\beta)d^{\beta}(\beta)f(\beta)^{1-\beta}\right]^{\frac{\sigma-1}{\sigma}} d\beta\right)^{\frac{\sigma}{\sigma-1}\lambda} + l_j \qquad (2).$$

By inspecting the utility function we may notice that each broadcasting demand is weighted by function $\gamma(\beta)d^{\beta}(\beta)f(\beta)^{1-\beta}$ specific to each genre. For the reason that individuals have no technology to unbundle advertising from an actual useful content and take shares of domestic and foreign content as given, we omit subscripts that refer to individuals. It is convenient to define $z(\beta)\equiv\left(\gamma(\beta)d^{\beta}(\beta)f(\beta)^{1-\beta}\right)^{-1}$. We refer to it as a virtual price. The higher the proportions of both foreign and domestic content are (or lower proportion of the advertising) and the higher the weight of a genre in the utility function compared to other genres is, the lower is the virtual price faced by consumer.

Each consumer is endowed with T units of time, therefore, his budget constraint is given by $\int_0^1 b_j(\beta)\beta + l_j = T$. Thus, the utility maximization problem is:

$$\max_{b_j} \frac{j}{\lambda}\frac{\sigma-1}{\sigma}\left(\int_0^1 \left[b_j(\beta)z(\beta)\right]^{\frac{\sigma-1}{\sigma}} d\beta\right)^{\frac{\sigma}{\sigma-1}\lambda} + l \ s.t. \int_0^1 b_j(\beta)d\beta + l_j = T \qquad (3).$$

For brevity define $\phi \equiv 1-\left(\frac{\lambda}{1-\lambda}\right)\frac{1}{\sigma-1}$. Then, solving the utility maximization problem yields individual broadcasting demands

$$b_j(\beta) = z(\beta)^{1-\sigma} V^{-\phi} \qquad (4),$$

where $V \equiv \int_0^1 z(\beta)^{1-\sigma} d\beta$ is an aggregate virtual price index (please note that all consumers face the same price for each genre; this will allow us to derive aggregate demands with ease). Please note that even though V is independent of each individual virtual price, it does depend on the aggregate level of prices.

This implies that marginal shocks in own price have no effect on the aggregate price level, however, discrete price changes do. Aggregating demands over all consumers yields

$$B(\beta) = \int b_j(\beta) h(j) dj = z(\beta)^{1-\sigma} V^{-\phi} H \quad (5),$$

where $H \equiv \int j^{\frac{1}{1-\lambda}} h(j) dj$. Without loss of generality we normalize H at one.

We assume that each genre served by stations (or firms) with identical cost structure so that market represents monopolistic competition. As we mentioned earlier, stations derive their revenues by bundling together "real" content and advertising and consumers cannot unbundle it.[18] Defining $B(\beta)$ to be the aggregate consumption of broadcasting of genre β, we have $B(\beta) = D(\beta) + F(\beta) + A(\beta)$, where $(D(\beta), F(\beta), A(\beta))$ are aggregate consumptions of domestic programming, foreign programming and advertising. Given public nature of broadcasting[19] we assume that broadcasters face same fixed costs $c > 0$.[20] The price of advertising is assumed to be increasing in the size of own audience and decreasing in the size of aggregate audience. More specifically, we assume that it takes the following form, $p(B(\beta)) \equiv \left(\frac{B(\beta)}{\mathrm{B}}\right)^{\theta}$, where $\mathrm{B} \equiv \int_0^1 B(\beta) d\beta$ is the aggregate demand for broadcasting by the whole radio broadcasting market. Parameter θ reflects the sensitivity of advertising pricing function to the relative size of the audience of each station,- the larger is θ the larger is the sensitivity. Thus, broadcaster's problem is

$$\max_{d(\beta), f(\beta)} \pi(\beta) \equiv p(B(\beta)) a(\beta) T - c, s.t. \pi(\beta) \geq 0 \quad (6),$$

where $a(\beta)T = (1 - d(\beta) - f(\beta))T$ is the total amount of advertising during the time period, in minutes

18 We essentially shine away from cases where consumers do have an ability to suppress advertising, for example, in the case of radio by recording radio stations and in the case of television using recording devices like TiVo.

19 We assume that broadcasting is non-excludable and non-rival or a pure public good.

20 Even costs which are normally considered variable in standard profit maximization problems, such as labor costs, are no longer variable in the case of broadcasting.

.[21] Broadcasters behave strategically and the industry reaches best reply equilibrium. We assume that individuals randomly tune in to each station i therefore, in light of the fact that marginal costs are zero, stations broadcast all the time.

Then, the first-order conditions for an interior solution for profit maximization for station serving genre β are:

$$\frac{\partial \pi(\beta)}{\partial d(\beta)} \propto \theta(\sigma-1)\beta\left(\frac{1-d(\beta)-f(\beta)}{d(\beta)}\right)-1=0 \qquad (7),$$

$$\frac{\partial \pi(\beta)}{\partial f(\beta)} \propto \theta(\sigma-1)(1-\beta)\left(\frac{1-d(\beta)-f(\beta)}{f(\beta)}\right)-1=0 \qquad (8).$$

For brevity define $u \equiv \theta(\sigma-1)$. Solving the system of equations (7) and (8) yields

$d^*(\beta)=\frac{\beta u}{1+u}$ and $f^*(\beta)=\frac{(1-\beta)u}{1+u}$. Thus, the virtual price is given by

$z^*(\beta)=\left(\beta^{\beta}(1-\beta)^{1-\beta}\gamma(\beta)\frac{u}{1+u}\right)^{-1}$.

The second-order conditions hold because $\frac{\partial \pi^2(\beta)}{\partial d(\beta)^2}=-u\beta\frac{1-f(\beta)}{d(\beta)^2}\le 0$,

$\frac{\partial \pi^2(\beta)}{\partial f(\beta)^2}=-u(1-\beta)\frac{1-d(\beta)}{f(\beta)^2}\le 0$, and

$\frac{\partial \pi^2(\beta)}{\partial d(\beta)^2}\frac{\partial \pi^2(\beta)}{\partial f(\beta)^2}-\left(\frac{\partial \pi^2(\beta)}{\partial d(\beta)\partial f(\beta)}\right)=\frac{u^2\beta(1-\beta)(1-d(\beta)-f(\beta))}{d(\beta)f(\beta)}\ge 0$.

Therefore, the aggregate consumption of domestic programming is given by

$$D^*=\left(\int_0^1 \beta\frac{u}{1+u}\left(\beta^{\beta}(1-\beta)^{1-\beta}\gamma(\beta)\frac{u}{1+u}\right)^{u}d\beta\right)*\left(\int_0^1\left(\beta^{\beta}(1-\beta)^{1-\beta}\gamma(\beta)\frac{u}{1+u}\right)^{u}d\beta\right)^{-\phi} \qquad (9).$$

21 Instead of smooth substitution of foreign and domestic content in the utility function and additive production function where from firm's standpoint foreign and domestic content are identically priced we might have used additive subutility from broadcasting of each genre and smooth substitution between foreign and domestic content in the production function. Such imperfect substitution can be justified by limited supply of high quality foreign and domestic music. This would be particularly true for certain genres, say French hip-hop or Kazakh folklore.

III. Effects of the Domestic Content Requirement.

The purpose of this paper is to analyze the effectiveness of the DCR in terrestrial radio broadcasting. There are economic and non-economic rationales for DCR. The former could be policymaker believing that there exist a positive externality from higher consumption of domestic programming, say higher tax revenue generated by the broadcasting industry, higher local wages, etc., and that the market fails to endogenize it. For example, the utility function might be augmented by function $E \equiv \mathrm{E}\left(\int \int_0^1 \left(eB_j\left(\beta\right) + \left(1-e\right)\right) d\left(\beta\right) h\left(j\right) d\beta dj \right)$ where $\mathrm{E}(.)$ is some positive and increasing function of its argument, $e \in [0,1]$. Modeled this way the externality is a weighted average of aggregate consumption of domestic programming in the whole economy and aggregate broadcasting of domestic programs. The non-economic rationale could be either poorly defined economic objectives, for example, higher incomes from tourism or sales of local books, and/or subjective belief by the policymaker in the good of production and consumption of domestically produced cultural goods, say, higher patriotism. For our purposes we just assume that a policymaker wants to increase consumption of domestically produced broadcasting.

Broadcasting is a public good that lacks direct pricing policymakers, therefore, one way a policymaker may reach his/her objective is to force producers to incorporate higher shares of domestic programming in their broadcastings by imposing a minimum proportion of domestic content in the total share of broadcasting. Presumably, as a result, consumers will increase their consumption of domestic broadcasted culture as well. We now address the validity of this conjecture that imposing a cultural DCR on broadcasters leads to higher consumption of domestic programming. To have feedback effects between stations to changes in DCR we impose a discrete DCR and then derive effects of marginal changes in DCR on aggregate consumption of domestic programming[22]. The most widely used tool to regulate

22 This is similar to the analysis of Mussa (1984).

terrestrial radio broadcasting is to impose a DCR so that domestic programming constitutes a minimum fraction of aggregate broadcasting. Formally, defining a policy instrument as δ we have DCR requiring $\delta \geq d(\beta)$.

When government imposes this DCR, constraint binds for all stations with genres β such that $d^*(\beta) \leq \delta$, i.e. for all $\beta \leq \delta\left(\frac{1+u}{u}\right)$. Therefore, each of these stations, instead of solving equations (7) and (8), now solves:

$$d(\beta) = \delta \qquad (10),$$

$$u(1-\beta)\left(\frac{1-d(\beta)-f(\beta)}{f(\beta)}\right)-1=0 \qquad (11).$$

The solution is $\hat{d}(\beta)=\delta$ and $\hat{f}(\beta)=\frac{u(1-\beta)(1-\delta)}{1+u(1-\beta)}$. Hereunder we refer to constrained solutions and functions thereof as hats and unconstrained as stars. We assume that fixed costs of production are negligible so that stations continue operation at any level of DCR. Combining solutions yields constrained virtual price $\hat{z}(\beta)=\left(\left(\frac{u(1-\beta)}{1+u(1-\beta)}\right)^{1-\beta}\gamma(\beta)\delta^{\beta}(1-\delta)^{1-\beta}\right)^{-1}$ with $\frac{\partial\hat{z}(\beta)/\partial\delta}{\hat{z}(\beta)}=\frac{\delta-\beta}{\delta(1-\delta)}$. This last expression has an ambiguous sign, however, when u is large and $\gamma'(\beta)<0$ is sufficiently negative, the region of importance is where β is small relatively to δ (this is because we will be integrating virtual prices and the highest density will be allocated to genres with low domestic content ratio, or more popular stations) . Therefore, we will loosely consider, where it is not crucial, that $\delta \geq \beta$. For our analysis we focus on the case where constraint is not binding for all stations, i.e. where $\delta < \frac{u}{1+u}$.

Define aggregate price indexes of constrained and unconstrained stations as $\hat{V} \equiv \int_{0}^{\delta\frac{1+u}{u}} \hat{z}(\beta)^{-u}\, d\beta$ and $V^{*} \equiv \int_{\delta\frac{1+u}{u}}^{1} z^{*}(\beta)^{-u}\, d\beta$. Then, aggregate demands for genre β when constraint is binding is given by $\hat{\mathrm{B}} \equiv \hat{B}(\beta)\left(\hat{V}+V^{*}\right)^{-\phi}$ and when constraint is not binding by $\mathrm{B}^{*} \equiv B^{*}(\beta)\left(\hat{V}+V^{*}\right)^{-\phi}$. Having defined aggregate demands allows us to derive aggregate consumption of domestic programming by the whole market:

$$\hat{D} = \int_{0}^{\frac{\delta(1+u)}{u}} \hat{d}(\beta)\hat{B}(\beta)\, d\beta + \int_{\frac{\delta(1+u)}{u}}^{1} d^{*}(\beta) B^{*}(\beta)\, d\beta \qquad (12).$$

Since stations that find constraint binding have to oblige by the regulation, we have $\hat{d}(\beta) = \delta\ \forall \beta \in \left[0, \delta\frac{1+u}{u}\right]$ while $d^{*}(\beta) = \beta\frac{u}{1+u}\ \forall \beta \in \left[\delta\frac{1+u}{u}, 1\right]$. We then can rewrite equation (12) as

$$\hat{D} = \delta \int_{0}^{\frac{\delta(1+u)}{u}} \hat{B}(\beta)\, d\beta + \int_{\frac{\delta(1+u)}{u}}^{1} \beta \frac{u}{1+u} B^{*}(\beta)\, d\beta \qquad (13).$$

Having defined aggregate consumption of domestic programming by the whole market, marginal shock in DCR for any given δ is given by the following equation:

$$\frac{\partial \hat{D}}{\partial \delta} = \int_0^{\frac{\delta(1+u)}{u}} \left[1 + \frac{\partial \hat{B}(\beta)}{\partial \delta}\frac{\delta}{\hat{B}(\beta)}\right]\hat{B}(\beta)\,d\beta$$

$$= \int_0^{\frac{\delta(1+u)}{u}} \left[1 - u\frac{\partial \hat{z}(\beta)}{\partial \delta}\frac{\delta}{\hat{z}(\beta)} - \phi\frac{\partial\left(\hat{V}+V^*\right)}{\partial \delta}\frac{\delta}{\hat{V}+V^*}\right]\hat{B}(\beta)\,d\beta$$

$$= \int_0^{\frac{\delta(1+u)}{u}} \left[1 - u\left(\frac{\delta-\beta}{1-\delta}\right) - \phi\left(\frac{\int_0^{\frac{\delta(1+u)}{u}} (-u)\left(\frac{\delta-j}{1-\delta}\right)\hat{z}(j)^{-u}\,dj}{\hat{V}+V^*}\right)\right]\hat{B}(\beta)\,d\beta$$

$$= \int_0^{\frac{\delta(1+u)}{u}} \left[1 - u\left(\frac{\delta-\beta}{1-\delta}\right)\right]\hat{B}(\beta)\,d\beta + \phi\left(\frac{\int_0^{\frac{\delta(1+u)}{u}} u\left(\frac{\delta-\beta}{1-\delta}\right)\hat{z}(\beta)^{-u}\,d\beta}{\hat{V}+V^*}\right) \qquad (14).$$

Define $\overline{\beta} \equiv \left(\int_0^{\frac{\delta(1+u)}{u}} \beta\hat{z}(\beta)^{-u}\,d\beta\right)\left(\int_0^{\frac{\delta(1+u)}{u}} \hat{z}(\beta)^{-u}\,d\beta\right)^{-1}$. It is an expected value of β over interval $\left[0, \frac{\delta(1+u)}{u}\right]$ and probably density function $\frac{\hat{z}(\beta)^{-u}}{\hat{V}}$. We know that because $\gamma'(\beta) \le 0$ we have $0 \le \overline{\beta} \le \frac{\delta}{2}\left(\frac{1+u}{u}\right)$. This implies that $\overline{\beta} \le \delta$ as long as $u \ge 1$. Moreover, define $\mu \equiv \frac{\hat{\mathrm{B}}}{\hat{\mathrm{B}}+\mathrm{B}^*} = \frac{\hat{V}}{\hat{V}+V^*}$ as the share of the constrained aggregate demand in the total demand for broadcasting services. Then, the equation (14) can be restated in the compact form as

$$\frac{\partial \hat{D}}{\partial \delta} = \hat{\mathrm{B}}\left(1 - (1-\phi\mu)\frac{u}{1-\delta}\left(\delta - \overline{\beta}\right)\right) \qquad (15),$$

Let us denote $G \equiv 1 - (1-\phi\mu)\frac{u}{1-\delta}\left(\delta - \overline{\beta}\right)$. Then, $\frac{\partial \hat{D}}{\partial \delta} = \hat{\mathrm{B}} * G$ where $\hat{\mathrm{B}} \ge 0$.

Proposition 1. The sets of parameters over which the effect of DCR on the aggregate consumption of the domestic programs is productive and counterproductive are non-empty. For any given elasticity of substitution, concavity parameter, sensitivity of the price of advertising to the share of the own's market, distribution of weights of each genre represented by function $\gamma(\beta)$, there exist a range of δ such that DCR policy is counterproductive, both with respect to the case where it is or it is not in place.

Proof: *By inspecting equation* (15) *we observe that when a policy is not in place then marginal changes in DCR have no effect on the consumption of the domestic programming, or the slope of* $\partial\hat{D}/\partial\delta = 0$. *This happens because* $\hat{\mathrm{B}} = 0$*, which is true by construction. Ultramarginal changes lead to* $\hat{\mathrm{B}} > 0$ *yet* $G \approx 1$ *because the second term of* G *will be miniscule (note that* $\delta - \overline{\beta} \approx 0$ *for any finite* u*). For such changes we have* $\hat{D} > D^{*}$. *Further, when* $\delta = 1$ *then* $\hat{D} = 0$ *by construction. Given that the set of* δ *is compact and that function* $\hat{D}$ *is continuous in* δ*, there exist at least one maximum and at least one point where constrained and unconstrained consumptions of domestic programs are equal. Therefore, the set of parameters over which DCR is productive (in both cases where DCR is or isn't in place before regulation) and the set over which DCR is counterproductive is not empty. Hence, there exist a range of values of* δ *where DCR reduces consumption of domestic programming ex-post whether ex-ante some DCR was or was not in force*■

By exploring this expression we may notice that the success of DCR policy depends on several key parameters. First, we require that elasticity of substitution, σ (please recall that $\theta(\sigma-1) \equiv u$), between genres being large enough to allow consumers reshuffle their consumption bundle away from constrained stations since the latter increase their virtual price in response to DCR (again, stations that find constraint just binding will actually reduce their price in response to DCR, however, for large elasticity of substitution there will be very few of those stations). Since consumers reshuffle away from "constrained" stations towards unconstrained stations (with higher proportions of domestic content) and leisure, we also need that ϕ be sufficiently small (or λ be sufficiently large) to guarantee that the flow of consumption from constrained stations does not channel only towards stations with large domestic content but to leisure. The other important factor is the size of DCR, namely, the value of δ. For large enough δ we have $G < 0$. This factor is essentially the function of the distance between the most-preferred ratio of domestic content and available content. When δ is small then such distance is small, hence, individuals do not adjust their consumption significantly in response to DCR. As a result, DCR shocks lead to small decrease in aggregate broadcasting demands so that the direct effect of increasing the share of domestic programs outweighs the indirect effect of adjustment of the consumption bundle to higher virtual prices. Conversely, when δ is large then the distance between the most-preferred domestic content and available content (or regulated value of domestic content) is large. In this case, if people are sensitive enough to virtual prices then those genres farthest from the most-preferred content experience substantial drop in the audience. Further, a consideration has to be given to the parameter θ. We require θ be not very small as not to offset large values of σ that lead to $\partial\hat{D}/\partial\delta \leq 0$. Since in our model parameters θ and σ work in tandem, $\theta \leq 1$ ($\theta \geq 1$) can be viewed as a dampener (amplifier) of the effects driven by the elasticity of substitution. Naturally, when θ is large then profits of stations become extremely sensitive to the size of audience (or broadcasting demands), therefore, policy leads to high changes in price, broadcasting demands, and consequently, drop in the consumption of domestic programs. When θ is small then decrease in demands due to DCR is small so that overall consumption of domestic programs increases.

The intuition for proposition 1 is straightforward. DCR has two effects,- the direct effect that increases shares of domestic content across all constrained stations, and the indirect effect that decreases the aggregate demand for broadcasting services due to the diminished by regulation appeal. To understand the intuition of the above result one might think of consumption of domestic programming as $\hat{D} = \delta\hat{\mathrm{B}} + s(\delta)\mathrm{B}^*$, where $s(\delta)$ is some average value of shares of domestic content of unrestricted stations. We know that $s'(\delta) \geq 0$ since the region over which the average is taken shrinks as δ increases. Taking this into account we can further simplify the expression of consumption of domestic programming as $\hat{D} = S(\delta)(\hat{\mathrm{B}} + \mathrm{B}^*)$ where $S(\delta)$ is the average value of share of domestic content over whole market. Again, $S'(\delta) > 0 \because s'(\delta) \geq 0$. Then, as δ increases so does the average share of domestic content over whole market. However, aggregate demand, given by $(\hat{\mathrm{B}} + \mathrm{B}^*)$, may fall. More specifically, the average appeal of content of stations that are constrained by the regulation falls while the average appeal of stations that are not affected by the regulation rises. When individuals assign higher weights to stations that are subject to regulation relative to stations that are not (this is where we use out notion of popular programming having large audience than folklore programming) then we may observe that decrease in consumption of constrained stations is not absorbed by increase in demand for unconstrained stations. This leads to the fall in the aggregate consumption of broadcasting.

Hence, the effect of DCR will depend on how fast the average share of domestic content increases and how fast aggregate consumption of broadcasting falls. When σ is small then aggregate demand changes little in response to DCR, thus, DCR is productive. Further, when ϕ is large (λ is small) then fall in constrained demand is largely absorbed by an increase in unconstrained demand so that aggregate demands falls insignificantly. In this case DCR is productive as well. In the opposite case we will have consumption of domestic program fall. Finally, we require sufficiently large right-skewness of distribution of $\gamma(\beta)$ [23] because it implies large value of μ and $(\delta - \bar{\beta})$.

A policymaker could be interested in the maximum value of domestic programming attainable by the market. The solution to equation $G = 0$ gives us such a value. Formally, define $\hat{\delta}_0 \equiv \{\delta | G(\delta) = 0\}$. This set may contain several values. When $\gamma(\beta)$ is sufficiently right-skewed then such set will be a singleton, however, this conjecture can only be asserted by numerical simulations.

23 We say that the distribution is right-skewed when it has longer tail on the right.

In the real world the DCR rarely exceeds 50% threshold, therefore, we focus on the case where $\delta \le 1/2$. To ensure that $\hat{\delta}_0 \le 1/2$ we assume that u is large enough and ϕ is small enough. Given this will be shown that for $\phi \le 1/2$, $\partial G/\partial \delta$ evaluated at δ_0 is negative so that δ_0 is indeed a maximum. Since we shown before that the slope of the function at $\delta = 0$ is zero then we know that $\hat{D}$ is not globally concave. Therefore, analytically it can only be shown that δ_0 is only a local maximum. In section IV we show that uniqueness of solution is predicated upon by the properties of $\gamma(\beta)$,- large right-skewness leads to δ_0 being unique, or $\hat{D}$ has global maximum.

In addition, the results above are based on the premise that firms do not exit the market even when they are severely constrained by the DCR. Essentially, we assume that fixed costs are negligible. When this is not the case then high levels of DCR will lead to firms leaving the broadcasting market. This will exacerbate the counterproductive results.

The next question of interest is the effect different parameters of the system have on the maximum value(s) of DCR. In order to sign comparative statics we need to impose more structure on our model. We assume that $\gamma(\beta) = (1-\beta)^r$ where $r \ge 1$. We normalize θ at unity and assume that $u \ge 2$ so that elasticity of substitution is large enough, or $\sigma \ge 3$. These assumptions are restrictive more than necessary, however, they allow us to focus on sufficient conditions. Now, before embarking on deriving comparative statics of $\hat{\delta}_0$ let us first postulate some properties of functions $\hat{z}^{-u}(\beta)$ and $z^*(\beta)^{-u}$ that are instrumental to the analysis to follow.

Lemma 1: $\hat{z}^{-u}(\beta)$ ***and*** $z^*(\beta)^{-u}$ ***are decreasing in*** β ***at the decreasing rate when*** $u \ge 2$ *and* $\delta \le 1/2$.

Proof. ***See Appendix 1*■**

Results stated in lemma 1 are driven by the assumption of $r \ge 1$. Therefore, the assumption that there exist 1-to-1 correspondence between proportion of type 1 (folklore) and type 2 (pop) content and proportions of domestic and foreign content and that individuals prefer type 2 content to type 1 content is instrumental. A very strong implication of lemma 1 is that stations that are more popular (thus, have lower share of domestic programming) charge lower prices. This is in part due to assumption that industry is not perfectly competitive (due to licensing of radio transmissions by broadcasting authorities) therefore each station is a monopoly on its own genre. Absent of such assumption we might expect to have prices in popular stations driven down by entry.

At this state, let us postulate some of the properties of $\bar{\beta}$ as a function of parameters of the system (u, δ, r).

Lemma 2: Function $\bar{\beta}$ is increasing in δ at the rate of less than one-half

Proof: ***See Appendix 2■***

Intuitively, when functional form of the distribution does not change in response to changes in the interval of the random variable, increasing the upper boundary of a function increases the mean by half of the amount for a uniform function. Therefore, increasing the upper boundary of a distribution function that is strictly decreasing will add density less than average density, therefore, the mean will increase by less than one-half. Even though increase in δ flattens $\hat{z}(\beta)^{-u}$ which implies increases $\bar{\beta}$, for large enough u such an increase is minimal.

Lemma 3: Function $\bar{\beta}$ is decreasing in u

Proof: See Appendix 3■

This makes sense intuitively. As u increases there are two effects,- (1) the upper boundary over which $\bar{\beta}$ is taken shrinks therefore so does $\bar{\beta}$, and (2) increase in u makes $\hat{z}^{-u}$ more convex or right-skewed, therefore, the higher weights are given to values of β closer to a lower boundary than an upper boundary. This causes $\bar{\beta}$ to fall even further.

To proceed with our analysis we will require to derive the following two properties of function G evaluated at point $\hat{\delta}_0$.

Lemma 4. $\frac{\partial G}{\partial \delta} \leq 0$ ***for all*** $\phi \leq \frac{1}{2}$**.**

Proof: ***See Appendix 4*■**

Lemma 5. $\frac{\partial G}{\partial u} \leq 0$ ***for all*** $\phi \leq \frac{1}{2}$**.**

Proof: ***See Appendix 5*■**

Lemma 4 confirms that the point $\hat{\delta}_0$ is a local maximum.

Proposition 2. The larger is the elasticity of substitution between genres, σ, the smaller is the maximum value attainable by DCR policy, $\hat{\delta}_0$

Proof: $\frac{\partial \hat{\delta}_0}{\partial \sigma} = -\frac{\partial G / \partial \sigma}{\partial G / \partial \delta}$. *By definition* $\frac{\partial u}{\partial \sigma} = \theta \approx 1$ *therefore combining lemmas 4 and 5 gives the result*■

This result says that the higher is the elasticity of substitution the lower is the critical value of $\hat{\delta}_0$. Positive changes in the elasticity of substitution have little effect on the direct effect but provide a significant upward boost to the second effect, i.e. the negative response of aggregate broadcasting demand to increases in policy are larger the larger is the elasticity of substitution. Again, we want to point out that this result holds only when the concavity parameter is large enough so that the outflow of consumption of programming from constrained stations towards leisure is sufficiently large relative to the outflow of consumption towards stations with high domestic content. Therefore, when elasticity of substitution is high for some given large value of concavity parameter, policymaker has to exercise caution in choosing the level of DCR over the existing level because it may very well be that the targeted level of DCR might lie in the decreasing portion of $\hat{D}$, the area where DCR is counterproductive.

Proposition 3. The larger is the sensitivity of the price of advertising to the size of the audience (reflected in parameter θ), the smaller is the maximum value attainable by DCR policy, $\hat{\delta}_0$

Proof: $\frac{\partial\hat{\delta}_0}{\partial\sigma}=-\frac{\partial G/\partial\theta}{\partial G/\partial\delta}$. *We have* $\frac{\partial G}{\partial u}\frac{\partial u}{\partial\sigma}\geq\frac{\partial G}{\partial u}\frac{\partial u}{\partial\theta}$ *because* $\frac{\partial u}{\partial\sigma}\approx 1$ *and* $\frac{\partial u}{\partial\theta}=u\geq 1$*, and*

because $\frac{\partial\phi}{\partial\theta}=0$. *Therefore,* $0\geq\frac{\partial G}{\partial\sigma}\geq\frac{\partial G}{\partial\theta}$ *implies that* $\frac{\partial\hat{\delta}_0}{\partial\sigma}\leq 0$ ■

This result is also intuitive because θ in our model corresponds to sensitivity of price of advertising to the size of the audience. This effect is similar in nature to the effect of increasing in elasticity of substitution. Large sensitivity of price of advertising to the size of the audience means that equilibrium demands are sensitive to prices so that policy shocks is more likely to cause large drop in broadcasting demand, and consequently, consumption of domestic programs. This translates to point δ_0 reached at smaller values of DCR.

Proposition 4. The larger is the concavity parameter, λ, that reflects the easy of substitution between broadcasting and leisure, the smaller is the maximum value attainable under DCR type 1 policy, $\hat{\delta}_0$

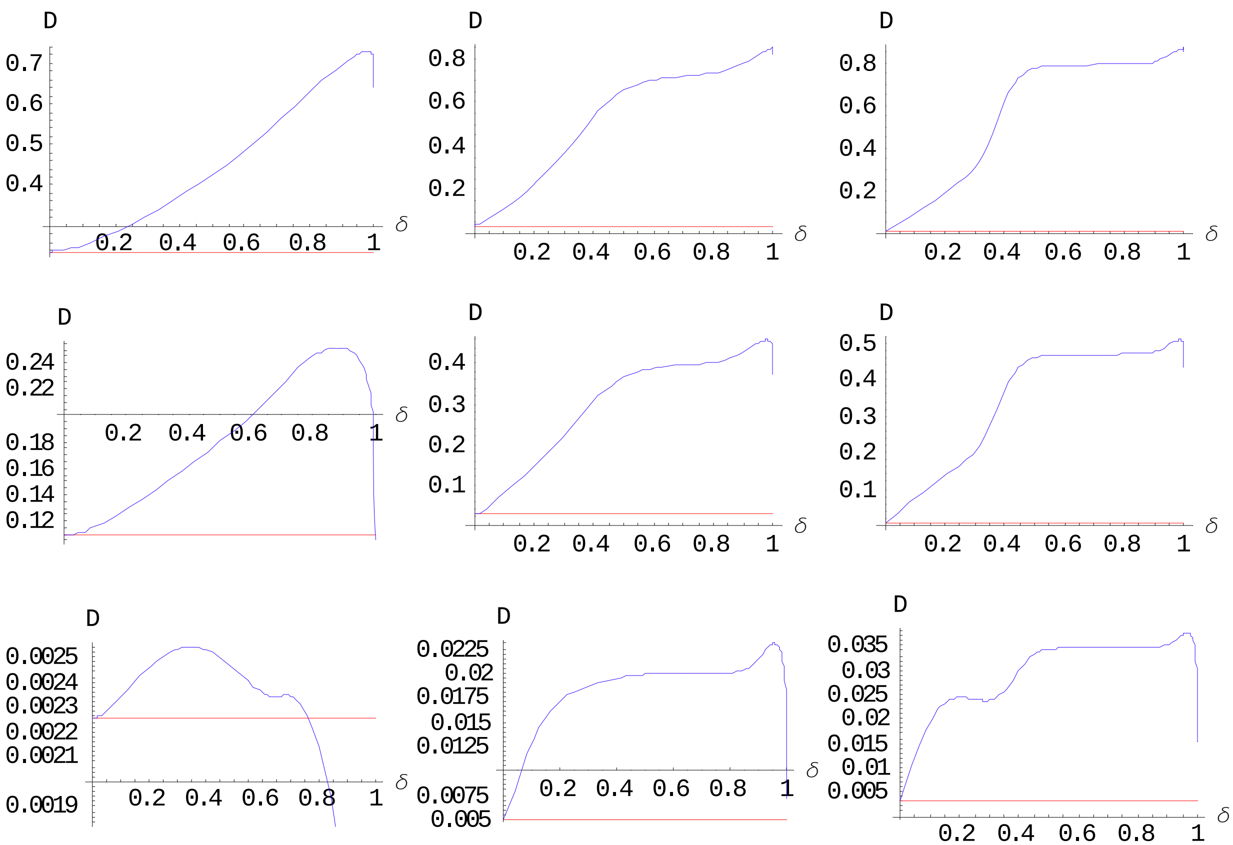
Proof: $\frac{\partial\hat{\delta}_0}{\partial\lambda}=-\frac{\partial G/\partial\lambda}{\partial G/\partial\delta}$. *We have* $\frac{\partial G}{\partial\lambda}=\frac{\mu u\left(\delta-\overline{\beta}\right)}{1-\delta}\frac{\partial\phi}{\partial\lambda}=-\frac{\mu\left(\delta-\overline{\beta}\right)}{1-\delta}\left(\frac{1}{1-\lambda}\right)^2\leq 0$ *because* μ *is independent of* ϕ*, therefore, independent of* λ ■

This result is not surprising either,- large concavity parameter implies low substitutability between genres and large substitutability between broadcasting and leisure. The former effect prevents individuals from replacing consumptions of low domestic content stations with consumption of high domestic content stations while the latter effect reduces overall demand for broadcasting, thus, overall consumption of domestic programming.

IV. Simulations

As we said in section III, we cannot ascertain analytically if the maximum we have derived is a local or global one. The type of maximum depends on the interplay of the parameters of the system. In this section, we will provide side by side graphs of the consumption of domestic programming in the constrained and unconstrained case. We consider two cases (i) the case where people have very weak preference of foreign music to domestic music and (ii) the case where individuals have strong preference of foreign produced music to domestically produced music. Assuming the simplest parametric form for $\gamma(\beta) = (1-\beta)^r$, case (i) corresponds to $r = 1/10$ and case (2) corresponds to $r = 11/10$.

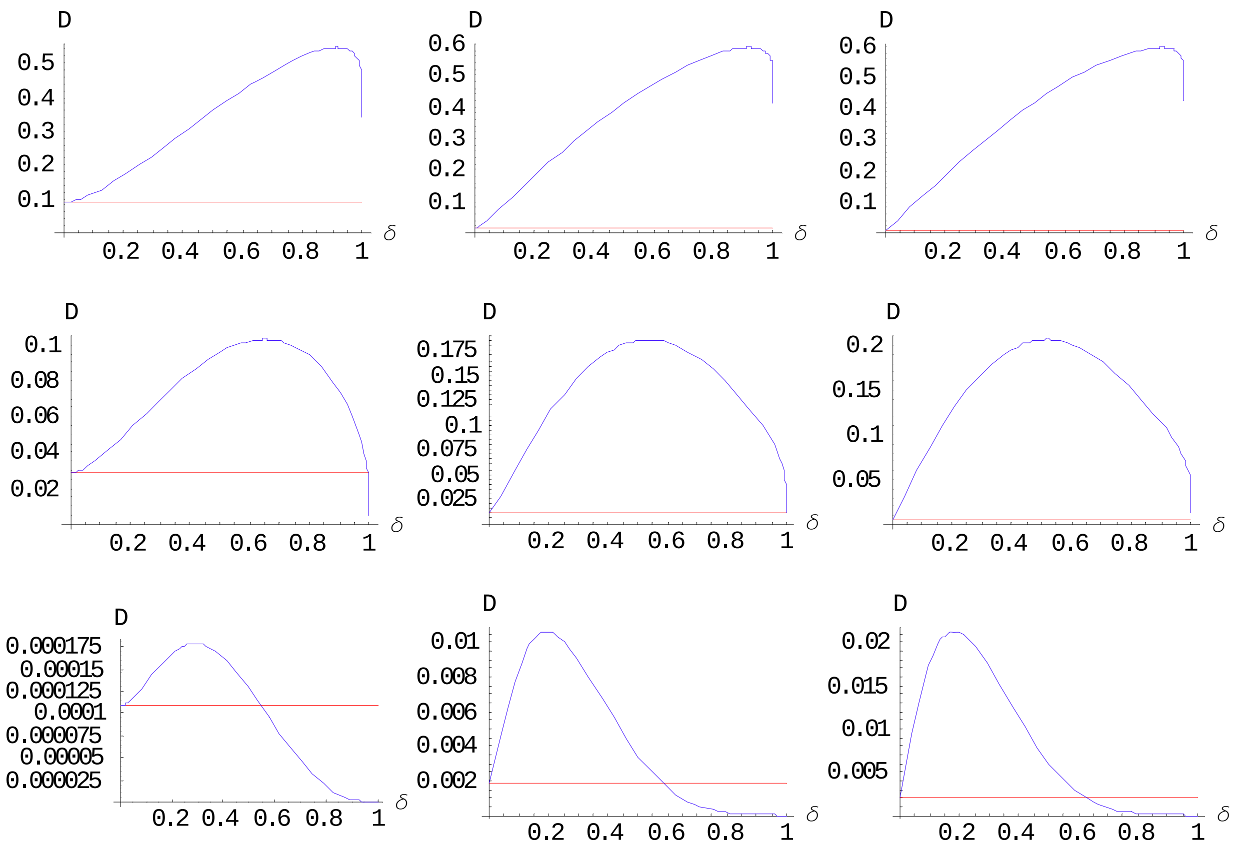
Case (i). $r = 1/10$



Graph 1. Graphs of constrained and unconstrained consumptions of domestic programming with nearly uniform distribution of genres' weights.

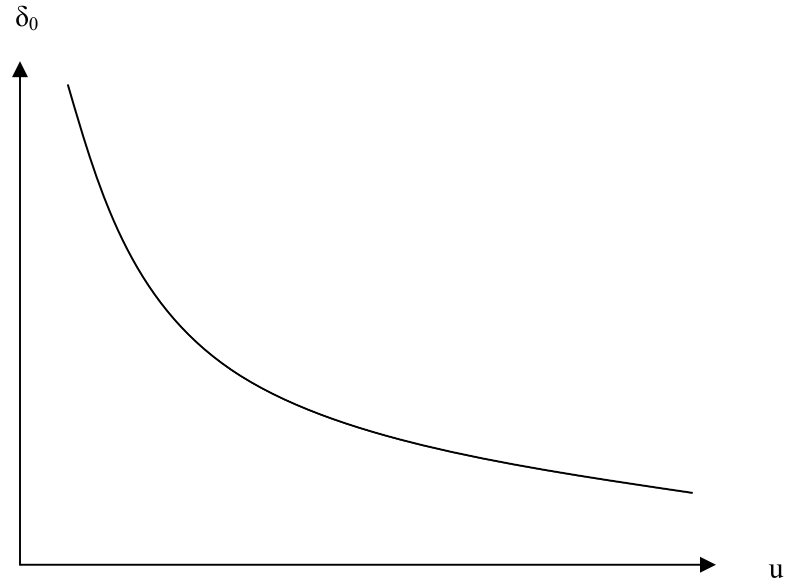
Rows correspond to $\lambda = 1/6$, $\lambda = 2/6$, and $\lambda = 5/6$, while columns correspond to $u = 2$, $u = 12$, and $u = 22$. Horizontal line corresponds to unconstrained consumption of domestic programming. As it can be seen from graph 1, because distribution of weights is nearly uniform, constrained consumption of domestic programming peaks at DCR nearly equal to unity (or the point where DCR becomes virtually binding for all stations) and then drops to zero. Therefore, in societies characterized by uniform or nearly uniform distribution of genres in preferences of the individuals, DCR almost always increases consumption of domestic programming. Furthermore, when both the elasticity of substation and concavity parameters are large, constrained consumption of domestic programming has several maximums.

Case (ii). $r = 11/10$



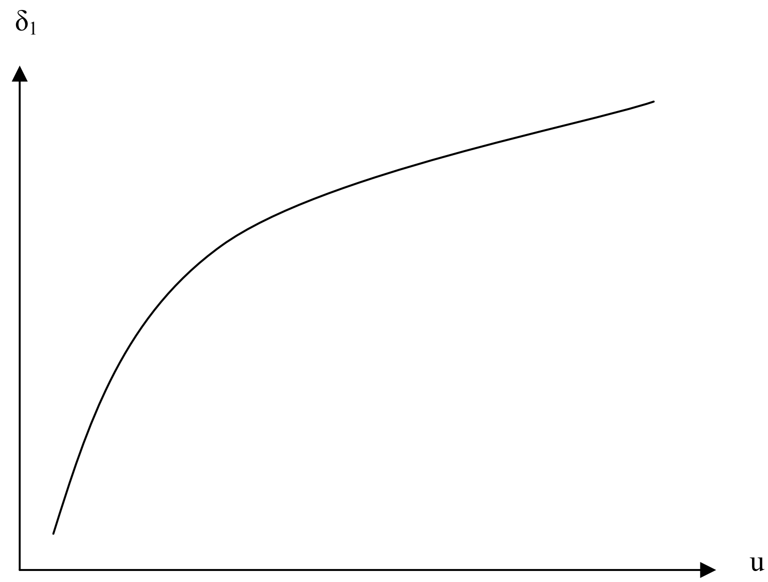
Graph 2. Graphs of constrained and unconstrained consumptions of domestic programming with strictly rightly skewed distribution of genres' weights.

Rows correspond to $\lambda = 1/6$, $\lambda = 2/6$, and $\lambda = 5/6$, while columns correspond to $u = 2$, $u = 12$, and $u = 22$. When weights are skewed towards stations with high foreign content then DCR has a unique global maximum even though constrained consumption of domestic programming is not globally concave. Furthermore, we see that such maximum is smaller, the larger is the elasticity of substitution, sensitivity of the price of advertising, and concavity parameter. This is in conformity with propositions 2, 3, and 4 laid out in the previous section. We can plot locus's of δ_0 and $\delta_1 \equiv \left\{ \delta \middle| \hat{D} = D^* \right\}$ as a function of either sensitivity of price of advertising or elasticity of substitution for the case where $\hat{D}$ reaches a unique global maximum .



Graph 3. The graph of δ_0 with strictly decreasing distribution of genres' weights as a function of u .

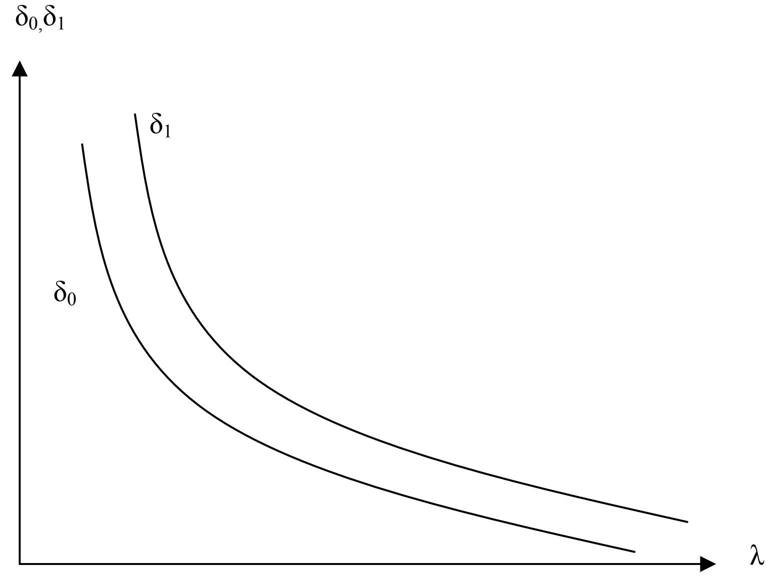
The above is the graph of the δ_0 as a function of u . We may observe that this confirms our propositions 2 and 3 which says that the maximum value reached by $\hat{D}$ falls in response to both the elasticity of substitution and sensitivity of the price of advertising to size of audience. Again, the intuition is that the higher are these parameters in the society where some DCR already in place, the more likely is the policy maker to overshoot in his/her choice of DCR.



Graph 4. The graph of δ_1 with strictly decreasing distribution of genres' weights as a function of u .

Surprisingly, the simulations show that the value of DCR where constrained and unconstrained consumptions of domestic programming are equal (besides the autarkic value) is increasing in the elasticity of substitution and sensitivity of price of advertising. Therefore, if the economy already has some ex-ante domestic content protection in place and government is weighting putting a new DCR, then the larger is the elasticity of substitution or sensitivity of the price of advertising, the less is the chance that policy intervention leads to counterproductive results ex-post.

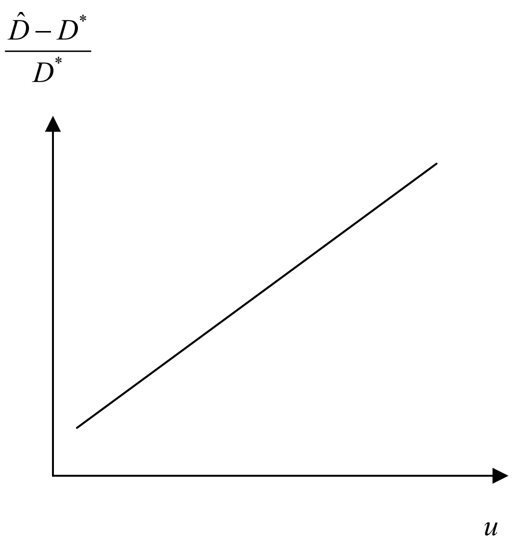
In a similar fashion, we can plot locus's δ_0 and δ_1 as a function of λ to obtain



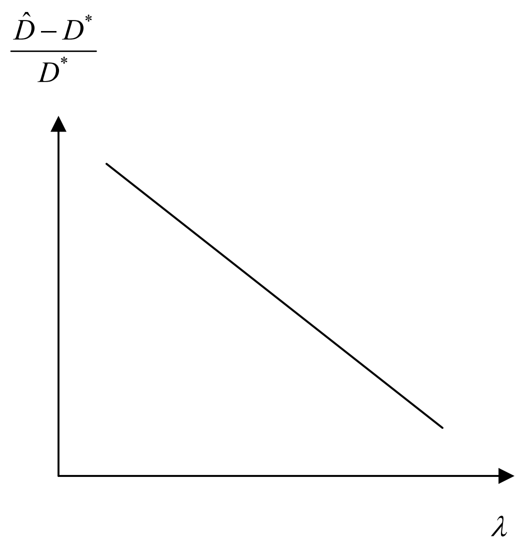
Graph 5. The graph of δ_0 and δ_1 with strictly decreasing distribution of genres' weights as a function of the concavity parameter.

As opposed to the simulations results above with respect to the elasticity of substitution, increases in concavity parameter lead to lower of both δ_0 and δ_1. Essentially, increases in concavity parameter shift the locus of $\hat{D}$ inward. This happens because concavity parameter can be viewed as a proxy for the opportunity cost of time. The higher is the opportunity cost of time the more picky consumers are and the more prone they are to switch to leisure at even small dissatisfaction. Therefore, even small DCR policy may lead to counterproductive results.

Finally, one may be interested in the proportional changes in the difference between the constrained consumption of domestic content and unconstrained consumption evaluated at δ_0 to changes in the elasticity of substitution and the concavity parameter. The results are summarized in the following graphs.



Graph 6. The graph of $\frac{\hat{D}-D^*}{D^*}$ with strictly decreasing distribution of genres' weights as a function of the elasticity of substitution.



Graph 7. . The graph of $\frac{\hat{D}-D^*}{D^*}$ with strictly decreasing distribution of genres' weights as a function of the concavity parameter.

Inspecting graph 6 we notice that the larger is the elasticity of substitution or sensitivity of preferences, the larger is the proportional difference between maximum constrained consumption of domestic programming and unconstrained consumption of domestic programming. The intuition is that the larger is either σ or θ, the higher is the competitive pressure on stations, therefore, they are forced to offer higher domestic and foreign content and lower advertising. This translated into lower prices and higher demand. Hence, consumption of domestic programming increases. However, when equilibrium is constrained then stations do not offer the best mixture of domestic and foreign content, therefore, under higher competitive pressure. This means that the rate of change in constrained demand evaluated at the maximum attainable value of domestic programming is higher that the rate of change in unconstrained consumption of domestic programming. Hence, the difference between constrained and unconstrained consumptions of domestic programs increases in both the elasticity of substitution and sensitivity of advertising.

On the contrary, as seen on graph 7, the larger is the concavity parameter the smaller is the ratio of constrained consumption of domestic programming to unconstrained one. Please recall that we have structured our model such that all virtual prices are greater than unity by assuming that $\gamma(\beta)$ is falling in β and $\gamma(0)=1$. This assumption drives the result that individual and aggregate demands are decreasing in concavity parameter. Since equilibrium virtual prices in both constrained and unconstrained cases are independent of the concavity parameter, consumption of domestic programs falls. Similarly to the intuition above, constrained station are more strained by the competitive pressure hence adjust their strategies more fervently in response to DCR. In other words, constrained consumption of domestic programming falls faster than unconstrained so that difference between the two also falls.

V. Conclusions and Extensions

We have established that a marginal DCR increment increases proportions of domestic content across all markets. However, by the nature of being constrained, it also increases virtual prices facing consumers for stations that are constrained by the policy. This leads to decrease in consumption of constrained stations' output and increase in unconstrained stations output. When consumers have preferences such that people tend to substitute undesirable consumption towards leisure rather than other station (for example, for reasons of either having high opportunity cost of time or large linguistic barrier), this may lead to an average decline in the consumption of broadcasting services. This, in turn, may lead to decline in the aggregate demand for domestically produced programs despite the increase in their relative share.

We have shown that for uniform or nearly uniform distribution of genre's preferences consumption of domestic programs peaks at very large values of DCR, therefore, a policymaker most certainly reaches its objective of increased consumption of domestic programs by imposing DCR. It is possible that constrained consumption of domestic programs has several maximums as so that a policymaker may be concerned about non-optimality of the DCR as compared to the case when policy is not in place. On the contrary, when distribution of genres' weights is highly skewed to the right (or strictly decreasing) then the maximum is likely to be global and unique and is reached at small or moderate values of the DCR. In this case, a policymaker has to exercise caution since his/her choice of DCR may lie over the region where aggregate consumption of domestic programs is declining. Worse than that, it might be over the region where aggregate consumption of domestic programs is smaller than the one when policy is not in place at all.

The irony of the DCR policy is that it is normally imposed on societies that have small autarkic consumption of domestic programming either due to the language barrier or high opportunity cost of time. We, however, have showed that the policy might not work in exactly the economies that are characterized by these two factors. Countries that fall into the first category are some of the countries of the former USSR (countries that are dominated by the Russian language), Canada (there are Canadians that do not speak French), New Zealand (support of Maori language). The latter might be developed countries where leisure is a highly valued commodity, like Canada, Australia, and European Union.

We have shown that the higher is the elasticity of the substitution between genres, sensitivity of price of advertising to size of audience, concavity parameter, the higher is the chance that the proposed DCR may fall in the region where aggregate consumption of domestic programs is declining. However, when policy is not in place, then the region over which DCR may lead to lower than autarkic consumption of domestic programs expands with elasticity of substitution, sensitivity of price of advertising, and shrinks with the concavity parameter.

Please note that the negative results of DCR are overly optimistic because we have not allowed firms to leave the market. When fixed costs of production are high then severely constrained station will abandon the market. This will further aggravate the counterproductive results of DCR policy.

The model developed in this section can also be applied to evaluation of DCR in TV broadcasting. A promising extension of our model would be solving social planner's problem and finding a tax or subsidy that could be levied upon broadcasters as to replicate the results that policy makes hopes to achieve by imposing DCR. Another potential extension would be to consider the effects of the DCR in a general equilibrium setup. We have taken the price of advertising as given, however, it is possible to build a model where preferences of individuals reflect both the time and income constraint and derive price of advertising by modeling explicitly the behavior of advertisers.

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Appendices

Appendix 1: ***Proof of Lemma 1.*** *We suppress argument* β *where necessary for notational simplicity.*

Differentiating $\hat{z}(\beta)^{-u}$ *with respect to* β *we have* $\frac{\partial \hat{z}^{-u}}{\partial \beta} = -u\hat{z}^{-u-1}\left(\frac{\partial \hat{z}}{\partial \beta}\right) = \hat{z}^{-u}\left(-u\hat{q}\right)$, *where*

$\hat{q} \equiv \frac{1}{1+u(1-\beta)} + \log\left(\frac{(1/\delta - 1)u(1-\beta)}{1+u(1-\beta)}\right) + \frac{r}{1-\beta}$. *We need to show that* $\frac{\partial \hat{z}^{-u}}{\partial \beta} \le 0$. *Differentiating* $\hat{q}$ *with respect to* β *yields* $\frac{\partial \hat{q}}{\partial \beta} = \frac{1}{1-\beta}\left(\frac{r}{1-\beta} + \frac{u(1-\beta)-1}{\left(1+u(1-\beta)\right)^2}\right) \ge 0$ *because* $r \ge 1$, *and*

$\frac{\partial \hat{q}}{\partial u} = \frac{1}{u\left(1+u(1-\beta)\right)^2} \ge 0$. *Therefore, for any given* β, $u \ge 2$, $r \ge 1$, *and* $\delta \le 1/2$ *we have* $\hat{q} \ge 1$.

Differentiating $\hat{z}(\beta)^{-u}$ *with respect to* β *twice gives*

$\frac{\partial^2 \hat{z}^{-u}}{\partial \beta^2} = -u\hat{z}^{-u}\left(-u\hat{q}\left(\frac{\partial \hat{z}}{\partial \beta}\frac{1}{\hat{z}}\right) + \frac{\partial \hat{q}}{\partial \beta}\right) = u\hat{z}^{-u}\left(u\hat{q}^2 - \frac{\partial \hat{q}}{\partial \beta}\right)$. *Since* $u \ge 1$, *then* $\frac{\partial^2 \hat{z}^{-u}}{\partial \beta^2} \ge 0$ *when* $u\hat{q}^2 - \frac{\partial \hat{q}}{\partial \beta} \ge 0$.

By inspecting this expression we notice that it is increasing in r *and decreasing in* δ, *thus, evaluated at* $r = 1$ *and* $\delta = 1/2$ *will yield the minimum. Further, differentiating* $u\hat{q}^2 - \frac{\partial \hat{q}}{\partial \beta}$ *with respect to* β *yields*

$\frac{\partial}{\partial u}\left(u\hat{q}^2 - \frac{\partial \hat{q}}{\partial \beta}\right) \ge \hat{q}^2 + 2\hat{q}^2\frac{1}{\left(1+u(1-\beta)\right)^2} + \frac{u\left(3-u(1-\beta)\right)}{(1-\beta)\left(1+u(1-\beta)\right)^3}$. *This expression is strictly positive for all* $u \ge 2$ *and* $0 \le \beta \le 1$. *Therefore,* $u\hat{q}^2 - \frac{\partial \hat{q}}{\partial \beta}$ *reaches its minimum at* $u = 2$. *Hence,*

$u\hat{q}^2 - \frac{\partial \hat{q}}{\partial \beta} \ge 2\left(\frac{1}{3-2\beta} + \frac{1}{1-\beta} + \log\frac{1-\beta}{3-2\beta}\right)^2 - \left(\frac{1}{1-\beta}\right)\left(\frac{1}{1-\beta} + \frac{1-2\beta}{(3-2\beta)^2}\right)$. *Solving this inequality yields*

$u\hat{q}^2 - \frac{\partial \hat{q}}{\partial \beta} > 0$ *for* $0 \le \beta \le 1$. *Therefore,* $\frac{\partial^2 \hat{z}^{-u}}{\partial \beta^2} \ge 0$. *Similarly, differentiating* $z^*(\beta)^{-u}$ *with respect to* β

yields $\frac{\partial\left(z^*\right)^{-u}}{\partial\beta}=-\left(z^*\right)^{-u}u\left(\frac{\partial z^*}{\partial\beta}\frac{1}{z^*}\right)=-\left(z^*\right)^{-u}uq^*$, *where* $q^*\equiv\log\left(\frac{1}{\beta}-1\right)+\frac{r}{1-\beta}\geq 0$. $\frac{\partial\left(z^*\right)^{-u}}{\partial\beta}\leq 0$ *when* $r\geq 1$. *We need to show that* $\frac{\partial^2\left(z^*\right)^{-u}}{\partial\beta^2}\geq 0$. *The second derivative of* $z^*\left(\beta\right)^{-u}$ *with respect to* β *is given by* $\frac{\partial^2\left(z^*\right)^{-u}}{\partial\beta^2}=-u\left(z^*\right)^{-u}\left(-uq^*\left(\frac{\partial z^*}{\partial\beta}\frac{1}{z^*}\right)+\frac{\partial q^*}{\partial\beta}\right)=u\left(z^*\right)^{-u}\left(u\left(q^*\right)^2-\frac{\partial q^*}{\partial\beta}\right)$. *We have* $\frac{\partial q^*}{\partial\beta}=\frac{1}{1-\beta}\left(\frac{r}{1-\beta}-\frac{1}{\beta}\right)$ *which has an ambiguous sign. When* $\frac{\partial q^*}{\partial\beta}\leq 0$ *then* $\frac{\partial^2\left(z^*\right)^{-u}}{\partial\beta^2}\geq 0$. *Otherwise, when* $\frac{\partial q^*}{\partial\beta}\geq 0$ *then a sufficient condition for* $\frac{\partial^2\left(z^*\right)^{-u}}{\partial\beta^2}\geq 0$ *is* $\left(q^*\right)^2-\frac{\partial q^*}{\partial\beta}\geq 0$ *because* $u\geq 2$. *This is implied by* $\log\left(\frac{1-\beta}{\beta}\right)+\beta^{-\frac{1}{2}}\left(1-\beta\right)^{-\frac{1}{2}}\geq 0$. *Therefore,* $\frac{\partial^2\left(z^*\right)^{-u}}{\partial\beta^2}\geq 0$. *Therefore,* $\hat{z}^{-u}\left(\beta\right)$ *and* $z^*\left(\beta\right)^{-u}$ *are decreasing and convex in* β *for all* $u\geq 2$ ■

Appendix 2. Proof of lemma 2. *Since* $\frac{\hat{z}\left(\beta,\delta\right)^{-u}}{\hat{V}\left(\delta\right)}$ *could be viewed as probability density function because it is always positive, finite and integrates to 1, we know that* $\overline{\beta}=E\left[\beta\middle|\delta\right]=\int_o^{\delta\frac{1+u}{u}}\beta\frac{\hat{z}\left(\beta\middle|\delta\right)^{-u}}{\hat{V}\left(\delta\right)}d\beta$ *or is a conditional expectation of* β. *Differentiating this expectation with respect to* δ *we obtain*

$$\frac{\partial\overline{\beta}}{\partial\delta}=\frac{\delta\frac{1+u}{u}\hat{z}\left(\delta\frac{1+u}{u}\middle|\delta\right)^{-u}\left(\frac{1+u}{u}-E\left[\beta\middle|\delta\right]\right)}{\hat{V}\left(\delta\right)}+\frac{u\,\mathrm{var}\left[\beta\middle|\delta\right]}{\delta\left(1-\delta\right)}.$$

We know that $0\leq E\left[\beta\middle|\delta\right]\leq\frac{\delta\left(1+u\right)}{2u}$ *because* $\hat{z}\left(\beta\middle|\delta\right)^{-u}$ *is decreasing in* β *by lemma. Since direct operations on* $\frac{\partial\overline{\beta}}{\partial\delta}$ *are not analytically feasible, we turn to approximations. Taking the first-order Taylor expansion around* $\beta=\frac{\delta\left(1+u\right)}{2u}$ *yields*

$$\hat{z}\left(\beta\middle|\delta\right)^{-u}\approx\hat{z}\left(\frac{\delta(1+u)}{2u}\middle|\delta\right)^{-u}\left(1-uq\left(\frac{\delta(1+u)}{2u}\middle|\delta\right)\left(\beta-\frac{\delta(1+u)}{2u}\right)\right),$$

$$\hat{z}\left(\beta\middle|\delta\right)^{-u}\approx\hat{z}\left(\frac{\delta(1+u)}{2u}\middle|\delta\right)^{-u}\left(\frac{\delta(1+u)}{2u}+\left(1-uq\left(\frac{\delta(1+u)}{2u}\middle|\delta\right)\frac{\delta(1+u)}{2u}\right)\left(\beta-\frac{\delta(1+u)}{2u}\right)\right),\text{ and}$$

$$\beta^2\hat{z}\left(\beta\middle|\delta\right)^{-u}\approx\hat{z}\left(\frac{\delta(1+u)}{2u}\middle|\delta\right)^{-u}\left(\left(\frac{\delta(1+u)}{2u}\right)^2+\frac{\delta(1+u)}{2u}\left(2-uq\left(\frac{\delta(1+u)}{2u}\middle|\delta\right)\frac{\delta(1+u)}{2u}\right)\left(\beta-\frac{\delta(1+u)}{2u}\right)\right)$$

Therefore, for Taylor expansion around $\beta=\frac{\delta(1+u)}{2u}$ *we have*

$\int_o^{\delta\frac{1+u}{u}}\hat{z}\left(\beta\middle|\delta\right)^{-u}d\beta\approx\hat{z}\left(\frac{\delta(1+u)}{2u}\middle|\delta\right)^{-u}\delta\frac{1+u}{u}$, $\int_o^{\delta\frac{1+u}{u}}\beta\hat{z}\left(\beta\middle|\delta\right)^{-u}d\beta\approx\hat{z}\left(\frac{\delta(1+u)}{2u}\middle|\delta\right)^{-u}\frac{1}{2}\left(\delta\frac{1+u}{u}\right)^2$, *and*

$\int_o^{\delta\frac{1+u}{u}}\beta^2\hat{z}\left(\beta\middle|\delta\right)^{-u}d\beta\approx\hat{z}\left(\frac{\delta(1+u)}{2u}\middle|\delta\right)^{-u}\frac{1}{4}\left(\delta\frac{1+u}{u}\right)^3$. *Therefore,* $E\left[\beta\middle|\delta\right]\approx\frac{\delta(1+u)}{2u}$ *and*

$E\left[\beta^2\middle|\delta\right]\approx\left(\frac{\delta(1+u)}{2u}\right)^2$. *Then,* $\frac{\partial\bar{\beta}}{\partial\delta}\approx\left[\frac{\delta\frac{1+u}{u}\hat{z}\left(\delta\frac{1+u}{u}\middle|\delta\right)^{-u}}{\hat{V}(\delta)}\right]\frac{1+u}{u}\left(1-\frac{\delta}{2}\right)$. *The term in the square brackets is the proportion of the rectangular area* $\delta\frac{1+u}{u}\hat{z}\left(\delta\frac{1+u}{u}\middle|\delta\right)^{-u}$ *to the total area of the integral over the same region. This proportion will be smaller than the proportion of the rectangular area to the area below the tangent line to curve* $\hat{z}(\beta)^{-u}$ *going through point* $\beta=\frac{\delta(1+u)}{u}$. *Therefore, we have*

$\frac{\delta\frac{1+u}{u}\hat{z}\left(\delta\frac{1+u}{u}\middle|\delta\right)^{-u}}{\hat{V}(\delta)}\le\frac{\delta(1+u)}{u+\frac{1}{2}u(1+u)\delta}$, *where we have set* $\hat{q}\left(\delta\frac{1+u}{u}\right)\approx 1$ *or its minimum. Taking this inequality into account yields that* $\frac{\partial\bar{\beta}}{\partial\delta}\le\frac{\delta(1+u)^2(1-\delta/2)}{u\left(u+\frac{1}{2}u(1+u)\delta\right)}\le\frac{1}{2}$ *for all* $u\ge 2$. *Intuitively,* $\frac{u\,\mathrm{var}\left[\beta\middle|\delta\right]}{\delta(1-\delta)}$

is the second-order effect therefore it is dominated by the term $\dfrac{\delta \frac{1+u}{u} \hat{z}\left(\delta \frac{1+u}{u} \middle| \delta \right)^{-u} \left(\frac{1+u}{u} - E\left[\beta \middle| \delta \right] \right)}{\hat{V}(\delta)}$.

The element that determines the magnitude of this element is $\dfrac{\delta \frac{1+u}{u} \hat{z}\left(\delta \frac{1+u}{u} \middle| \delta \right)^{-u}}{\hat{V}(\delta)}$. *For large* u *the function* $\hat{z}(\beta)^{-u}$ *is very convex (or decreasing very fast in* β *) so that the proportion of the rectangular area* $\delta \frac{1+u}{u} \hat{z}\left(\delta \frac{1+u}{u} \middle| \delta \right)^{-u}$ *to the total area of the integral is very small. Therefore, when* $u \geq 2$ *(or is sufficiently large) we have* $\dfrac{\partial \overline{\beta}}{\partial \delta} \leq \dfrac{1}{2}$ ■

Appendix 3. Proof of lemma 3.

$$\frac{\partial \overline{\beta}}{\partial u} = -\frac{\frac{\delta \hat{z}^{-u}}{u^2}\left(\delta \frac{1+u}{u} - \overline{\beta} \right)}{\int_0^{\frac{\delta(1+u)}{u}} \hat{z}(\beta)^{-u} \, d\beta} + \frac{\int_0^{\frac{\delta(1+u)}{u}} \beta \left(\hat{z}(\beta)^{-u} \right)' d\beta - \overline{\beta} \int_0^{\frac{\delta(1+u)}{u}} \left(\hat{z}(\beta)^{-u} \right)' d\beta}{\int_0^{\frac{\delta(1+u)}{u}} \hat{z}(\beta)^{-u} \, d\beta}$$

$$= -\frac{\frac{\delta \hat{z}^{-u}}{u^2}\left(\delta \frac{1+u}{u} - \overline{\beta} \right)}{\int_0^{\frac{\delta(1+u)}{u}} \hat{z}(\beta)^{-u} \, d\beta} + \frac{\int_0^{\frac{\delta(1+u)}{u}} \beta \hat{z}(\beta)^{-u} \left(-\varphi(\beta) \right) d\beta - \overline{\beta} \int_0^{\frac{\delta(1+u)}{u}} \hat{z}(\beta)^{-u} \left(-\varphi(\beta) \right) d\beta}{\int_0^{\frac{\delta(1+u)}{u}} \hat{z}(\beta)^{-u} \, d\beta}$$

where $\varphi(\beta) \equiv \frac{\partial \hat{z}}{\partial u} \frac{u}{\hat{z}} + \log(\hat{z}) = -\frac{1}{u + 1/1-\beta} + \log(\hat{z})$ *and* $\frac{\partial \varphi(\beta)}{\partial \beta} \geq 0$ *(recall that* $\frac{\partial \hat{z}}{\partial \beta} \geq 0$*). Again, one can view* $\frac{\hat{z}(\beta)^{-u}}{\hat{V}}$ *as a distribution function of* β *over the interval* $\left(0, \delta \frac{1+u}{u} \right)$. *Hence, we can rewrite*

$$\frac{\partial\overline{\beta}}{\partial u}=-\frac{\frac{\delta\hat{z}^{-u}}{u^2}\left(\delta\frac{1+u}{u}-\overline{\beta}\right)}{\int_0^{\frac{\delta(1+u)}{u}}\hat{z}(\beta)^{-u}\,d\beta}+E(\beta)E(\varphi(\beta))-E(\beta\varphi(\beta))=-\frac{\frac{\delta\hat{z}^{-u}}{u^2}\left(\delta\frac{1+u}{u}-\overline{\beta}\right)}{\int_0^{\frac{\delta(1+u)}{u}}\hat{z}(\beta)^{-u}\,d\beta}-\operatorname{cov}(\beta,\varphi(\beta))$$ *The first term of the above equation is positive because* $0\le\overline{\beta}\le\delta\frac{1+u}{u}$ *while the second term is positive because* $\frac{\partial\varphi(\beta)}{\partial\beta}\ge 0$. *Therefore,* $\frac{\partial\overline{\beta}}{\partial u}\le 0$ ■

Appendix 4. Proof of lemma 4. *Differentiating* G *with respect to* δ *we obtain*

$$\frac{\partial G}{\partial\delta}=-u\left[\frac{\delta-\overline{\beta}}{(1-\delta)^2}\left(1-\phi\left(\mu+(1-\delta)\frac{\partial\mu}{\partial\delta}\right)\right)+\frac{(1-\phi\mu)\left(1-\partial\overline{\beta}/\partial\delta\right)}{(1-\delta)}\right]$$

$$=-u\left[\frac{1}{(1-\delta)^2}\left(\left(\delta-\overline{\beta}\right)\left(1-\phi\left(\mu+(1-\delta)\frac{\partial\mu}{\partial\delta}\right)\right)+\frac{1}{2}(1-\phi\mu)(1-\delta)\right)+\frac{(1-\phi\mu)\left(1/2-\partial\overline{\beta}/\partial\delta\right)}{(1-\delta)}\right]$$

The second term in the square brackets is positive by lemma 2. The sign of the first term is not so obvious because $\frac{\partial\mu}{\partial\delta}\ge 0$ *and is potentially unbounded. More specifically,*

$$\frac{\partial\mu}{\partial\delta}=\frac{\frac{1+u}{u}z^{-u}}{\hat{V}+V^*}-\frac{u(1-\mu)\int_0^{\delta\frac{1+u}{u}}\hat{z}(\beta)^{-u}\left(\frac{\delta-\beta}{\delta(1-\delta)}\right)d\beta}{\hat{V}+V^*}=\frac{\frac{1+u}{u}z^{-u}}{\hat{V}+V^*}-\frac{u\mu(1-\mu)\left(\delta-\overline{\beta}\right)}{\delta(1-\delta)}$$

$$=\frac{1}{\delta}\left(L-\frac{u\mu(1-\mu)\left(\delta-\overline{\beta}\right)}{(1-\delta)}\right)$$

where $L\equiv\frac{\delta\frac{1+u}{u}z^{-u}}{\hat{V}+V^*}$. *Function* $L\in\left[0,\frac{1}{2}\right]$ *because* $z^{-u}(\beta)$ *is decreasing and convex in* β *(to see this one may construct a triangle with a hypotenuse formed by a line tangent to curve* $z^{-u}(\beta)$ *at a point* $\delta\frac{1+u}{u}$. *Then, the area of the rectangle* $\delta\frac{1+u}{u}z\left(\delta\frac{1+u}{u}\right)^{-u}$ *is smaller than half the area of such a right triangle. Given that* $z^{-u}(\beta)$ *is convex the area of the right triangle is smaller than the area of the* *integral of this function. Hence,* $L \in \left[0, \frac{1}{2}\right]$*). As* $\delta \to 0$ *then* $\frac{\partial \mu}{\partial \delta} \to \infty$*. If* δ *is large then*

$1-\phi\left(\mu+(1-\delta)\frac{\partial \mu}{\partial \delta}\right) \geq 0$ *which implies that first term is positive and* $\frac{\partial G}{\delta \delta} \leq 0$*. However, when* δ *is*

small then $1-\phi\left(\mu+(1-\delta)\frac{\partial \mu}{\partial \delta}\right) \leq 0$*. Focusing on the case where* $1-\phi\left(\mu+(1-\delta)\frac{\partial \mu}{\partial \delta}\right) \leq 0$ *we have*

$(\delta-\bar{\beta})\left(1-\phi\left(\mu+(1-\delta)\frac{\partial \mu}{\partial \delta}\right)\right) \geq \delta\left(1-\phi\left(\mu+(1-\delta)\frac{\partial \mu}{\partial \delta}\right)\right)$*, because* $\bar{\beta} \in \left[0, \frac{\delta(1+u)}{2u}\right]$*. Therefore, the*

first term of equation $\frac{\partial G}{\partial \delta}$ *is strictly smaller than*

$\delta\left(1-\phi\left(\mu+(1-\delta)\frac{\partial \mu}{\partial \delta}\right)\right)+\frac{1}{2}(1-\phi\mu)(1-\delta)=\frac{1}{2}(1-\phi\mu)(1+\delta)-\delta(1-\delta)\phi\frac{\partial \mu}{\partial \delta}$*. Substituting for* $\frac{\partial \mu}{\partial \delta}$ *and*

rearranging yields $\frac{1}{2}(1-\phi\mu)(1+\delta)-(1-\delta)\phi\left(L-u\mu(1-\mu)(\delta-\bar{\beta})\right) \leq 1-\delta-\phi\left(\mu(1+\delta)+1-\delta\right)$

because $L-u\mu(1-\mu)(\delta-\bar{\beta}) \leq \frac{1}{2}$*. Given that* $1 \geq \mu \geq \delta\frac{1+u}{u}$ *(this is because* $\frac{\partial \hat{z}^{-u}}{\partial \beta} \leq 0$*) we have*

$\mu(1+\delta)+1-\delta \leq 2$ *(substituting for maximum possible value of* μ *equal to one yields the result).*

Therefore, a sufficient condition for $\frac{\partial G}{\partial \delta} \leq 0$ *is* $\phi \leq \frac{1}{2}$ ■

Appendix 5. Proof of lemma 5. *Differentiating* G *with respect to* u *gives*

$\frac{\partial G}{\partial u} \propto -\left[1-\phi\left(\mu+u\frac{\partial \mu}{\partial u}\right)\right](\delta-\bar{\beta})+u(1-\phi\mu)\frac{\partial \bar{\beta}}{\partial u}+\mu u(\delta-\bar{\beta})\frac{(1-\phi)}{u}$ *where*

$\frac{\partial \phi}{\partial \sigma}=\frac{\partial}{\partial \sigma}\left(1-\left(\frac{\lambda}{1-\lambda}\right)\frac{1}{\sigma-1}\right)=\left(\frac{\lambda}{1-\lambda}\right)\left(\frac{1}{\sigma-1}\right)^2=\frac{1-\phi}{u}$*. Rearranging the above equation yields*

$$\frac{\partial G}{\partial u} \propto -\left[1-\phi\left(\mu+u\frac{\partial\mu}{\partial u}\right)+\mu(1-\phi)\right]\left(\delta-\overline{\beta}\right)+u(1-\phi\mu)\frac{\partial\overline{\beta}}{\partial u}$$

$$\propto -\left[\mu+1-\phi\left(2\mu+u\frac{\partial\mu}{\partial u}\right)\right]\left(\delta-\overline{\beta}\right)+u(1-\phi\mu)\frac{\partial\overline{\beta}}{\partial u}$$

$$\leq -\left[1-\frac{u}{2}\frac{\partial\mu}{\partial u}\right]\left(\delta-\overline{\beta}\right)+u(1-\phi\mu)\frac{\partial\overline{\beta}}{\partial u}$$

where the latter inequality comes from the assumption that $\phi \leq \frac{1}{2}$. *Differentiating the share of the constrained demand in the aggregate demand with respect to* u *yields*

$\frac{\partial\mu}{\partial u} = -\mu(1-\mu)\left[O_1+uO_2+O_3\right]$, *where* $O_1 \equiv \frac{\delta z^{-u}}{u^2\mu(1-\mu)\left(\hat{V}+V^*\right)}$,

$$O_2 = \frac{\int_0^{\frac{\delta(1+u)}{u}} \hat{z}(\beta)^{-u}\left(\frac{\partial\hat{z}(\beta)/\partial u}{\hat{z}(\beta)}\right)d\beta}{\int_0^{\frac{\delta(1+u)}{u}} \hat{z}(\beta)^{-u}\,d\beta} - \frac{\int_{\frac{\delta(1+u)}{u}}^{1} z^*(\beta)^{-u}\left(\frac{\partial z^*(\beta)/\partial u}{z^*(\beta)}\right)d\beta}{\int_{\frac{\delta(1+u)}{u}}^{1} z^*(\beta)^{-u}\,d\beta},$$

$$O_3 = \frac{\int_0^{\frac{\delta(1+u)}{u}} \hat{z}(\beta)^{-u}\log\hat{z}(\beta)\,d\beta}{\int_0^{\frac{\delta(1+u)}{u}} \hat{z}(\beta)^{-u}\,d\beta} - \frac{\int_{\frac{\delta(1+u)}{u}}^{1} z^*(\beta)^{-u}\log z^*(\beta)\,d\beta}{\int_{\frac{\delta(1+u)}{u}}^{1} z^*(\beta)^{-u}\,d\beta}.$$

We need to show that $\frac{\partial G}{\partial u} \leq 0$, *however, signing* $\frac{\partial\mu}{\partial u}$ *directly is not feasible. If* $\frac{\partial\mu}{\partial u} \leq 0$ *then* $\frac{\partial G}{\partial u} \leq 0$. *If* $\frac{\partial\mu}{\partial u} \geq 0$ *then it is sufficient to put an upper boundary on it. We may immediately observe that* $O_1 \geq 0$.

Expression $\frac{\partial\hat{z}(\beta)/\partial u}{\hat{z}(\beta)} = -\frac{1-\beta}{u(1+u(1-\beta))} \leq 0$ *and* $\frac{\partial z^*(\beta)/\partial u}{z^*(\beta)} = -\frac{1}{u(1+u)}$ *therefore*

$O_2 \geq -\frac{1-\frac{\delta(1+u)}{2u}}{u\left(1+u\left(1-\frac{\delta(1+u)}{2u}\right)\right)}+\frac{1}{u(1+u)} = \frac{\delta}{u(2-\delta)} \geq 0$. *Further,* $O_3 \leq 0$ *because* $\log\hat{z}(\beta)$ *and*

$\log z^*(\beta)$ *both increase in* β *and by the mean value theorem the average value of*

$(\log \hat{z}(\beta))_{mean} \le (\log z^*(\beta))_{mean}$. *Since* $\hat{z}(\beta)^{-u}$ *and* $z^*(\beta)^{-u}$ *are decreasing in* β *by lemma 1, we have*

$O_3 \ge \log \frac{\hat{z}(0)}{z^*\left(\frac{1}{2}+\frac{\delta(1+u)}{2u}\right)}$ *We are interested in the minimum possible value attainable by* O_3. *It is*

potentially unbounded. Since $\hat{z}(\beta)^{-u}$ *and* $z^*(\beta)^{-u}$ *are decreasing in* β *by lemma 1, we have*

$$O_3 \ge \log \frac{\hat{z}(0)}{z^*\left(\frac{1}{2}+\frac{\delta(1+u)}{2u}\right)}$$
$$\ge \log\left[\left(\frac{1}{2}+\frac{\delta(1+u)}{2u}\right)^{\frac{1}{2}+\frac{\delta(1+u)}{2u}}\left(\frac{1}{2}-\frac{\delta(1+u)}{2u}\right)^{\frac{1}{2}-\frac{\delta(1+u)}{2u}+r}\frac{1}{1-\delta}\right].$$

The right hand side is increasing in δ. *We assume that* $\delta \le 2/3$ *which implies that* $O_3 \ge \frac{2}{3}-\frac{r}{5}$.

Combining the above inequalities yields $\frac{\partial \mu}{\partial u} \le -\mu(1-\mu)\left(\frac{2}{3}-\frac{r}{5}\right)$. *Define* $\delta(1+u)q \equiv \omega$ *and*

$m \equiv \frac{\frac{1}{2}\left((1+\omega)z^{-u}+z^{-u}\right)\delta\frac{1+u}{u}}{\frac{1}{2}(1+\omega)z^{-u}\left(\frac{1+\omega}{uq}\right)}$ *where* z^{-u} *is either* $\hat{z}^{-u}$ *or* $(z^*)^{-u}$ *evaluated at* $\beta = \delta\frac{1+u}{u}$ *since both are*

equal to each other at this point (for this reason we omit hats and stars from virtual prices). Please note

that $\partial\hat{q}/\partial\beta \ge \partial q^*/\partial\beta$ *evaluated at* $\beta = \delta\frac{1+u}{u}$ *even though* $\hat{q} = q^*$ *when evaluated at* $\beta = \delta\frac{1+u}{u}$; *since*

we deriving the lower boundary we chose the smaller, or $\partial q^*/\partial\beta$. *A tangent line to function* $z^{-u}(\beta)$ *is*

given by equation $(1+\omega)z^{-u}$. *With this in mind, function* m *is the ratio of the area formed by tangent*

line for $\beta \in \left[0, \delta\frac{1+u}{u}\right]$ *and tangent line for* β *from zero to the point where tangent line crosses zero*

line. We use this function to approximate $\mu(1-\mu)$, *namely, we assume that* $m(1-m) \approx \mu(1-\mu)$.

Therefore, we shall have $\mu(1-\mu) \le 4\left[\left(\frac{1}{1+\omega}\right)^2 - \left(\frac{1}{1+\omega}\right)^4\right]$. *Given this we have*

$1-\frac{u}{2}\frac{\partial\mu}{\partial u} \ge 1-2u\left(\frac{1}{1+\omega}\right)^2\left(\frac{2}{3}-\frac{r}{5}\right)$. *The right-hand side of this inequality increasing in* r *because*

$\partial\omega/\partial r \propto \partial q/\partial r \ge 0$, *thus, setting* $r=1$ *yields the minimum value of it. Further,*

$\omega = \delta(1+u)\left(\log\left(\frac{2u}{1+u}-1\right)+\frac{r}{1-\delta\frac{1+u}{u}}\right)$, *therefore,* $u \ge 2$ *implies that* $\omega \le \delta(1+u)\left(\frac{ru}{u-\delta(1+u)}-1\right)$,

therefore, $1-\frac{u}{2}\frac{\partial\mu}{\partial u} \ge 1-\frac{7u}{30}\left(\frac{u-\delta(1+u)}{u-\delta(1+u)+(1+u)^2(1-\delta)^2}\right)^2$. *Again, the right-hand side of this*

inequality is decreasing in δ, *therefore, setting* $\delta = \frac{1}{1+u}$ *(please note that* $0 \le \phi\mu \le 1$ *and*

$0 \le \beta \le \frac{\delta(1+u)}{2u}$ *imply that* $\frac{1}{1+u} \le \delta \le \frac{4}{3+u}$ *from the definition of* $G=0$*) yields*

$1-\frac{u}{2}\frac{\partial\mu}{\partial u} \ge 1-\frac{7u}{30}\left(\frac{u-1}{u-1+u^2}\right)^2$ *which reaches its minimum at* $1/40$. *Combining this and lemma 3 yields*

$\frac{\partial G}{\partial u} \le 0$ ■